Chance in a Branching World

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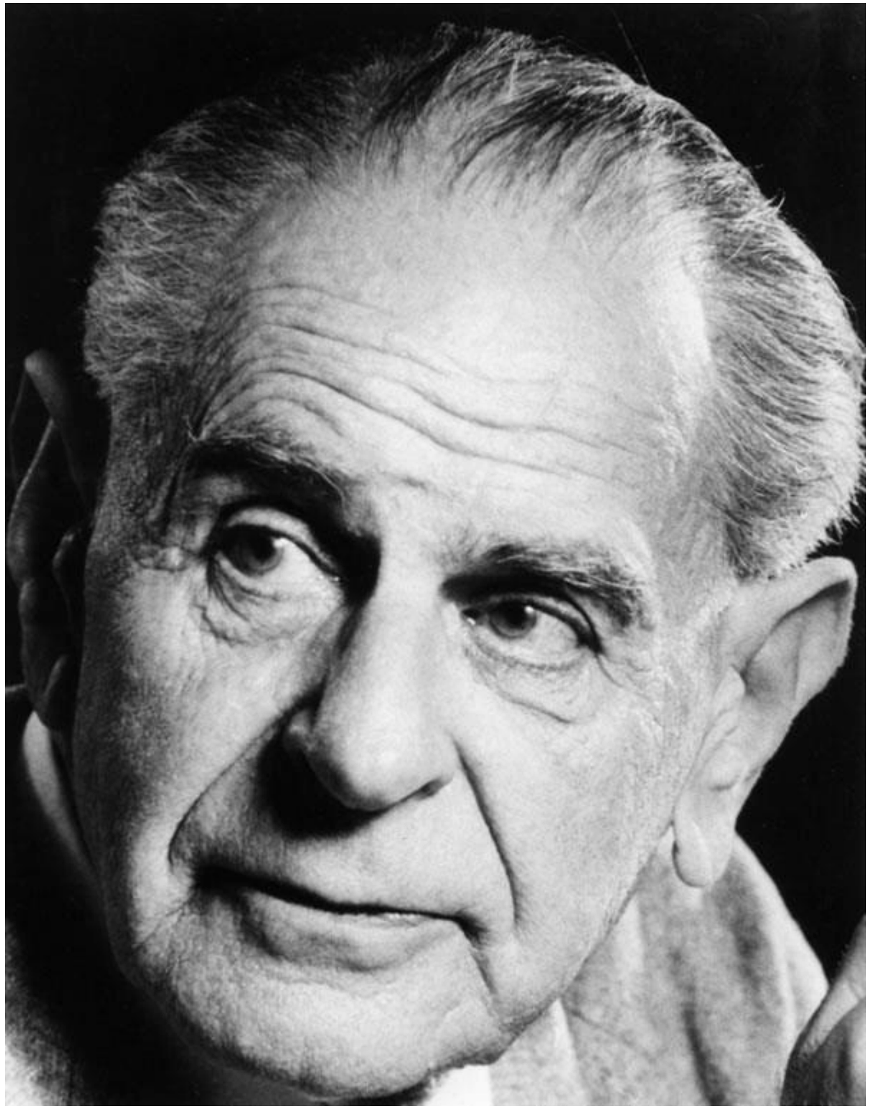
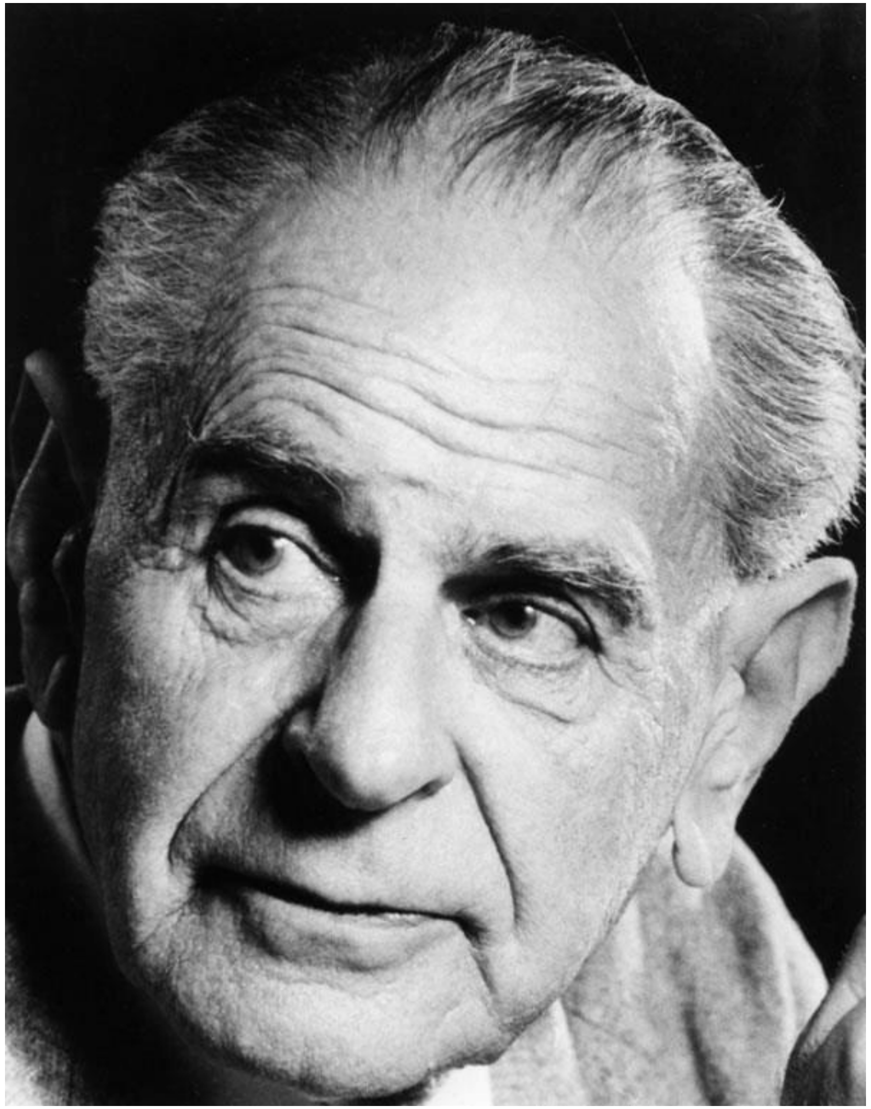
"There are two important fields in modern physics in which physicists have allowed subjectivism not only to enter, but to play an essential role: Boltzmann's theory of the subjectivity of the direction of time, and Heisenberg's interpretation of the indeterminacy formulae as determining a lower limit to the effect of the observer's interference with the observed object."

