The physical, statistical, and computational challenges of Pulsar Timing

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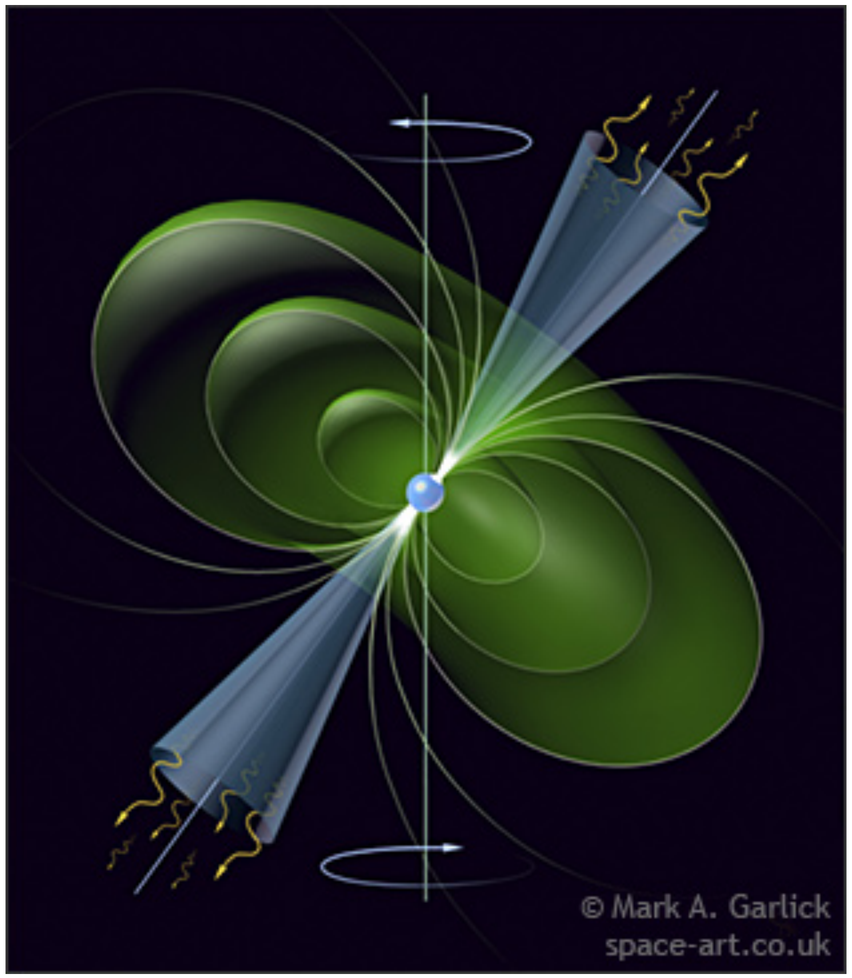
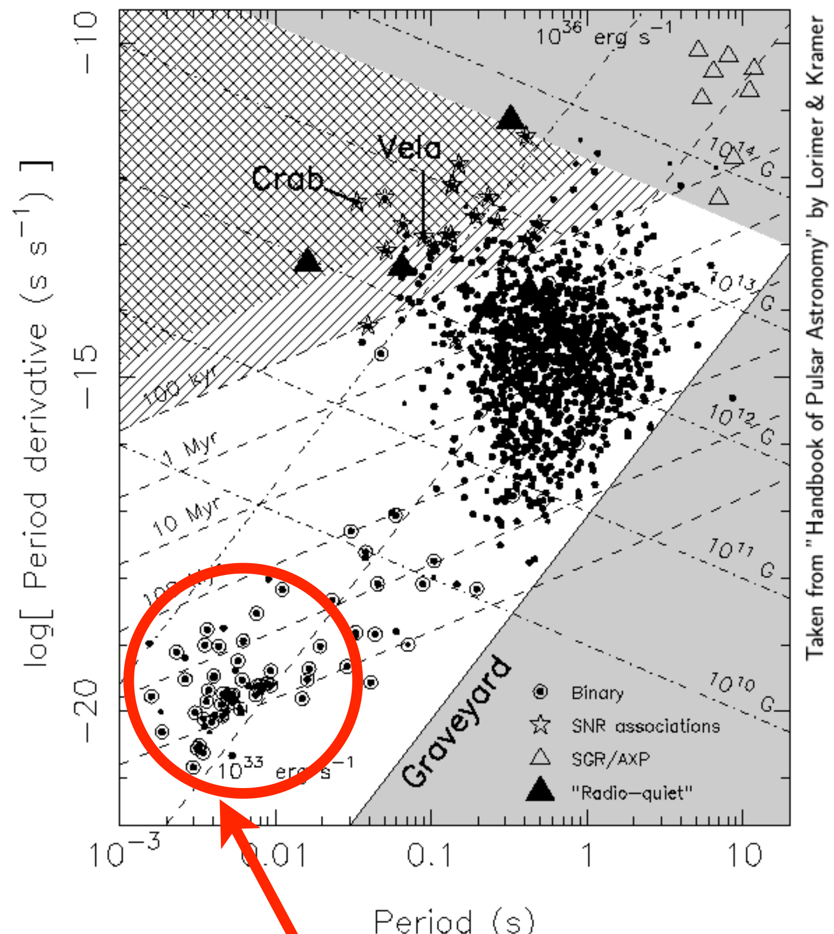
ICTS
March 22, 2017

