Electronic transport properties of the α-$\mathcal{T}_3$ lattice

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Plan of this talk

Basic information of Dice lattice: pseudospin-1 Dirac-Weyl system

Evolution of *S=1/2* (honeycomb) to *S=1* pseudospin (dice) lattice: the concept of $\alpha - \mathcal{T}_3$ lattice

Magnetotransport properties: the role of a variable Berry phase

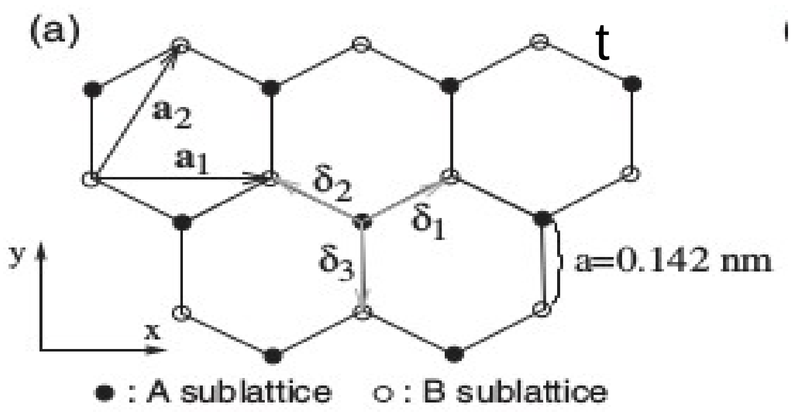
Haldane-like model of dice lattice: Anomalous Hall conductivity

Haldane-like model of dice lattice in presence of quantizing magnetic field

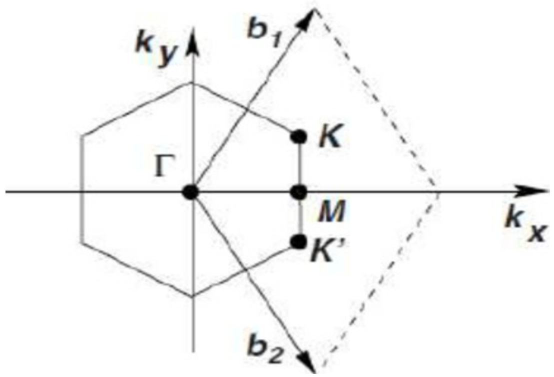
Summary and conclusions

Dirac-Weyl systems

Honeycomb lattice: graphene monolayer



Brillouin zone

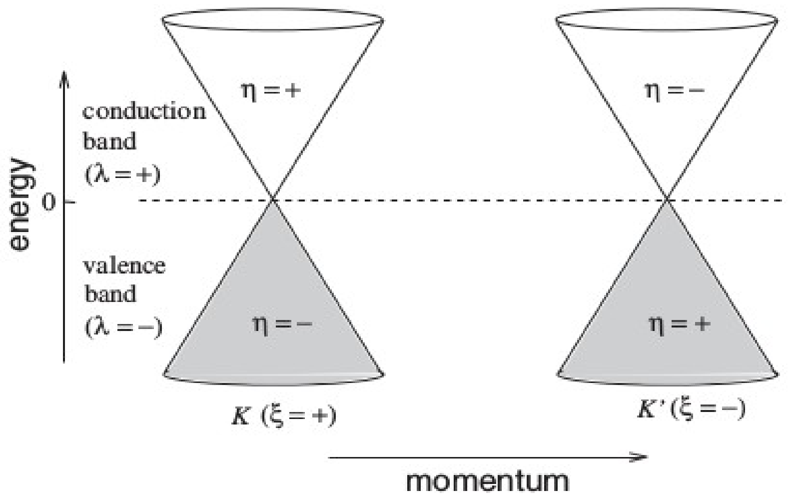


Two inequivalent K points

Dirac-like Hamiltonian: $H = v_f(\xi \sigma_x k_x + \sigma_y k_y)$

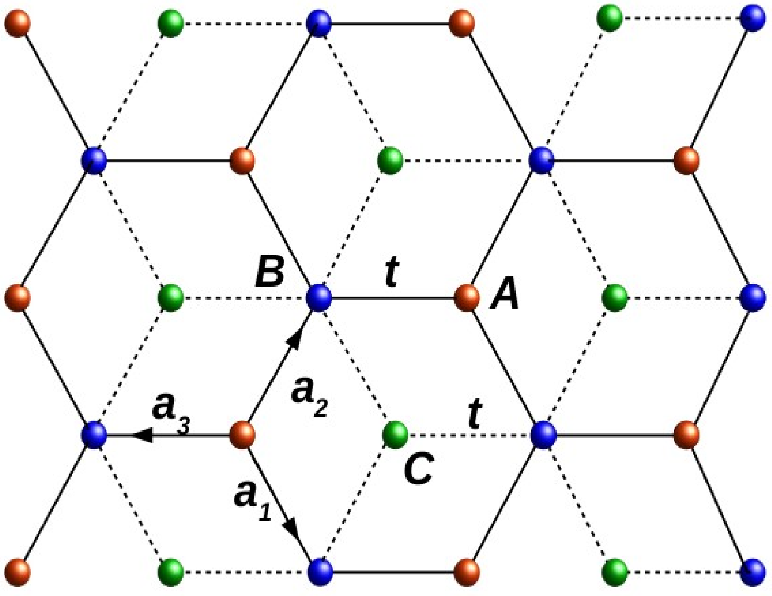
Low-energy modes: $E_\lambda = \lambda \hbar v_f \sqrt{k_x^2 + k_y^2}$

A state makes a closed loop encloses the Dirac points, it acquires the Berry phase: $\gamma_B^{\lambda,\xi} = \lambda\, \pi$