-- Karl Popper, *Objective Knowledge*

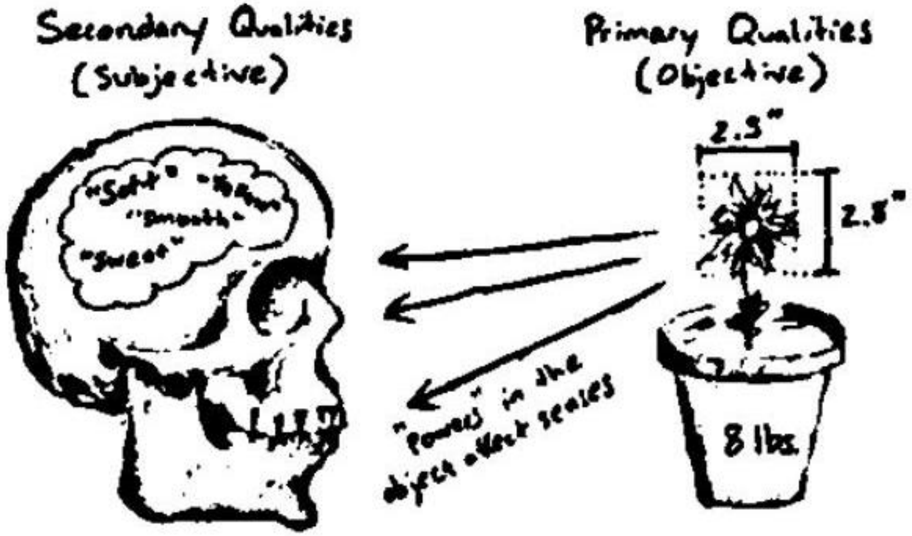


“There was also another intrusion of the subject, or of the observer, when Einstein brought in the observer in a number of imaginary thought experiments intended to elucidate relativity; but this is a field from which **the observer was exorcised**, slowly but steadily, by Einstein himself.”