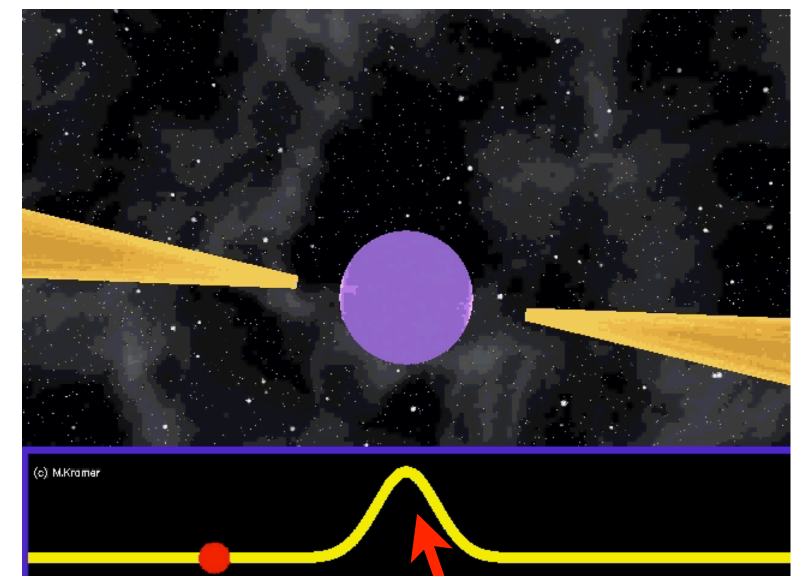
Pulsar Timing Preliminaries



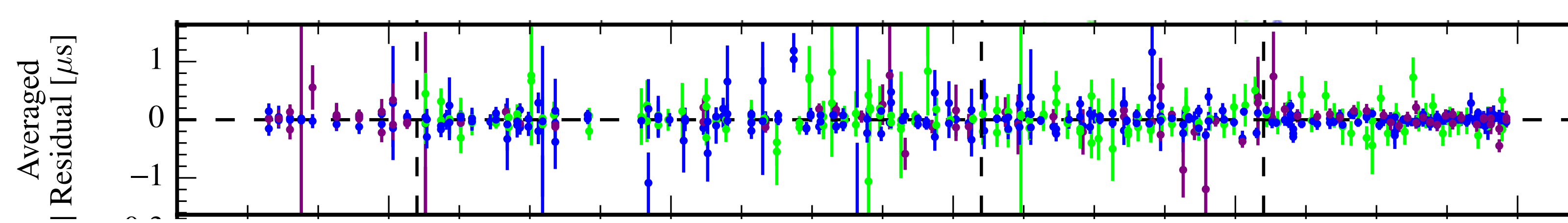
Highly magnetized rotating neutron star



Extremely stable clocks



time-of-arrival (TOA)



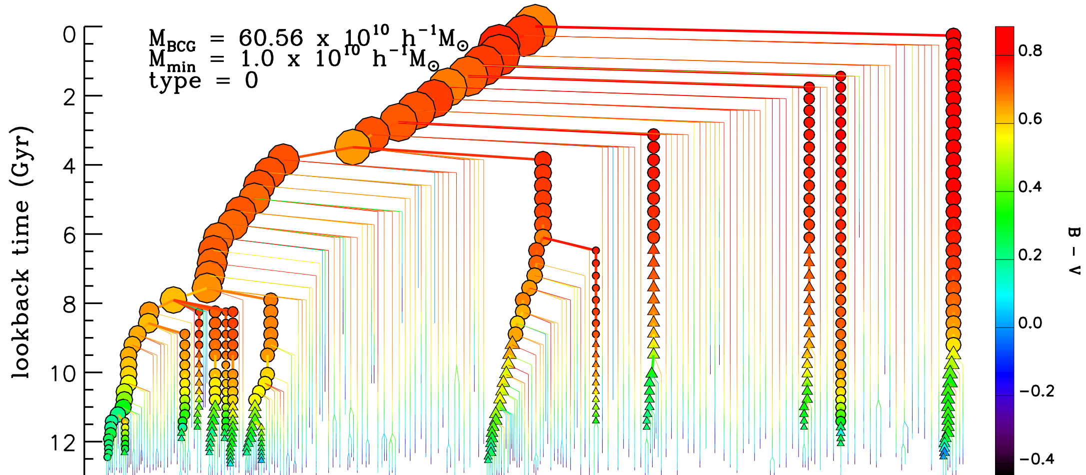
$$\delta t = t_{\text{measured}} - t_{\text{model}}$$

Pulsar Timing "Residual"

