Moisture gradients and low-frequency variability in the tropics

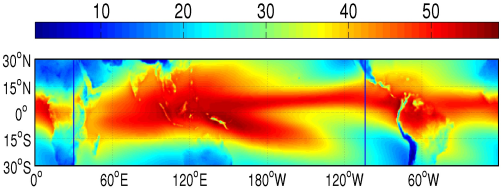
Jai Sukhatme

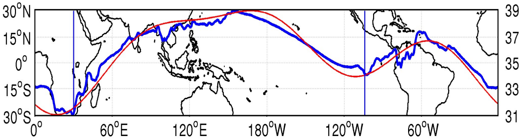
January 23, 2013

- Data.
  - The horizontal structure of tropical precipitable water (PW).
  - PW gradients and moist convective activity: a correspondence.
- Theory.
  - First baroclinic mode with water vapour.
  - Horizontal gradients of the saturation profile.
  - Non-rotating case.
  - Equatorial β-plane.

- Modern Era Retrospective-analysis for Research and Applications (MERRA) data.
- $\frac{1}{2}^\circ \times \frac{2}{3}^\circ$ lat-long resolution.
- Data for precipitable water (PW) spanning 2001-2010.

Mean PW


10
20
30
40
50
30°N
15°N
0°
15°S
30°S
0°
60°E
120°E
180°W
120°W
60°W


30°N
15°N
0°
15°S
30°S
0°
60°E
120°E
180°W
120°W
60°W
39
37
35
33
31

Mean PW in the two hemispheres

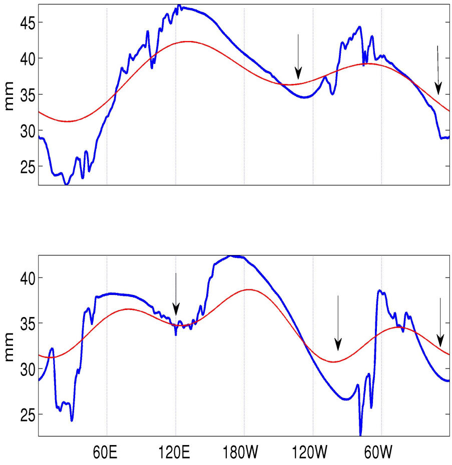


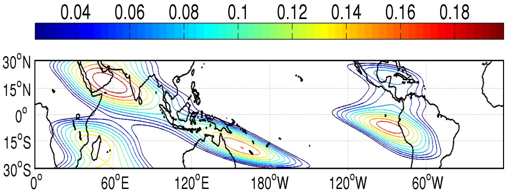
Figure: NH (upper panel), SH (lower panel).

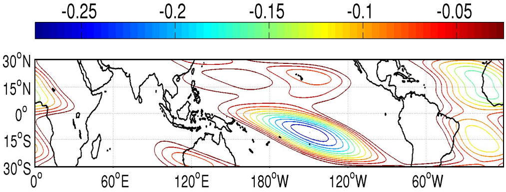
Tropical PW envelopes

- Subtropical anticyclones. Systematic depletion of PW $\Rightarrow$ dominant wavenumber two (three) profile of PW in the NH (SH) respectively (Dry Import + Downward motion).
- Structure accentuated by the regional monsoon systems.
- The two factors are not completely independent, i.e. they act in concert to shape the mean tropical PW.