-- Karl Popper, *Objective Knowledge*

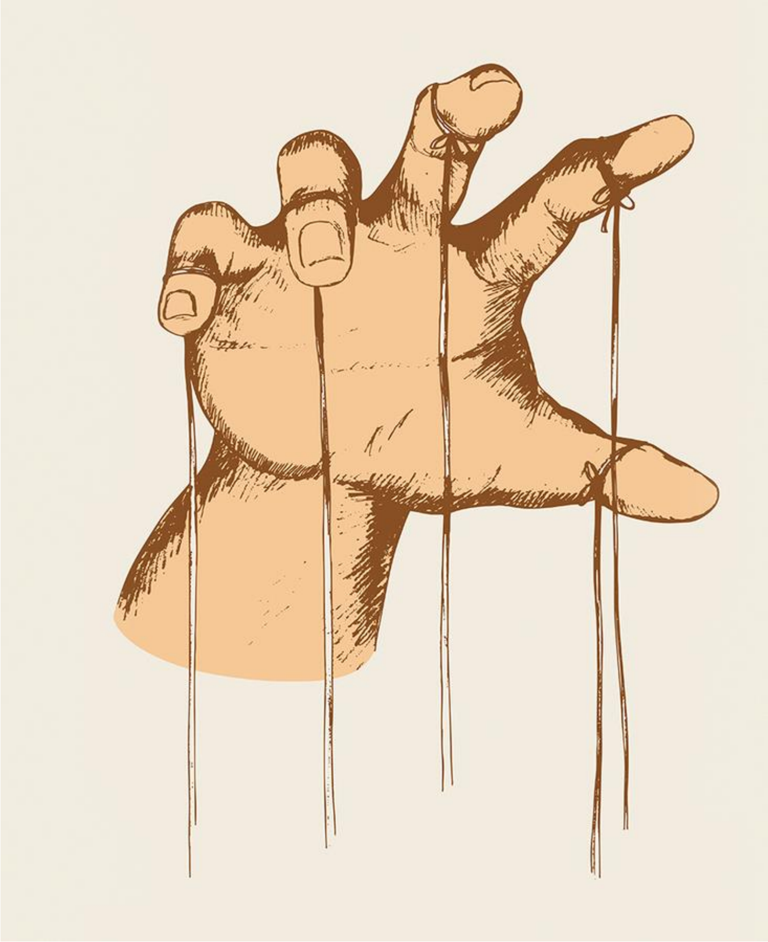


Subject Naturalism



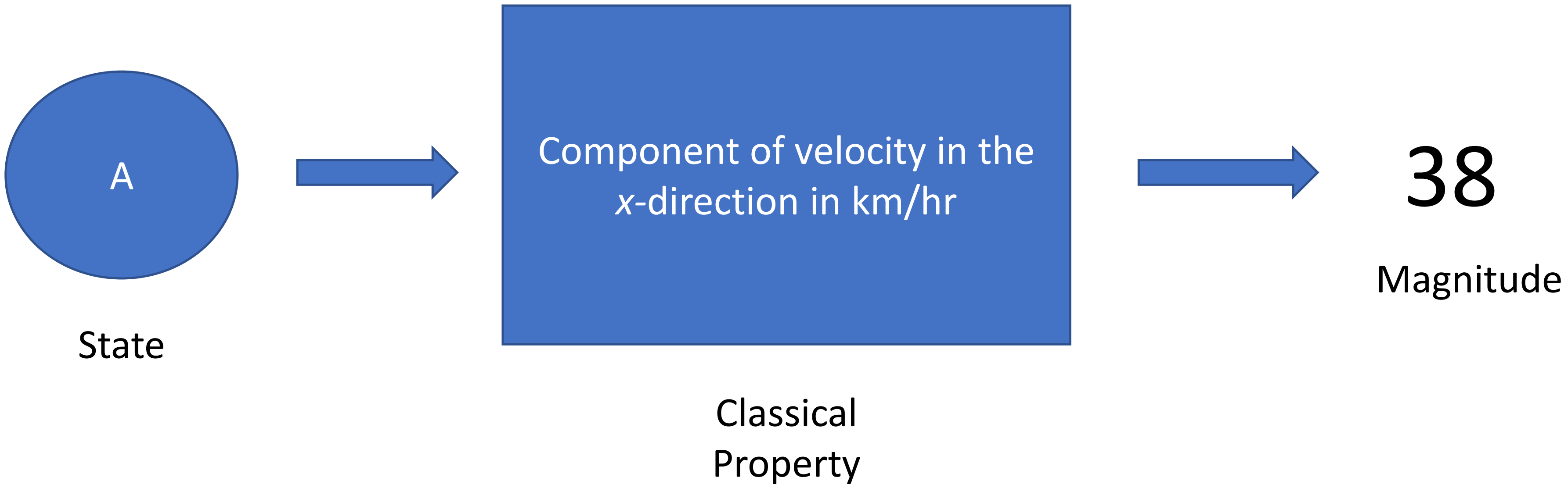
Observation: Our observation of the physical world must be understood as an interaction between physical systems, governed by the same basic laws that operate in generic interactions. Properly understanding the evidential significance of our observations for a physical theory requires locating humans within the theory.

Subject Naturalism

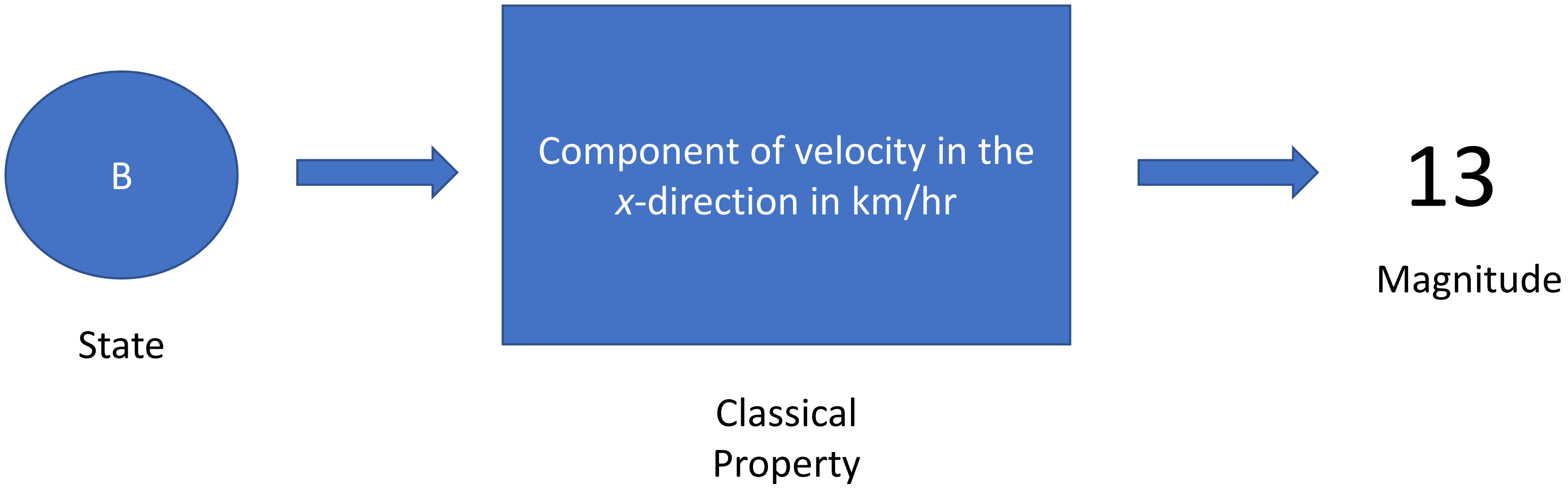


Action: Scientific knowledge is not meant to be purely descriptive, but also to provide effective strategies for action. This requires it to be geared towards use by agents like us, with our particular capacities for learning about and manipulating the world.