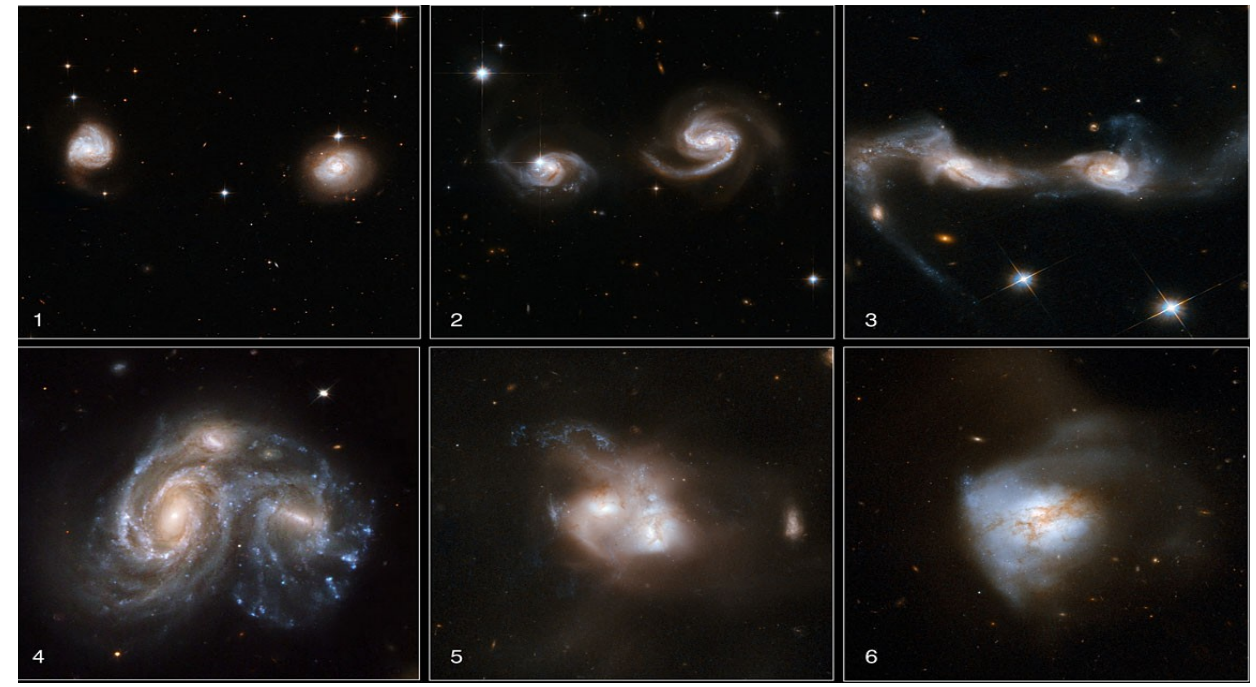
Measured Pulse time-of-arrival (TOA)

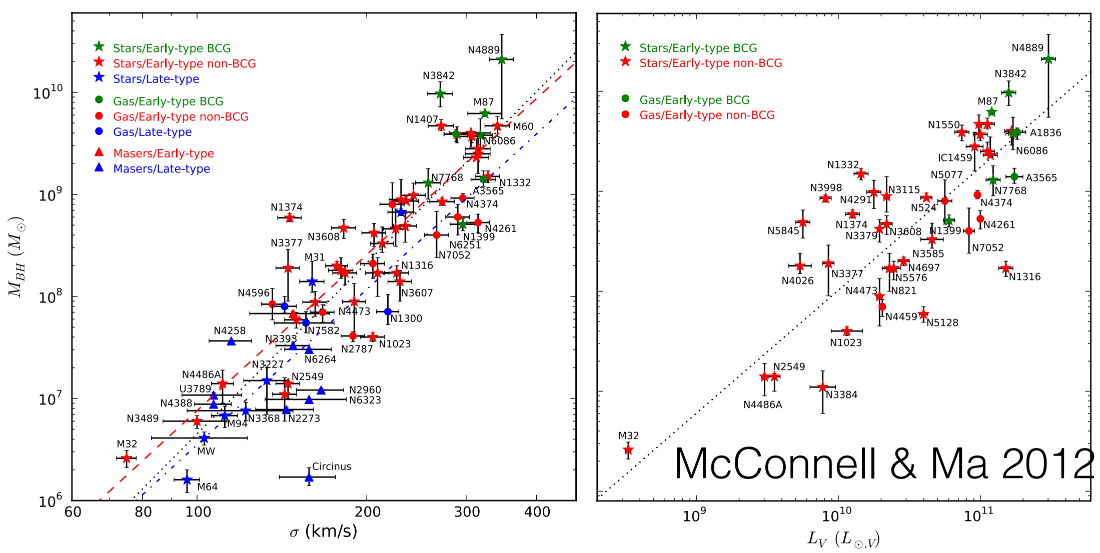
Model that accounts for many delay factors but not GWs

