Searching for New Physics BSM in Electric Dipole Moments.

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(arXiv:1201.4252)

Contents

1. Introduction

EDMs cover over huge range of physics and chemistry.

The targets are particles (quarks, leptons, neutron, protons), atoms (paramagnetic and diamagnetic atoms), molecules, ions, solid states etc.

The main theoretical problems in EDM are:

1. The problem how to predict (unambiguously) the EDMs in theories of New Physics.
2. The problem how to extract fundamental parameters from the observed or the upper bounds values of EDMs using old and new physics.

Why do we need New Physics BSM ?

Surely BSM

massive neutrinos and their mixings. Dark matters and Dark energy

Could be BSM

Muon g-2, Top quark forward-backward assymetry, $H \to 2\gamma \quad etc.$

CP phase in CKM matrix is insufficient for baryon asymmetry.

We need more Higgs if accept electro-weak baryogenesis.

Internal insufficiencies: big hierarchy problem of Higgs mass, so many parameters, gauge coupling unification, …

There are many models. Two Higgs doblet model (2HDM), Higgs Triplet (HTM), Left-Right Symmetric Model, Little Higgs, Technicolor, GUT (non SUSY and SUSY).

EDM is very sensitive to these New Physics

2. EDM in the SM

particle	Loop number	EDM value
quark	3	$d_d \approx 10^{-34}$ e cm
neutron	2(?)	$d_n \approx 10^{-32}$ e cm
W boson	3	$d_W \approx 8 \times 10^{-30}$ e cm
lepton	4	$d_e \approx 8 \times 10^{-41}$ e cm

Gauge-invariant effective dipole operator

$$\overline{\psi}_i \left(A_L^{ij} P_L + A_R^{ij} P_R \right) \sigma_{\mu\nu} \psi_j F^{\mu\nu}$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\begin{cases} F^{0i} = -E^i \\ F^{ij} = -\epsilon^{ijk} B^k \end{cases}$$

nonrelativistic limit ($\vec{p}_\psi = \vec{0}$)

$$2(A_L^{ij} + A_R^{ij})\overline{\psi} \begin{pmatrix} \vec{S}\cdot\vec{B} & 0 \\ 0 & \vec{S}\cdot\vec{B} \end{pmatrix} \psi - 2i(A_R^{ij} - A_L^{ij})\overline{\psi} \begin{pmatrix} \vec{S}\cdot\vec{E} & 0 \\ 0 & \vec{S}\cdot\vec{E} \end{pmatrix} \psi$$

$i = j$ $i = j$

Magnetic Dipole Moment

$$\mu_\psi \equiv \frac{eQ_\psi}{2m_\psi} g_\psi = -2\,\mathrm{Re}(A_R^{ii} + A_L^{ii})$$

Electric Dipole Moment

$$d_\psi = 2\,\mathrm{Im}(A_R^{ii} - A_L^{ii})$$

$\dfrac{e}{2m_\psi}$: Bohr magneton

$g_\psi^{\text{SM-tree}} = 2$: tree level in the SM

$a_\psi \equiv \dfrac{g_\psi - 2}{2}$: anomalous MDM

cf. flavor changing ($i \neq j$) decay

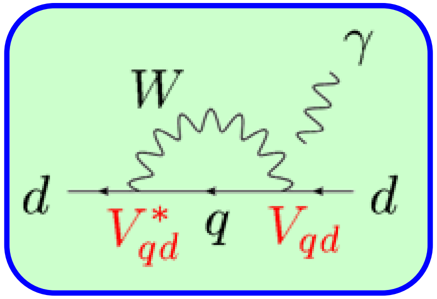
$$\Gamma(\psi_i \to \psi_j \gamma) \simeq \frac{m_{\psi_i}^3}{4\pi} \left(|A_L^{ij}|^2 + |A_R^{ij}|^2 \right)$$

Quark EDM in the SM

$V_{\rm CKM}$ is the source of the imaginary part (CP violation)

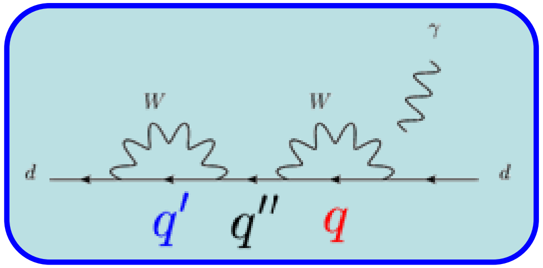
$$d_\psi = 2\,\mathrm{Im}(A_R^{ii} - A_L^{ii})$$

1-loop :

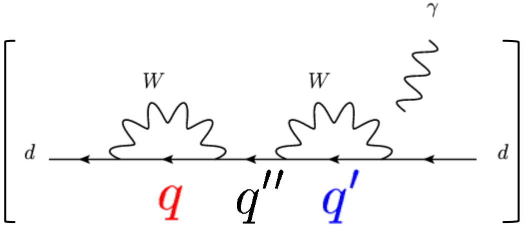


$|V_{qd}|^2$ is real ⟶ **no EDM at 1-loop**

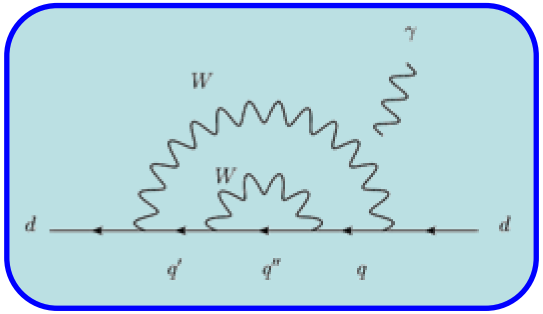
2-loop :



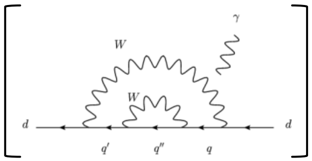
$=$



* ⟶ $\sum_{q,q',q''}$ is real



$\sum_{q,q',q''}$

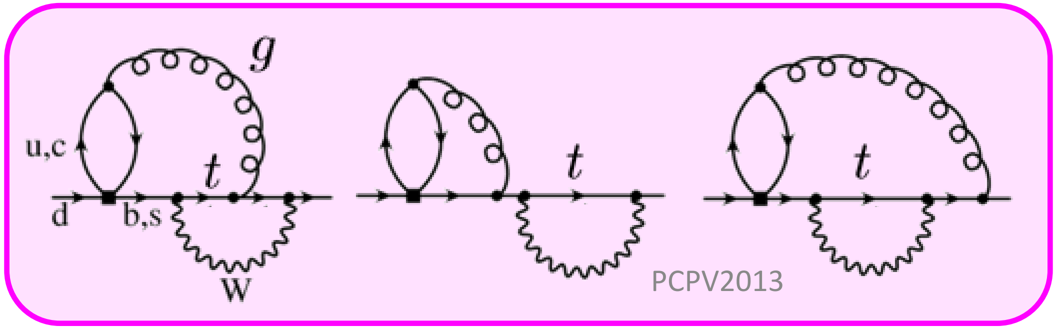


is real surprisingly !!

no large EDM at 2-loop

Sov.J.Nucl.Phys. **28** 75 (1978)

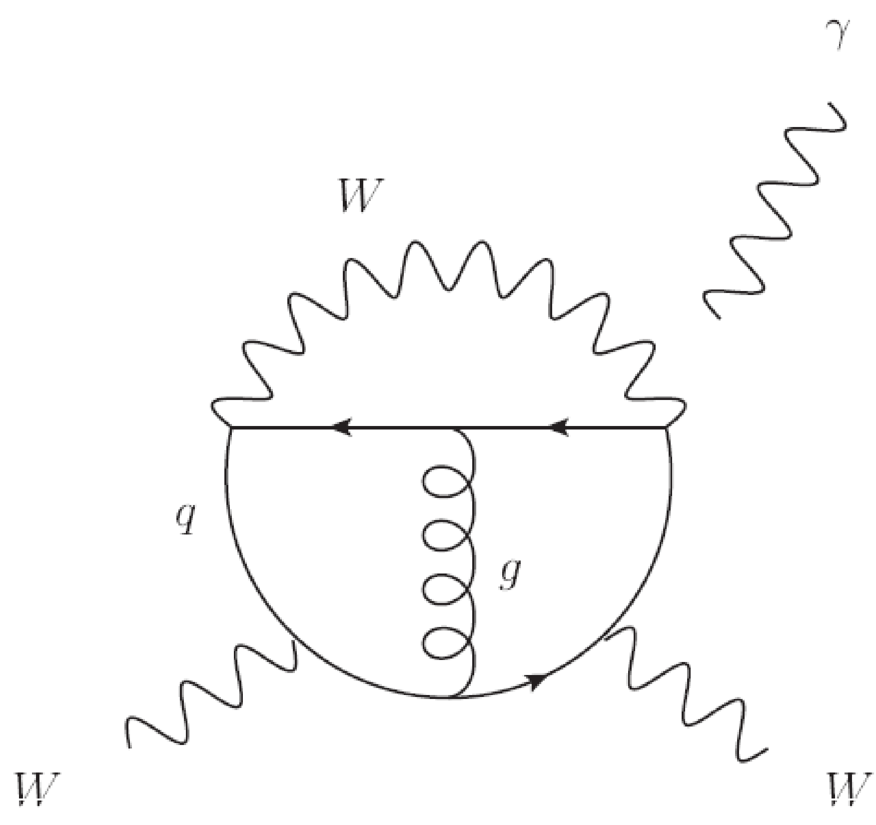
3-loop :



