Nano-optics & Mesoscopic Optics Laboratory

ICTS Colloquium, ICTS, October 2017

Anderson localization and Levy sums in random lasers

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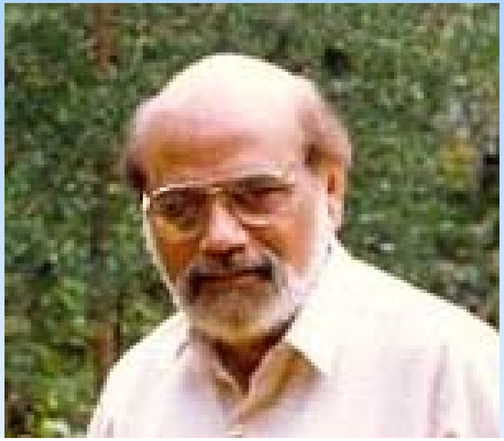
Nano-optics & Mesoscopic Optics Laboratory

Homage

Prof Narendra Kumar. 28 August 2017.



RAPID COMMUNICATIONS

PHYSICAL REVIEW B   VOLUME 50, NUMBER 13   1 OCTOBER 1994-I

Localization of light in coherently amplifying random media

Prabhakar Pradhan and N. Kumar*
Department of Physics, Indian Institute of Science, Bangalore 560 012, India

(Received 25 April 1994)

We derive and analyze the statistics of reflection coefficient of light backscattered coherently from an amplifying and disordered optical medium modeled by a spatially random refractive index having a uniform imaginary part in one dimension. We find enhancement of reflected intensity owing to a synergy between wave confinement by Anderson localization and coherent amplification by the active medium. This is not the same as that due to enhanced optical path lengths expected from photon diffusion in the random active medium. Our study is relevant to the physical realizability of a mirrorless laser by photon confinement due to Anderson localization.

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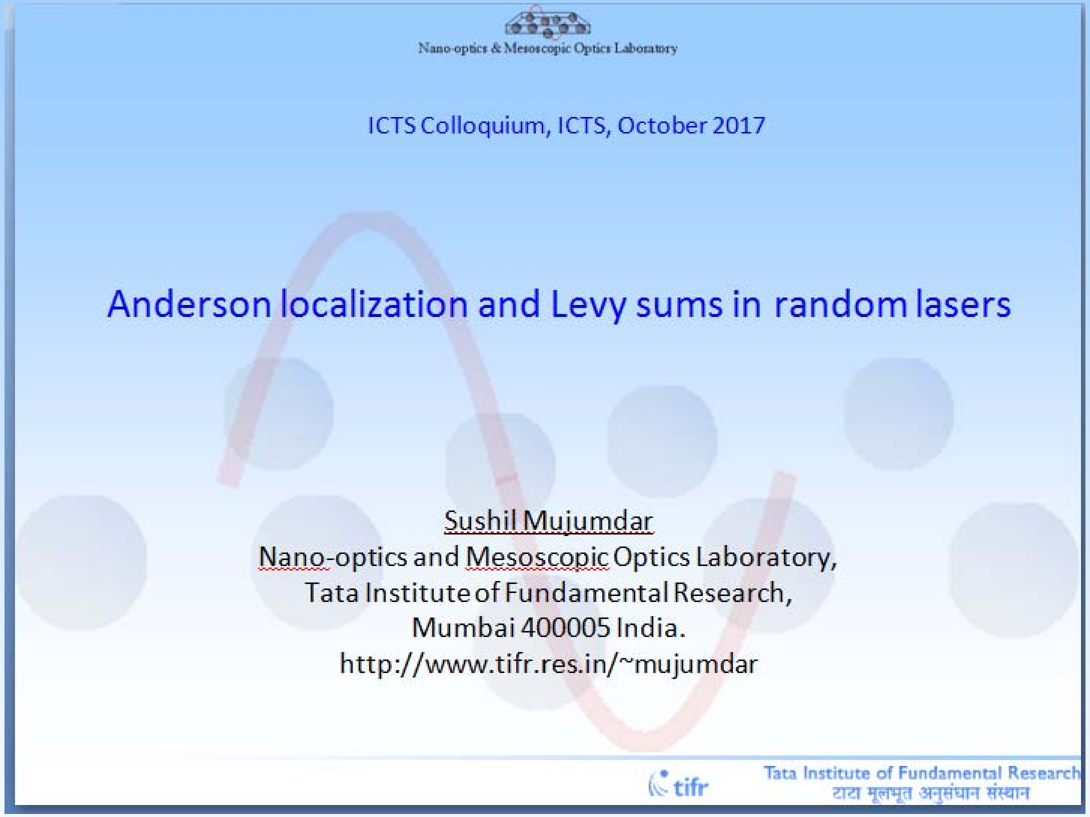
Acknowledgements



Anjani Kumar Tiwari (Ex-PhD),
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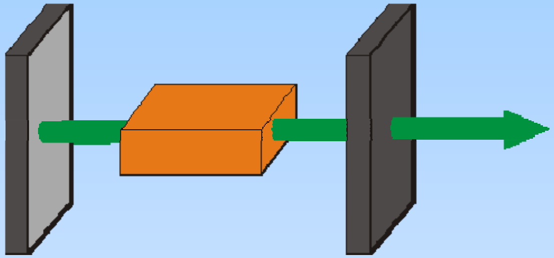
Sreeman Kumar (SA)

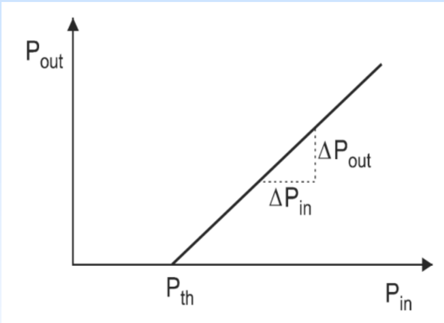
Government of India
Department of Atomic Energy

Department of Science & Technology
Ministry of Science & Technology

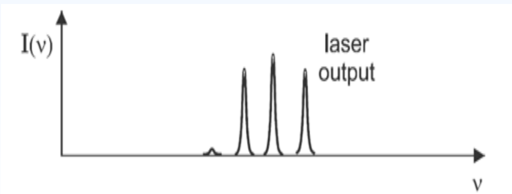
Conventional ~~Random~~ lasers



System



Lasing threshold.



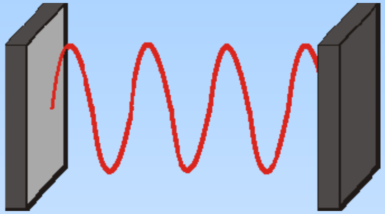
Lasing modes.

Conventional lasers exploit resonators to increase the dwell-time of light in the amplifying medium.

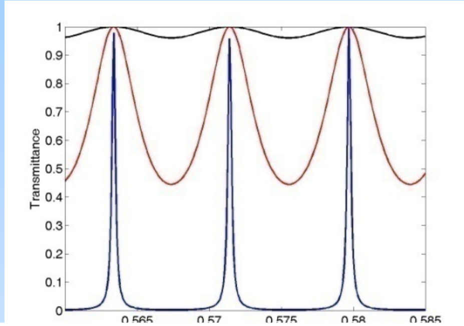
At threshold excitation, optical gain overcomes loss, and the output intensity diverges.

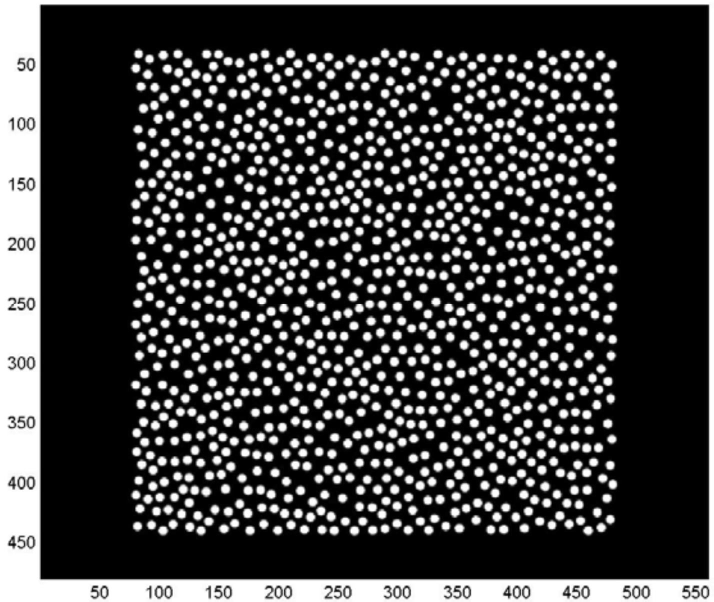
The output comprises well-defined narrow peaks in the spectrum, which are significantly narrower than the fluorescence profile.

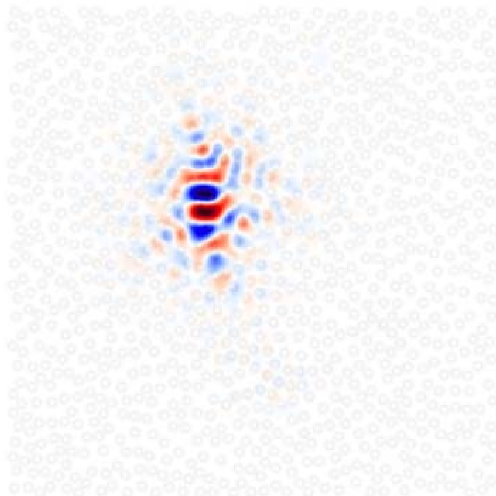
Quasimodes



Loss in the resonator causes broadening.



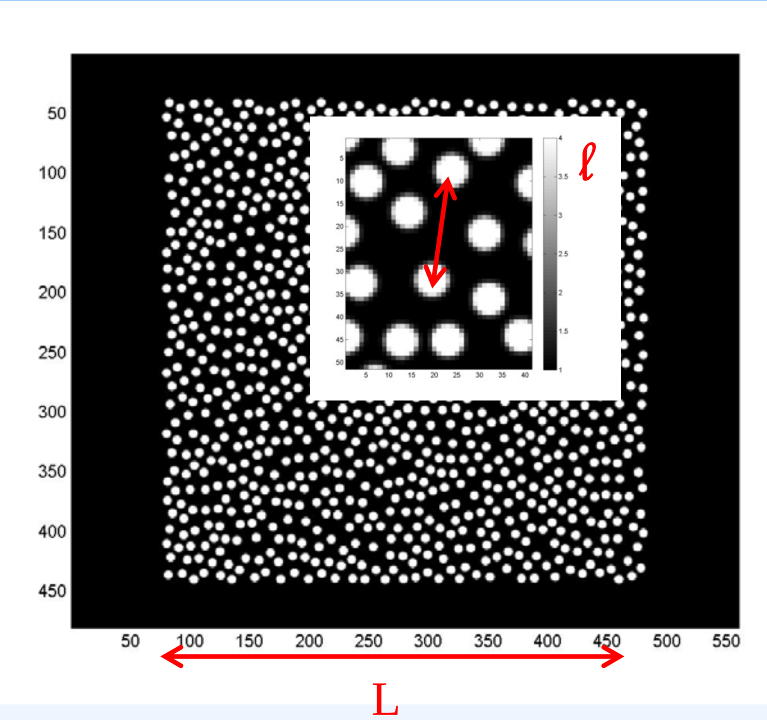




'Random' resonance.

What if we add gain to the structure?

Quasimodes

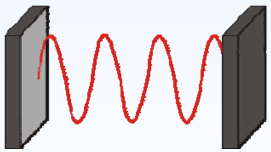


$\lambda \sim \ell; \ell << L$ Localization regime. Strong quasimodes.

$\lambda < \ell; \ell << L$ Diffusion regime. Weak quasimodes.

$\lambda < \ell; \ell \sim L$ Single scattering regime. Very weak quasimodes.

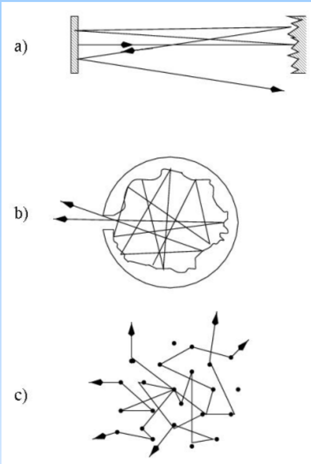
Dimensionality matters!!!



1D, 2D : Strong quasimodes, always localized! But size dependence.

3D: Weak quasimodes. Localization very very hard. Only diffusion!

~~Conventional~~ Random lasers



System

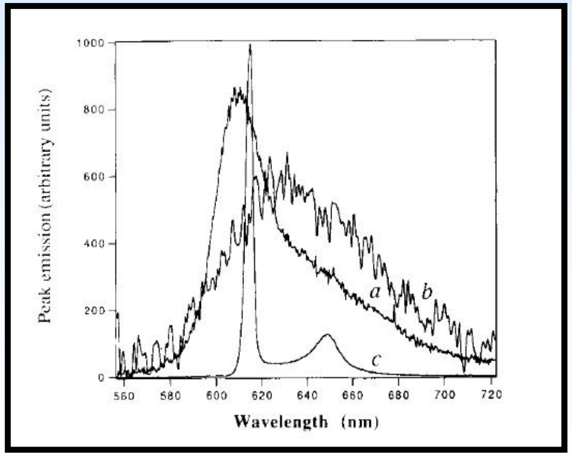
Letokhov, JETP(1968)

Generation of light using non-resonant feedback in a medium with negative absorption.

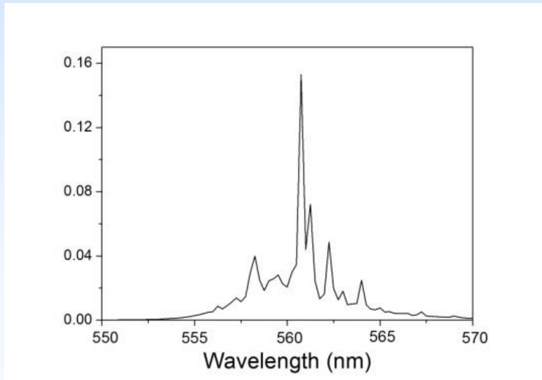
First-order temporal coherence.