Longitudinal PW gradient

Positive gradients (upper panel), Negative gradients (lower panel)

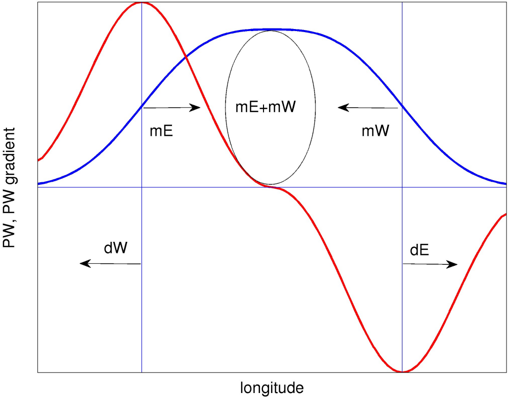




An observation

- Rough correspondence between the longitudinal gradient of mean tropical precipitable water (PW), and the geographical regions of genesis, and convective activity, of large-scale tropical systems.
- Regions with positive PW gradients, i.e. they have larger PW to the east, are the very zones that are implicated in the formation, and show high levels of convective activity, of eastward moving systems.
- Regions with negative gradients are the very regions where genesis, and maxima in variance, of westward moving systems.

Schematic



Waves & PW gradients

- Why?
- In a sense, a horizontal form of conditional instability.
- Competition between planetary vorticity gradient and PW (or moisture) gradient.
- Sobel, Nilsson and Polvani, JAS, 2001.

Single baroclinic mode

- Closely follow Gill, GAFD, 1982, Frierson, Majda & Paulius, CMS, 2004, Majda & Stechmann, PNAS, 2009.

$$\frac{\partial u}{\partial t} = \frac{gH_0}{\theta_0}\frac{\partial \theta'}{\partial x},$$
$$\frac{\partial v}{\partial t} = \frac{gH_0}{\theta_0}\frac{\partial \theta'}{\partial y},$$
$$\frac{\partial \theta'}{\partial t} - \frac{\theta_0 N^2 H_0}{g}\nabla.\mathbf{u} = \mathcal{S}.$$

Here H_0, θ_0 are the mean depth and a representative value for the potential temperature of the lower layer. $\mathbf{u} = (u, v)$ is the horizontal velocity field and N is the buoyancy frequency. θ' denotes the non-dimensional potential temperature perturbation from a linear background.

- Prescription for $\mathcal{S}$.

$$\mathcal{S} \propto \frac{(q - q_s)}{\tau_c} H(q - q_s).$$

$q(\vec{x}, t)$, in mm, is the total column water vapour or approximately the PW, while $q_s(\vec{x})$ is the prescribed background saturation profile of PW. Also, τ_c is the timescale of condensation and $H(\cdot)$ represents the Heaviside step function. [B. Khouider's talk, S. Stechmann's talk, Dias & Paulius, JAS, 2009]

- Structure of q_s.

$$q_s = Q_0 - \alpha_q x - \beta_q y.$$

α_q and β_q are the linear gradients of q_s in the zonal and meridional directions. $[\alpha_q \pm, \beta_q +]$

The non-dimensional system

$$\frac{\partial u}{\partial t} = \frac{\partial \theta'}{\partial x},$$
$$\frac{\partial v}{\partial t} = \frac{\partial \theta'}{\partial y},$$
$$\frac{\partial \theta'}{\partial t} - \nabla.\mathbf{u} = \mathcal{R}\frac{\tau_a}{\tau_c} q' H(q'),$$
$$\frac{\partial q'}{\partial t} - \frac{u}{R_{m_\alpha}} - \frac{v}{R_{m_\beta}} - \nabla.\mathbf{u} = -\frac{\tau_a}{\tau_c} q' H(q').$$

Typically, $\mathcal{R} < 1$ (Gill 1982, Frierson, Majda & Paulius 2004). We insist on $\mathcal{R} < 1$ to eliminate conditionally unstable modes. Further, $R_{m_{\alpha,\beta}} = Q_0/(\beta_q, \alpha_q)L$. $Q_0 \approx 40$ mm. $\therefore 1 \leq R_{m_{\alpha,\beta}} \leq 4$ is reasonable. Further, $R_{m_{\alpha,\beta}} \uparrow \ \Rightarrow$ PW gradient $\downarrow$.