⟶ $d_d \simeq 10^{-34}$ e cm

PRL **78** 4339 (1997)

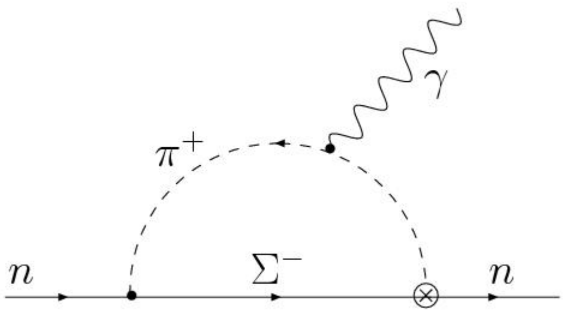
d_W and d_l



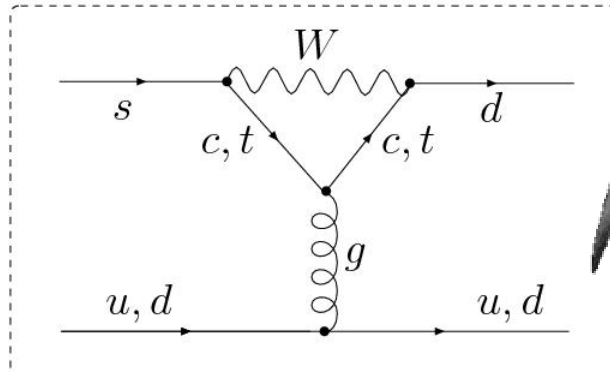
Neutron EDM in the SM

$V_{\rm CKM}$ is the source of the imaginary part (CP violation)

$$d_\psi = 2\,{\rm Im}(A_R^{ii} - A_L^{ii})$$



"strong penguin"



$${\rm Im}\,(V_{cs}^* V_{cd}) = -{\rm Im}\,(V_{ts}^* V_{td})$$
$$= s_{13}\, c_{23}\, s_{23}\, \sin\delta$$

phenomenological Hamiltonian

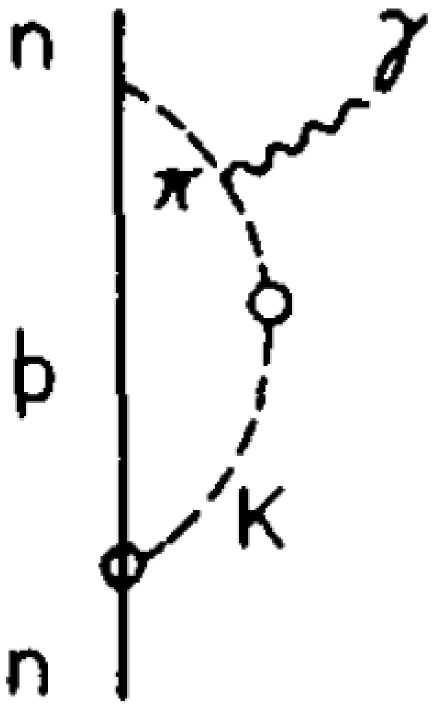
$$J_{CP} = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin\delta$$

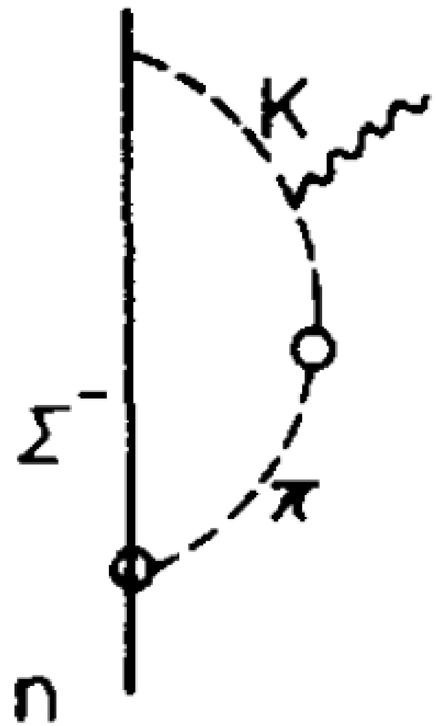
$$H = iG_F m_\pi^2\, \overline{u}_n\, (-1.93 + -0.65\gamma^5)\, u_\Sigma\, \varphi_\pi$$

$$d_n \simeq 10^{-32}\ {\rm e\ cm}$$

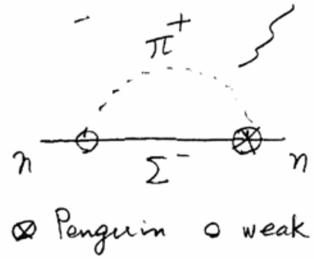
Khriplovich-Zhitnitskey PLB **109** 490 (1982)

$$V \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} ;$$





He et al. P.L.B197 (1987)



(a)

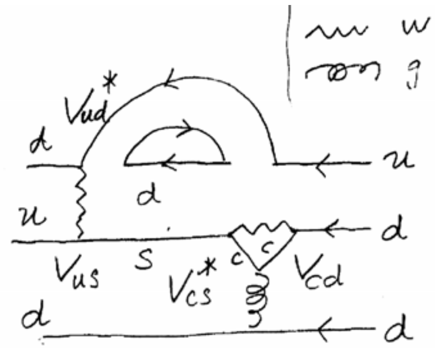


Fig. 1

(b)

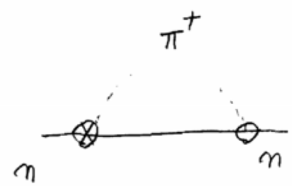
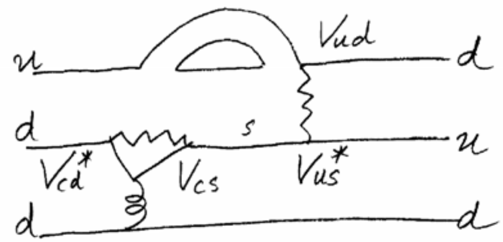
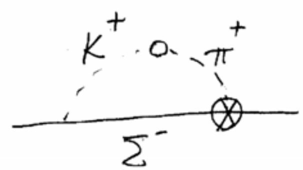


Fig. 2 (a)



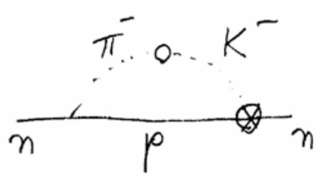
(b)



(a)

Fig. 3

(b)



(a)

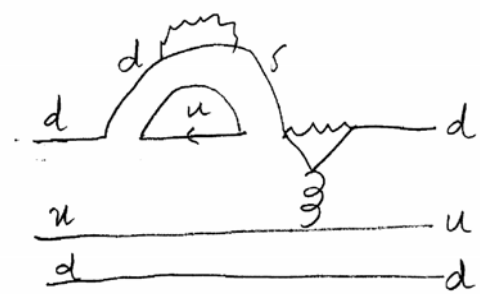


Fig. 4

(b)

$$\bar{d}^{i} V_{ud}^{*} u^{i'} \; \bar{u}^{j} V_{us} P_s^{\ell} P_c^{\ell} (\lambda_a)^{\ell j'} P_c^{j'} V_{cd} d^{j'}$$

$$\times \bar{d}^{k} (\lambda_b)^{kk'} d^{k'} \epsilon_{ijk} \epsilon_{i'j'k'}$$

$$\times \text{(Boson propagator (denominators))}$$

where $\cdots (1)$.

$$P_s^{\ell}(q) = (\gamma q + m_s)\,\delta_{\ell\ell}$$

strange quark projection operator.

(λ_a) : SU(3) Gelman matrix.