Hui Cao et al, PRL (1999, 2001, 2003)

S Mujumdar et al, PRL(2004)



Lawandy et al, Nature(1994)

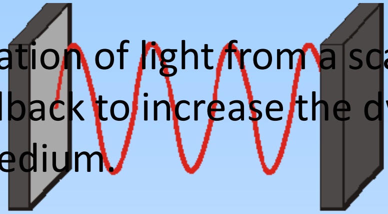


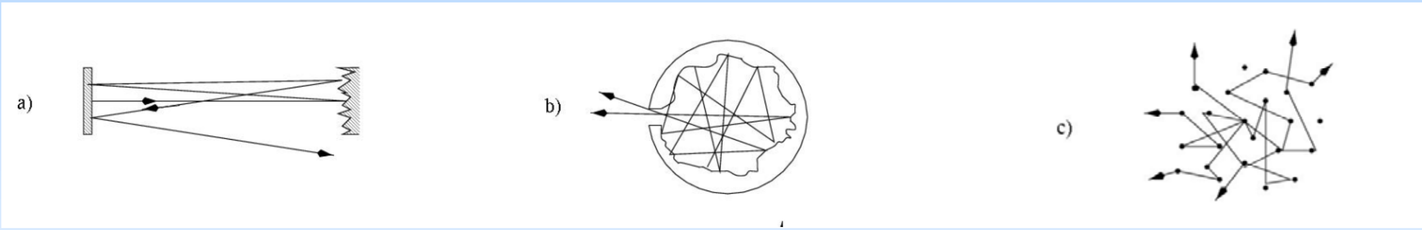
Hui Cao, J. Phys. A(2005)

DS Wiersma, Nat. Phys. (2008)

Random lasers: Diffusion with gain

Proposed by Letokhov: Generation of light from a scattering medium with negative absorption. Nonresonant feedback to increase the dwell-time of light, without realizing a resonator in the amplifying medium.

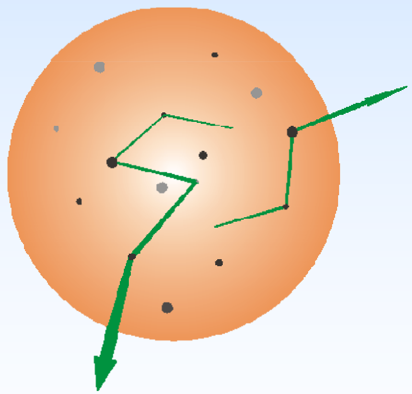




Light diffusion with gain.

$$\frac{\partial I(\mathbf{r},t)}{\partial t} = D\nabla^2 I(\mathbf{r},t) + \frac{v}{l_i} I(\mathbf{r},t) + S(\mathbf{r},t)$$

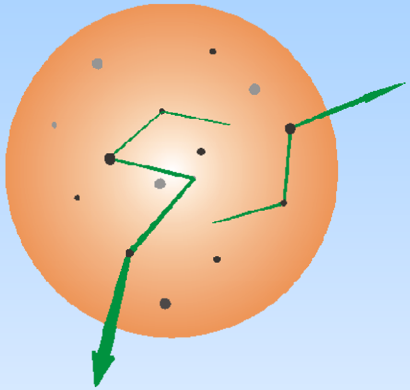
Volume gain > Surface loss ➔ threshold

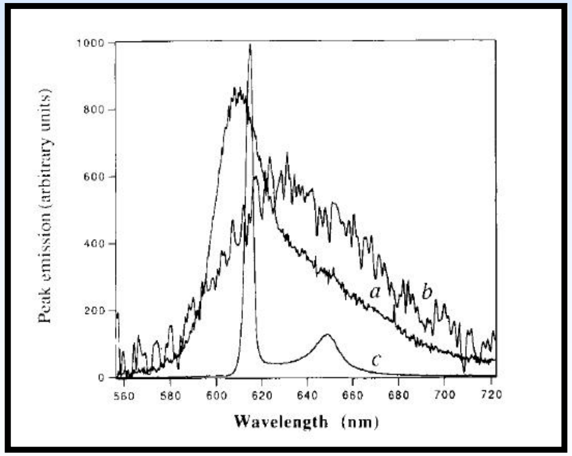


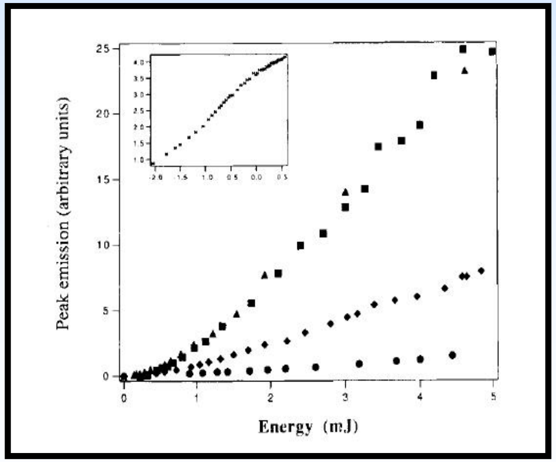
Sov. Phys. JETP,**26**, 835 (1968)

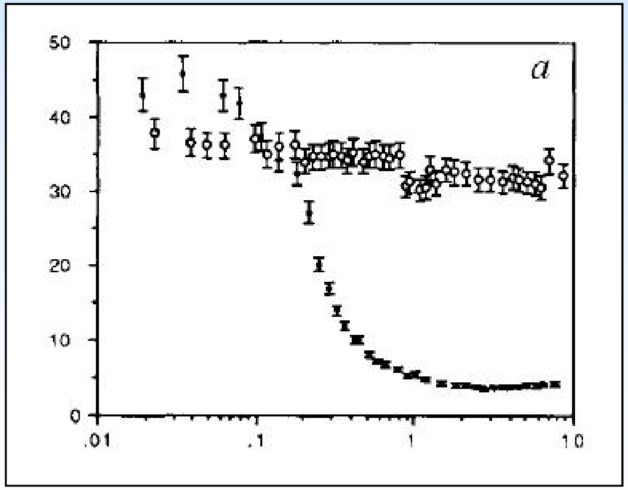
Random lasers: Diffusion with gain

Laser action in strongly scattering media. Nature, **368**, 436 (1994)





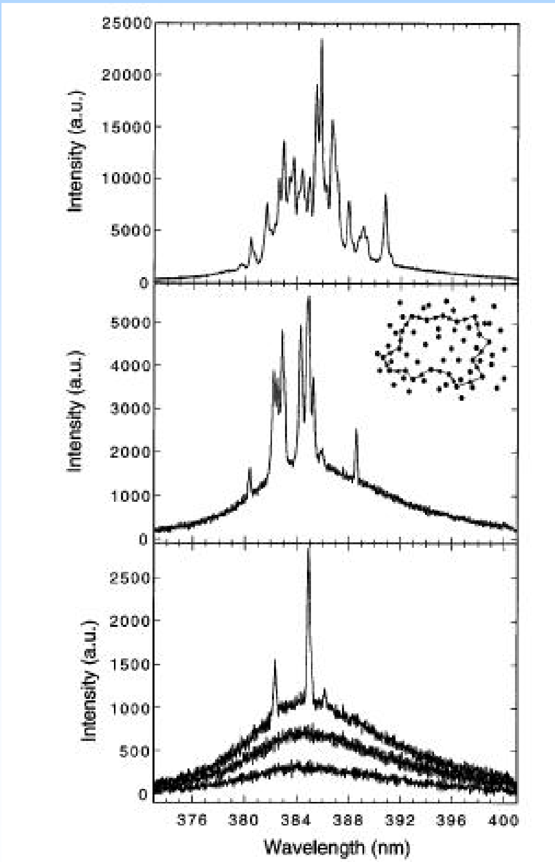




Bandwidths of a few (~5-8 nm): Diffusive random lasing

Random lasers: Localization with gain??

Random lasing action in semiconductor powders. Cao et al PRL, **82**, 2278 (1999)

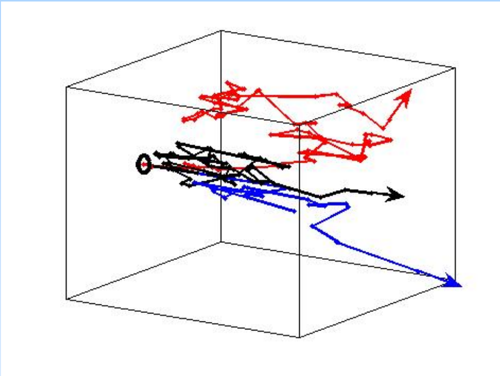


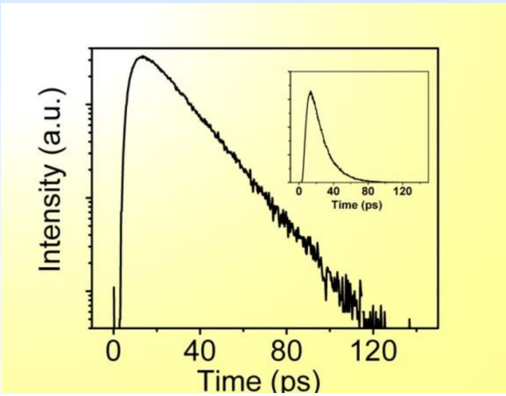
Sub-nanometer bandwidths : Coherent random lasing.
First order coherence.

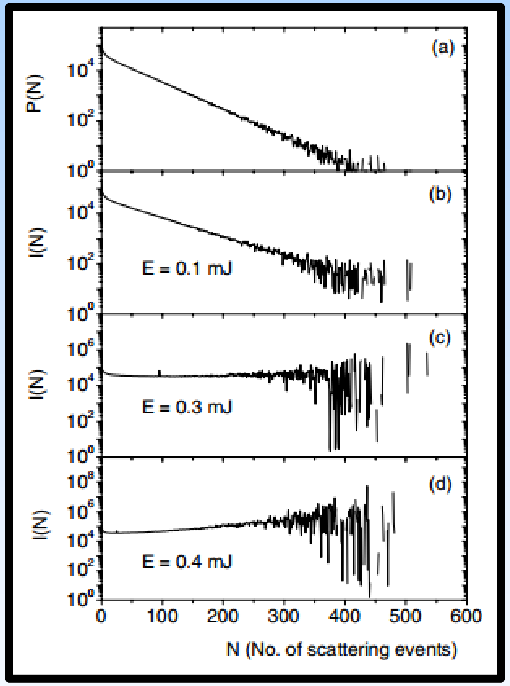
Anderson localization???

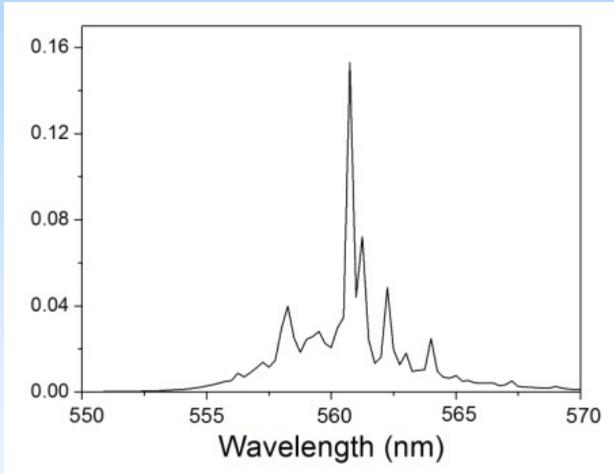
Random lasers: Diffusion with gain

Amplified extended modes in random lasers,
S Mujumdar, M Ricci, R Torre, D Wiersma, Phys. Rev. Lett. 93, 053903 (2004)



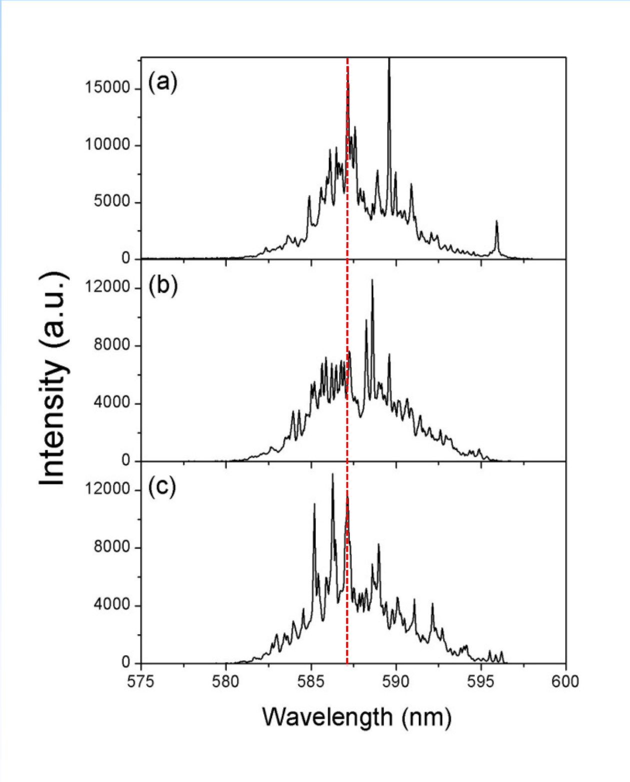




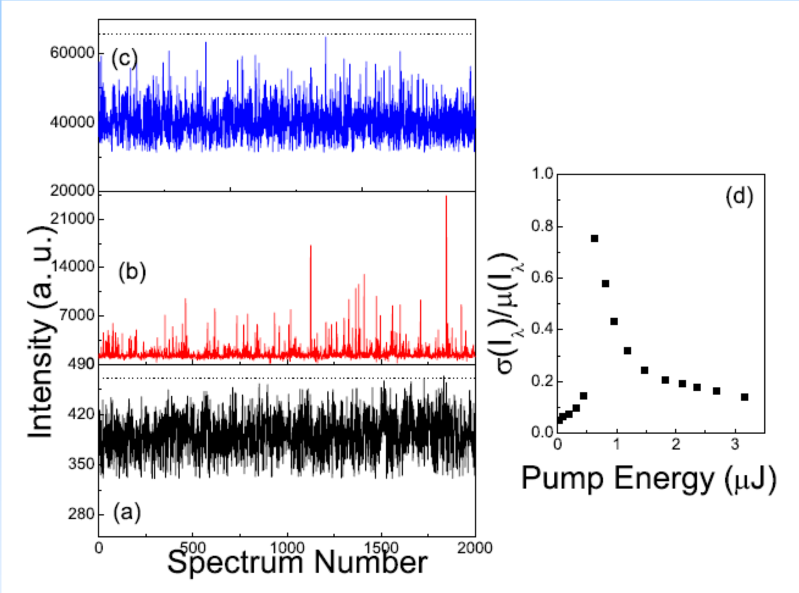


The distribution of pathlengths in a diffusive environment is exponential.
The gain experienced by the photons is exponentially proportional to their pathlengths.

