12-1-2014

Puri: Advanced School

Lecture 3

Overview

1) Construction of $\tilde{O}$ in the CFT
   a) limiting the set of observables
   b) the no-annihilation condition
   c) defining the $\tilde{O}$
2) Resolving all paradoxes
   a) strong subadditivity via complementarity
   b) locality via vanishing commutators
   c) lack of a left-inverse via sparseness
   d) $N_a = 0$ paradox by explicit construction
3) the Frozen vacuum
   a) near equilibrium states
   b) modifying the prescription for near eq. states

1) Today, I will explicitly construct operators that obey our fundamental equation, and use them to resolve all paradoxes

First, let us try and understand what we can actually measure in the CFT.

The first thing to do is to discretize a little. Let us put the CFT on S^3, in which case we have some angular momentum modes specified by $\vec{m}$.

In addition, let us discretize the frequencies, so that we have

$$O^{(i)}_{\omega_n, \vec{m}}$$

we may have different kind of operators (i) and say we take even e^N different frequencies bounded, of course, by

$$\omega_{max} << N^2$$

2) Now, we can measure correlators but with a number of insertions K, so that $K << N^2$

[Discuss later what happens]

Also, total energy $<< N^2$. i.e. $\langle O_{\omega_1} .. O_{\omega_n} \rangle$ with $\sum \omega_i << N^2$.

3) Consider the set of all polynomials in these modes ②

$$A_\alpha = \sum c(\alpha(n,m,i)) \left(O^{(i)}_{\omega_n, m}\right)^{\alpha(n,m,i)}$$

this set of polynomials forms a linear space, with

$$D_A \ll e^{N^2}$$

4) Now consider some generic state $|\psi\rangle$, with energy $E \gg N^2$. This corresponds to a big B.H. in the bulk. Then this state satisfies

$$A_\alpha |\psi\rangle \neq 0; \ \forall \alpha$$

Said another way, if we choose a linearly independent basis for $\{A_\alpha\}$, then the action of this basis on $|\psi\rangle$ generates linearly independent vectors

Now, define

$$\tilde{O}^{(i)}_{\omega,m} A_\alpha |\psi\rangle = A_\alpha e^{-\beta\omega/2} (O^i_{\omega,m})^\dagger |\psi\rangle$$

this leads to a set of linear equations for $\tilde{O}$.

These linear equations are consistent because $A_\alpha |\psi\rangle \neq 0 \ \forall \alpha$.

It is always possible to find an operator with a specified action on a set of linearly independent vectors

5) Let us consider another equivalent definition of the tilde operators. Consider the anti-linear map:

$$S : H_\psi \to H_\psi$$

with

$$S A_\alpha |\psi\rangle = A^\dagger_\alpha |\psi\rangle$$

H_ψ is the space spanned by acting with all polynomials A_α on $|\psi\rangle$.

This S-map also appears in the Tomita-Takesaki theory of modular automorphisms of von Neumann algebras

6) For any polynomial A_α, we can define

$$\tilde{A}_\alpha = S\, e^{-\beta H/2} A_\alpha\, e^{\beta H/2} S.$$

This is equivalent to our previous definition for $\tilde{O}_{\omega, k}$

7) Now, let us check that our fundamental equation for a smooth horizon is satisfied:

$$\tilde{O}_{\omega_2'} \tilde{O}_{\omega_1'} O_{\omega_1} O_{\omega_2} O_{\omega_3} |\psi\rangle$$
$$= \tilde{O}_{\omega_2'} O_{\omega_1} O_{\omega_2} O_{\omega_3} (O_{\omega_1'})^{+} |\psi\rangle \cdot (e^{-\beta \omega_1'/2})$$
$$= O_{\omega_1} O_{\omega_2} O_{\omega_3} (O_{\omega_1'})^{+} (O_{\omega_2'})^{+} |\psi\rangle \cdot (e^{-\beta(\omega_1' + \omega_2')/2})$$

which is precisely what we need.

<u>Exercise</u>: Use the S-definition to derive the same result.

8) So, this and our construction of local operators leads us to predict a <u>smooth horizon</u> in AdS/CFT

9). Now, we describe how to resolve each of the Mathur- AMPS paradoxes.

First, consider strong subadditivity.

The resolution is that the inside operators, described by $\underline{C}$, are a scrambled subset of the exterior operators which describe $\underline{A}$!