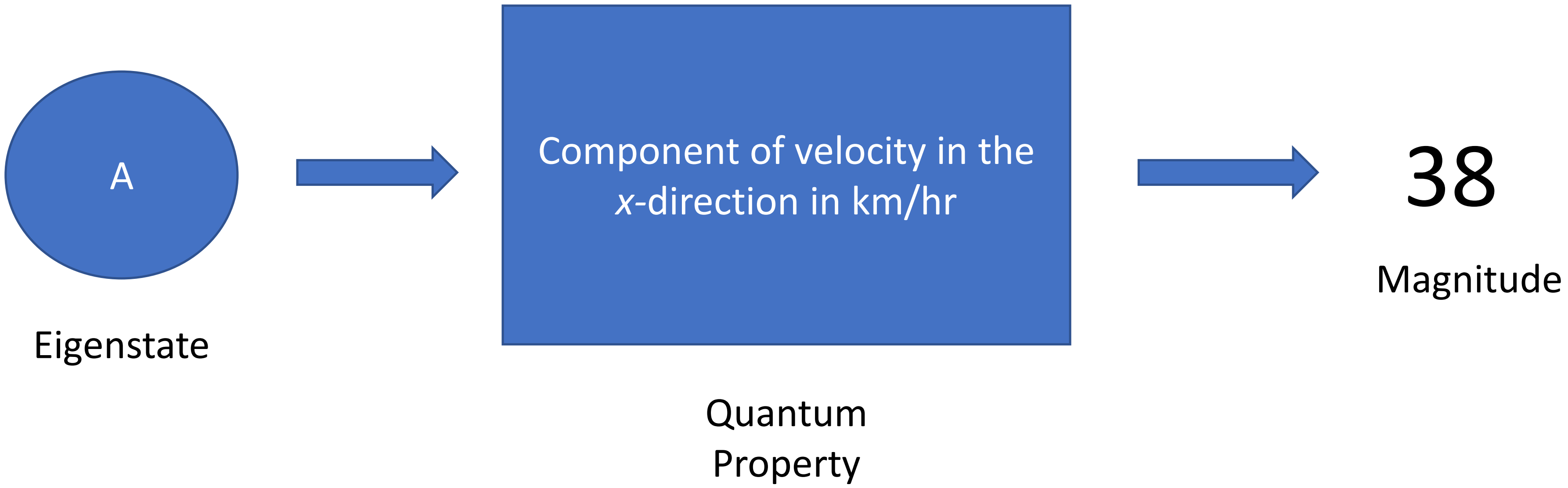
Classical Properties



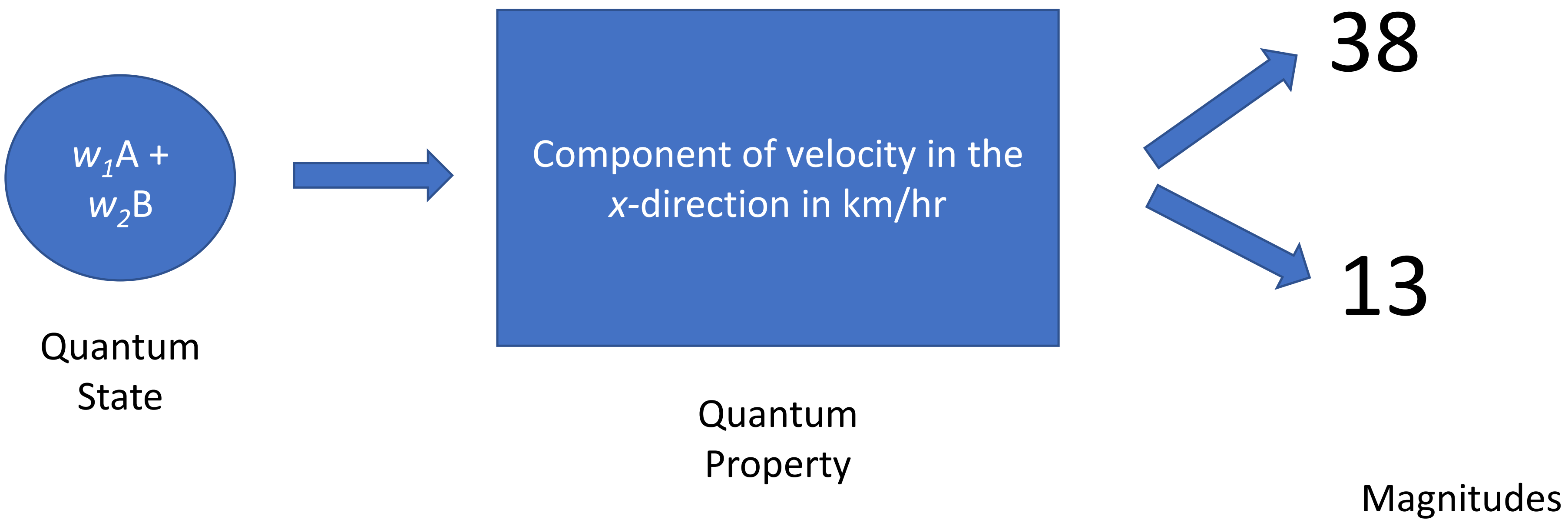
Classical Properties



Quantum Properties



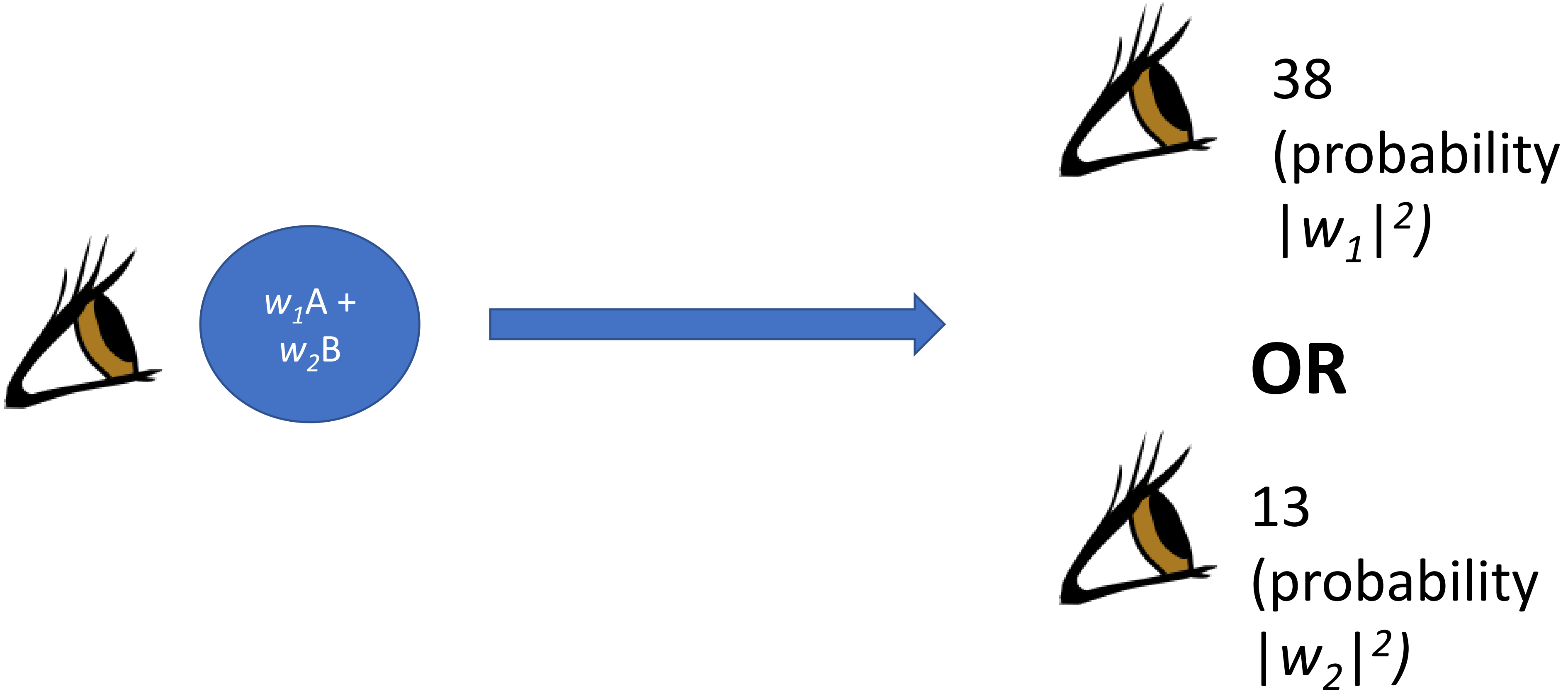
Quantum Properties



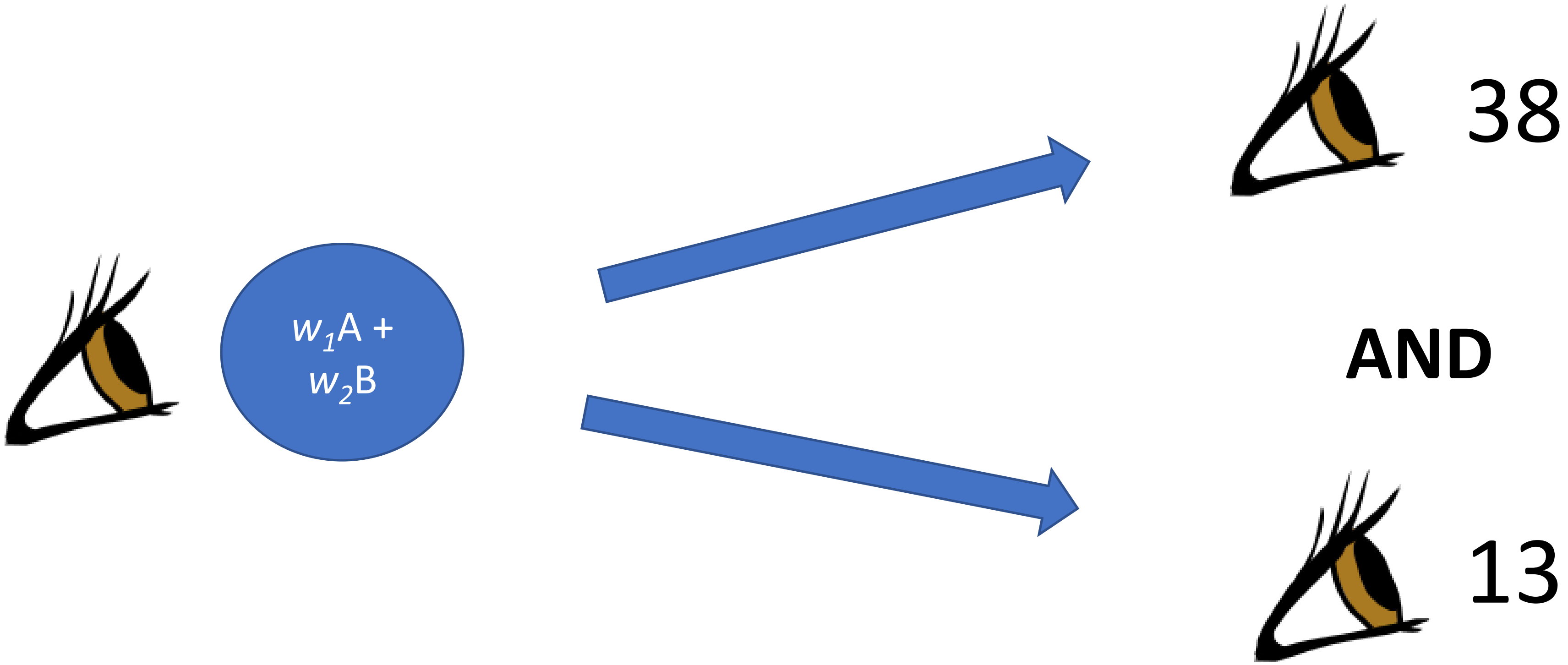
Schrodinger Equation

$$H(t)\,|\,\psi(t)\rangle = i\hbar\frac{d}{dt}\,|\,\psi(t)\rangle$$

Born Rule



Schrodinger Evolution



The Measurement Problem

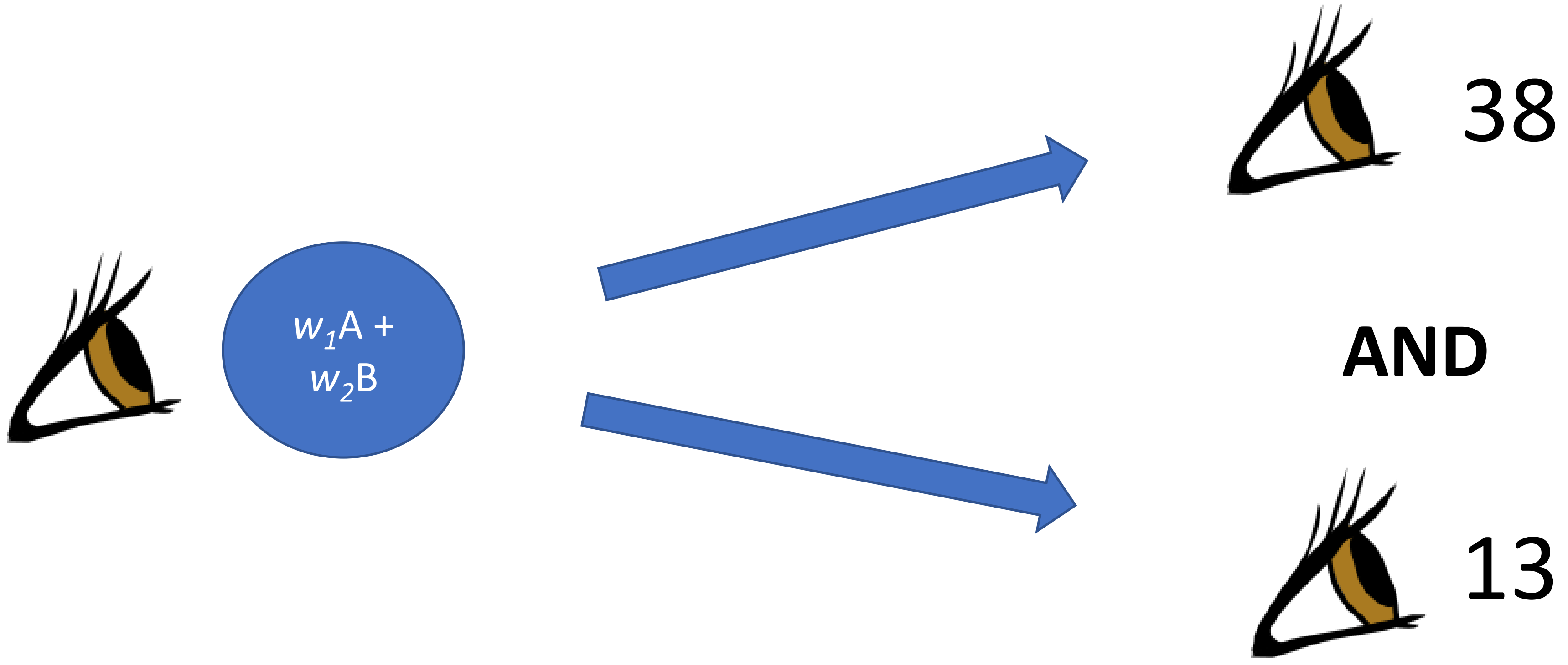
The following three claims are inconsistent:

1. The quantum description of a system is complete.
2. The quantum state always evolves in accord with a deterministic dynamical equation.
3. Measurements have single outcomes.