Randomness = Fluctuations!



Levy statistics



Modes exhibit diverging variance, indicating Levy statistics.

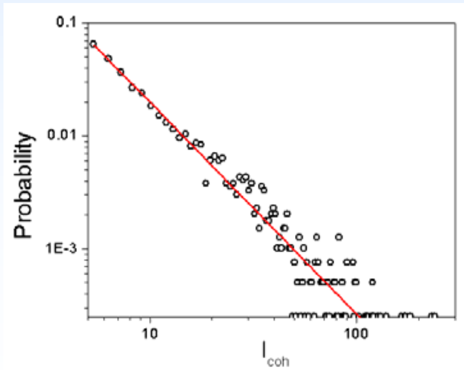
The intensity at a particular wavelength is Gaussian distributed below the critical intensity, and Levy, above.

$$I = I_0 exp(l/l_g)$$

$$P(l) = 1/\langle l \rangle exp(-l/\langle l \rangle)$$

$$P(I) = I_0^{\mu}/I^{\mu+1}$$

$$\mu = l_g/\langle l \rangle$$



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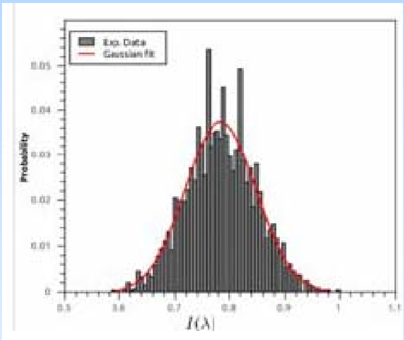
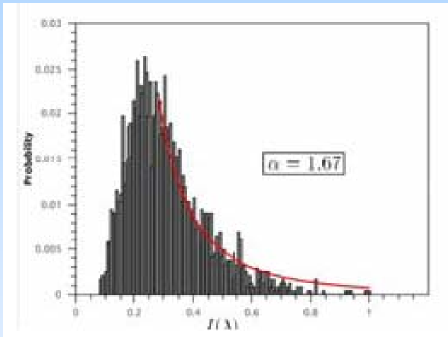
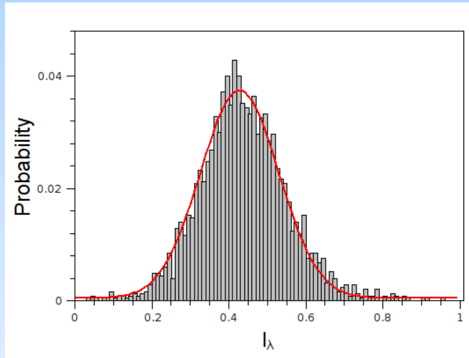
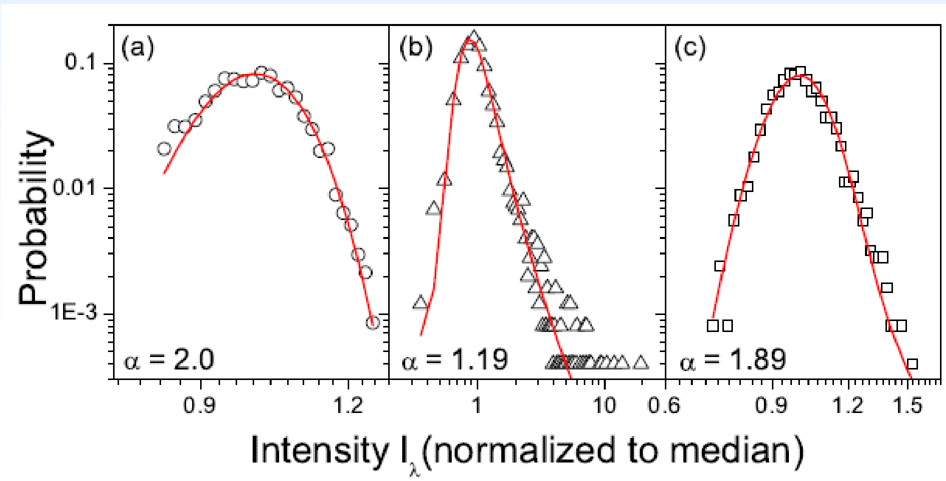
Gaussian-Levy(-Gaussian) Crossover

Individual modes follow Levy statistics because of exponential lifetimes, and exponential gain.









Econophysical model, based on Koutrouvelis algorithm.

α can define the random lasing threshold.

RU, AKT and SM, Opt. Lett. (2012)

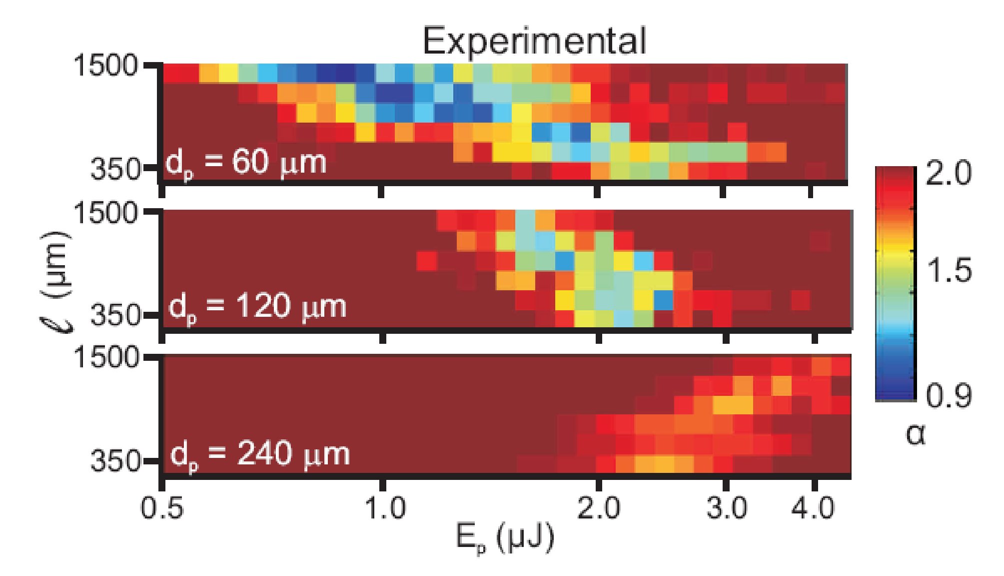
Gaussian-Levy(-Gaussian) Crossover

Using an α-stable fit, we map out the statistical regimes.

L~60 um

L~120 um

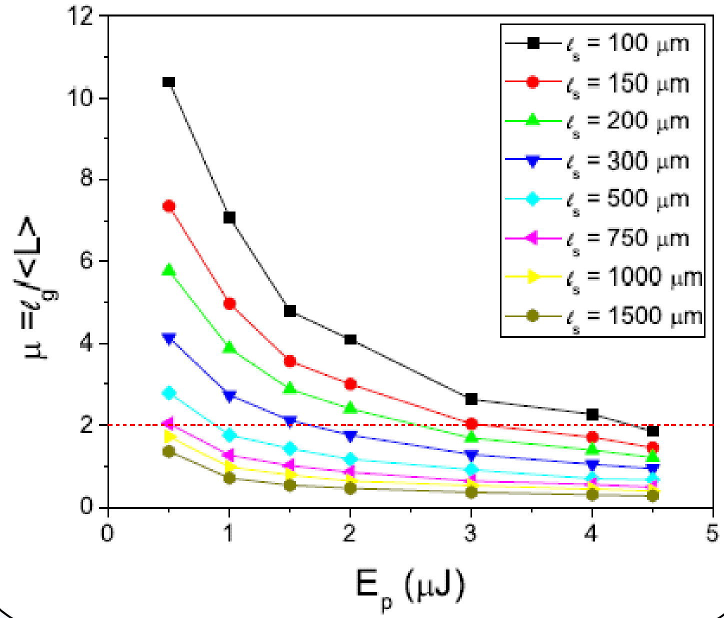
L~250 um

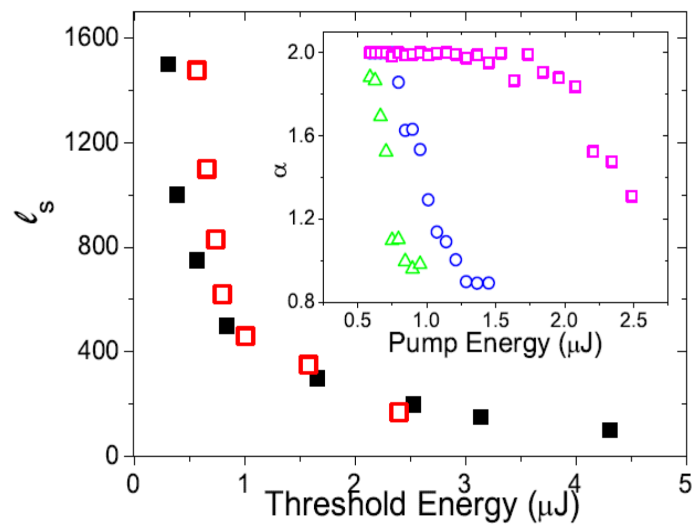


RU, AKT and SM, Opt. Lett. (2012)

Gaussian-Levy Transition

Map the first transition using theory by estimating <L> and lg:
Diffusion equation based theory.



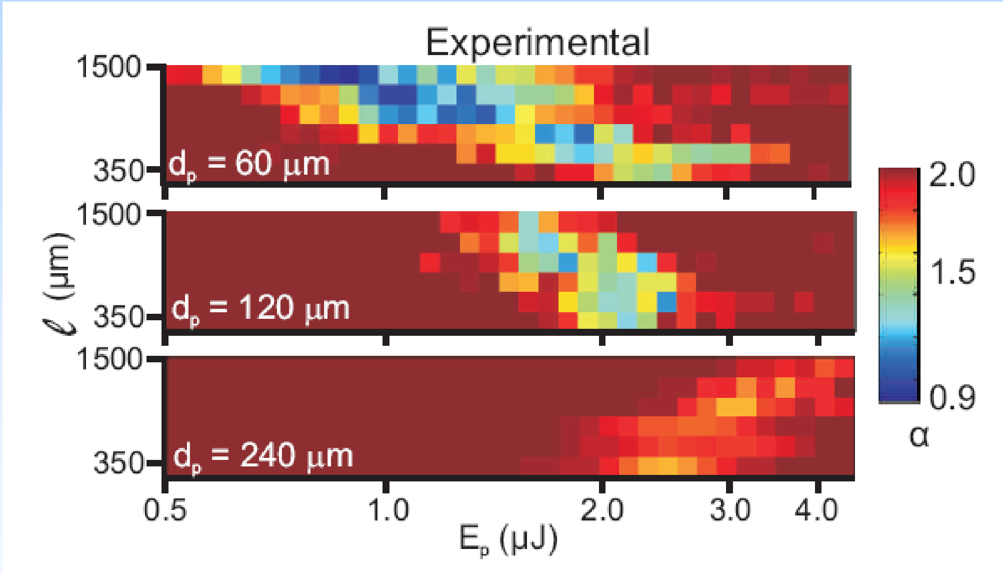


Excellent agreement between the two, in terms of trends and the rate of the transition.

RU and SM, PRA (2013)

How do you explain the complete behavior?

Gaussian-Levy-Gaussian Crossover



Truncated Levy Flights

VOLUME 73, NUMBER 22 PHYSICAL REVIEW LETTERS 28 NOVEMBER 1994

Stochastic Process with Ultraslow Convergence to a Gaussian: The Truncated Lévy Flight

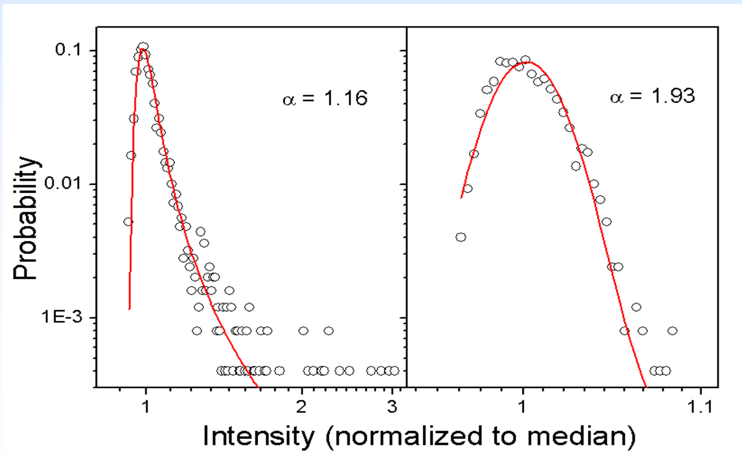
Rosario N. Mantegna and H. Eugene Stanley

Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215

(Received 18 May 1994)

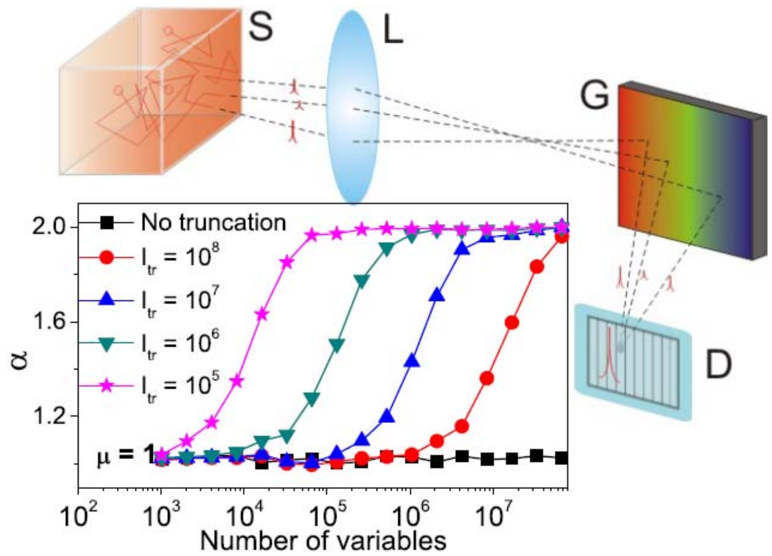
We introduce a class of stochastic process, the *truncated* Lévy flight (TLF), in which the arbitrarily large steps of a Lévy flight are eliminated. We find that the convergence of the sum of n independent TLFs to a Gaussian process can require a remarkably large value of n—typically $n \approx 10^4$ in contrast to $n \approx 10$ for common distributions. We find a well-defined crossover between a Lévy and a Gaussian regime, and that the crossover carries information about the relevant parameters of the underlying stochastic process.