Isotropic Dirac cones at two inequivalent K points

Dice or $\mathcal{T}_3$ lattice



One atom at each center of the honeycomb lattice

The center sites are bonded to the alternate corners of the hexagon

Equal hopping amplitude *t*

Dice lattice exhibits nodes with unequal connectivity:

Hub sites (B): 6 coordination number

Rim sites (A,C): 3 coordination number

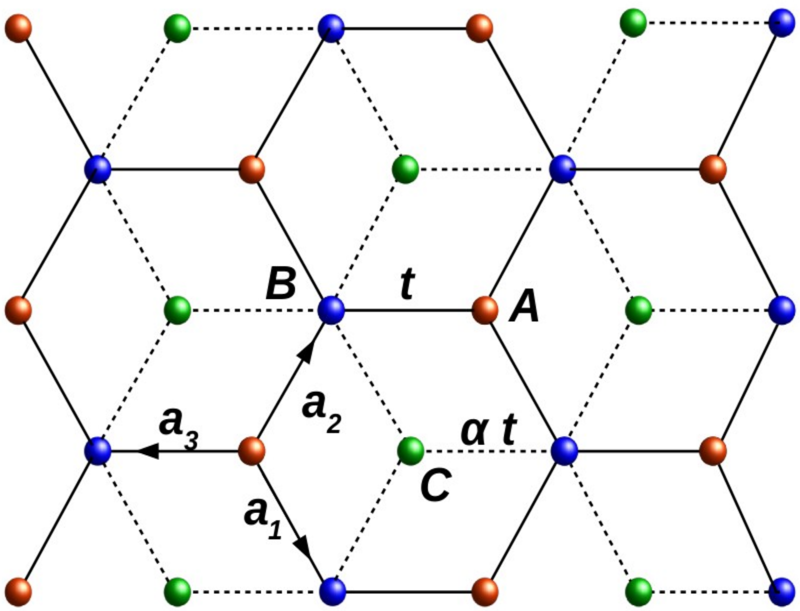
Unit cell is the same as that of a honeycomb lattice

Three inequivalent atoms (A, B, C) per unit cell

1st Brillouin zone is also a hexagon with two independent K points

Dice lattice has larger pseudospin 1 as compared to pseudospin-1/2 for honeycomb lattice

Concept of $\alpha - \mathcal{T}_3$ lattice



Hopping between C and B sites: $\alpha\ t$

It exhibits nodes with unequal hopping amplitude: t ***and*** αt

$\alpha = 0$: **C atom is disconnected**

$\alpha = 0$: graphene **(Pseudospin-1/2)**

$\alpha = 1$: dice **(Pseudospin-1)**

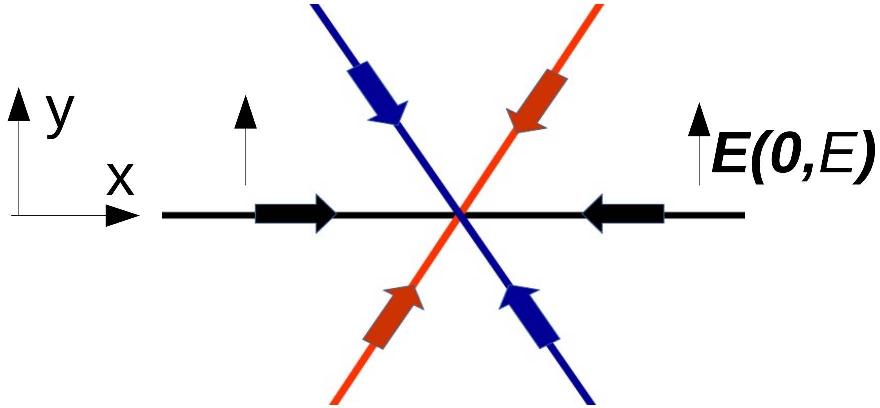
Continuous tuning of α**: one can go from honeycomb to dice lattice**

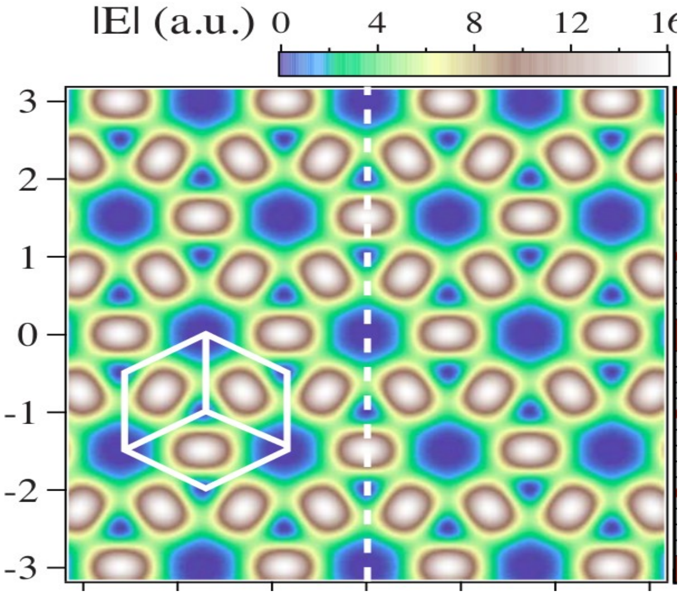
Three inequivalent atoms (A, B, C) per unit cell

1st Brillouin zone is also a hexagon with two independent K points

Raoux et al, PRL 112, 026402 (2016)

How to realize Dice lattice and $\alpha - \mathcal{T}_3$ lattice?