Resolving the Measurement Problem?

- *Induced Collapse* – When a superposition state interacts with a macroscopic object/measuring device/conscious observer, it collapses into an eigenstate.
- *Spontaneous Collapse* – Every superposition state has some chance of spontaneously collapsing into an eigenstate at any moment.
- *Hidden Variables* – The quantum state doesn't fully describe the system's state. Even if the quantum state is a superposition state, the system itself is in an eigenstate.
- *Instrumentalism* – Don't worry about what quantum theory says about the world, just use it to make predictions.

All Observers See Single Outcomes



The Measurement Problem?

The following three claims are ***not*** inconsistent:

1. The quantum description of a system is complete.
2. The quantum state always evolves in accord with a deterministic dynamical equation.
3. Individual observers observe single outcomes.

Everett / Many-Worlds Interpretation

Probability in the Everett Interpretation

$$\sqrt{\frac{2}{3}}|\uparrow_z\rangle + \sqrt{\frac{1}{3}}|\downarrow_z\rangle$$

Incoherence Problem: Why talk about probabilities at all, since there is seemingly no genuine uncertainty, and no alternate possibilities?

Probability in the Everett Interpretation

$$\sqrt{\frac{2}{3}}|\uparrow_z\rangle + \sqrt{\frac{1}{3}}|\downarrow_z\rangle$$