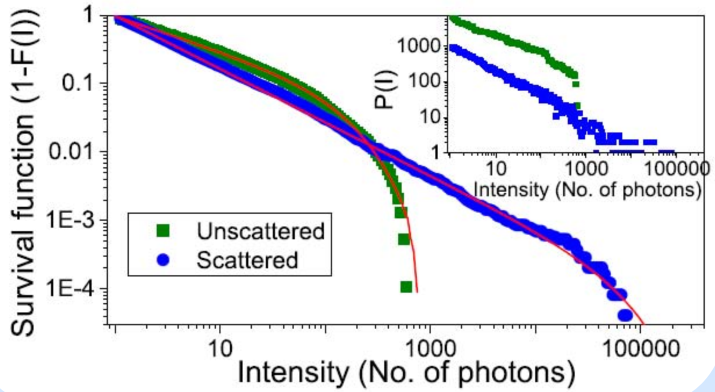
PACS numbers: 05.40.+j, 02.50.–r



Can we map the random lasing process onto a Truncated Levy Flight?

Truncated Levy Sums





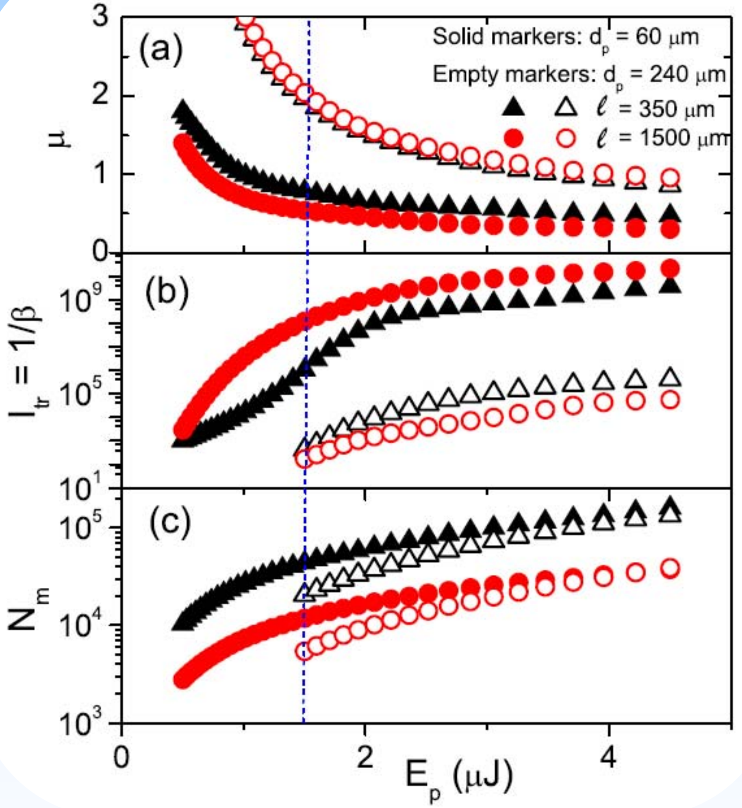
$$\sim x^{-\mu} \exp^{-\beta x} \qquad 1/\beta \sim I_{tr}$$

Experiment measures sums of Levy variables.

MC simulations of gain of photons,
and estimation of μ and Itr.

Can we map the random lasing process onto a Truncated Levy Flight?

Truncated Levy sums



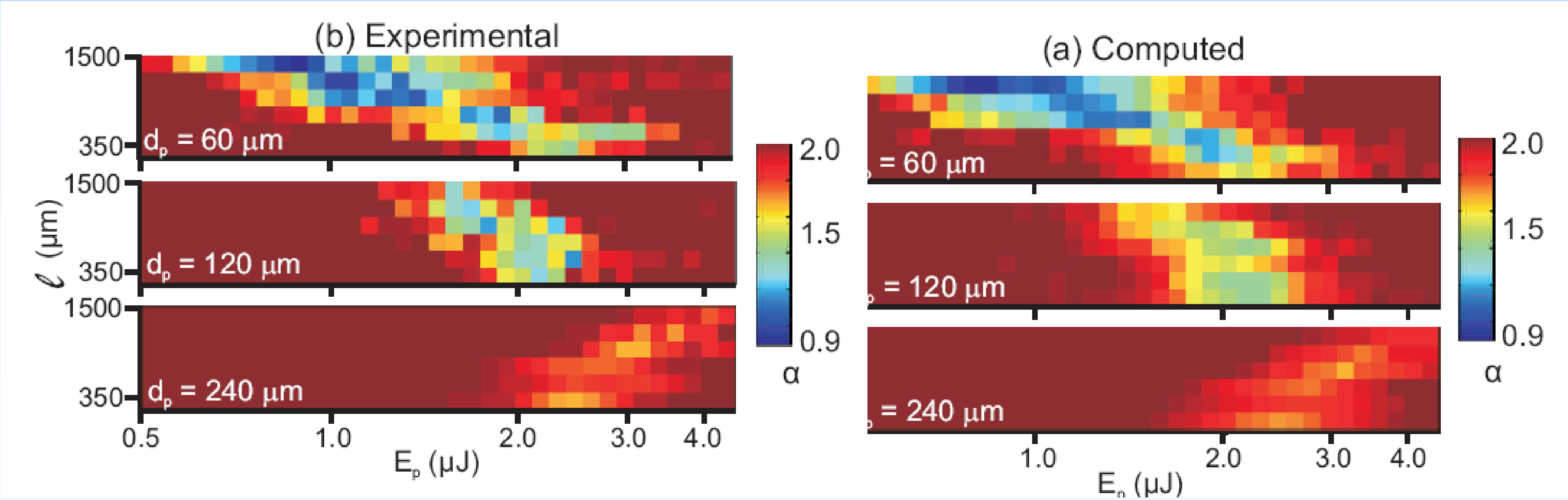
Pick up a random truncated Levy variable with the right parameters μ and ltr.

Sum up Nm of these variables.

Check the distribution.

Truncated Levy sums

A perfect fit!



Complete fit for the entire behavior of the random laser using Exponentially-tempered Truncated Levy Sums.

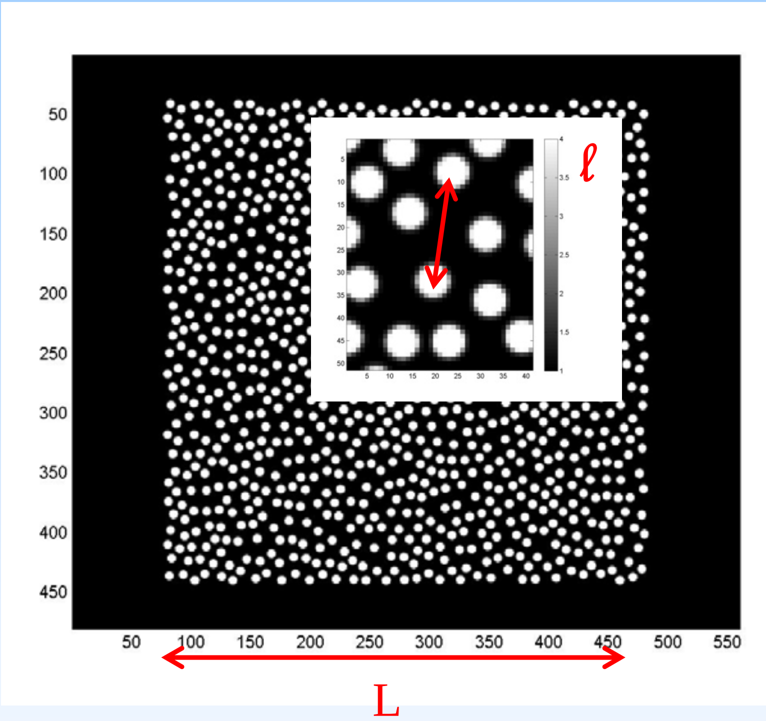
RU and SM, PRL (2015)

Part I: Conclusions

• Intensity fluctuations in random lasers are Levy.
• A truncated Levy flight can explain the statistical behavior of the system, including the two transitions.
• First-time demonstration of truncated Levy flights in optical systems.

• Coherent modes are the physical manifestation of extreme events in random lasers.
•Extreme value statistics shows the way to maximise the efficiency of random lasers.

Back to Quasimodes

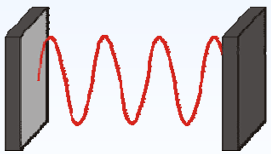


$\lambda \sim \ell; \ell << L$ Localization regime. Strong quasimodes.

$\lambda < \ell; \ell << L$ Diffusion regime. Weak quasimodes.

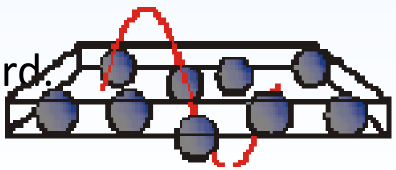
$\lambda < \ell; \ell \sim L$ Single scattering regime. Very weak quasimodes.

Dimensionality matters!!!

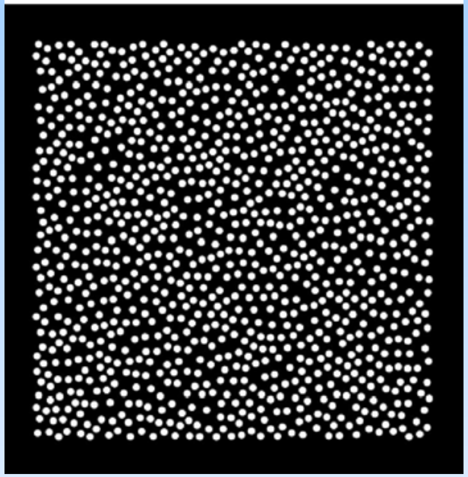


1D, 2D : Strong quasimodes, always localized! But size dependence.

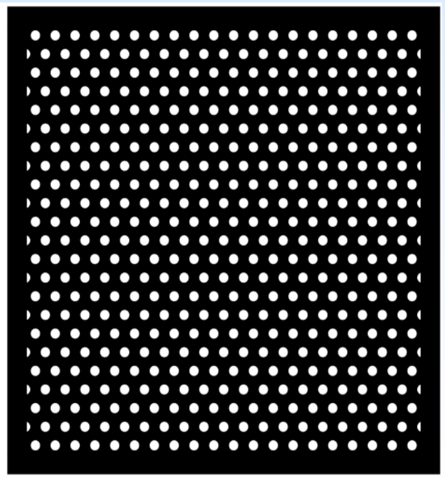
3D: Weak quasimodes. Localization very very hard.
Only diffusion!



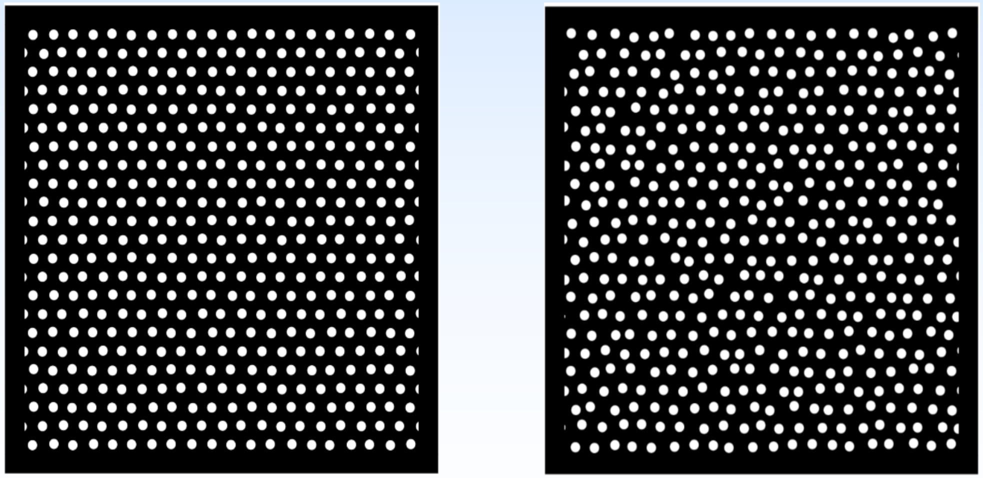
Random, and random



Random (completely)