Sources of GWs: SMBHBs



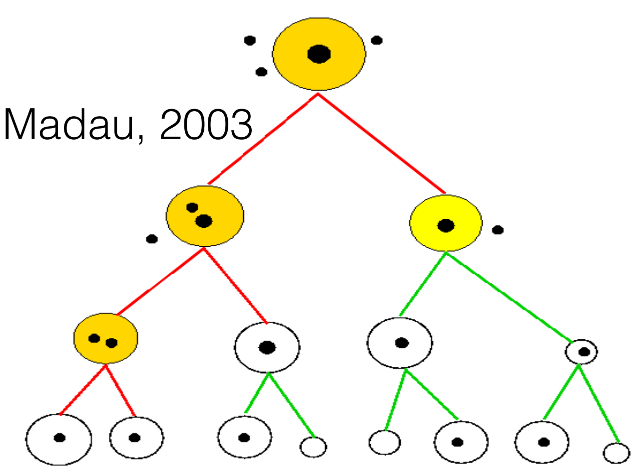
De Lucia et al, 2006

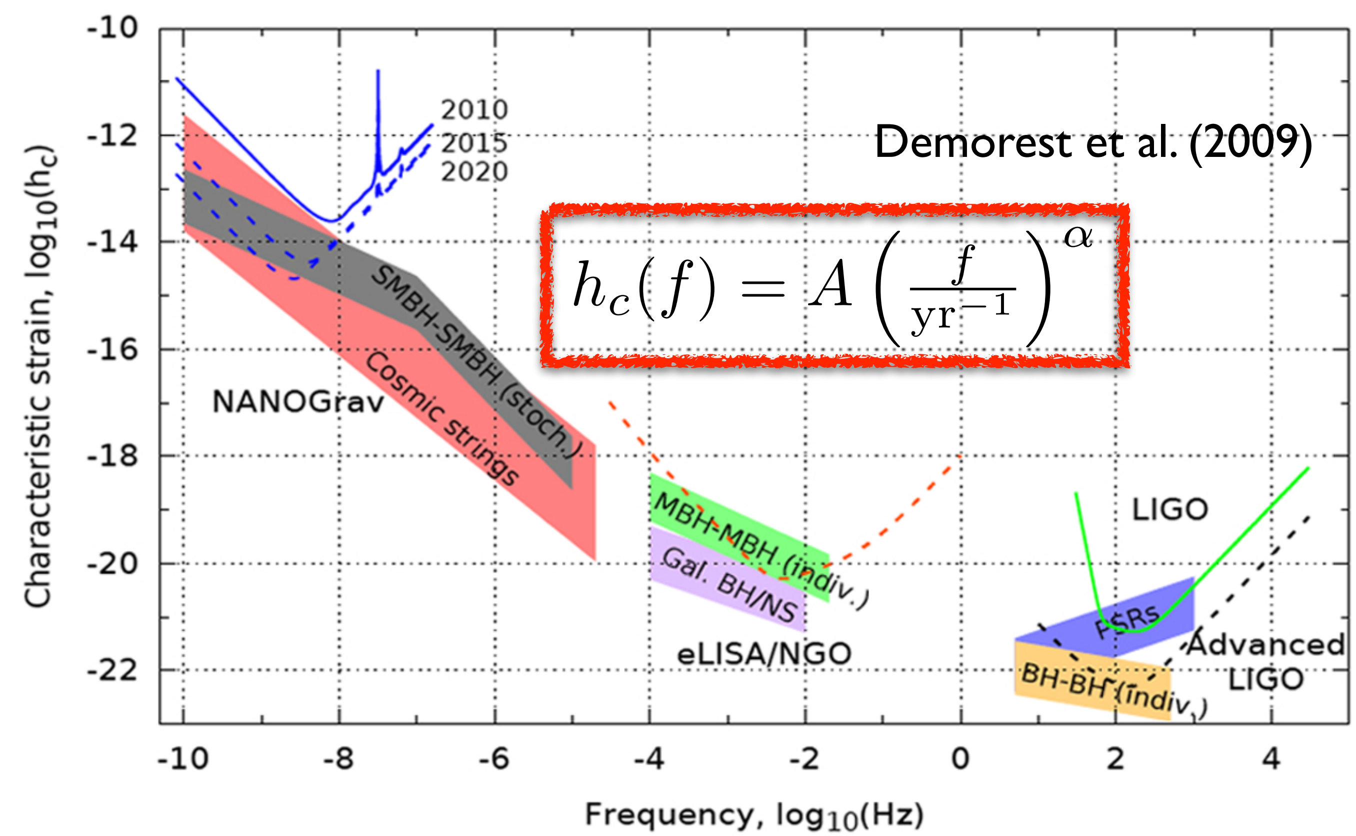




McConnell & Ma 2012

Vononteri, Hardt & Madau, 2003





Demorest et al. (2009)