$i, j, k \quad i', j', k'$ are color indicies

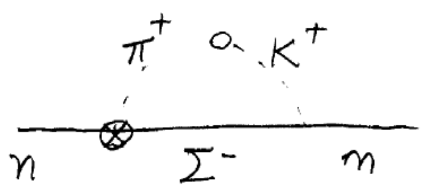


Fig. 5.

See also Mannel and Uraltsev (arXiv:1202.6270).

π -pole is dominant but the most dominant among it is $\pi^- p$ and not $\pi^+\Sigma^-$.

$$L_{\pi K} = h\left(\partial_\mu K^+ \partial^\mu \pi^- e^{-i\theta} + \partial_\mu K^- \partial^\mu \pi^+ e^{i\theta}\right)$$

current algebra relation

$$h m_K^2 = \sqrt{2} i f_\pi \langle \pi^{;} \pi^- | H_W | K_s \rangle$$

$$1.4 \times 10^{-31} < d_n < 9.9 \times 10^{-33} \quad \text{[e cm]}$$

Rumor: Pospelov calculated d_n=1¥times 10^{-29} in the SM.

Experimental bounds on EDM

$$d_e = (6.9 \pm 7.4) \times 10^{-28}\ \mathrm{e\ cm}$$ B.C. Regan *et al.* @ *BNL*, PRL**88** 071805 (2002)

$$\begin{cases} d_{\mu^-} = (-0.1 \pm 0.7) \times 10^{-19}\ \mathrm{e\ cm} \\ d_{\mu^+} = (-0.1 \pm 1.0) \times 10^{-19}\ \mathrm{e\ cm} \end{cases}$$ Muon g-2 collab. @ BNL, PRD**80** 052008 (2009)

$$d_n = (0.2 \pm 1.5(\mathrm{stat.}) \pm 0.7(\mathrm{sys.})) \times 10^{-26}\ \mathrm{e\ cm}$$

C.A. Baker *et al.* @ Institut Laue-Langevin, PRL**97** 131801 (2006)

Experimental bound on muon anomalous MDM

$$a_\mu \equiv \frac{g_\mu - 2}{2} = (11659208.0 \pm 5.4(\mathrm{stat.}) \pm 3.3(\mathrm{sys.})) \times 10^{-10}$$

Muon g-2 collab. @ BNL, PRD**73** 072003 (2006)

Experimental bounds (90%CL) on $\psi_i \to \psi_j \gamma$

$$\mathrm{BR}(\mu \to e\gamma) < 2.4 \times 10^{-12}$$ MEG collab., PRL**107** 171801 (2011)

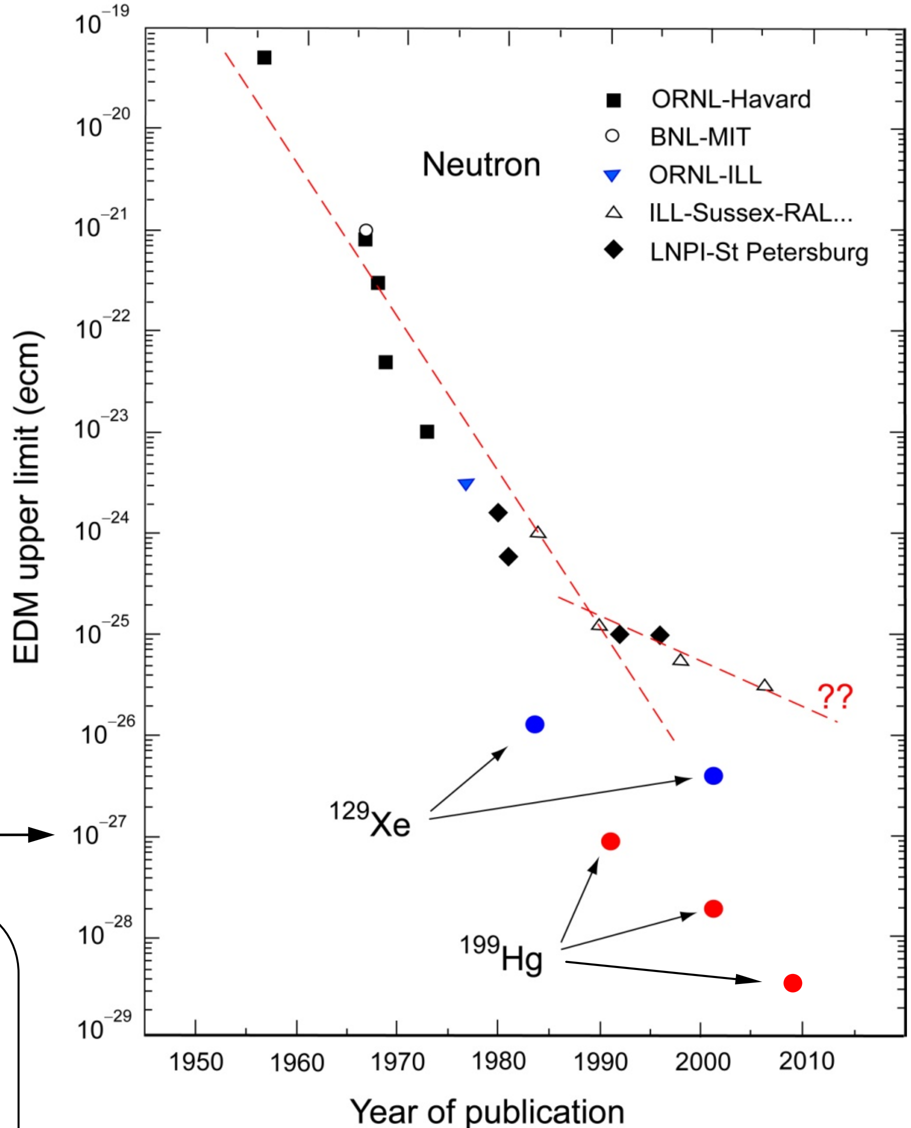
$$\begin{cases} \mathrm{BR}(\tau \to e\gamma) < 3.3[12.0] \times 10^{-8} \\ \mathrm{BR}(\tau \to \mu\gamma) < 4.4[4.5] \times 10^{-8} \end{cases}$$

Babar collab. @ SLAC, PRL**104** 021802 (2010)

[Belle collab. @ KEK, PLB**666** 16 (2008)]

$d = 10^{-27}$ $e\cdot$cm
$E = 10$ kV/cm

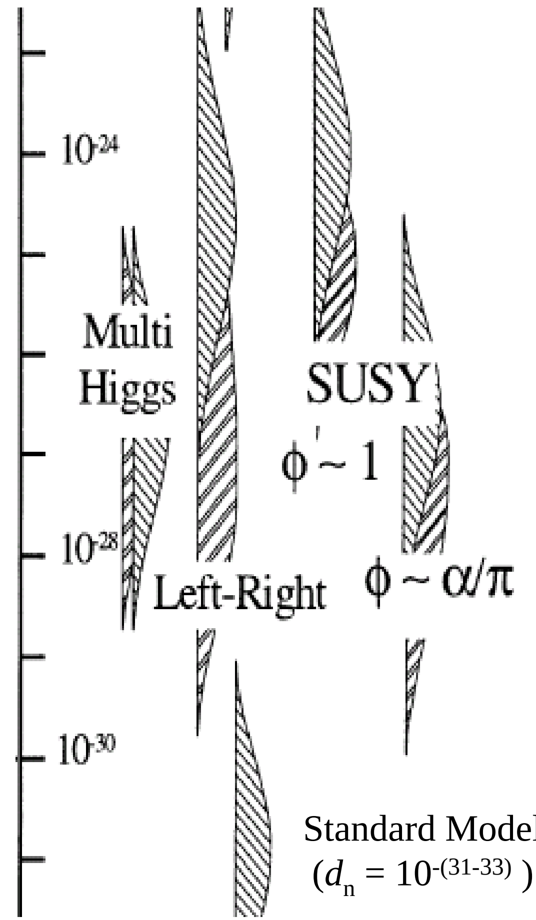
$\Delta\nu = 10$ nHz
（ $\Delta\omega \approx 1^\circ$ /day）



- $d(^{199}\text{Hg}) < 3.1 \times 10^{-29}$ $e$cm
  Grifith *et al.*, *PRL* **102** (2009) 101601

- $d(^{129}\text{Xe}) < 4.1 \times 10^{-27}$ $e$cm
  Rosenberry and Chupp, *PRL* **86** (2001) 22

