Bangalore School on Statistical Physics VIII
Course 2: CRITICAL DYNAMICS
Problem set 2

1. *Gaussian relaxational models in the ordered phase.*
For $T < T_c$ ($\tau < 0$), introduce the fluctuation fields $\pi^\alpha(\vec{x}, t) = S^\alpha(\vec{x}, t)$ for $\alpha = 1, \ldots, n-1$ and $\sigma(\vec{x}, t) = S^n(\vec{x}, t) - \sqrt{6|\tau|/u}$, linearize the resulting coupled Langevin equations for the relaxational models A and B for non-conserved and conserved order parameter, respectively, and thus compute the dynamic response $\chi^{\alpha\beta}(\vec{q}, \omega)$ and correlation functions $C^{\alpha\beta}(\vec{q}, \omega)$ in the ordered phase in the Gaussian approximation.

2. *Reversible mode-coupling terms and equilibrium condition.*
Confirm that the ansatz

$$F^\alpha_{\rm rev}[S^\alpha](\vec{x}) = -\int d^d x' \sum_{\alpha'} \left[Q^{\alpha\alpha'}(\vec{x}, \vec{x}') \, \frac{\delta H[S^\alpha]}{\delta S^{\alpha'}(\vec{x}')} - k_{\rm B} T \, \frac{\delta Q^{\alpha\alpha'}(\vec{x}, \vec{x}')}{\delta S^{\alpha'}(\vec{x}')} \right]$$

with antisymmetric $Q^{\alpha\alpha'}(\vec{x}, \vec{x}') = \left\{ S^\alpha(\vec{x}), S^{\alpha'}(\vec{x}') \right\} = -Q^{\alpha'\alpha}(\vec{x}', \vec{x})$ satisfies the equilibrium condition

$$\int d^d x \sum_\alpha \frac{\delta}{\delta S^\alpha(\vec{x})} \left(F^\alpha_{\rm rev}[S^\alpha](\vec{x}) \, e^{-H[S^\alpha]/k_{\rm B}T} \right) = 0 \ .$$