Potential landsacpe

Dice lattice: 3 counterpropagating identical laser beams on a plane with $\lambda = 3a/2$

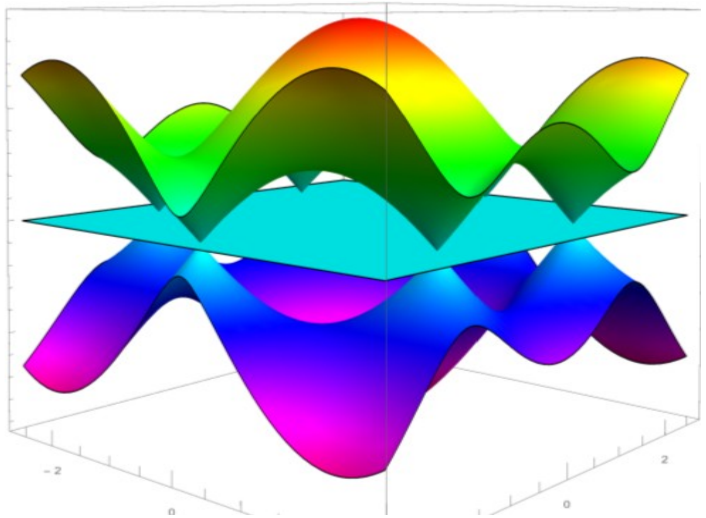
$\alpha - \mathcal{T}_3$ lattice : **Dephase one of the three pairs of laser beams to get** $\alpha \neq 1$

Rizzi et al, PRB (2006), Bercioux et al, PRA (2009)

Hamiltonian for $\alpha - \mathcal{T}_3$ lattice

Tight-binding approx. (within nearest-neighbour hopping)

$$H_0(\mathbf{k}) = \begin{pmatrix} 0 & tf^*(\mathbf{k})\cos\phi & 0 \\ tf(\mathbf{k})\cos\phi & 0 & tf^*(\mathbf{k})\sin\phi \\ 0 & tf(\mathbf{k})\sin\phi & 0 \end{pmatrix}$$



$$\alpha = \tan\phi \qquad f(\mathbf{k}) = \sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{3}} \mathbf{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{a}_\mathbf{j}}$$

$$\text{duality}: \ \alpha \to 1/\alpha. \ \ \alpha : \{0-1\}$$

Three level system: $E_\pm(\mathbf{k}) = \pm t|f(\mathbf{k})|, \quad E_0(\mathbf{k}) = 0.$

A flat band is sandwitched between two dispersive bands

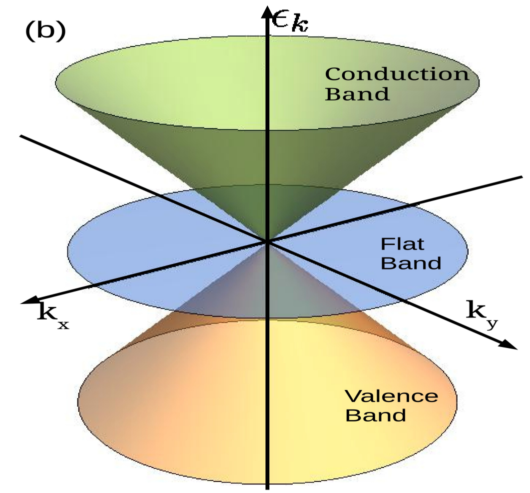
Dispersive bands are exactly the same as that of graphene

Three levels are α **independent, but the states do depend on** α

Vivcencio et al PRL 114 (2015) & Mukherjee, PRL 114 (2015)

The low-energy Hamiltonian around the Dirac points

$$H^{\xi}(\mathbf{k}) = \hbar v_f[\xi S_x(\alpha)k_x + S_y(\alpha)k_y]$$



α **-dependent spin-1 like matrices**

$$S_x(\alpha) = \begin{pmatrix} 0 & \cos\phi & 0 \\ \cos\phi & 0 & \sin\phi \\ 0 & \sin\phi & 0 \end{pmatrix} \quad S_y(\alpha) = \begin{pmatrix} 0 & -i\cos\phi & 0 \\ i\cos\phi & 0 & -i\sin\phi \\ 0 & i\sin\phi & 0 \end{pmatrix}$$

Low-energy spectrum around the Dirac points:

$E_{\pm}(\mathbf{k}) = \pm\hbar v_f|\mathbf{k}|$ and $E_0 = 0$ for all values of ***k***

Dice lattice: $\alpha = 1(\phi = \pi/4)$: $H^{\xi}(\mathbf{k}) = \hbar v_f(\xi S_x k_x + S_y k_y)$

$$\mathbf{S} = (S_x, S_y, S_z)$$

Standard spin-1 matrices

Spin-1 Dirac-Weyl Hamiltonian for the dice lattice

Variable Berry phase

For graphene: the Berry phase is π (Independent of valleys)

$\alpha > 0,$ **Valley-dependent Berry phase:**

Conduction and valence bands $\gamma_B^{\pm 1,\xi} = \pi\xi\left(\frac{1-\alpha^2}{1+\alpha^2}\right)$

Flat band $\gamma_B^{0,\xi} = -2\pi\xi\left(\frac{1-\alpha^2}{1+\alpha^2}\right)$