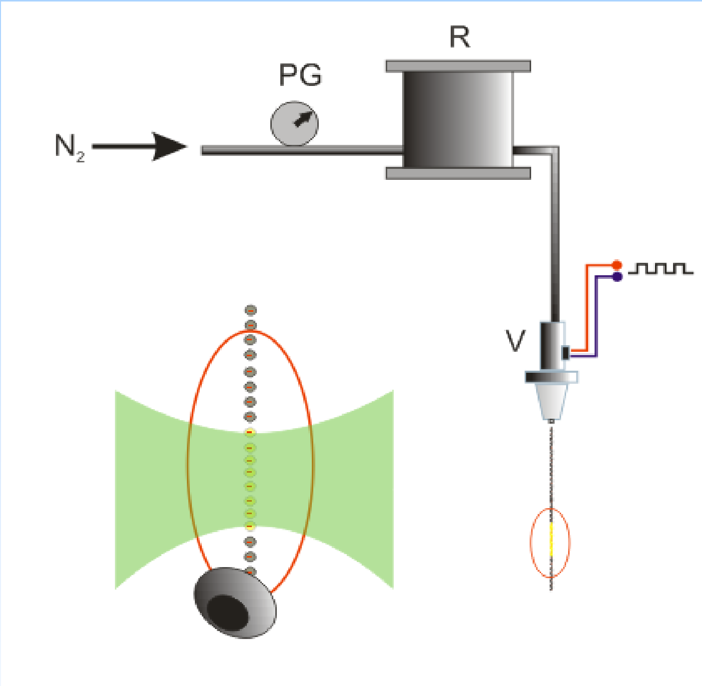


Periodic



Random, periodic-on-average

How? Vibrating Orifice Aerosol Generator



Fluid under pressure

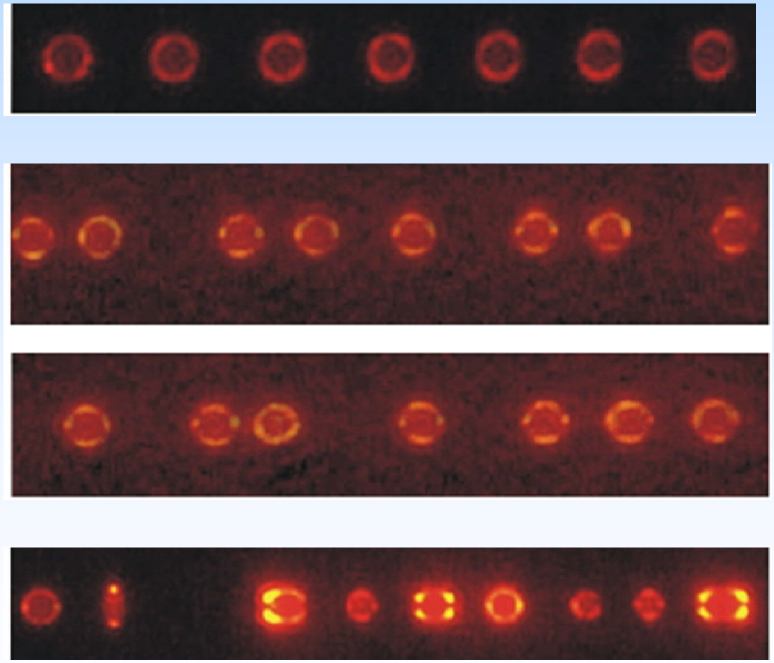
⇩

Through a microcapillary

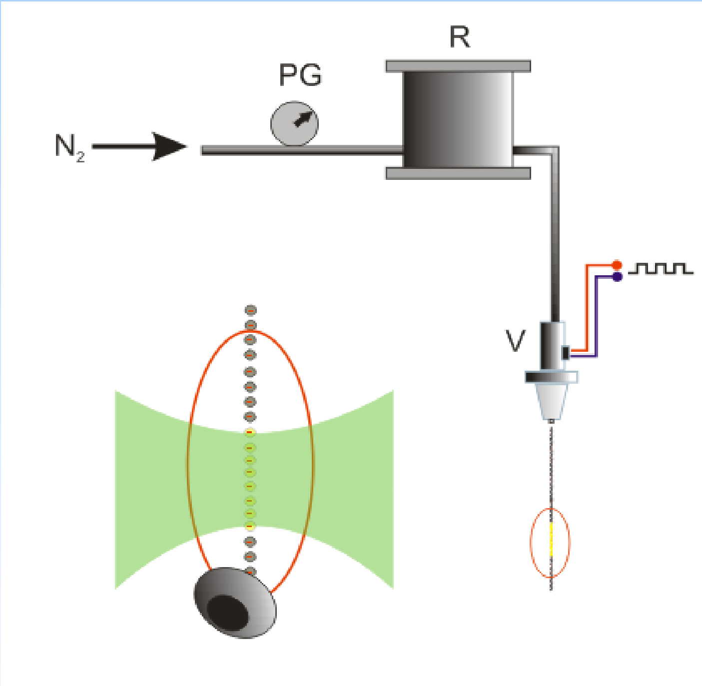
⇩

With a periodic perturbation (piezo-driven)

Right combination of pressure and frequency gives the monodispersity.



How? Vibrating Orifice Aerosol Generator



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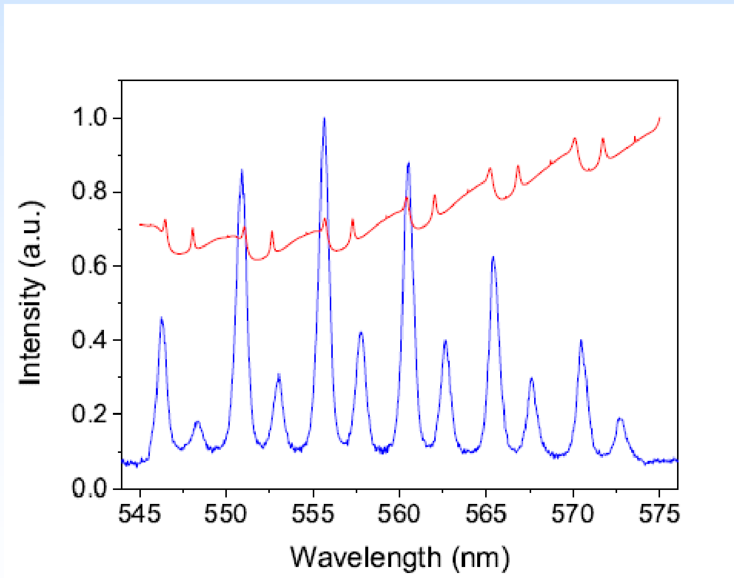
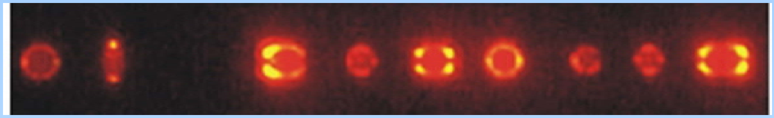
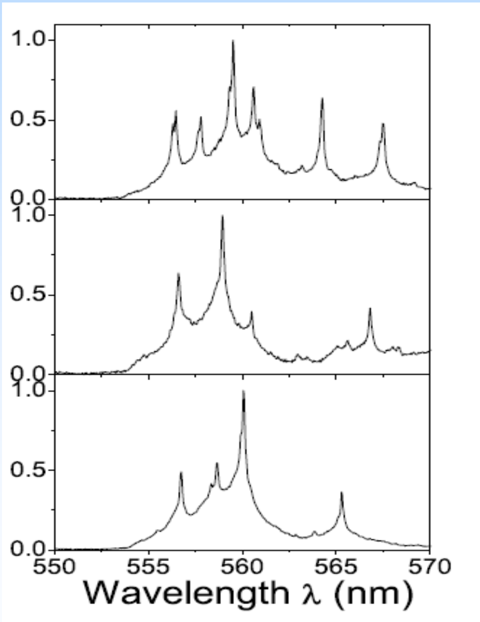
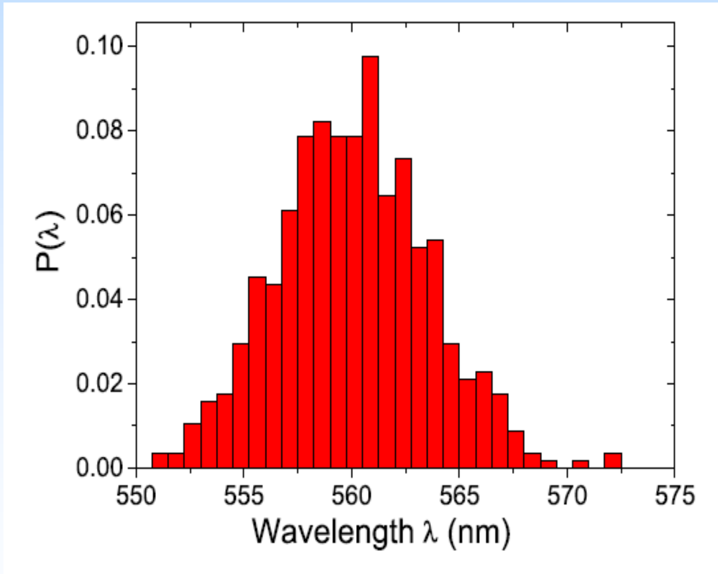


Spectra: Polydisperse random





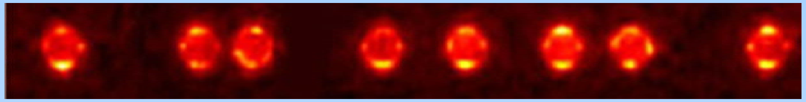
Spectra

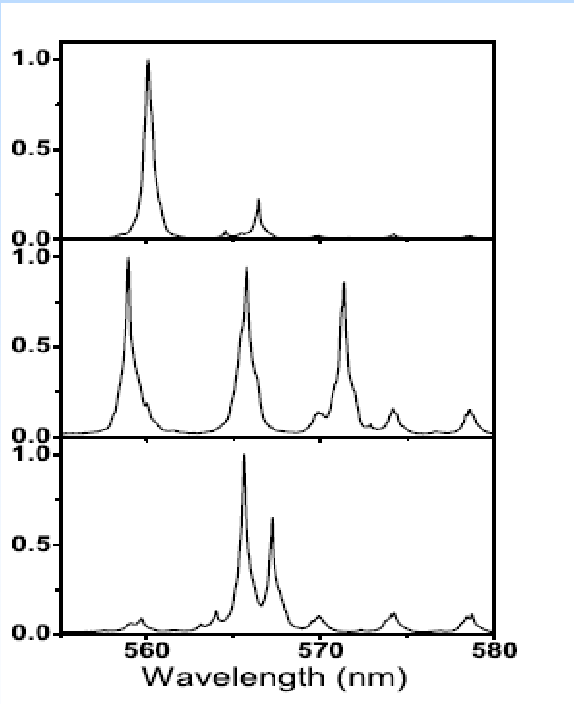


Distribution of lasing wavelengths

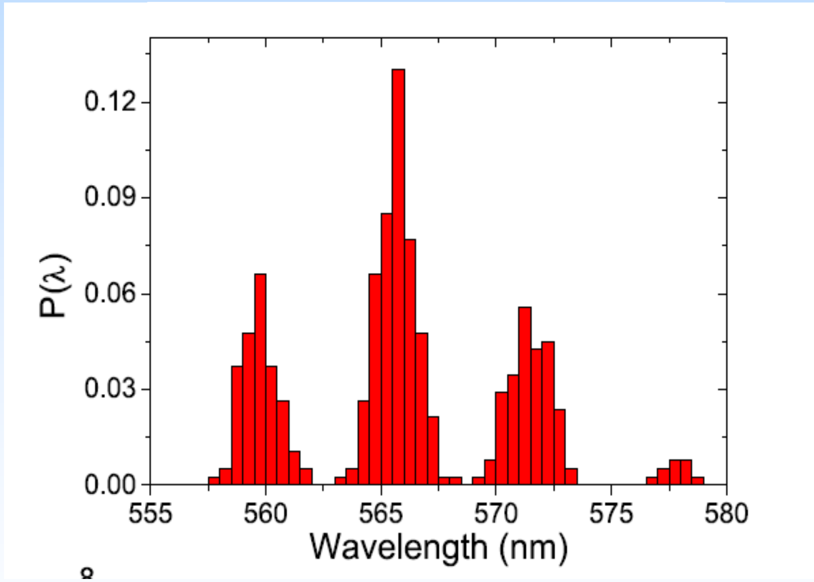
AKT, BC, RU and SM, Opt. Lett. (2012)

Spectra: Monodisperse aperiodic





Spectra



Distribution of lasing wavelengths

AKT, BC, RU and SM, Opt. Exp. (2012)

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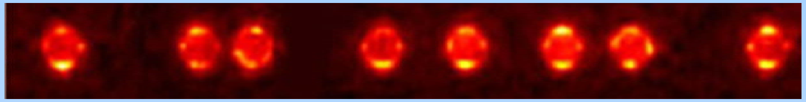
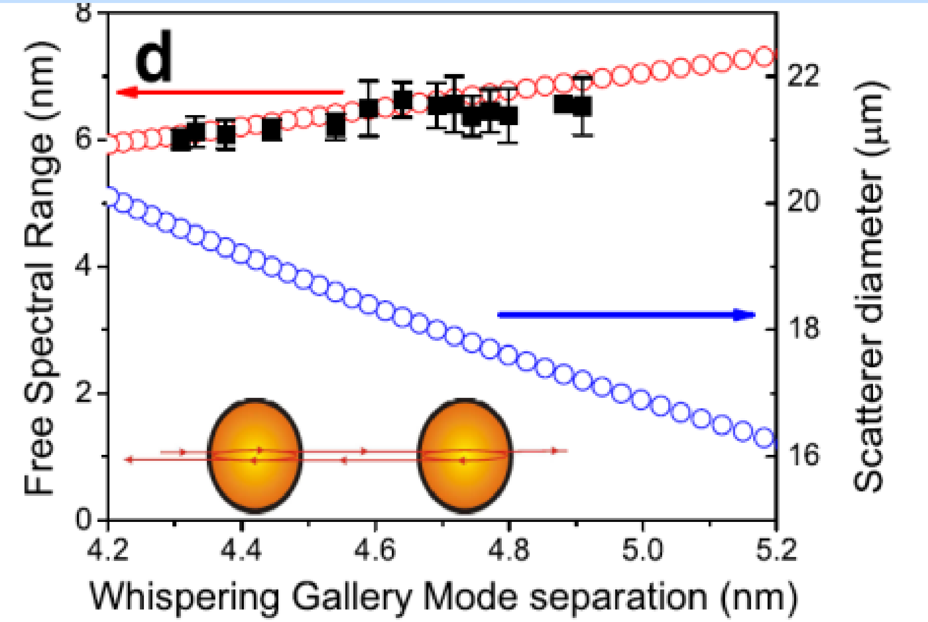
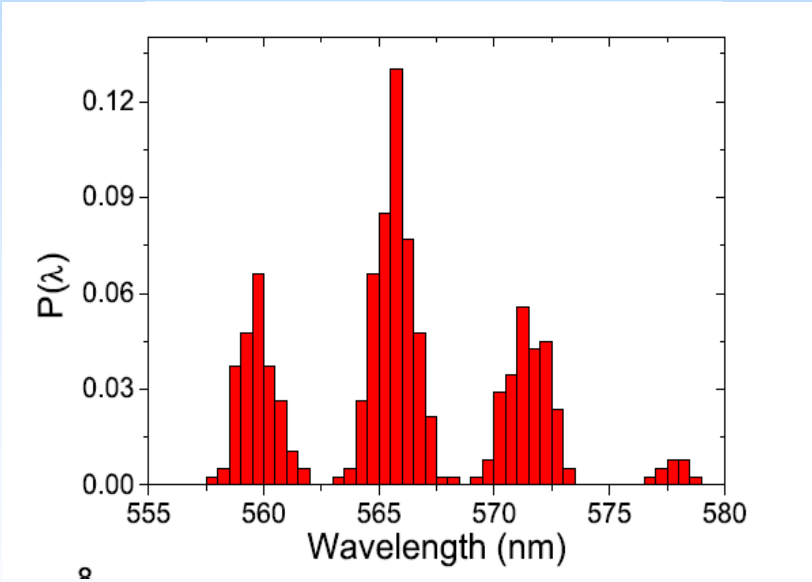
Spectra: Monodisperse aperiodic