Neutron EDM predicted values



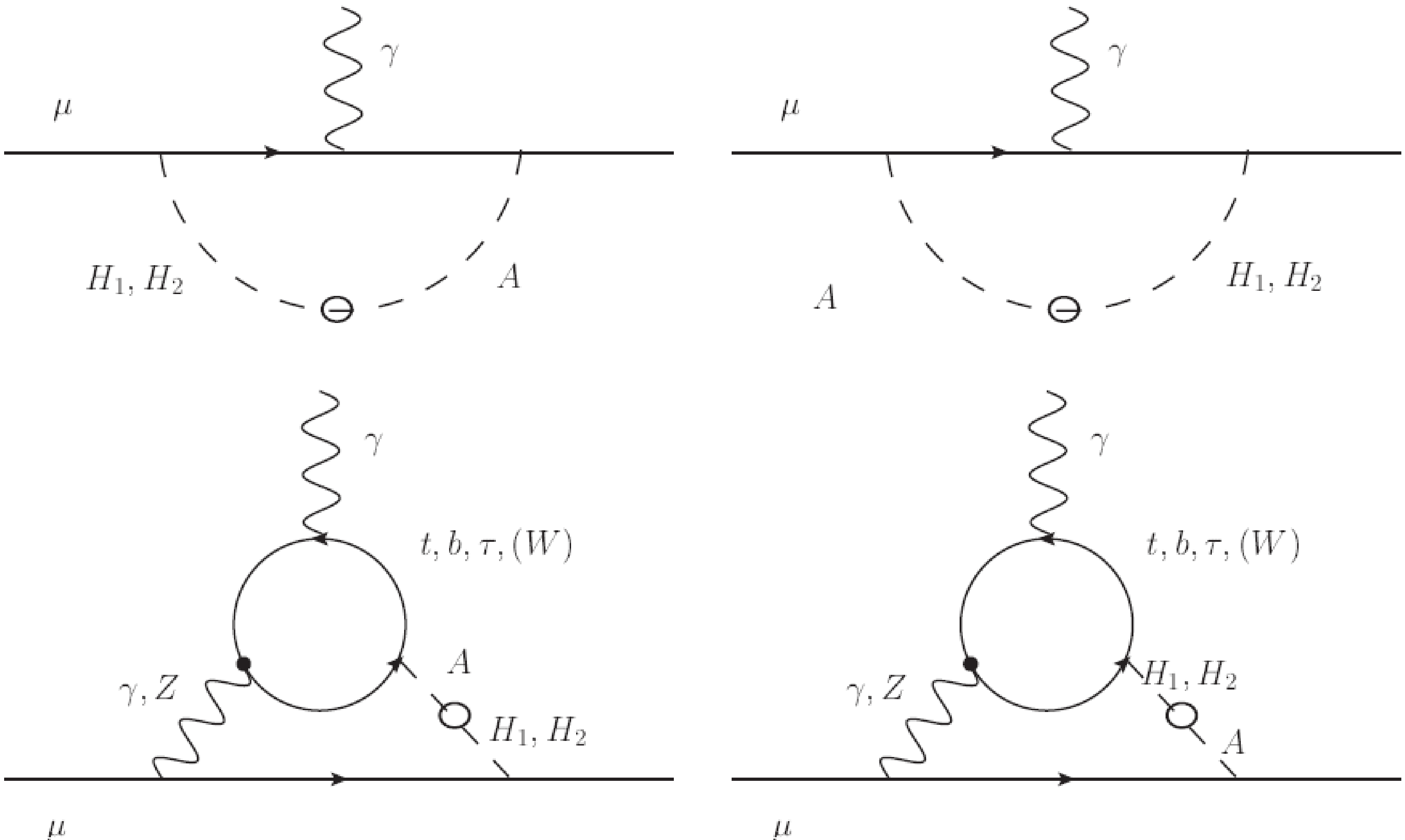
[Pendlebury and Hinds, *NIM* A **440** (00) 471]

3. EDM in New Physics

Two Higgs Doublet Model

type I (SM-like) :	ϕ_1 couples with all fermions
	ϕ_2 decouples with fermions
type II (MSSM-like) :	ϕ_1 couples with down-type quarks and charged leptons
	ϕ_2 couples with up-type quarks
type III (general) :	both of Higgs doublets couple with all fermions
etc. etc.	

Diagrams contributing to EDMs in 2HDM.



Weinberg, P.R. D42, Barger et al. P.R. D55,

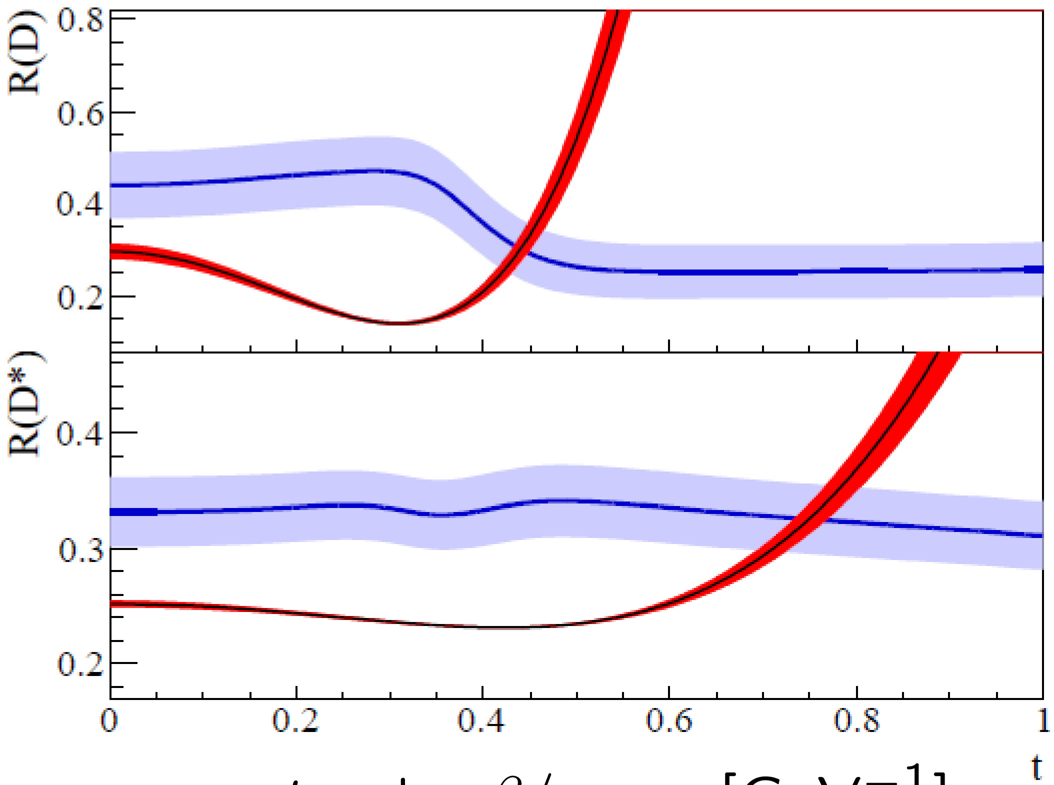
$$d_\mu^{1-\text{loop}} = \frac{em\sqrt{2}G_F \tan^2\beta}{(4\pi)^2}(m^3/m_0^2)[\ln(m^2/m_0^2) + 3/2](\text{Im}Z_0 + \text{Im}\tilde{Z}_0),$$

$$\frac{1}{v_1^* v_2}\langle \phi_2^0 \phi_1^{0*}\rangle_q \equiv \frac{\sqrt{2}G_F Z_0}{q^2 - m_H^2}, \quad \frac{1}{v_1 v_2}\langle \phi_2^0 \phi_1^0\rangle_q \equiv \frac{\sqrt{2}G_F \tilde{Z}_0}{q^2 - m_H^2}$$

The estimated values are in units of $\Im Z_0$. However, it is probable that $|\Im Z| \ll 1$. Indeed, the masses of neutral and charged Higgses and phases are tightly constrained from $R_b \equiv \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to hadrons)}$, $\Gamma(b \to s\gamma)$, $\overline{B}^0 - B$ mixing, ρ parameter etc., and we should take those constraints into account.

$$R(D) = \frac{Br(\overline{B} \to D\tau^-\overline{\nu}_\tau)}{Br(\overline{B} \to Dl^-\overline{\nu}_l)}$$

3.4σ deviation from SM.



$$t = \tan\beta/m_{H+}\ [\mathrm{GeV}^{-1}]$$

Figure 5. The comparison results of this paper (blue band) with predictions that include a charged Higgs boson of type II 2HDM (red band). The SM corresponds to $tan\beta/m_{H^+} = 0$.

BARBAR Collaboration

Left-Right (LR) Symmetric Model

There are many LR models.