$$h_c(f) = A\left(\frac{f}{\mathrm{yr}^{-1}}\right)^{\alpha}$$

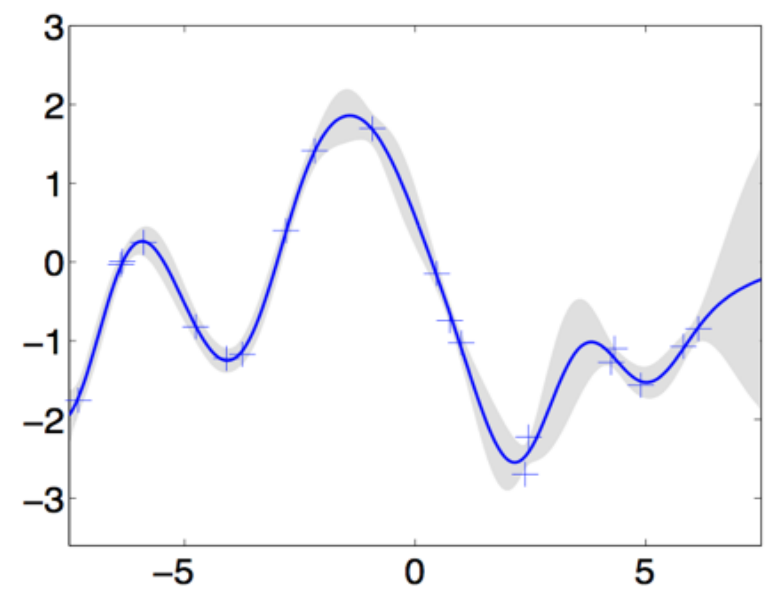
Challenges

- **Physical:**
  - Do we understand the noise in our data?
  - Are we confident in our GWB signal model?
- **Statistical:**
  - How do we model several different kinds of noise processes simultaneously?
  - Are the data really gaussian?
  - How do we assess detection significance?
  - How do we sample the large parameter spaces?
- **Computational:**
  - How do we model and sample all processes simultaneously while still allowing a decent run time?

A PTA noise *model*: everything is a Gaussian process

timing residuals = radiometer noise (white) + timing-model errors + jitter noise (white, epoch) + DM + timing noise (red) + GWs

van Haasteren & MV
PRD 90, 104012 (2014)



Basis picture
Search over basis coefficients and hyperparameters

$$y_{\mathrm{gp}} = F a$$

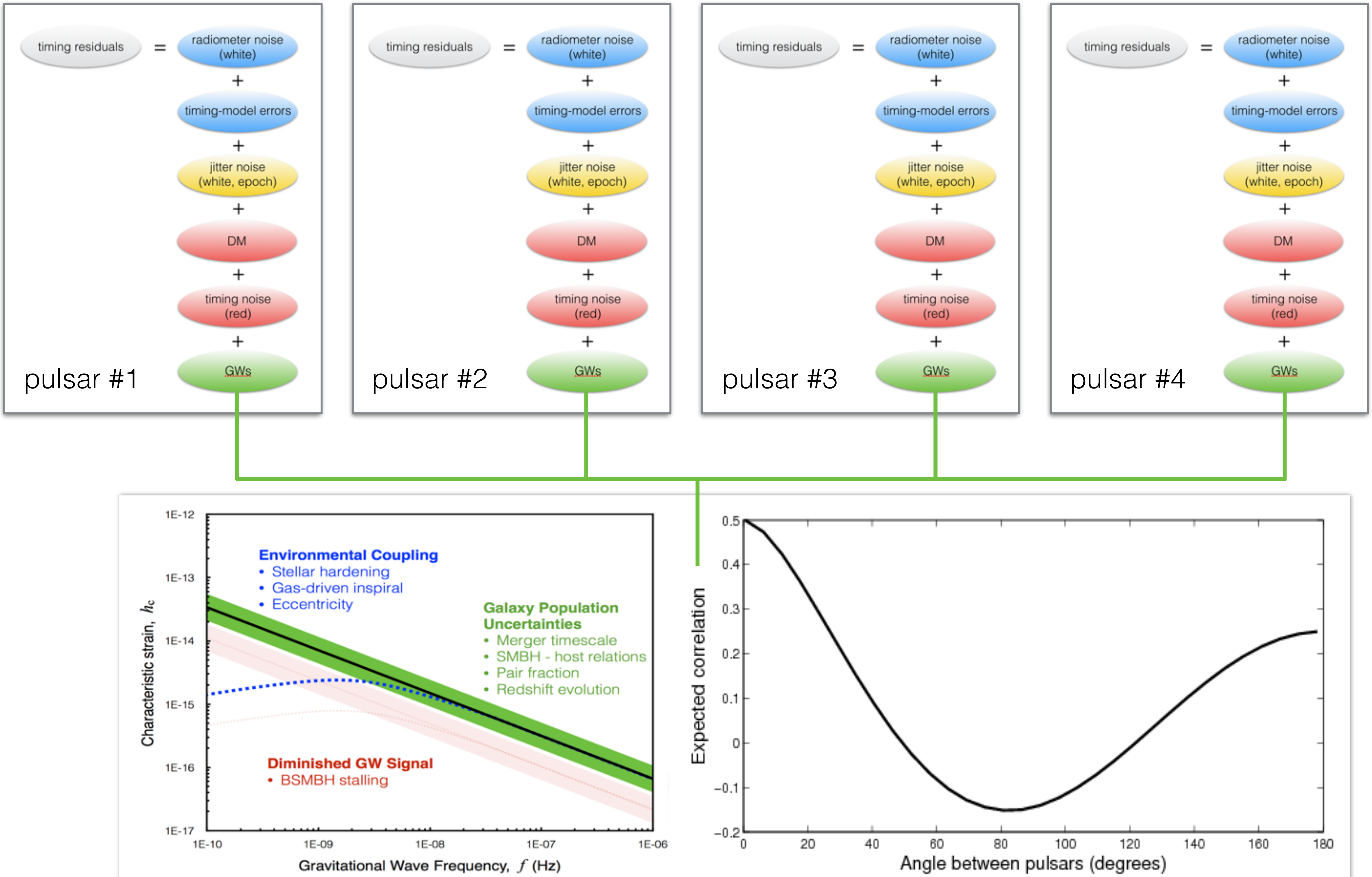
$$p(a) \propto e^{-a^T \Phi(\theta)^{-1} a/2}$$