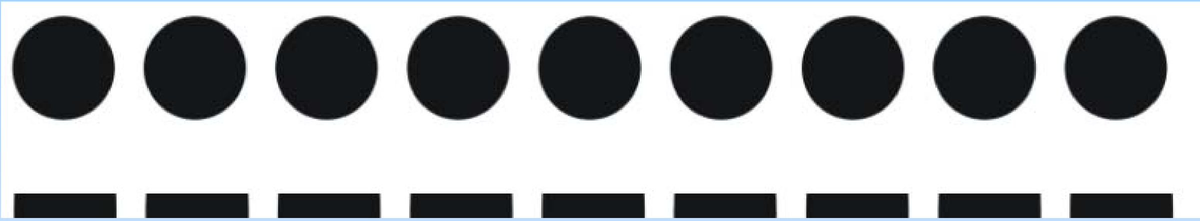
Bunch separation

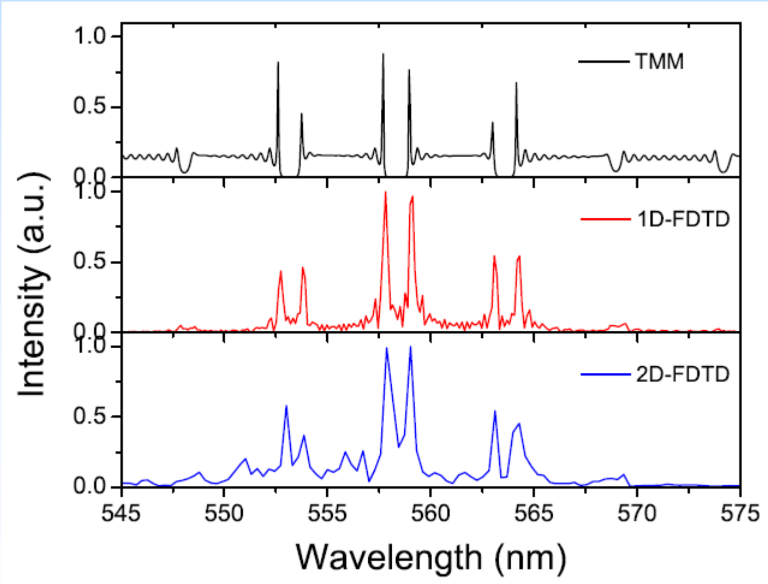


Distribution of lasing wavelengths

AKT, BC, RU and SM, Opt. Exp. (2012)

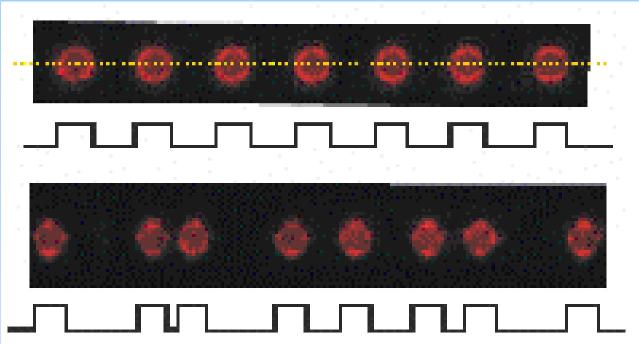
Calculations: Transfer Matrix Method

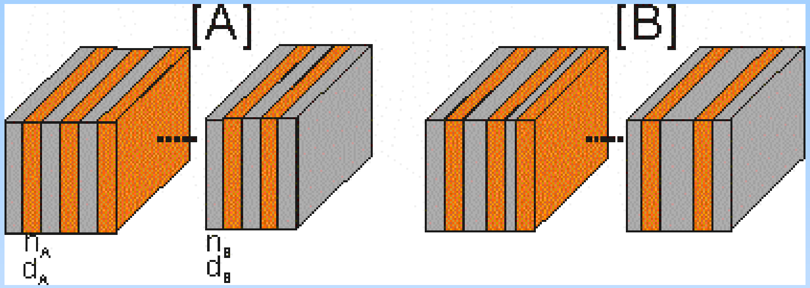


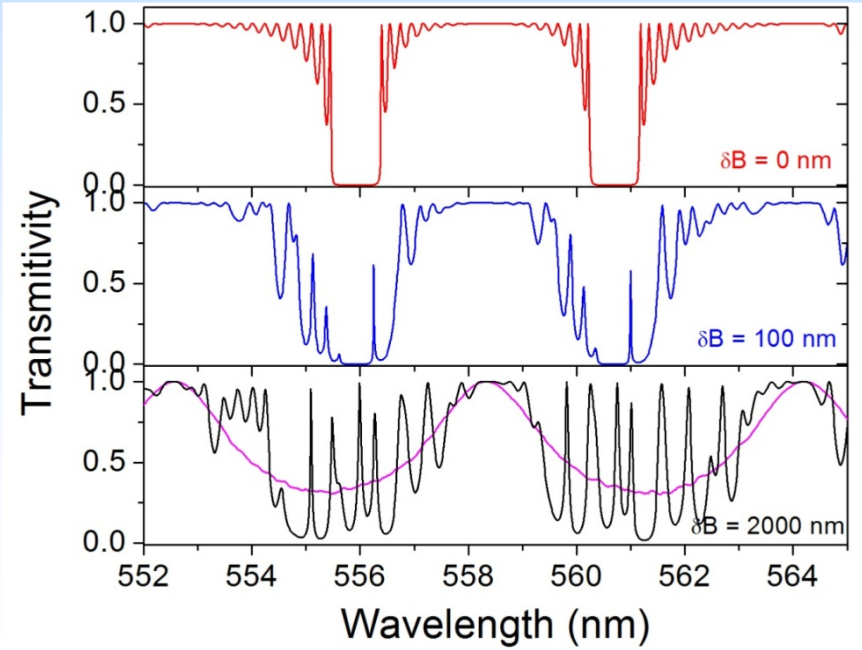


1D approximation

Calculations: Transfer Matrix Method







Periodic

On

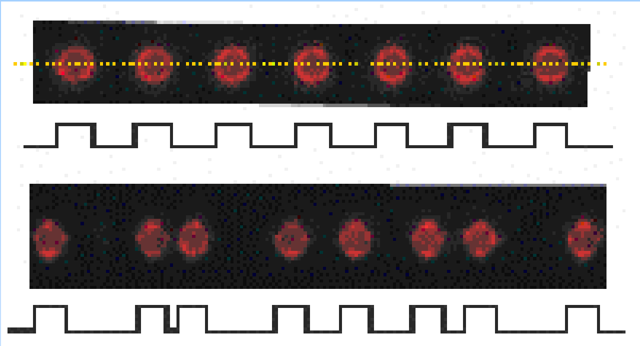
Average

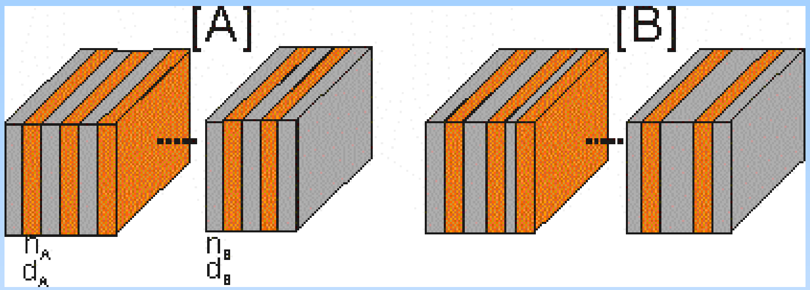
Random

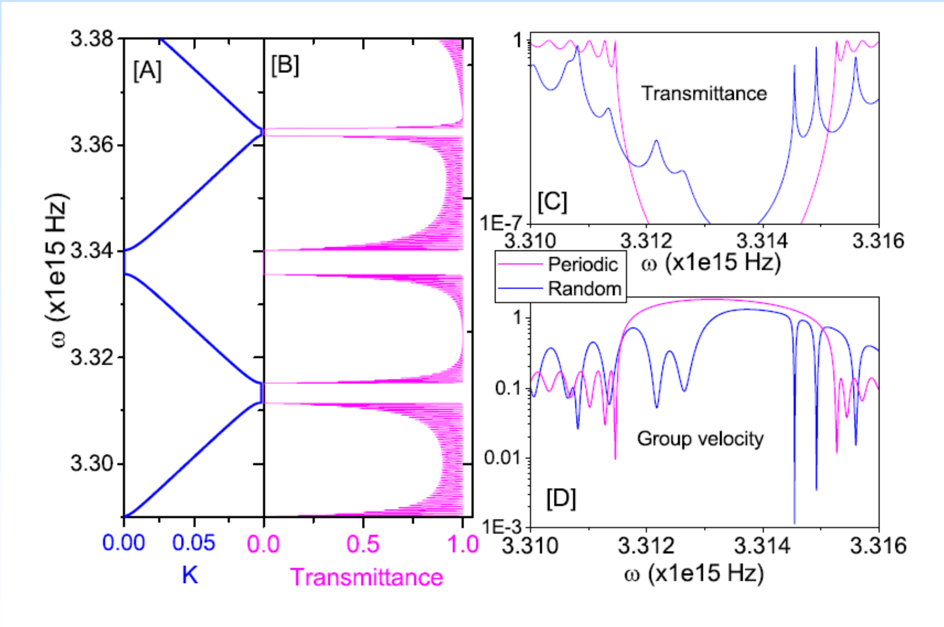
System

(PARS)

Calculations: Transfer Matrix Method







Periodic

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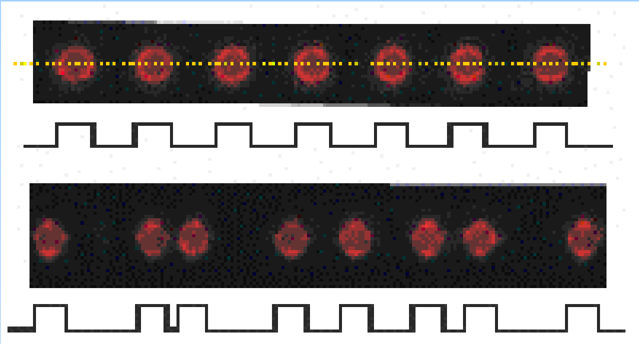
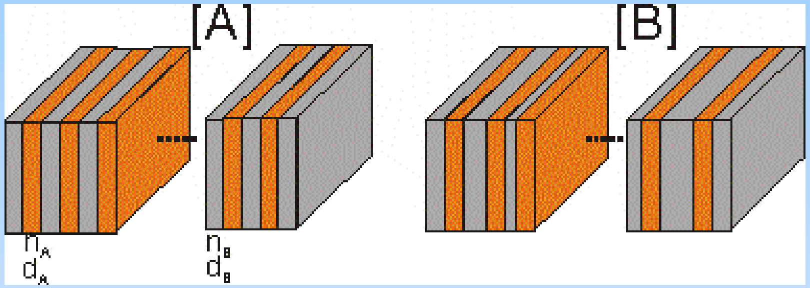
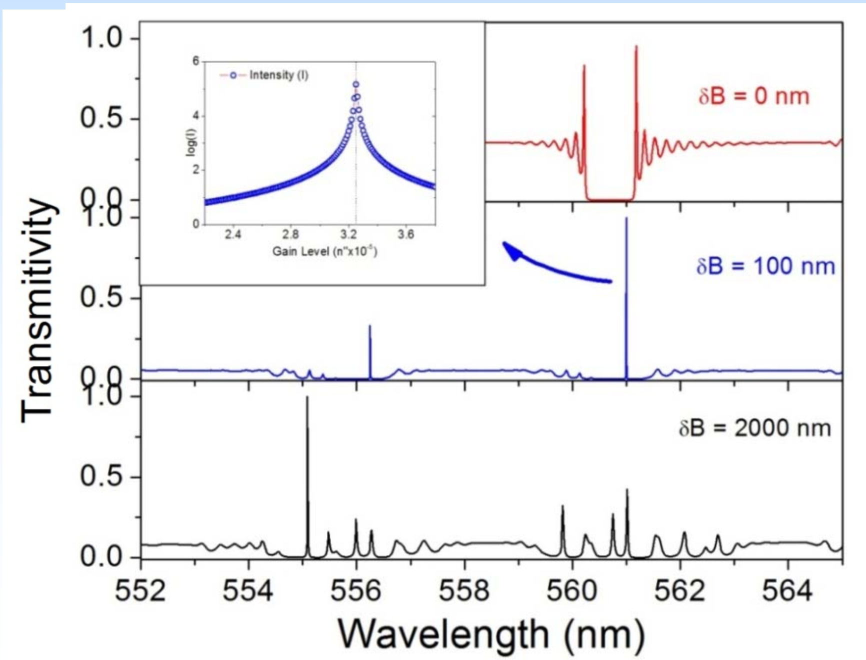
Calculations: Transfer Matrix Method