If we consider it as a remnant from SO(10), $SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R$, it satisfies at v_{PS} energy scale

$$g_L = g_R$$

and PS model is unified at M_{GUT} as

$$\frac{M_4}{\alpha_4} = \frac{M_{2L}}{\alpha_{2L}} = \frac{M_{2LR}}{\alpha_{2R}} = \frac{M_{1/2}}{\alpha_{GUT}}.$$

Also mixing matrices of left-handed and right-handed fermions are same. Of course these constraints are realized at v_{PS} but deviate from as energy goes down to the SM scale by renormalization effects.

Pati, Marshak-Mohapatra

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Then charge quantization

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B-L)$$

$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} = (1,2,2,0)$$

$$Q_L = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} = (3,2,1,1/3) \qquad Q_R = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix} = (3,1,2,1/3)$$

$$L_L = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} = (3,2,1,1/3) \qquad L_R = \begin{pmatrix} N_R^i \\ e_R^i \end{pmatrix} = (1,1,2,-1)$$

Φ couples with $\overline{Q}_L Q_R$, so $B-L(\Phi)=0$.

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$$

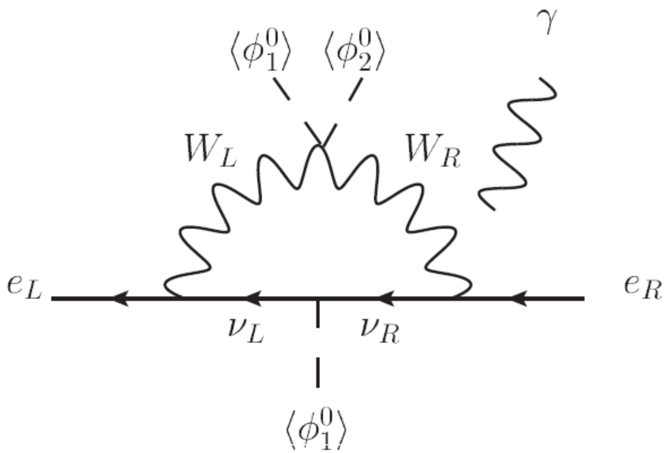
leads to $U(1)_{I_{3L}+I_{3R}} \times U(1)_{B-L}$ and not to $U(1)_Q$.

So we need additionally, for instance,

$$\Delta_L = (3,1,2), \quad \Delta_R = (1,3,2)$$

$$\begin{pmatrix} \frac{1}{2}g^2(\kappa^2 + \kappa'^2 + 2v_L^2) & g^2\kappa\kappa' \\ g^2\kappa\kappa' & \frac{1}{2}g^2(\kappa^2 + \kappa'^2 + 2v_R^2) \end{pmatrix}$$

$$\tan 2\zeta = \frac{2\kappa\kappa'}{v_R^2 - v_L^2} \approx \frac{M_{W_L}^2}{M_{W_R}^2}$$



$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix},$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

and

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}$$

$$d_e = \frac{eG_F}{8\sqrt{2}\pi^2} I_1(\frac{M_D^2}{M_W^2}, 0)\sin 2\zeta \Im(M_D)$$

$$= 2.1 \times 10^{-24} I_1(\frac{M_D^2}{M_W^2}, 0)\sin 2\zeta(\Im(M_D)/1\text{MeV})\text{e cm}$$

$$(\bar{\nu}^c, \bar{N})_R \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}_L + H.C.$$

I_1 takes a values 2-1/2

Inserting the observed upper limit of

$$M_{W_R} > 2.15 \times 10^3 \text{ GeV } 95\%\text{C.L.}$$

$$|\sin 2\zeta| < 0.008 \text{ or } \zeta \approx \frac{M_{W_L}^2}{M_{W_R}^2}$$

$$|d_e| < \begin{cases} 8.2 \times 10^{-27} \dfrac{|\Im(m_D)|}{\text{MeV}} \text{ e cm} & \text{for } \left(\dfrac{m_R}{m_W}\right)^2 \gg 1, \\ 3.3 \times 10^{-26} \dfrac{|\Im(m_D)|}{\text{MeV}} \text{ e cm} & \text{for } \left(\dfrac{m_R}{m_W}\right)^2 \ll 1, \end{cases}$$

Structure of matter multiplets

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$u^c = (\, u_1^c \quad u_2^c \quad u_3^c \,) \sim (\bar{3}, 1, \frac{-2}{3})$$

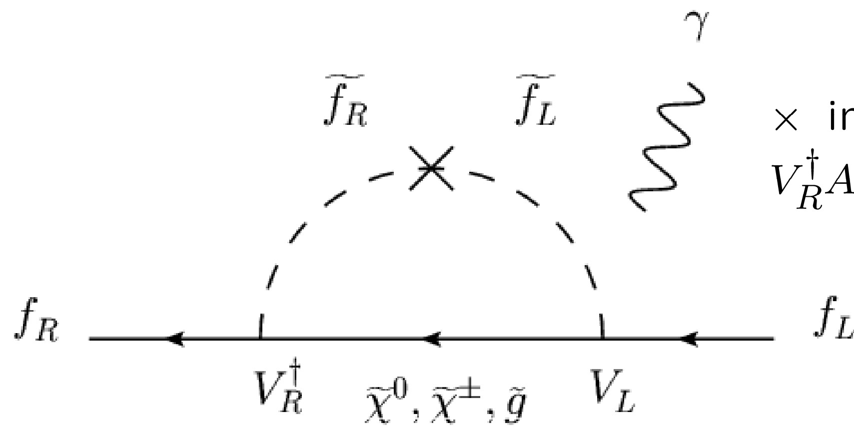
$$d^c = (\, d_1^c \quad d_2^c \quad d_3^c \,) \sim (\bar{3}, 1, \frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

$$e^c \quad \sim (1, 1, +1)$$

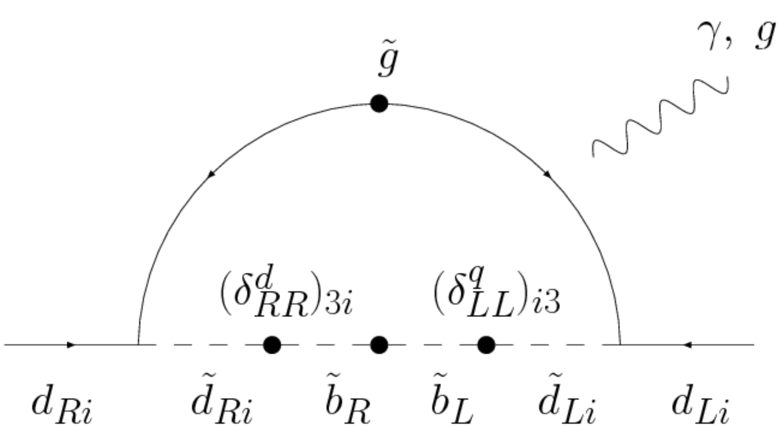
$$\nu^c \quad \sim (1, 1, 0)$$

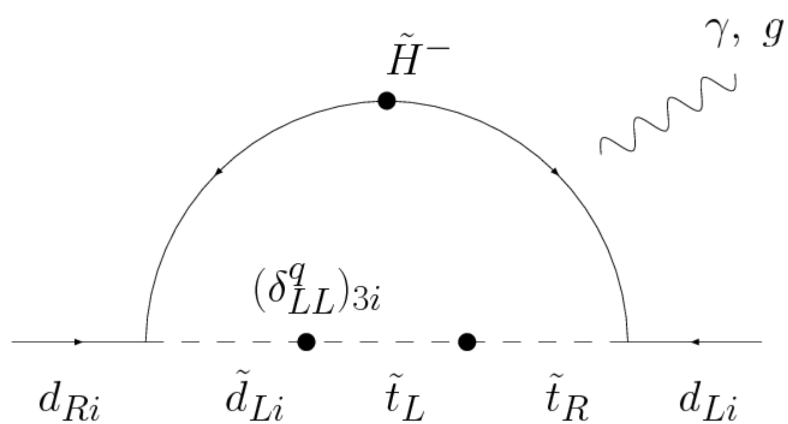
SUSY



$\times$ implies the trilinear A_{ij} term. If $A \propto Y_{ij}$, $V_R^\dagger A V_L$ becomes real and no EDM contrbution.