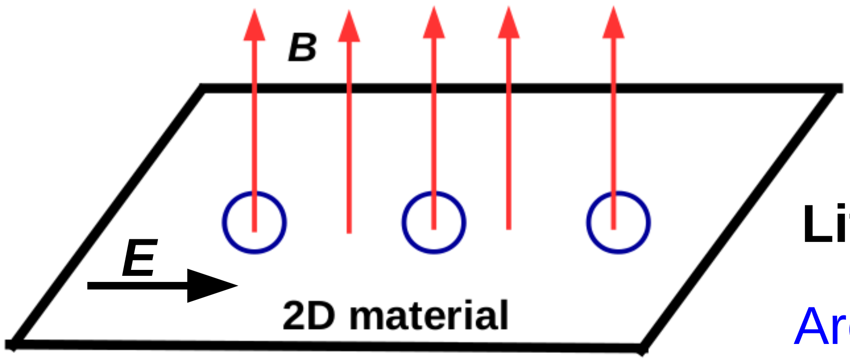
For dice lattice, the Berry phase is zero for all the bands

The Berry phase is smoothly decreasing with α **and becomes zero for dice lattice**

The Berry phase is different in two K valleys except for graphene and dice lattice

Valley dependent Berry phase will be reflected on magnetotransport properties

Honeycomb lattice in presence of magnetic field



Applying uniform magnetic field normal to the lattice: Landau levels are formed

Lifshitz-Onsager semiclassical quantization:

Area of cyclotron orbit in k-space is quantized

$$S_n = \frac{2\pi e B}{\hbar}(n + 1/2),\ n = 0, 1, 2$$

For linear spectrum: $E_n = \hbar\omega_c\sqrt{n+1/2}$

Cyclotron motion in k-space, acquires Berry phase. It must appears in Landau levels

Modified Lifshitz-Onsager semiclassical quantization: $S_n = \frac{2\pi e B}{\hbar}(n + 1/2 - \gamma_B^{\xi}/2\pi)$

Berry phase dependent Landau levels $E_n = \hbar\omega_c\sqrt{n + 1/2 - \gamma_B^{\xi}/2\pi} = \hbar\omega_c\sqrt{n}$

This is the reason why zero-energy Landau level exist in graphene. Existence of zero-energy LL shows up in the Hall conductivuty measurement

Graphene: Carbon in 2D, M I Katsnelson

$\alpha - \mathcal{T}_3$ lattice in presence of magnetic field

Applying uniform magnetic field normal to the lattice: Landau levels are formed

For α-$\mathcal{T}_3$ lattice, Berry phase dependent Landau levels (using operator method and modified LO semiclassical quantization condition:

$$E_{n,\xi}^{\lambda} = \lambda\hbar\omega_c\sqrt{n + 1/2 - \gamma_B^{\xi}/2\pi}, \; n = 0, 1, 2...$$

$E_n^{\text{flat}} = 0$ **(semiclassical quantization condition fails here)**

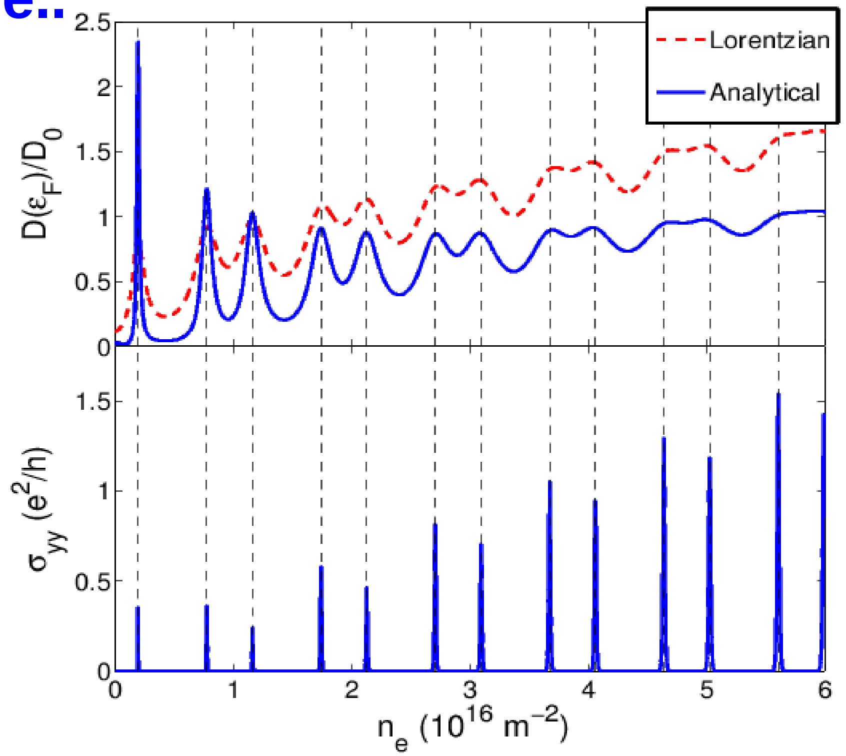
Valley degeneracy of Landau levels is removed $0 < \alpha < 1$