Periodic

On

Average

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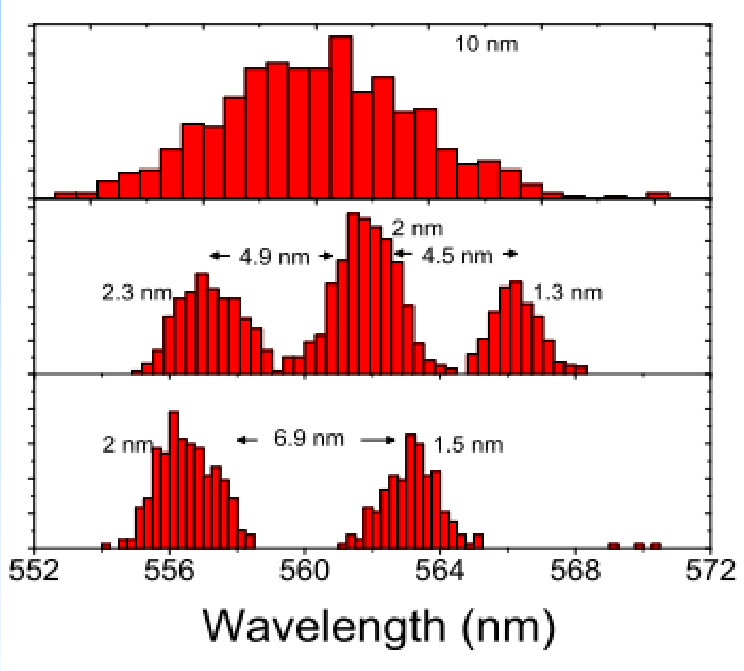
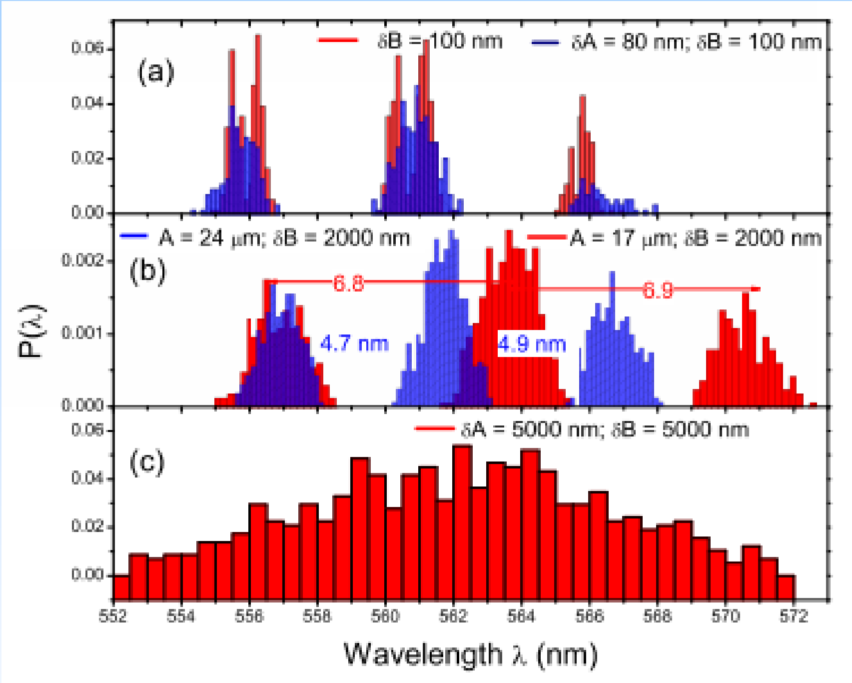
System

(PARS)

Calculations and Experiments



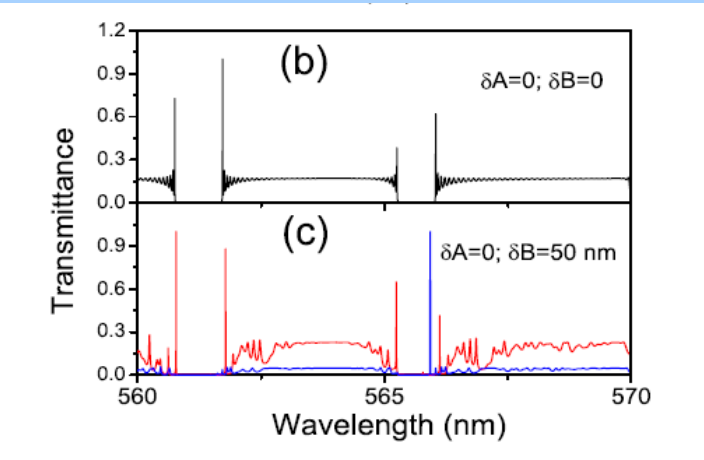


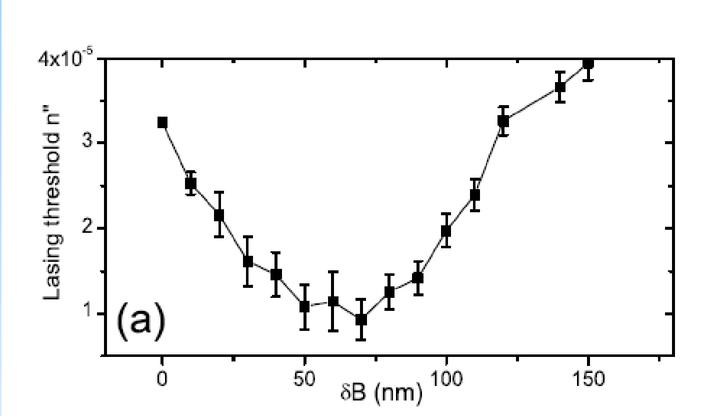
Gap-state random lasing from the PARS structure.

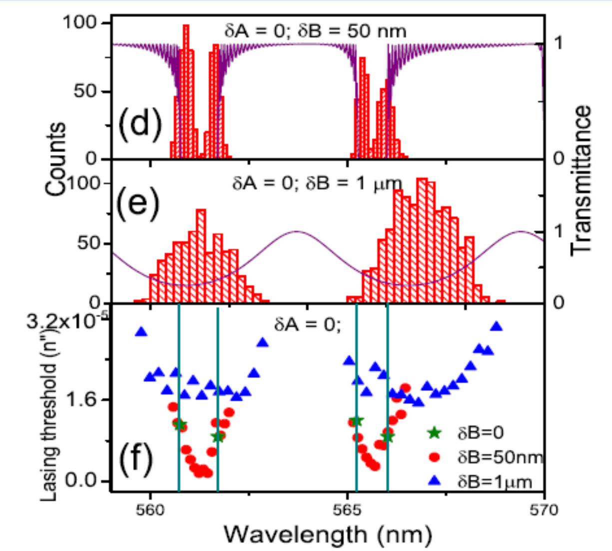
AKT and SM, PRL (2013)

Gap States and Bandedge states

Surprise: Quality factor of gap modes can be higher than the band-edge modes.





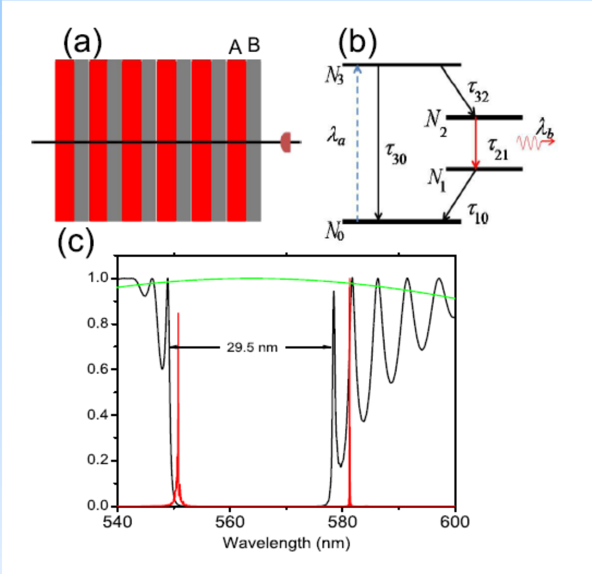


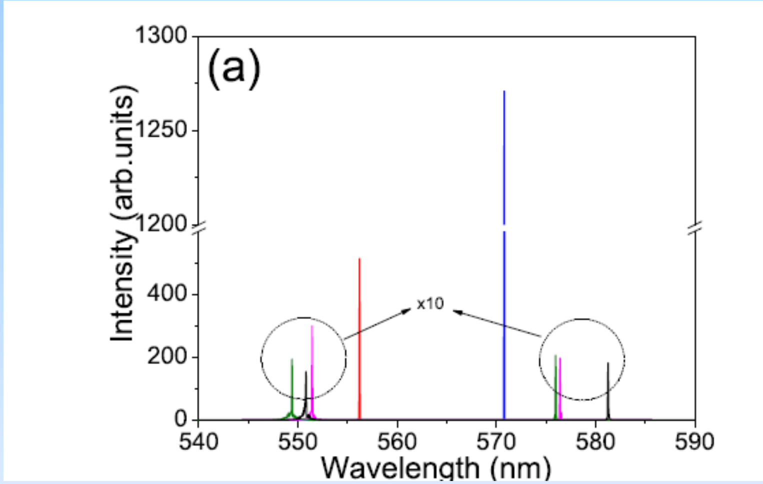
Threshold of lasing does lower in the presence of randomness.

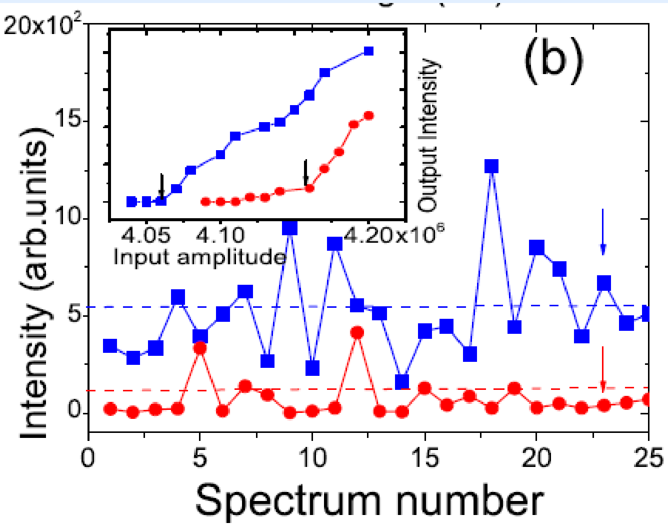
Hard to measure threshold, what about emission intensity?

KSA, RK and SM, PRA (2015)

Gap States and Bandedge states





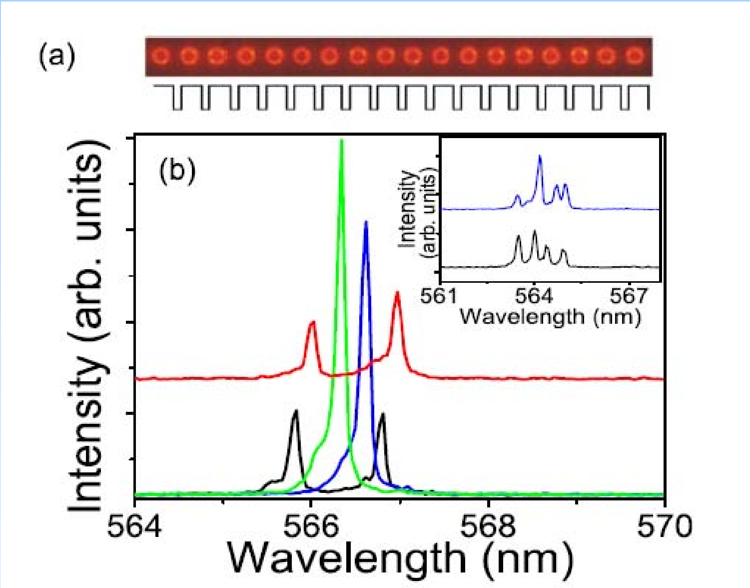


Lower emission intensity for the band-edges, while higher intensity in the random modes.

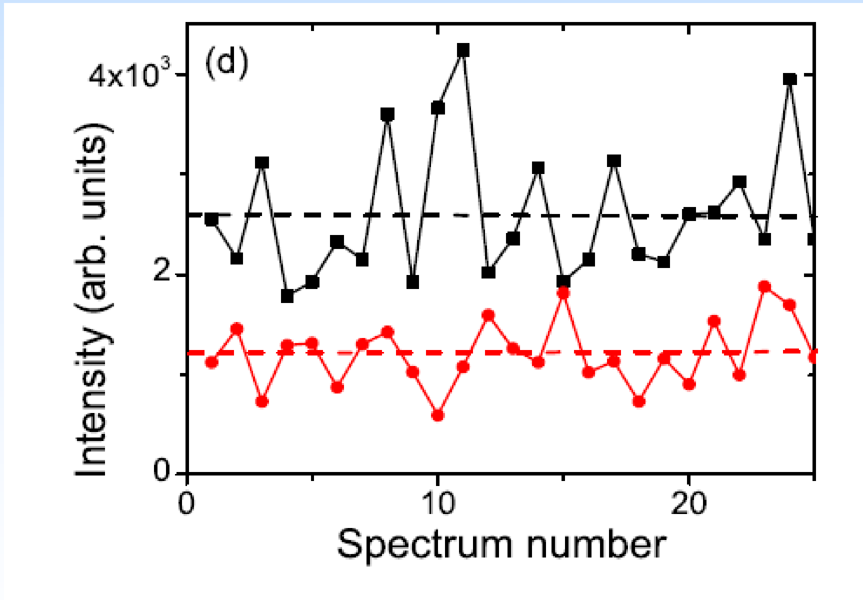
Statistically consistent, over multiple configurations.

KSA, RK and SM, PRA (2015)

Gap States and Bandedge states



Experimental confirmation on the microdroplet array.