Kernel picture
Marginalize over basis coefficients, search over hyperparameters

$$p(y_{\mathrm{gp}}) \propto e^{-y_{\mathrm{gp}}^T K(\theta)^{-1} y_{\mathrm{gp}}/2}$$

$$K(\theta) = F \Phi(\theta) F^T$$

Stochastic GWs as correlated Gaussian process

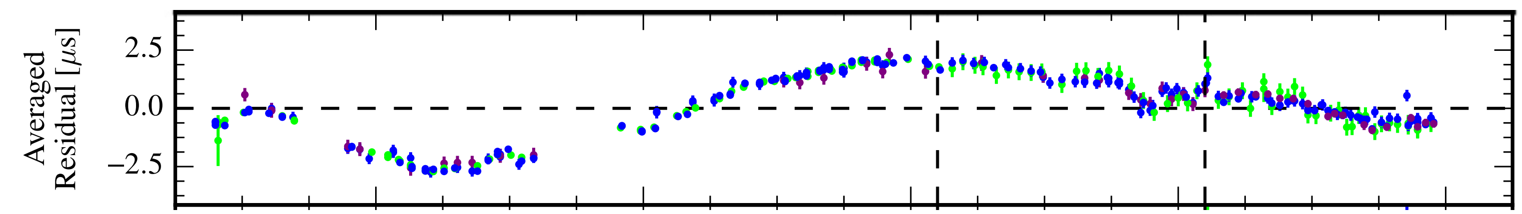


[Burke-Spolaor 2015]

[Jenet et al. 2015]

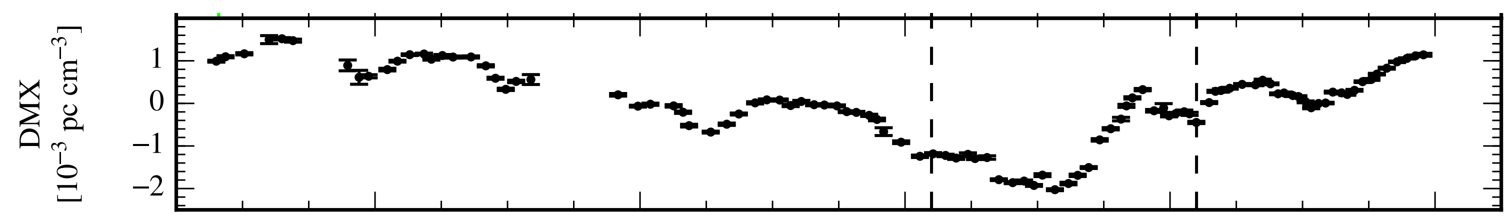
All the red noise processes!