The way to see this is to appreciate that

$$[\tilde{O}_\omega, O_{\omega'}] \neq 0.$$

This must be the case because if you consider operators

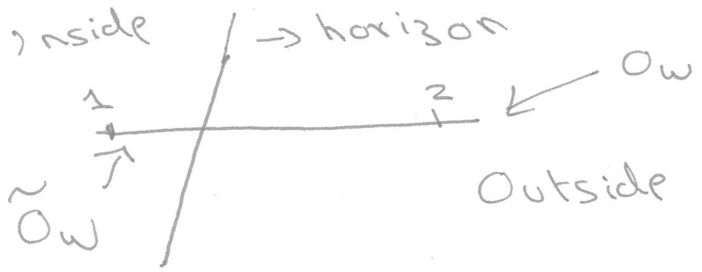
$$O_{\omega_1}, O_{\omega_2} \ldots O_{\omega_n}$$

with the fine discretization that we have and consider all polynomials $P(O_{\omega_i})$ (without the restriction that total insertions $< k$), and moreover if we take polynomials with energy $E \gg N^2$ (again, recall A_α was restricted to the set with energy ~~A_α~~ $E(A_\alpha) \ll N^2$) then, $P(O_{\omega_i})$ generates the full Hilbert space, just by counting.

So, <u>as</u> <u>an</u> <u>operator</u>

$$[\tilde{O}_\omega, O_{\omega'}] \neq 0$$

10) But, now we also need to preserve locality



If we take points 1 and 2 as shown above, then we need

$$[\phi(x_1), \phi(x_2)] \sim 0$$

if x_1, x_2 are spacelike separated

This is ensured by

$$\tilde{O}_\omega O_{\omega'} A_\alpha |\Psi\rangle$$

$$= O_{\omega'} A_\alpha O_\omega^\dagger |\Psi\rangle \cdot e^{-\beta\omega/2}$$

and,

$$O_{\omega'} \tilde{O}_\omega A_\alpha |\Psi\rangle$$

$$= O_{\omega'} A_\alpha O_\omega^\dagger |\Psi\rangle.$$

So, the commutator

$$[\tilde{O}_\omega, O_{\omega'}]$$

<u>annihilates</u> <u>$|\Psi\rangle$</u> and all descendants of $|\Psi\rangle$ produced by acting with excitations bounded in energy and no. of insertions.

11) So, the commutar vanishes <u>effectively</u>

$$[\tilde{O}_\omega, O_{\omega'}] \doteq 0$$

and locality is preserved unless one measures N^2-pt correlators

12) Now, we resolve the lack of a left inverse paradox. The answer goes along the same lines. We have

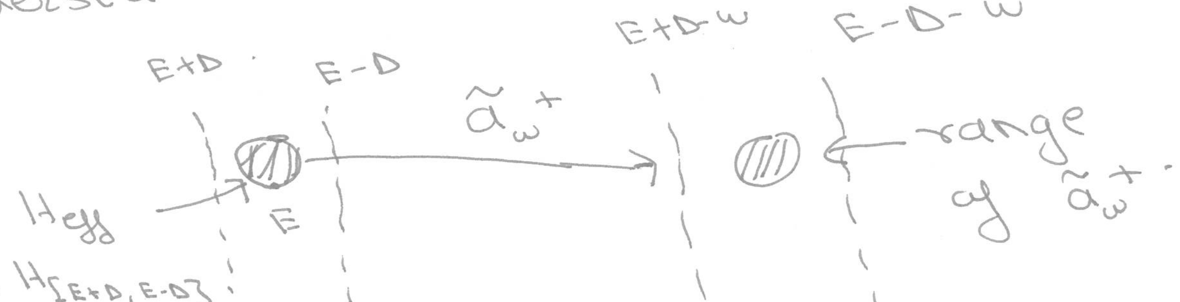
$$[\tilde{a}_\omega, \tilde{a}_\omega^+] \doteq 1$$

but as an operator

$$[\tilde{a}_\omega, \tilde{a}_\omega^+] \neq 1$$

and so $\tilde{a}_\omega^+$ does not need to have a left-inverse

More precisely consider some state $|\psi\rangle$ of energy E, and let us understand how $\tilde{a}_\omega^+$ acts.