Non-rotating modes

- Rapid condensation ($\tau_c \to 0$) + No horizontal gradients ($R_{m_{\alpha,\beta}} \to \infty$) $\Rightarrow$ "Usual" slow moist gravity waves.
- Case I : Only a meridional gradient (i.e. $R_{m_\alpha} = \infty$).
- Case II : Only a zonal gradient (i.e. $R_{m_\beta} = \infty$).

Case I: Meridional gradient

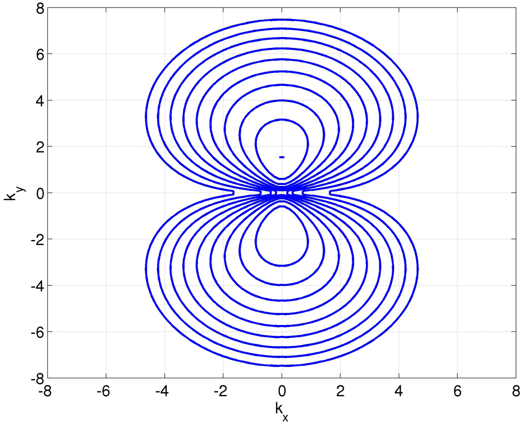


Figure: Contours of $\Im(\sigma) > 0$. $\mathcal{R} = 0.8, R_{m_\beta} = 4$.

Case I: Meridional gradient

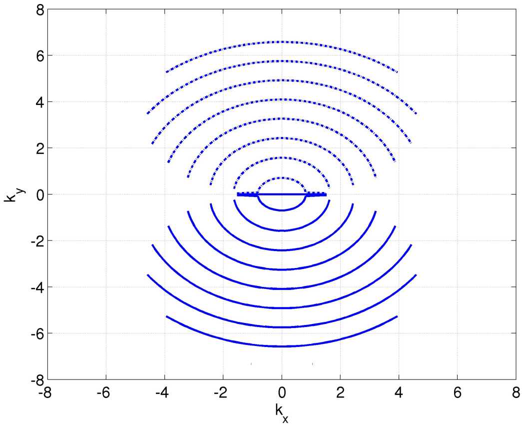


Figure: $\Re(\sigma)$ for $\Im(\sigma) > 0$. Solid (dashed) curves are for positive (negative) contours. $\mathcal{R} = 0.8$, $R_{m_\beta} = 4$.

Case I: Meridional gradient

- Large-scale unstable gravity modes. Localized around $k_x = 0$.
- Growth rates hint at an intraseasonal timescale.
- Robust with respect to reasonable $\mathcal{R}, R_{m_\beta}$ and outside rapid condensation, i.e. when $\epsilon = \frac{\tau_a}{\tau_c} \nrightarrow 0$.
- Symmetry! Unstable modes have east-west symmetry. [B. Khouider's talk, Majda, Stechmann and Khouider, PNAS, 2007]

Case II: Zonal gradient

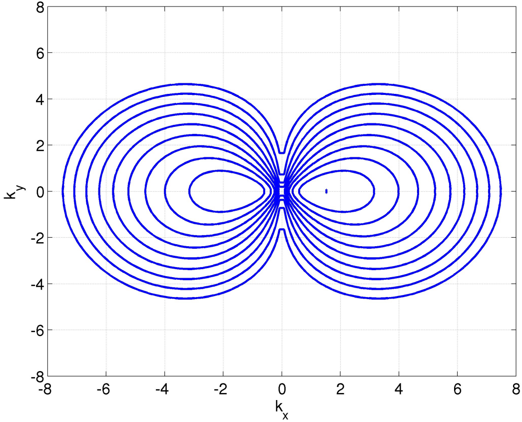


Figure: Contours of $\Im(\sigma) > 0$. $\mathcal{R} = 0.8, R_{m_\alpha} = 4$.