Intrinsic Red Noise



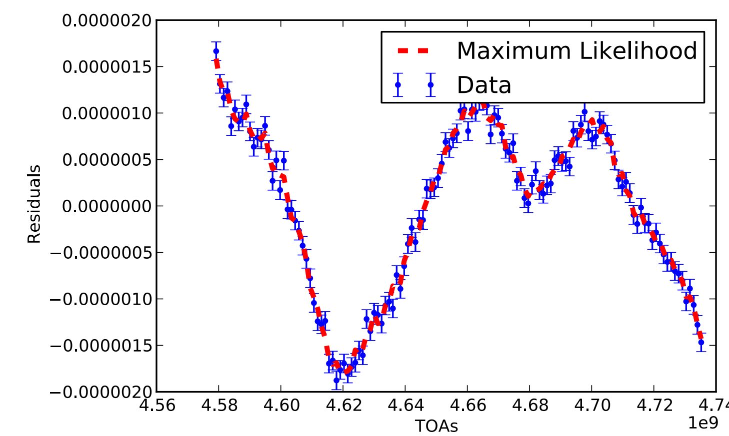
DM variations

ν^{-2}



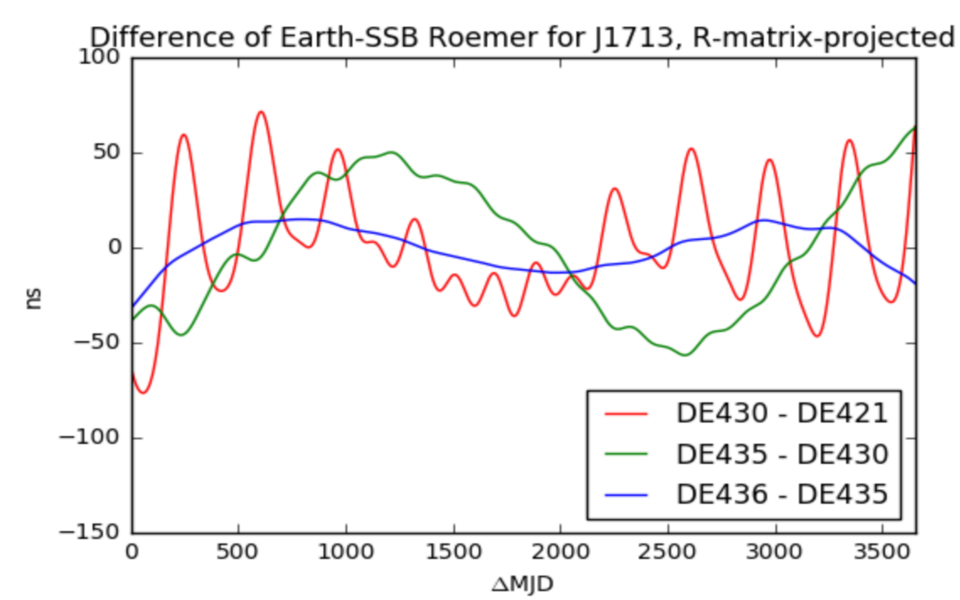
GWB

α_{ab}



Solar System Ephemeris Error

γ_{ab}



Hierarchical likelihood

Computationally cheap (0.07 s)

Large number of parameters (10,000)

(all for NANOGrav 9-year dataset)

$$p(y_i|w_\mu, \mathrm{GP}) = \frac{\mathrm{e}^{-\frac{1}{2}\sum_{i,j}\left(y_i - \sum_\mu \phi_\mu(x_i) w_\mu\right)(N_{ij})^{-1}\left(y_j - \sum_\mu \phi_\mu(x_j) w_\nu\right)}}{\sqrt{(2\pi)^n \det N}}$$

$$\times \frac{e^{-\frac{1}{2}\sum_{\mu\nu} w_\mu (\Sigma_{\mu\nu})^{-1} w_\nu}}{\sqrt{(2\pi)^m \det \Sigma}}$$

Integrate over Gaussian-process coefficients

Marginalized likelihood

Computationally cheap (2 s)

Smaller number of parameters (400)

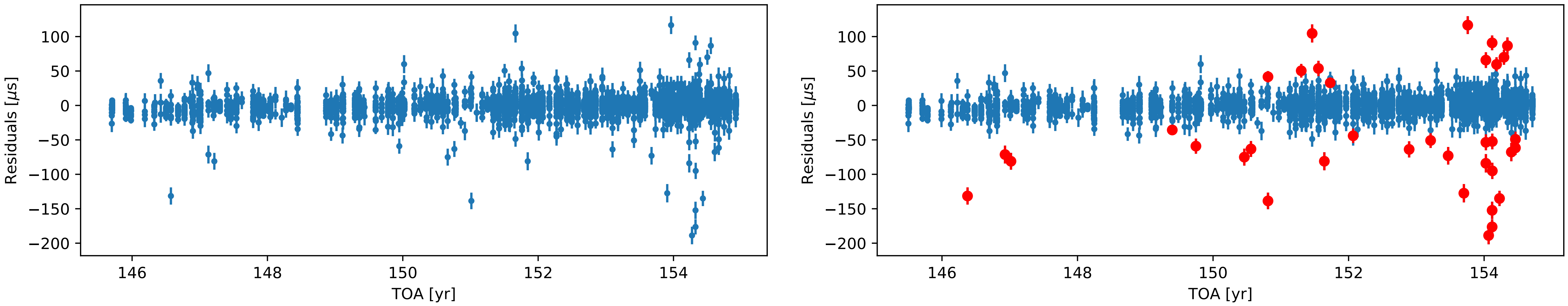
$$p(y_i|\mathrm{GP}) = \frac{\mathrm{e}^{-\frac{1}{2}\sum_{i,j} y_i \left(N_{ij} + K_{ij}\right)^{-1} y_j}}{\sqrt{(2\pi)^n \det(N + K)}}$$

$$K_{ij} = k(x_i, x_j) = \sum_{\mu\nu} \phi_\mu(x_i) \Sigma_{\mu\nu} \phi_\nu(x_j)$$