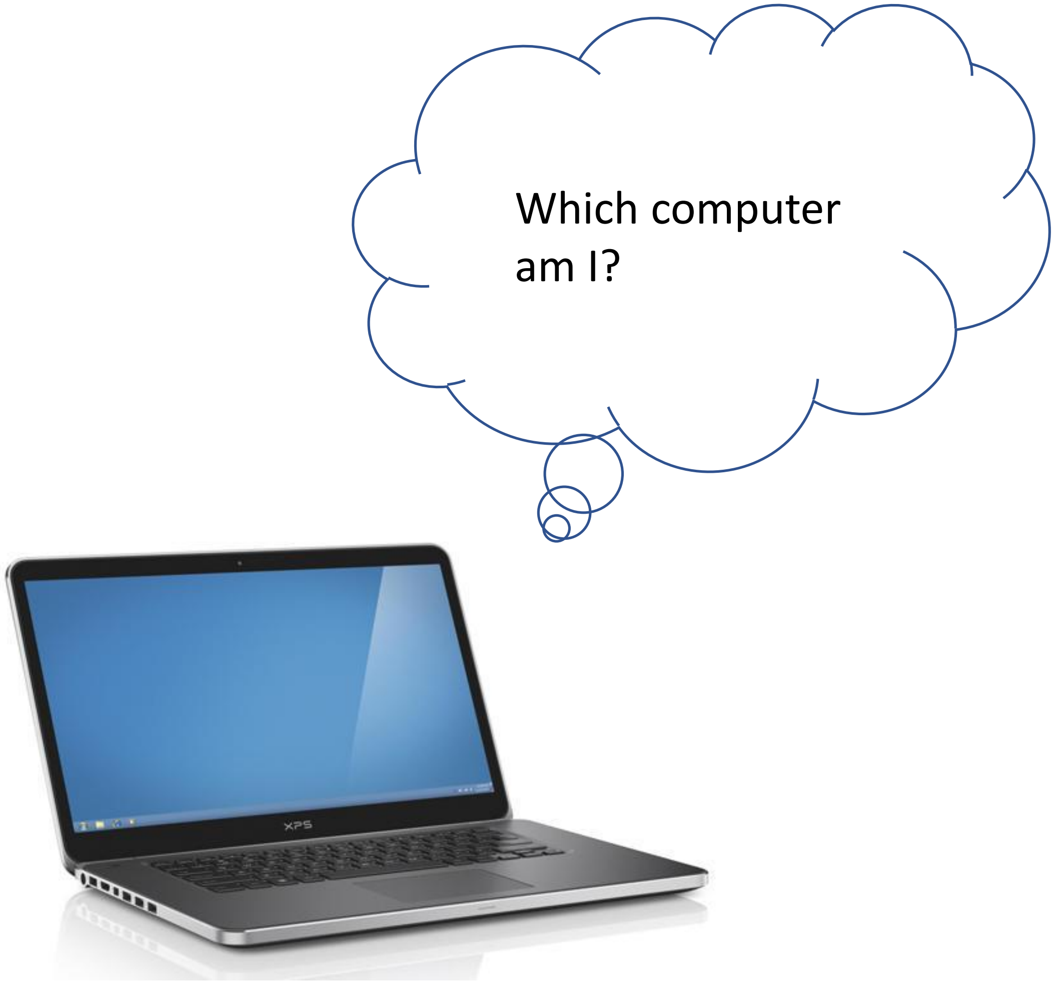
Quantitative Problem: Why are the probabilities given by the Born Rule?

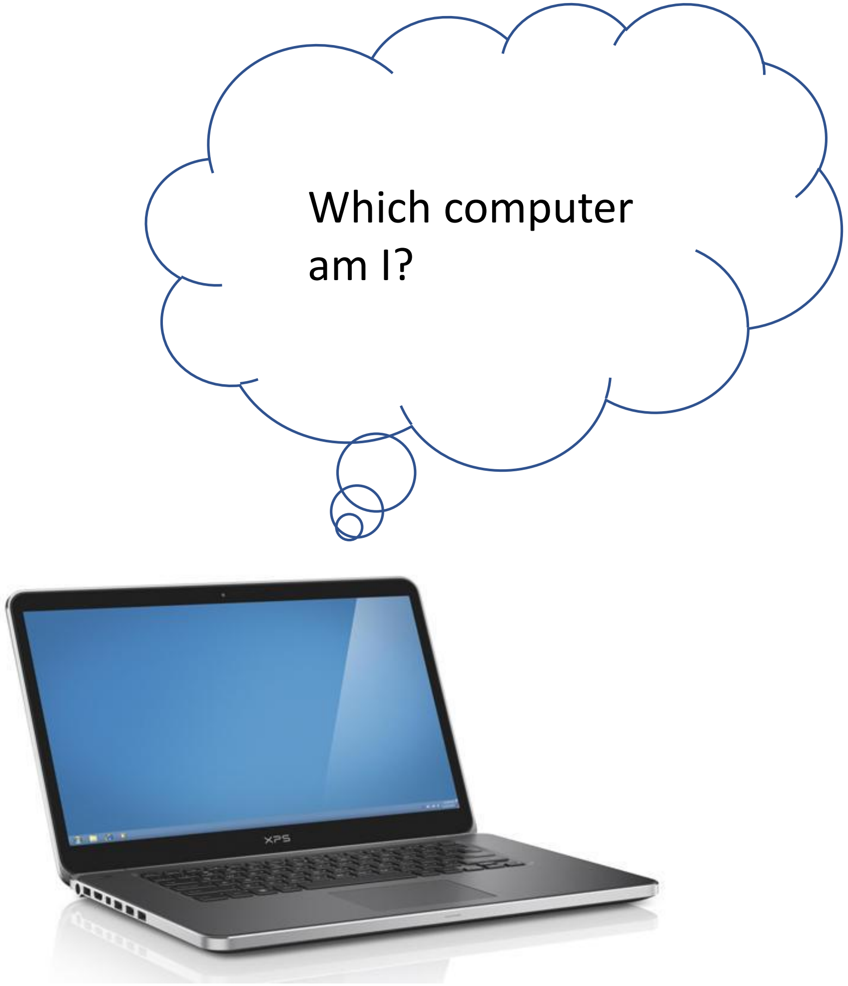
Probability in the Everett Interpretation

$$\sqrt{\frac{2}{3}}|\uparrow_z\rangle + \sqrt{\frac{1}{3}}|\downarrow_z\rangle$$