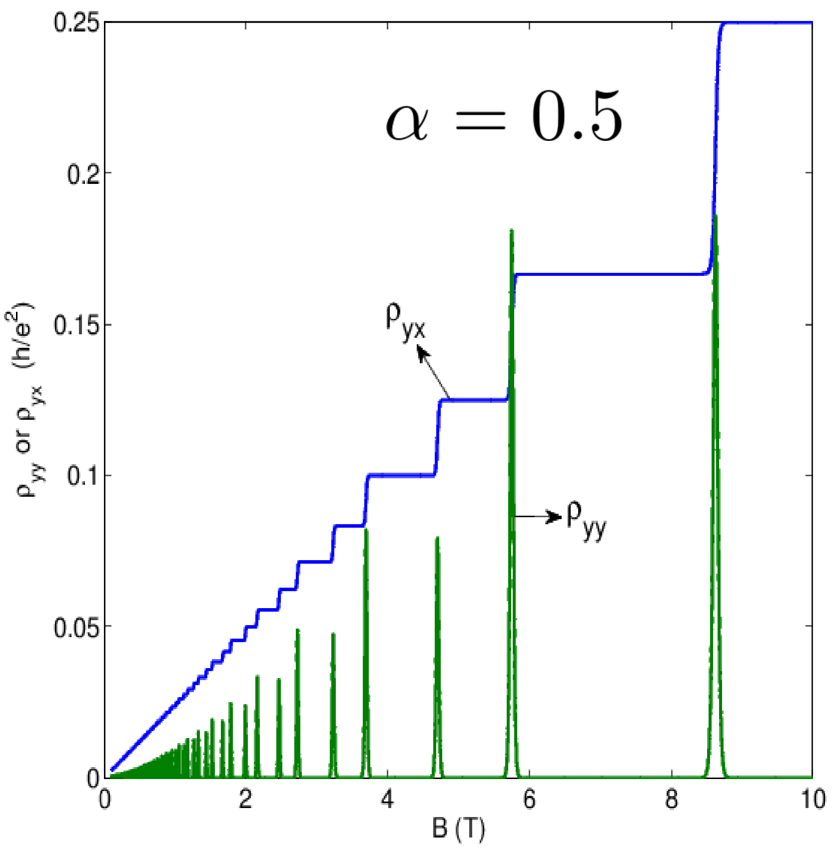
Zero-energy Landau level does not exist for $0 < \alpha \leq 1$

Subnikov de-Haas (SdH) oscillations and quantized Hall resistivity

Valley-dependent DoS through Berry phase..

$$D_\xi(\epsilon_f) \sim \cos[2\pi\epsilon_f^2 - 1/2 + \gamma_B^\xi/2\pi]$$

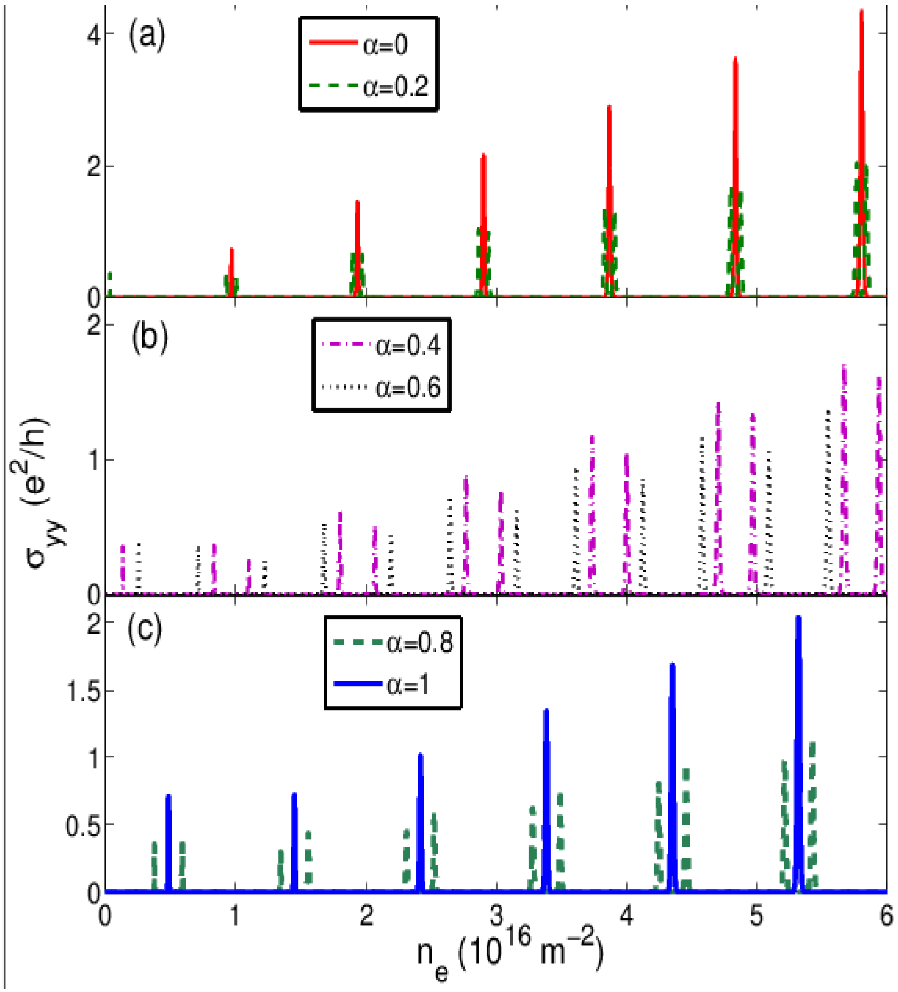




Each valley contributes differently, giving rise to two closely spaced peaks.

Tutul and TKG, JP: CM (2016)

Longitudinal conductivity



$$D_\xi(\epsilon_f) \sim \cos[2\pi\epsilon_f^2 - 1/2 + \gamma_B^\xi/2\pi]$$

Oscillatory behaviour consisting of a number of peaks

A Peak splits into two unequal peaks due to valley-dependent DoS

Two splitted peaks move in opposite direction as α increases.