New and improved outlier model

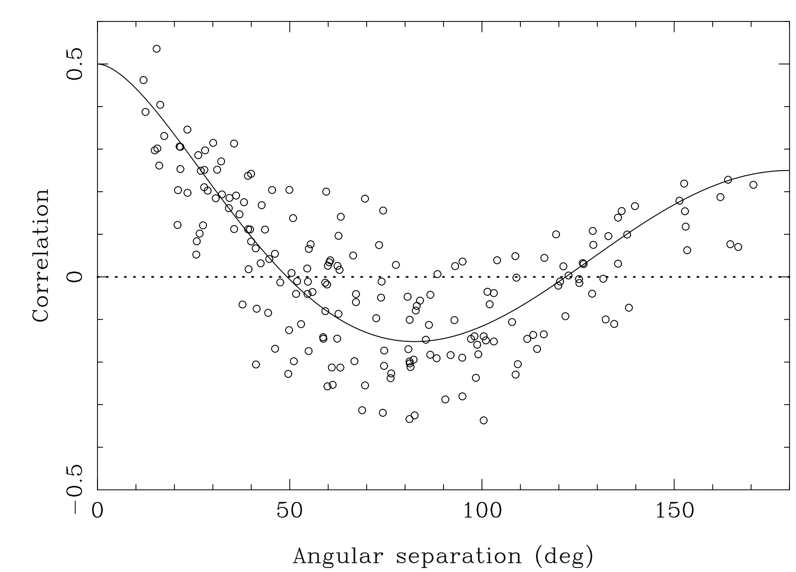
- Previous outlier model was gaussian mixture model (Vallisneri van Haasteren 2016) required several complicated coordinate transformation + custom NUTS HMC sampler (required coding gradients of likelihood)

- New method (see Tak's talk) is gaussian + t-distribution. Uses simple Gibbs sampler and requires minimal changes to already existing MH based codes.



Assessing detection significance

- "smoking gun" of GWB is specific cross correlation pattern



- Model 1: $S_{ab}(f) = \alpha_{ab} P(f)$

- Model 2: $S_{ab}(f) = \delta_{ab} P(f)$

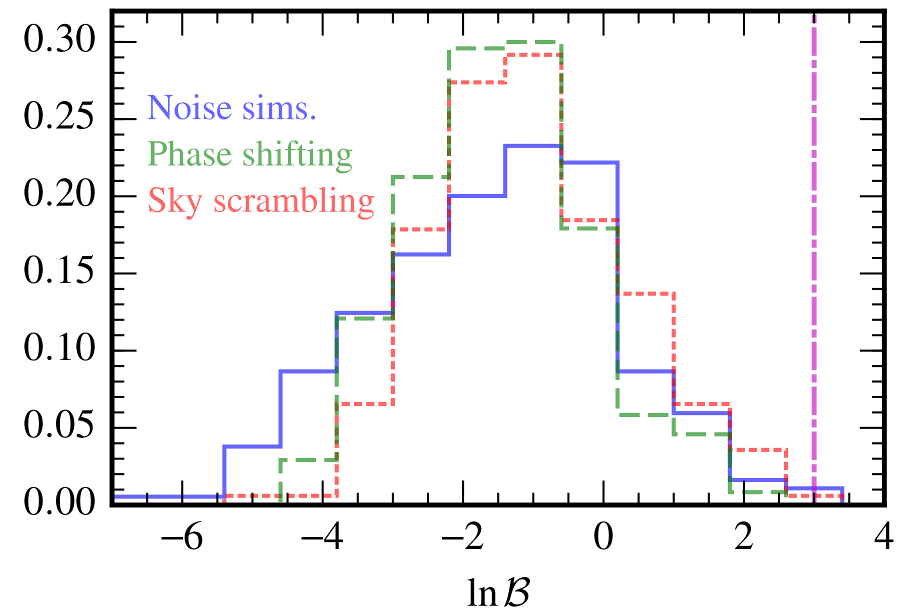
- Want to compare Models 1 and 2 in order to separate spectral information from spatial correlation information.

- There exist both Bayesian and frequentist versions of this test.

Assessing detection significance (2)

$$\mathbf{C}_{\rm gwb} = \mathbf{F}\varphi_{\rm gwb}\mathbf{F}^{\rm T}$$

Phase Shifting — Sky Scrambles



- Compute BF for real data
- Draw new random sky positions for pulsars and use in constructing correction coefficients. Repeat this for many scrambles.
- Add random phases to Fourier components. Repeat this for many shifts.
- Because of finite number of pulsars there are only a few 100-1000 independent sky scrambles.
- Can be extremely computationally expensive; however can also be used for less expensive frequentist techniques.

Frequentist statistics and nuisance parameters

- Statistic $X(d, \theta)$ that depends on data d and noise parameters θ.
- Standard method is to use ML values $\hat{\theta}_{\rm ML}$ and compute statistic.
- What if nuisance parameters are poorly constrained and/or correlated with parameters of interest (i.e. red noise and GWB)?
- Possible solution: Compute $X(d, \theta)$ drawing θ from posterior distribution $p(\theta|d)$ and compute a "meta-statistic" of this distribution.
- What methods exist for dealing with this kind of problem?

Final thoughts/challenges

- Currently our samplers use adaptive metropolis (AM), single component adaptive metropolis (SCAM) and differential evolution (DE) along with parallel tempering. What other kinds of proposals can help improve mixing.

- Kernel or Basis picture?

- Other ways of determining detection significance for stochastic and deterministic GWs.

- We have several frequentist statistics that are good proxies for the more expensive Bayesian runs, in principle. How do we properly construct these statistics in the presence of many nuisance parameters?