$\doteq H_\psi \cap H_{[E+D, E-D]}$

= intersection of descendants of $|\psi\rangle$ with $H_{[E+D, E-D]}$

$H_{[E+D, E-D]}$ = subset of states with energy in this range

$\tilde{a}_\omega^+$ needs to act correctly only on descendants of $|\psi\rangle$ produced by finite number of insertions. So, it may be a <u>sparse</u> <u>operator</u>.

13) Now, let us consider the

$$N_\alpha \neq 0$$

paradox.

First, we see that explicitly $N_\alpha |\psi\rangle = 0$ because we have

$$N_\alpha = \alpha_1^+ \alpha_1 + \alpha_2^+ \alpha_2$$

with

$$\alpha_1 = a - e^{-\beta\omega/2} \tilde{a}^+$$

$$\alpha_2 = \tilde{a} - e^{-\beta\omega/2} a^+$$

Clearly $\alpha_1 |\psi\rangle = \alpha_2 |\psi\rangle = 0$

What is wrong with the trace argument?

$\tilde{a}$, and $\tilde{a}^+$ are <u>state-dependent</u> operators.

For such state-dependent operators we can't change basis.

Let me give an example. Consider a product system

and some state

$$|\psi\rangle \in H_A \otimes H_B$$

Now say we want

$$S_A = -\mathrm{tr}(\rho_A \ln \rho_A)$$

Let me carry out the Marolf-Polchinski argument. To evaluate the trace, we go to a <u>basis of product states</u>

$$|v_1\rangle = |A_1\rangle \otimes |B_1\rangle$$

$$\vdots$$

$$|v_n\rangle = |A_n\rangle \otimes |B_n\rangle$$

In each vector:

$$\langle v_m| \rho \ln \rho |v_m\rangle = 0$$

So, $S_A = 0$??.

Clearly this is absurd, because ρ is <u>state</u>-<u>dependent</u>

14) Finally we turn to another issue
the "frozen vacuum".
We argued that

$$N_\alpha |\psi\rangle = 0$$

but this is a little too strong.
We can always choose to shine a laser
on the B.H. in which case the
infalling observer will see some particles

So, let us ask a little more carefully
when we expect $N_a |\psi\rangle = 0$.
We expect this in an equilibrium state.

15) At first order, we can classify
equilibrium states by demanding

$$\langle \psi | O(t_1) O(t_2) .. O(t_n) | \psi \rangle$$

$$= \langle \psi | O(t_1+T) O(t_2+T) .. O(t_n+T) | \psi \rangle$$

as long as $T << e^{N^2}$ and then the
time-translational invariance holds to
an excellent approximation $(e^{-N^2/2})$

16) Now consider some near-equilibrium states, obtained by:

$$|\Psi_{ne}\rangle = U|\Psi\rangle$$

where

$$U = e^{i(A_\alpha + A_\alpha^\dagger)}$$

For such a state OO correlators are not time-translationally invariant but given $|\Psi_{ne}\rangle$, we can uniquely write

$$|\Psi_{ne}\rangle = U|\Psi_0\rangle$$

Hence, for such states $|\Psi_{ne}\rangle$, basically we need to define $\tilde{O}_\omega$ on $|\Psi_0\rangle$ or alternately, we write:

$$\tilde{O}_\omega A_\alpha |\Psi\rangle_{n.e.} = e^{-\beta\omega/2} A_\alpha U O_\omega^\dagger U^\dagger |\Psi_{n.e.}\rangle.$$

17). Open questions

This brings us to the central open question here. How does one really classify equilibrium states?

The demand that

$$\langle \psi | O(t_1) \ldots O(t_n) | \psi \rangle$$

is time-translationally invariant is clearly insufficient.

For example, consider

$$|\psi'\rangle = e^{i\tilde{O}_\omega} |\psi\rangle.$$

this $\tilde{O}$ commutes with the O's -

and so

$$\langle \psi' | O(t_1) \ldots O(t_n) | \psi' \rangle$$
$$= \langle \psi | O(t_1) \ldots O(t_n) | \psi \rangle$$

So, $|\psi'\rangle$ also has the property that correlators of ordinary operators O are time-trans invariant. But we would like to interpret $|\psi'\rangle$ as a non-eq state with an excitation behind the horizon. How do we detect that $|\psi'\rangle$ is out of equilibrium — OPEN QUESTION!