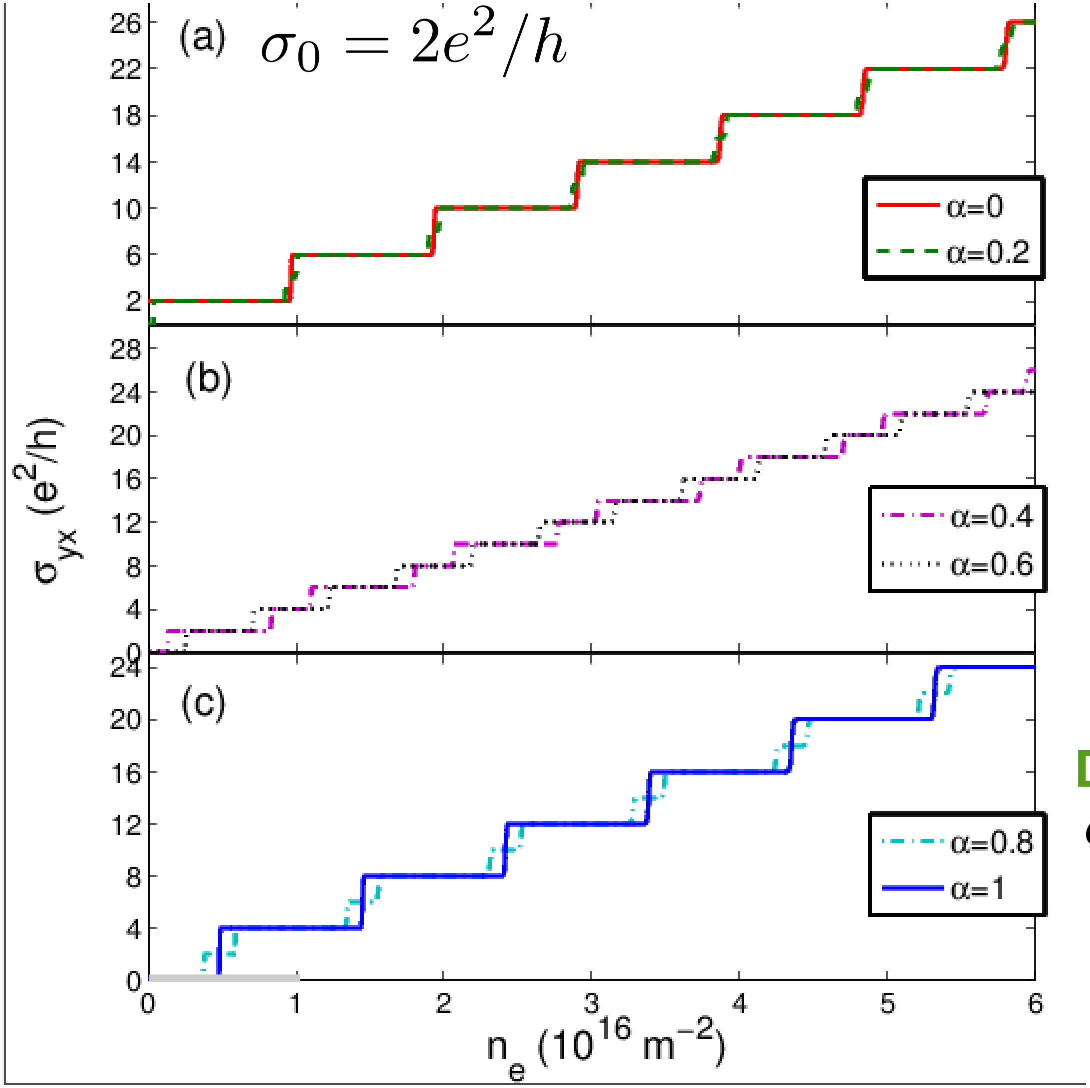
Right-split of *N*-th peak and left-split of (*N*+1)-th peak merges together when α =1

For dice lattice, a new set of conductivity peaks whose positions are completeley different from graphene

Tutul Biswas and TKG, JP: CM (2016)

Anomalous quantized Hall conductivity

***N*: number of Landau levels below Fermi energy (excluding zero energy LL)**



graphene

$$\alpha = 0: \ \sigma_{xy} = \sigma_0(2N+1)$$

$$0 < \alpha < 1: \sigma_{xy} = \sigma_0(N+0)$$

Each valley contributes separately, giving additional Hall plateaus

Dice lattice

$$\alpha = 1: \ \sigma_{xy} = \sigma_0(2N+0)$$

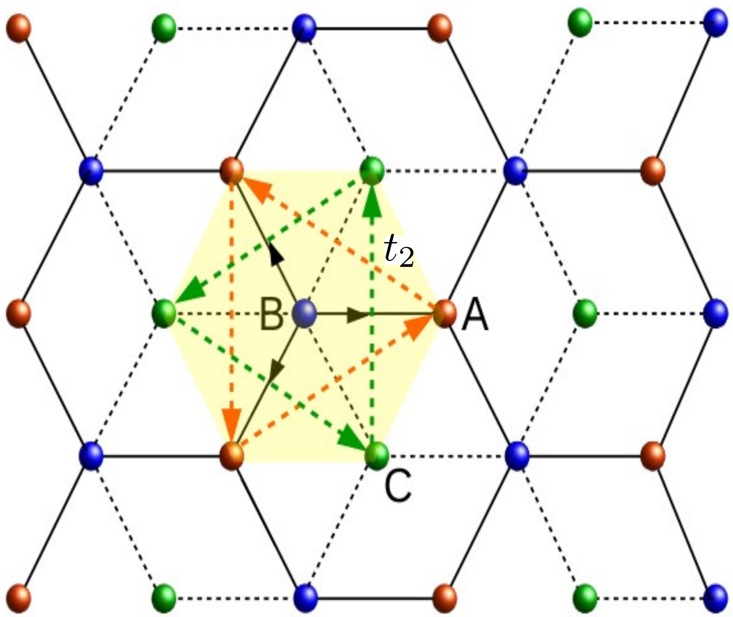
(Similar to bilayer graphene)

Dice lattice can be thought of as a "zero-separation" bilayer graphene

Tutul Biswas and TKG, JP: CM (2016)

Haldane-like model of dice lattice

Spatially periodic magnetic flux through hexagonal unit cell centered around any hub stite (B) vanishes