$$\delta^q_{LL} = \frac{(m^2_{\tilde{q}})_{ij}}{m^2_{\tilde{q}}} \quad etc.$$





Hisano et al. Phys.Rev.D80

In general soft breaking terms are

$$
\begin{aligned}
L_{SB}^{(1)} &= \mu_{Qij}^2 \tilde{Q}_{Li}^\dagger \tilde{Q}_{Lj} + \mu_{uij}^2 \tilde{u}_{Ri}^* \tilde{u}_{Rj} + \mu_{dij}^2 \tilde{d}_{Ri}^* \tilde{d}_{Rj} + \mu_{Lij}^2 \tilde{L}_{Ri}^\dagger \tilde{L}_{Rj} \\
&+ \mu_{eij}^2 \tilde{e}_{Ri}^* \tilde{e}_{Rj} + \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \mu_S^2 S^* S \\
&+ A_{uij} \tilde{u}_{Ri}^* \tilde{Q}_{Li} H_1 + A_{dij} \tilde{d}_{Ri}^* \tilde{Q}_{Li} H_2 + A_{eij} \tilde{e}_{Ri}^* \tilde{Q}_{Li} H_2
\end{aligned}
$$

$$
L_{SB}^{(2)} = \frac{1}{2}\left(M_3 \tilde{g}^a \tilde{g}^a + M_2 \tilde{W}^a \tilde{W}^a + M_1 \tilde{B}^a \tilde{B}^a + c.c. \right)
$$

MSSM+universal boundary condition (CMSSM)

$$\mathbf{m}_Q^2 = \mathbf{m}_{\overline{u}}^2 = \mathbf{m}_{\overline{d}}^2 = \mathbf{m}_{\overline{L}}^2 = \mathbf{m}_{\overline{l}}^2 = m_0^2 \mathbf{1}_3,$$
$$m_{H_u} = m_{H_d} = m_0$$
$$\frac{M_3}{g_3^2} = \frac{M_2}{g_2^2} = \frac{M_1}{g_1^2} = \frac{M_{1/2}}{g_u^2}$$
$$\mathbf{a}_u = A_0 \mathbf{Y}_u, \quad \mathbf{a}_d = A_0 \mathbf{Y}_d \quad \mathbf{a}_u = A_0 \mathbf{Y}_e.$$

$$\left(\Delta m_{\tilde{\ell}}^2\right)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2} \left(Y_\nu^\dagger L Y_\nu\right)_{ij} \ ,$$

where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix $L = \log[M_{\mathrm{GUT}}/M_{R_i}]\delta_{ij}$.

	minimal SO(10) model	simple 5D SO(10) model
m_0	415	272
$M_{1/2}$	790	420
A_0	0	0
$\tan\beta$	45	45
h_0	119	115
H_0	786	449
A_0	787	449
$H^\pm$	791	457
$\tilde{g}$	1756	981
$\tilde{\chi}^0_{1,2,3,4}$	$333, 631, 928, 938$	$171, 324, 535, 548$
$\tilde{\chi}^\pm_{1,2}$	$631, 938$	$324, 548$
$\tilde{d}, \tilde{s}_{R,L}$	$1576, 1645$	$898, 934$
$\tilde{u}, \tilde{c}_{R,L}$	$1582, 1643$	$901, 931$
$\tilde{b}_{1,2}$	$1409, 1473$	$784, 849$
$\tilde{t}_{1,2}$	$1266, 1475$	$698, 864$
$\tilde{\nu}_{e,\mu,\tau}$	$667, 667, 619$	$386, 386, 355$
$\tilde{e}, \tilde{\mu}_{R,L}$	$511, 672$	$317, 395$
$\tilde{\tau}_{1,2}$	$342, 642$	$186, 392$
$\mathrm{BR}(b \to s\gamma)$	3.27×10^{-4}	2.36×10^{-4}
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	1.04×10^{-8}	4.95×10^{-9}
Δa_μ	12.0×10^{-10}	37.7×10^{-10}
Ωh^2	0.113	

Motivation of SUSY SO(10) GUT

- SUSY + GUT

Ø Experimental evidence: three gauge couplings unification with MSSM particle contents.

- SO(10) GUT

Ø SO(10) fundamental representation include all the matter in the MSSM plus right-handed neutrinos.

$$\begin{aligned} \mathbf{16} &= \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1} \\ &= (Q, u^c, e^c) + (L, d^c) + (\nu^c) \end{aligned}$$

Ø Experimental evidence: very tiny neutrino masses.

It can be explained via the "seesaw mechanism", and it works well in SO(10) GUT in the presence of the right-handed neutrinos. (Yanagida, Gell-Mann et al. (79'))

Minimal SO(10) model

(Babu-Mohapatra (93'); Fukuyama-Okada (01'))

- Two kinds of symmetric Yukawa couplings

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

$$W_Y = Y_{10}^{ij} \mathbf{16}_i H_{10} \mathbf{16}_j + Y_{126}^{ij} \mathbf{16}_i H_{126} \mathbf{16}_j$$

- Two Higgs fields are decomposed to

$$\begin{aligned} \mathbf{10} &\rightarrow (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}) \\ \overline{\mathbf{126}} &\rightarrow (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{15}, \mathbf{2}, \mathbf{2}) \\ &+ (\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}) + (\mathbf{10}, \mathbf{1}, \mathbf{3}) \end{aligned}$$

- SU(4) adjoint **15** have a basis, $\mathrm{diag}(1, 1, 1, -3)$ so as to satisfy the traceless condition. Putting leptons into the 4th color, we get, so called, 'Georgi–Jarslkog' factor, -3 leptons.

Mass relation

- All the mass matrices are descried by only two fundamental matrices.

$$M_u = c_{10} M_{10} + c_{126} M_{126}$$
$$M_d = M_{10} + M_{126}$$
$$M_D = c_{10} M_{10} - 3 c_{126} M_{126}$$
$$M_e = M_{10} - 3 M_{126}$$
$$M_R = c_R M_{126}$$
$$\rightarrow M_e = c_d \left(M_d + \kappa M_u \right) \quad (\text{'GUT relation'})$$

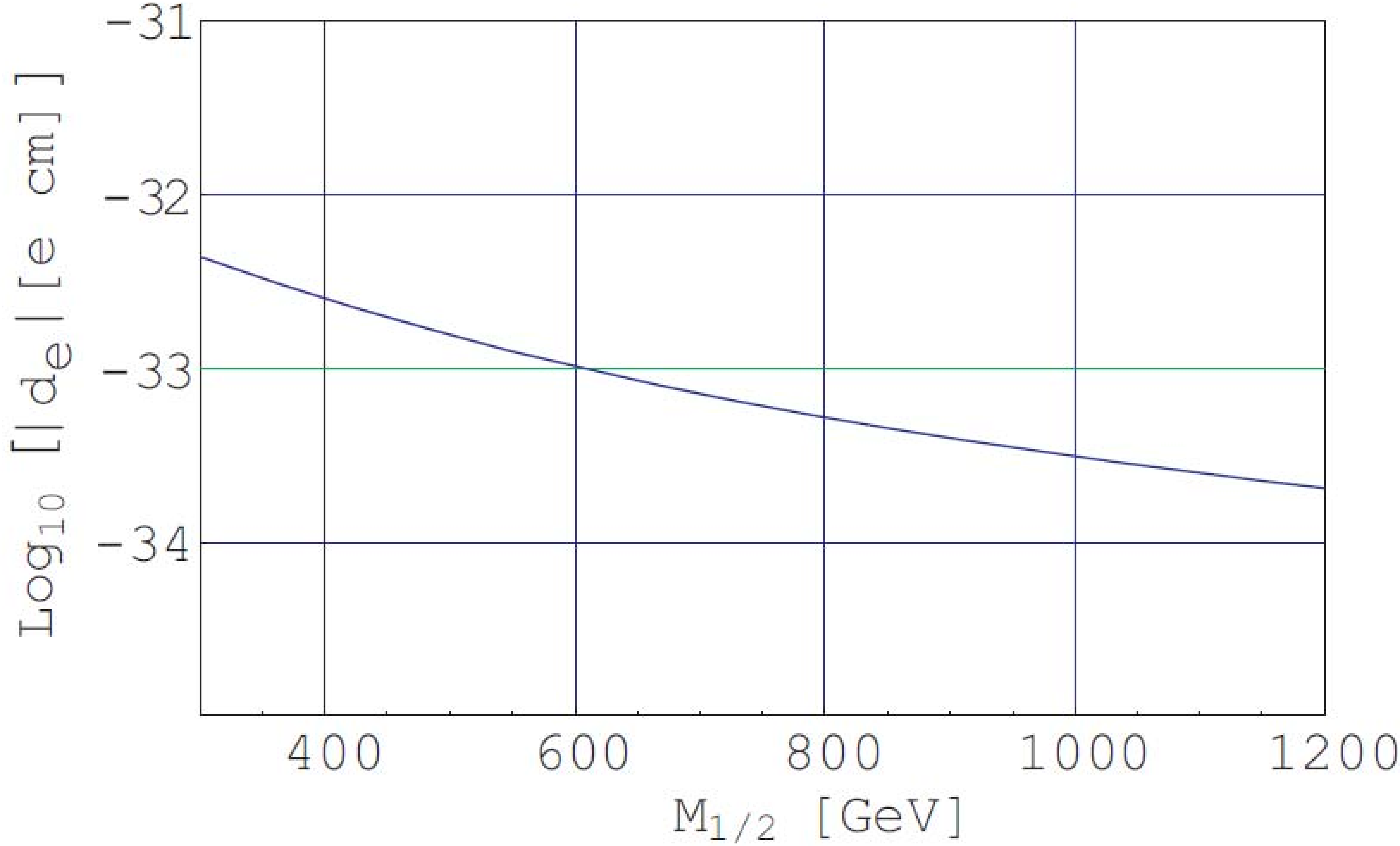
$$M_\nu = -M_D^T \frac{1}{M_R} M_D$$

- 13 inputs : 6 quark masses, 3 angles + 1 phase in CKM matrix,
  3 charged-lepton masses.