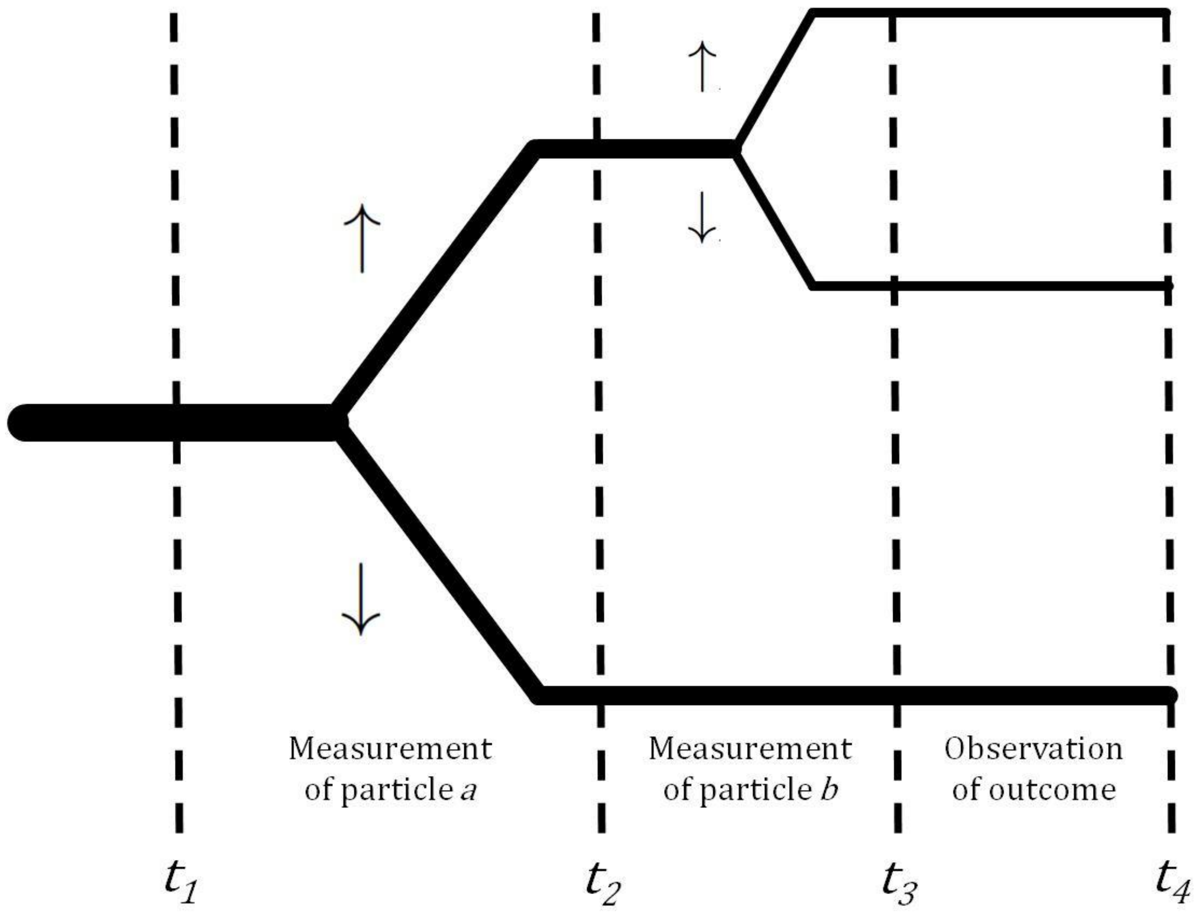
Epistemic Problem: How does our statistical evidence confirm quantum mechanics?

Self-Locating Uncertainty





Self-Locating Uncertainty



From Carroll and Sebens, "Self-Locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics"

Epistemic Separability Principle

"The credence one should assign to being any one of several observers having identical experiences is independent of the state of the environment."

Born Rule

Before the Measurement?

If probabilities in quantum theory are entirely based on post-measurement uncertainty, why should I reason probabilistically before the measurement is made?

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Reflection Principle: If at time $t < t'$, I know that I will assign probability p to some event A at t', and I know this will not be due to temporary irrationality, then I should assign probability p to event A at time t.

But Reflection Principle is controversial in cases of self-location. Example: Sleeping Beauty paradox.

Probability without Uncertainty

Expected Utility

	Light Traffic (P = 0.2)	Heavy Traffic (P = 0.8)
Drive	Arrive early (U = 45)	Arrive very late (U = 10)
Take Train	Arrive a little late (U = 30)	Arrive a little late (U = 30)

EU(Drive) = $0.2 \times 45 + 0.8 \times 10 = 17$

EU(Take Train) = $0.2 \times 30 + 0.8 \times 30 = 30$

A rational agent will follow the action with the higher expected utility.

Savage's Representation Theorem

Given a set of rationality constraints on a subject's preference structure, there is a unique probability measure over states of the world and a unique utility function over consequences of action such that an act A will be preferred over an act B if and only if the expected utility of A is greater than the expected utility of B.

Deutsch-Wallace Representation Theorem

Given a preference ordering over rewards, there is a unique (up to affine transformations) utility function over the rewards. If the agent is to use standard decision theory to reason about which actions to choose in order to maximize her expected utility, and we want the expected utilities of the actions to reflect the agent's given preferences over those sequences, then the probability distribution over outcomes we use when calculating the expected utility of actions **must be given by the Born rule**.

Some Key Assumptions

- The agent only cares about the macroscopic state of the world.
- The agent's preferences are *diachronically consistent* – they should not change without any new information being learned.
- The agent does not care about mere branching. She doesn't have a preference for creating more branches if there is no other reward associated with it.