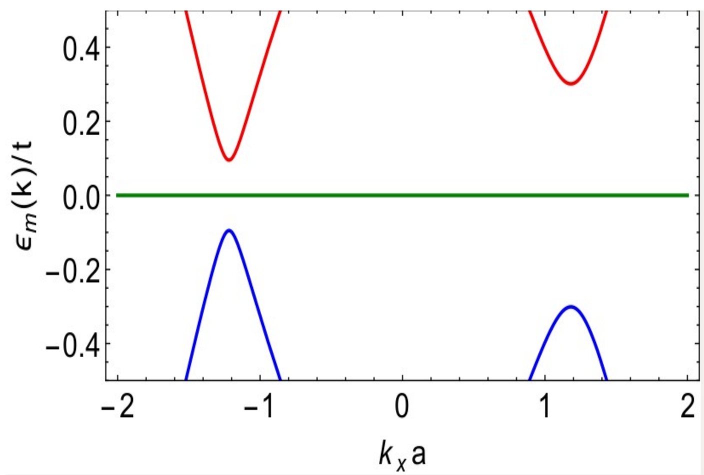
On-site energies M at A and $-M$ at C sublattices

Low-energy Hamiltonian at a partcular magnetic field:

$$H_\xi = \hbar v_f(\xi S_x k_x + S_y k_y) + (M - \xi\tilde{t}_2)S_z$$

$$E_\pm = \pm\sqrt{(\hbar v_f k)^2 + (M - \xi\tilde{t}_2)^2}; E_0 = 0$$

Both space inversion and time reversal symmetries are broken



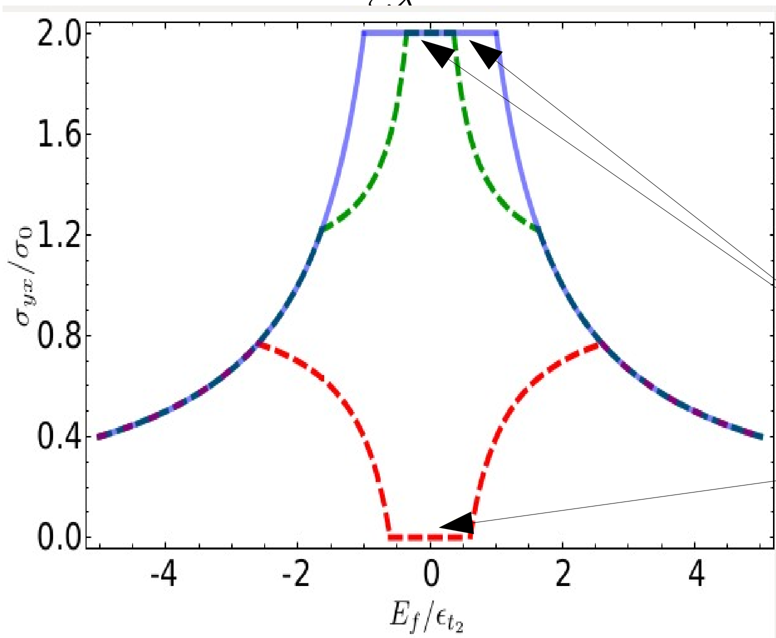
It opens up a valley-dependent gap

Bashab Dey, PK,OP and TKG, in preparation

Anomalous Hall conductivity in the Haldane model of dice lattice

$$\sigma_{yx}(E_f) = \frac{e^2}{\hbar}\sum_{\xi,\lambda}\int_0^\infty \Omega_\lambda^\xi(\mathbf{k})d^2k$$

$$\Omega_\pm^\xi(\mathbf{k}) = \pm\frac{\xi m_\xi}{(k^2+m_\xi^2)^{3/2}};\Omega_0^\xi(\mathbf{k}) = 0$$



When Fermi energy lies in the gap and

$M < \tilde{t}_2 : \sigma_{xy} = 2e^2/h.$ Topological Insulator

$M > \tilde{t}_2 : \sigma_{xy} = 0.$ trivial insulator

Varying mass M, one can go from trivial insulator to topological insulator with Chern number 2

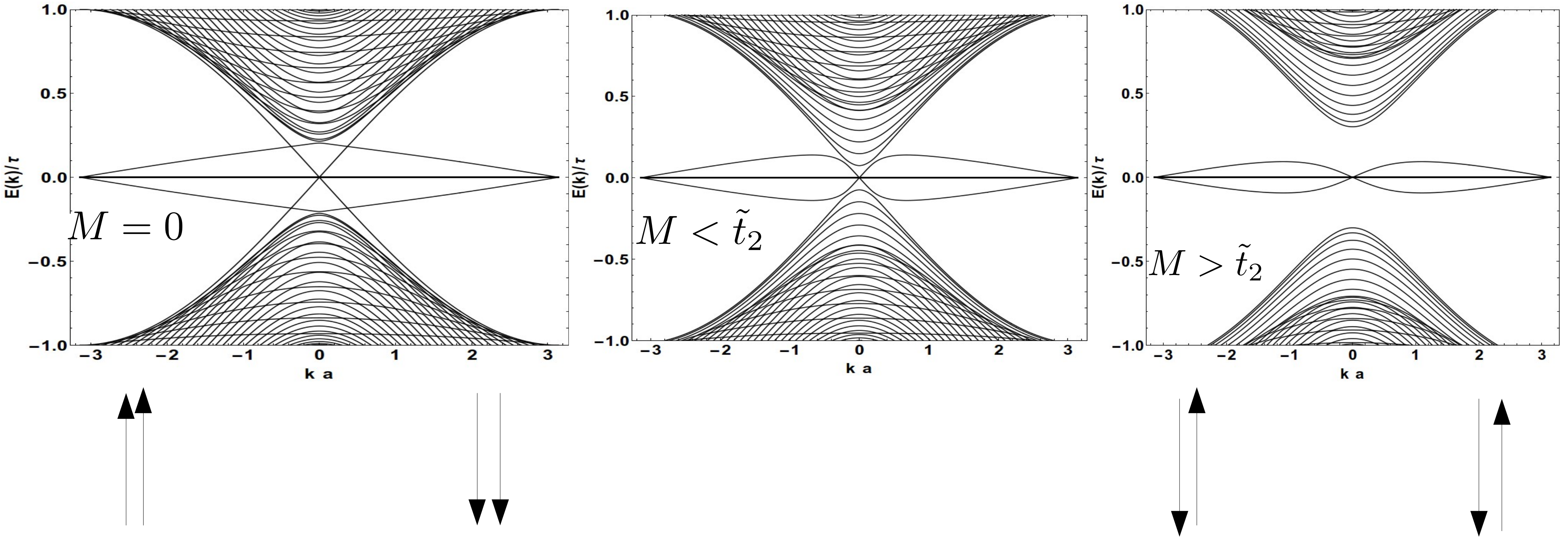
If Fermi energy lies in the gap, where is the conducting channel?