  ⇒ fix $|c_d|$ and κ

  ⇒ predictions in the parameters in the neutrino sector!

Prediction of electron EDM



4. Problem due to many-body system in atoms and molecules.

for paramagnetic atom

$$K = d_{atom}/d(e)$$
$$= 2e \sum_M \frac{\langle 0| \sum_{i=1}^N (\gamma_0^i - 1)\mathbf{\Sigma}^i \cdot \mathbf{E}_{int}^i |M\rangle\langle M| \sum_{i=1}^N z^i |0\rangle}{E_0 - E_M}$$

for diamagnetic atom

$$\mathbf{d}_{atom} = \sum_n \frac{\left\langle 0|e\sum_i^Z \mathbf{r}_i|n\right\rangle \left\langle n|e\sum_i^Z \left(\Phi(\mathbf{r}_i) - \frac{1}{Ze}\langle \mathbf{d}_{nucleus}\rangle \cdot \nabla\Phi(\mathbf{r}_i)\right)|0\right\rangle}{E_0 - E_n} + h.c.$$

To estimate Schiff moment consists of several ingredients.
(1) is to estimate fundamental interactions like $\overline{\theta}$ and cEDM:

$$\overline{\theta}\frac{\alpha_s}{8\pi}G\tilde{G}+\sum_{q=u,d,s}i\frac{\tilde{d}}{2}\overline{q}g_s(G\sigma)\gamma_5 q+...$$

(2) is how to evaluate $\mathbf{S}$ from the above interactions, including many-body treatment and core polarization.
(3) is how to sum up the intermediate energy eigenstates.

The Schiff's Theorem

$$H_0 = \sum_i c\boldsymbol{\alpha}\cdot\boldsymbol{p}_i + \beta_i mc^2 + V_{nucl}(r_i) + \sum_{i<j} V_C(r_{ij}),$$

$$H_{PTV} = -\sum_i \mathbf{d}_e^i \cdot \mathbf{E}_{int}^i - \sum_i \mathbf{d}_e^i \cdot \mathbf{E} - e\sum_i \mathbf{r}_i \cdot \mathbf{E}.$$

$$E_m^1 = -\sum_i \langle m_0|\mathbf{d}_e^i|m_0\rangle \cdot \mathbf{E}$$

$$E_m^2 = \sum_{n\neq m}\sum_i \left\{ \frac{\langle m_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|n_0\rangle\langle n_0|e\mathbf{r}^i\cdot\mathbf{E}|m_0\rangle}{E_m^0 - E_n^0} + \frac{\langle m_0|e\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle}{E_m^0 - E_n^0} \right\}.$$

$$\mathbf{d}' = \sum_i \langle m_0|\mathbf{d}_e^i|m_0\rangle - \sum_{n\neq m}\sum_i \left\{ \frac{\langle m_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|n_0\rangle\langle n_0|e\mathbf{r}^i|m_0\rangle}{E_m^0 - E_n^0} + \frac{\langle m_0|e\mathbf{r}^i|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle}{E_m^0 - E_n^0} \right\}.$$

$$\langle m_0|e\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle = ie\langle m_0|\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot[\mathbf{p}^i,\ H_0]|m_0\rangle$$

$$= ie\langle m_0|\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{p}|m_0\rangle(E_m^0 - E_n^0)$$

$$\mathbf{\Sigma}\cdot\mathbf{E}_{int} = [\mathbf{\Sigma}\cdot\nabla,\ H_0]$$

$$\Delta E = -d_e\langle m_0|(\beta-1)\mathbf{\Sigma}\cdot\mathbf{E}|m_0\rangle - 2d_e\sum_{n\neq m}\frac{\langle m_0|\mathbf{r}\cdot\mathbf{E}|n_0\rangle\langle n_0|(\beta-1)\mathbf{\Sigma}\cdot\mathbf{E}_{int}|m_0\rangle}{E_m - E_n}$$

$$= -\mathbf{d}(atom)\cdot\mathbf{E}$$

The Schiff's moment

$$H_{atom} = H_{electron} + H_{nucleus} + \sum_{i=1}^{Z} (e\Phi(\mathbf{r}_i) - e\mathbf{r}_i \cdot \mathbf{E}) - \mathbf{d}_{nucleus} \cdot \mathbf{E},$$

$$-i\left[\sum_{i=1}^{Z} \mathbf{p}_i, H_{atom}\right] = -e\sum_{i=1}^{Z} \nabla_i \Phi(\mathbf{r}_i) + Ze\mathbf{E},$$

$$V = \langle \mathbf{d}_{nucleus} \rangle \cdot \mathbf{E} - \frac{1}{Z}\sum_{i=1}^{Z} \langle \mathbf{d}_{nucleus} \rangle \cdot \nabla_i \Phi(\mathbf{r}_i)$$

$$-\mathbf{d}_{nucleus} \cdot \mathbf{E} \rightarrow -(\mathbf{d}_{nucleus} - \langle \mathbf{d}_{nucleus} \rangle) \cdot \mathbf{E}.$$

$$\Phi(\mathbf{r}) - \frac{1}{Ze} \langle \mathbf{d}_{nucleus} \rangle \cdot \nabla \Phi(\mathbf{r})$$

$$\mathbf{d}_{atom} = \sum_n \frac{\left\langle 0|e\sum_i^Z \mathbf{r}_i|n\right\rangle \left\langle n|e\sum_i^Z \left(\Phi(\mathbf{r}_i) - \frac{1}{Ze}\left\langle \mathbf{d}_{nucleus}\right\rangle \cdot \nabla\Phi(\mathbf{r}_i)\right)|0\right\rangle}{E_0 - E_n} + h.c.$$

$$\int \rho(\mathbf{x})d^3x \;=\; Z|e|, \quad \int \mathbf{x}\rho(\mathbf{x})d^3x = \left\langle \mathbf{d}_{nucleus}\right\rangle,$$

$$\int x^2\rho(\mathbf{x})d^3x \;=\; Z|e|\left\langle x^2\right\rangle_{ch}, \quad \int (x_k x_{k'} - \frac{1}{3}\delta_{kk'}x^2)\rho(\mathbf{x})d^3x = Z|e|\left\langle Q_{kk'}\right\rangle \quad \text{etc}$$

$$\left\langle 0_N \middle| e\Phi(\mathbf{r}) - \frac{1}{Z}\left\langle \mathbf{d}_{nucleus}\right\rangle \cdot \nabla\Phi(\mathbf{r}) \middle| 0_N \right\rangle = -\frac{Ze^2}{|\mathbf{r}|} + 4\pi e\mathbf{S}\cdot\nabla\delta(\mathbf{r}) + \ldots$$

$$\mathbf{S}^{ch} = \frac{e}{10}\sum_{p=1}^{Z}\left(r_p^2 - \frac{5}{3}\langle r^2\rangle_{ch}\right)\mathbf{r}_p.$$

$$\langle 0_N|S|0_N\rangle = \sum_{i\neq 0}\frac{\langle 0|S|i\rangle\langle i|V_{PT}|0\rangle}{E_0 - E_i}$$

$$\mathbf{S} = \frac{1}{10} \sum_{N}^{A} \sum_{i} e_i \left((\mathbf{r}_N + \boldsymbol{\rho}_i)^2 - \frac{5}{3} \langle r^2 \rangle_{ch} \right) (\mathbf{r}_N + \boldsymbol{\rho}_i).$$

$$\sum_i e_i = e_N, \quad \sum_i e_i \boldsymbol{\rho}_i = \mathbf{d}_N.$$

$$\mathbf{S} = \mathbf{S}^{ch} + \mathbf{S}^{nucl},$$

$$\mathbf{S}^{nucl} = \frac{1}{6} \sum_{N}^{A} \mathbf{d}_N (r_N^2 - \langle r^2 \rangle_{ch}) + \frac{1}{5} \sum_{N}^{A} \left(\mathbf{r}_N (\mathbf{r}_N \cdot \mathbf{d}_N) - \frac{1}{3} \mathbf{d}_N r_N^2 \right).$$

$$\langle \Psi_a | S^z | \Psi_a \rangle \equiv S = (a_0 + b) G_{\pi NN} g^{(0)} + a_1 G_{\pi NN} g^{(1)} + (a_2 - b) G_{\pi NN} g^{(2)}$$