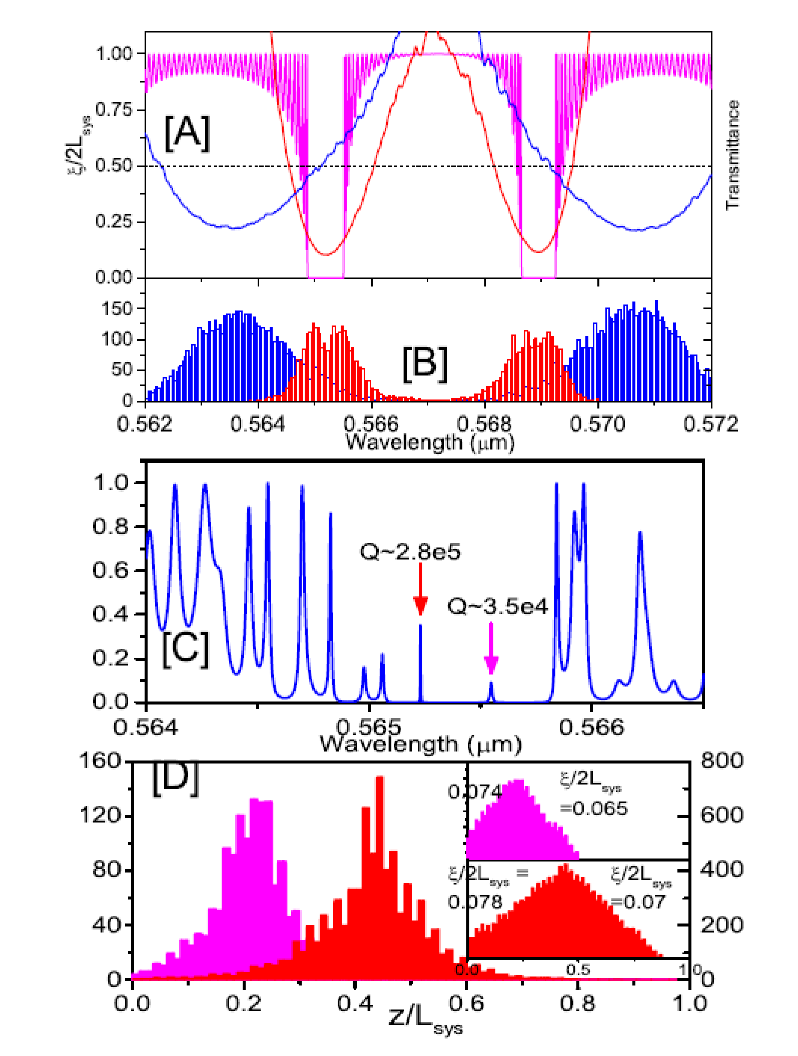
Configuration-independent result.

KSA, RK and SM, PRA (2015)

Gap States and Localization

Passive PARS

$$\xi = -\frac{L}{\langle \ln T \rangle}$$

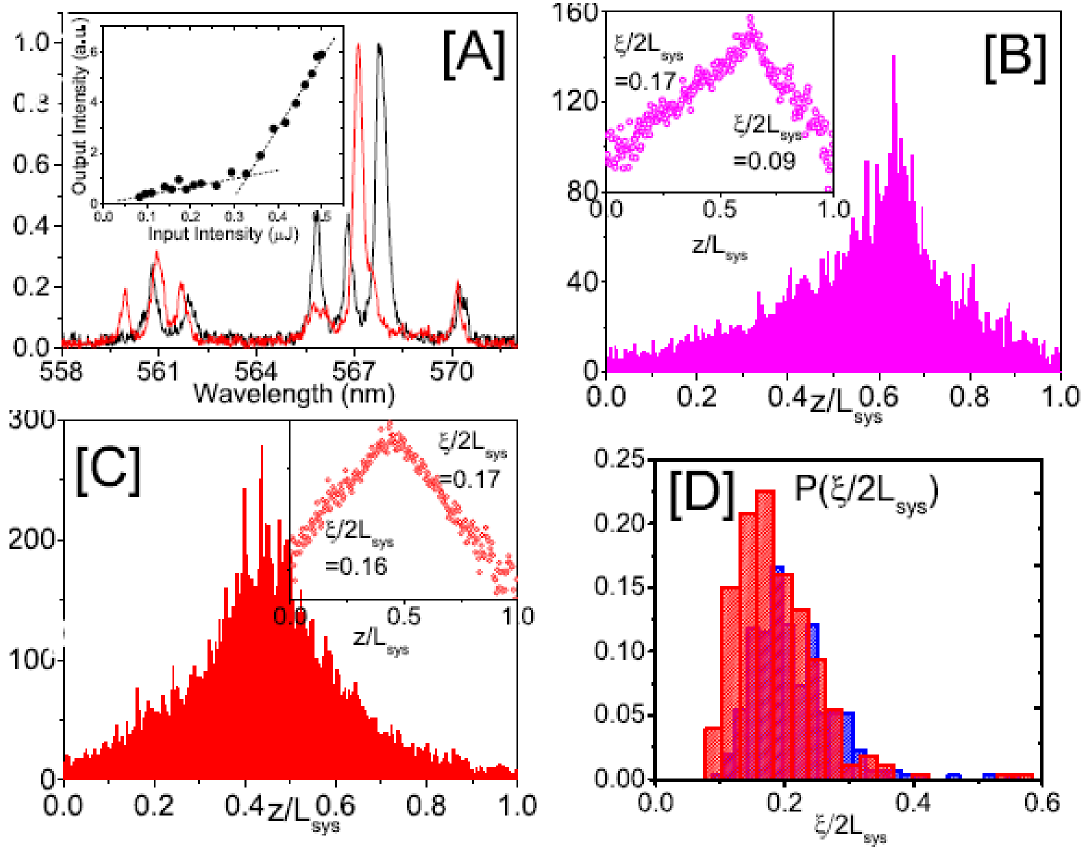


Anderson Localization Laser!

System Size: 40 unit cells

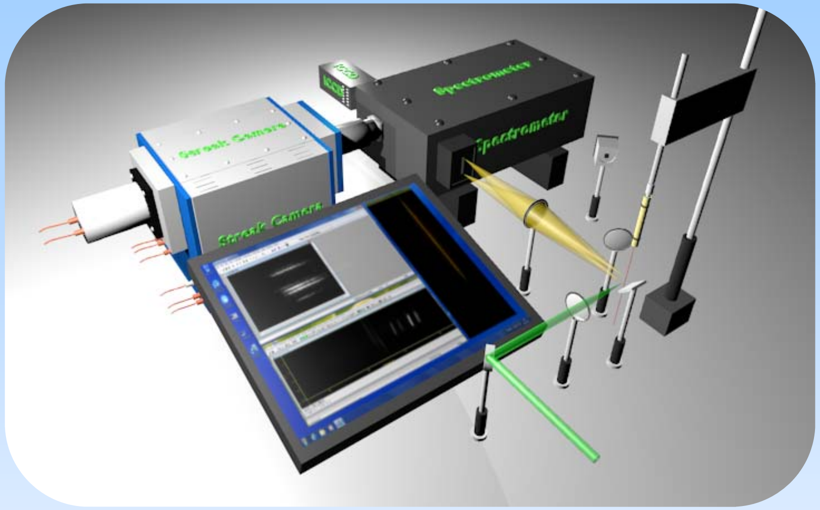
Loss length (in passive system): 46 unit cells

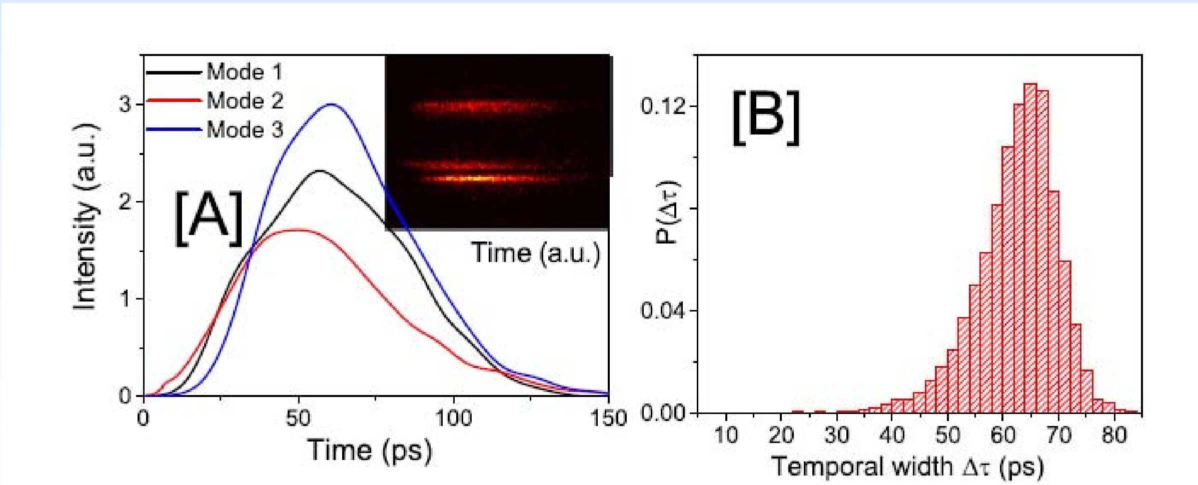
Localization length: 11 unit cells



Direct observation of random lasing over Anderson states.

Pulsewidths of Anderson Localized Modes





Left skewed distribution of pulsewidths.

Pulsewidths of Anderson Localized Modes

Modified coupled emitter-cavity rate equations:

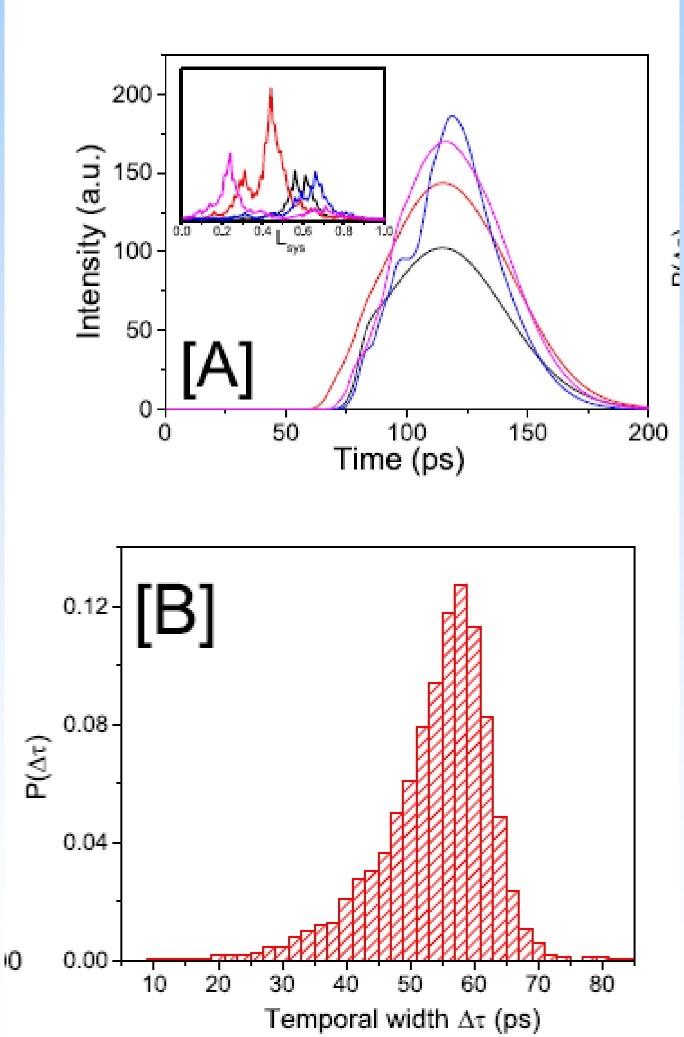
$$\frac{dn}{dt} = KN_2(n+1) - \gamma_c n$$

$$N_{PhotonOut} = \gamma_c n$$

$$\frac{dN_2}{dt} = R_p - KN_2 n - \gamma_2 N_2$$

$$K = \frac{3}{4\pi^2}\frac{\omega_a \gamma_{rad}\lambda^3}{\Delta\omega_a V_c}$$

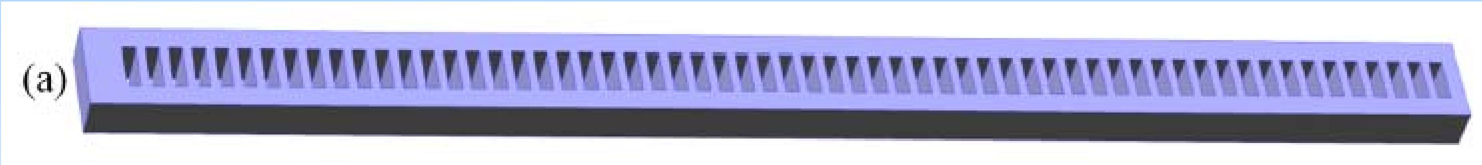
$$V_c = \frac{\int \epsilon E^2 dx^3}{max(\epsilon E^2)}$$

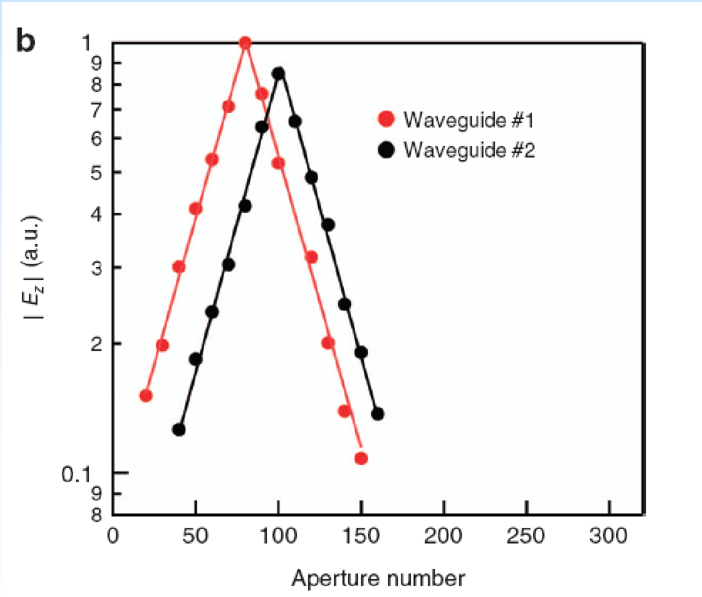


Under Review. RK, MB, KSA and SM, PRA Rapid Comm.

Plasmonic localization at terahertz wavelengths

Anderson Localization of plasmons in coupled resonators.





SP, BG, SM, and AN, Light :Science and App (Nature) (2017)

Part II - Conclusions

Localization-induced lasing from a PARS system (resonator arrays).
Probability distribution of pulsewidths has been measured.

Outlook

Much more to study with PARS systems, Intensity fluctuations of localized modes, correlations in localized modes…

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Featured in Virtual Journal of Biomedical Optics, (2012).

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Nano-optics & Mesoscopic Optics Laboratory

ICTS Colloquium, ICTS, October 2017

Anderson localization and Levy sums in random lasers

Thank you for your attention!

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