Bashab Dey, PK,OP and TKG, in preparation

Edge states of haldane-like model of dice lattice with armchair edges

Topological insulator

Trivial insulator



Left edge

Right edge

Two pairs of co-propgating edge states; finite net current

Left edge

Right edge

Two pairs of counter-propgating edge states; zero net current

Bashab Dey, PK,OP and TKG, in preparation

Haldane model of dice lattice in presence of quantizing magnetic field

$$H_\xi = \hbar v_f[\xi S_x(k_x + eA_x) + S_y(k_y + eA_y)] + (M - \xi\tilde{t}_2)S_z$$

Valley-dependent exact Landau energy levels are obtained

$$\epsilon_{nj}^{\xi} = 2\sqrt{\frac{-\beta}{3}}\cos[\frac{1}{3}\cos^{-1}(\frac{3\gamma}{2\beta}\sqrt{\frac{-3}{\beta}}) - \frac{2\pi j}{3}];\ j = 0, 1, 2. \quad n \geq 1$$

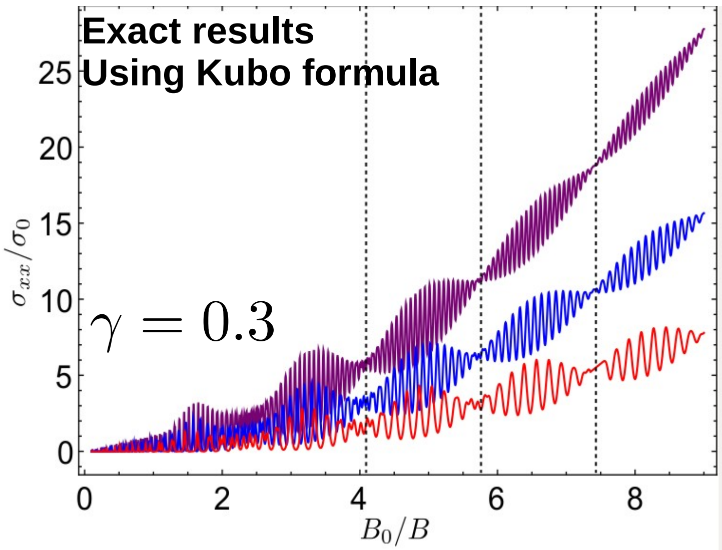
$$\beta = (m_\xi v_f^2)^2 + (2n+1)\epsilon_c^2,\ \gamma = \xi m_\xi v_f^2 \epsilon_c^2$$

$$\epsilon_0^{\xi} = [-\xi m_\xi v_f^2 \pm \sqrt{(m_\xi v_f^2)^2 + 4\epsilon_c^2}]/2$$

$$\epsilon_{00}^{\xi} = -\xi m_\xi v_f^2$$

Bashab Dey, PK,OP and TKG, in preparation

Magnetotransport in Haldane model of dice lattice



Semiclassical: $\sigma^{\xi}_{xx} \propto \cos[l_0^2 S^{\xi} - \gamma_B^{\xi}(\epsilon_f)]$

S^{ξ} : valley depependent area of the Fermi circle

$\gamma_B^{\xi}(\epsilon_f)$: energy dependent Berry phase

$$\sigma_{xx} = (\sigma^{+}_{xx} + \sigma^{-}_{xx})$$

Location of the j−th node : $\frac{B_0}{B_j} = \frac{2j-1}{4\gamma} - \frac{1}{\epsilon_f}$; $j = 1, 2, ...$ $\quad \gamma = M/\tilde{t}_2$

difference between two beat nodes : $\Delta = \frac{1}{2\gamma}$

Number of oscillations between two successive nodes: $N_{\rm osc} = \frac{\epsilon_f^2 - \gamma^2 - 1}{4\gamma}$

Results from simple analysis match very well with the exact results based on Kubo formula

On-site energy (*M*) and next nearest neighbour hopping energy can be extracted from this analysis.

Bashab Dey, PK,OP and TKG, in preparation

Summary and conclusion

Magnetotransport properties of the $\alpha - \mathcal{T}_3$ lattice

Haldane-like model of dice lattice: topological Chern insulator in three-band system

Shubnikov-de Haas oscillations in Haldane-like model of dice lattice

Magnetotransport coefficents (SdH oscillations and quantized Hall conductivity are directly related to the Berry phase acquired during cyclotron motion

Thank you for your kind attention