TABLE I: Calculated coefficients a_i and b for ^{199}Hg. The units are e fm^3. The last two references include the Skyrme interaction SkO′. Five results of Ban et.al. are due to Hartee-Fock and Hartree-Fock-Bogoliubov approximations. SLy4, SIII et al. indicate several Skyrme interactions.

	a_0	a_1	a_2	b
Dmitriev-Sen'kov 2003 [97]	−0.0004	−0.055	0.009	–
de Jesus-Engels (averaged) [96]	0.007	0.071	0.018	–
Ban et al [98]				
SLy4(HF)	-0.013	0.006	0.022	-0.003
SIII(HF)	-0.012	-0.005	0.016	-0.004
SV(HF)	-0.009	0.0001	0.016	-0.002
SLy(HFB)	-0.013	0.006	0.024	-0.007
SkM*(HFB)	-0.041	0.027	0.069	-0.013

$S(^{225}Ra)$	5.06	10.4	− 10.1

Engel et al, Phys.Rev C68

Polar molecule case

The enhancement factor is given by

$$K = \sum_n \frac{\langle\psi|2ic\beta_i\gamma_5\mathbf{p}_i^2|\phi_n\rangle\langle\phi_n|\sum_i ez_i|\psi\rangle}{E - E_n} + h.c.$$

(B.P.Das, Lec.Note Chem. **50**) So the detailed calculations are reduced to those of electron wave functions in atoms and molecules. For molecular case, unfortunately, only H_2^+ can be solved in the Born-Oppenheimer approximation

5. NP after the discovery of Higgs

at tree level

$$m_h < M_Z|\mathrm{cos}(2\beta)| \text{ and } M_Z = 91.2\mathrm{GeV}$$

One loop correction to m_h in CMSSM is

$$m_h^2 \approx M_Z^2 \mathrm{cos}^2 2\beta + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\mathrm{ln}\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

where

$$M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}, \quad X_t = A_t - \mu \mathrm{cot}\beta, \quad v = 174\mathrm{GeV}$$

with the trilinear Higgs-stop coupling constant $A_t \Rightarrow$ No Simple CMSSM.

Is 126 GeV too small from the SM since naively

$$m_h \approx 4\pi v_{EW} = 3TeV$$

In analogy of chiral dynamics where scalar meson mass

$$f_0(980\text{MeV}) \approx 4\pi f_\pi$$

However 226 GeV is a little bit too large from SUSY since it naively requires

$$m_{\tilde{t}} \geq \text{a few TeV.}$$

This condition may be relaxed by the large trilinear term though it
enforce a new SUSY breaking mechanism.

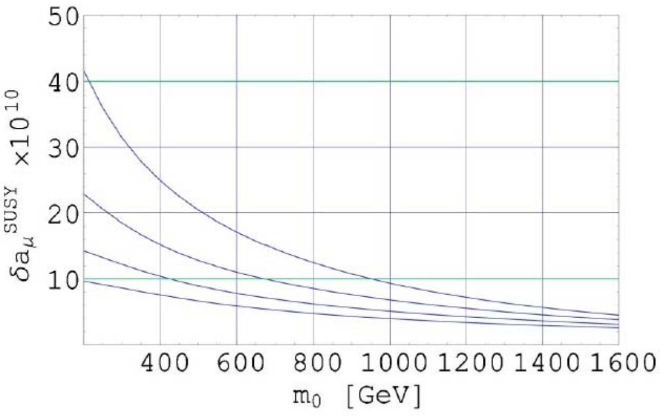


FIG. 6. The SUSY contribution to the muon $g-2$ in units of 10^{-10} as a function of m_0 (GeV) for $M_{1/2}$ =400,600,800,1000 GeV (from top to bottom) with $A_0=0$ and $\mu>0$.

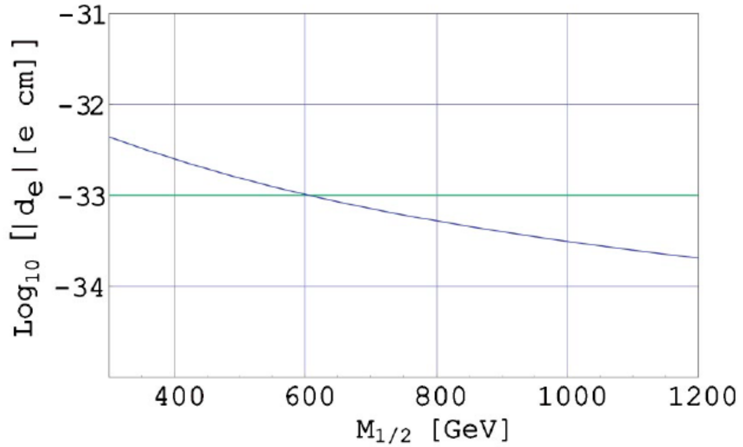


FIG. 10. The electron EDM, $\log_{10}[|d_e|(e\ \mathrm{cm})]$, as a function of $M_{1/2}$ (GeV) along the cosmological constraint of Eq. (18).

$$\Gamma(\ell_i \to \ell_j \gamma) \sim \frac{e^2}{16\pi} m_{\ell_i}^5 \times \frac{\alpha_2}{16\pi^2} \frac{\left|\left(\Delta m^2_{\tilde{\ell}}\right)_{ij}\right|^2}{M_S^8} \tan^2\beta\ ,$$

$$\left(\Delta m^2_{\tilde{\ell}}\right)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2} \left(Y_\nu^\dagger L Y_\nu\right)_{ij}\ ,$$

Hisano et al. P.R.D53

Non-zero triliniear term (A_0) changes the prediction;

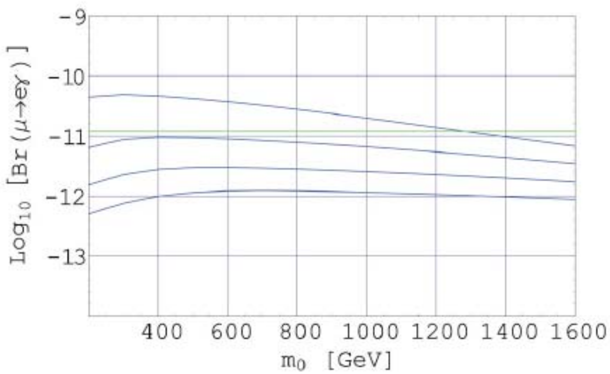


FIG. 1. The branching ratio, $\log_{10}[\mathrm{Br}(\mu\to e\gamma)]$, as a function of m_0 (GeV) for $M_{1/2}$=400,600,800,1000 GeV (from top to bottom) with $A_0=0$ and $\mu>0$.

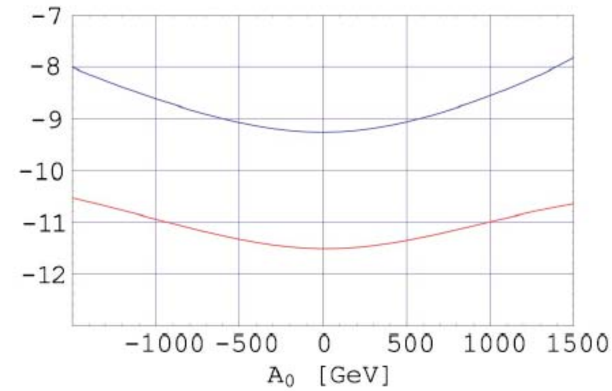


FIG. 5. The branching ratios, $\log_{10}[\mathrm{Br}(\tau\to\mu\gamma)]$ (top) and $\log_{10}[\mathrm{Br}(\mu\to e\gamma)]$ (bottom) as functions of A_0 (GeV) for m_0 =600 GeV and $M_{1/2}$=800 GeV.

$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$ (MEG Collaboration)

But large A_0 enforces new SUSY breaking mechanism. SUSY breaking mechanism is not established yet.

Final goal is still far away but LHC results are very（positively) important for (against) SUSY.

All participants hope Fountain Hotel becomes a memorial place of new phase of CP-violation.

Thank you.