Case II: Zonal gradient

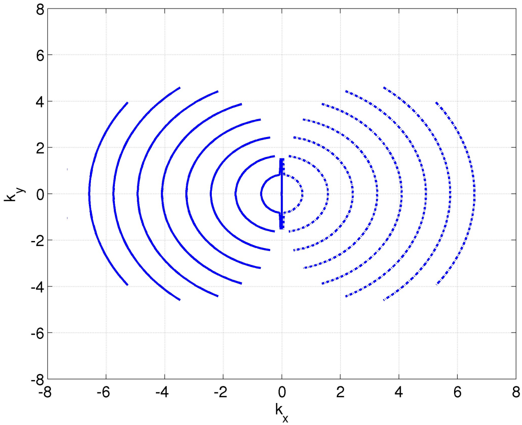


Figure: $\Re(\sigma)$ for $\Im(\sigma) > 0$. Solid (dashed) curves are for positive (negative) contours. $\mathcal{R} = 0.8, R_{m_\alpha} = 4$.

Case II: Zonal gradient

- Large-scale unstable gravity modes. Localized around $k_y = 0$.
- Robust with respect to reasonable $\mathcal{R}, R_{m_\alpha}$ and outside rapid condensation.
- Broken Symmetry! Unstable modes do not have east-west symmetry.
- As background PW increases from east to west (i.e. $\alpha+$), only westward propagating unstable modes (and vice versa). Schematic!

Equatorial β-plane

- Augment momentum equations with $-\beta y v$ and $\beta y u$ respectively.
- New non-dimensionalization.
- In the limit of rapid condensation, after assiduous manipulation...

$$\partial_t^2[v_{tt} - (1-\mathcal{R})(v_{yy} + v_{xx}) - \frac{\mathcal{R}}{R_{m_\beta}} v_y - \frac{\mathcal{R}}{R_{m_\alpha}} v_x + y^2 v]$$

$$+\partial_t[-\frac{\mathcal{R}}{R_{m_\alpha}} v - \frac{y\mathcal{R}}{R_{m_\alpha}} v_y - (1-\mathcal{R}) v_x + \frac{y\mathcal{R}}{R_{m_\beta}} v_x] = 0.$$

- Setting $v(x,y,t) = v(y) \exp \mathrm{i}[kx - \sigma t] \Rightarrow$ polynomial eigenvalue problem.

Equatorial β-plane

- $\mathcal{R}, R_{m_\beta}, R_{m_\alpha}$: vertical, meridional and zonal moisture gradients.
- $\mathcal{R} = 0, R_{m_\beta}, R_{m_\alpha} = \infty$. The "dry" equatorial β-plane SWE.
- $\mathcal{R} \neq 0, R_{m_\beta}, R_{m_\alpha} = \infty \Rightarrow$ "traditional" moist SWE. Slow Kelvin, Rossby, mixed Rossby-gravity and gravity waves.
- Case I : $\mathcal{R} \neq 0, R_{m_\alpha} = \infty$: Only meridional gradient.
- Case II : $\mathcal{R} \neq 0, R_{m_\beta} = \infty$: Only zonal gradient.

Equatorial β-plane: Case I

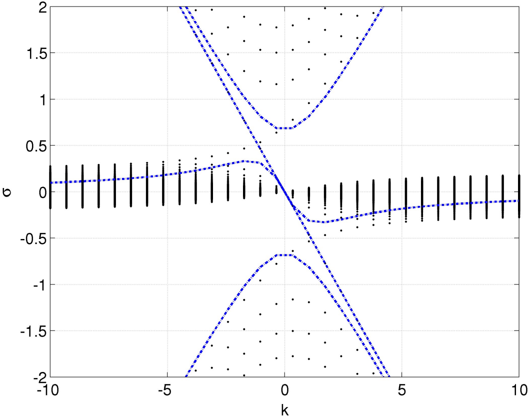


Figure: $\sigma(k)$ vs k for $\mathcal{R} = 0.8, R_{m_\beta} = 2, R_{m_\alpha} = \infty$. The superposed dashed curves are analytical estimates of the dispersion curves in the zero gradient case with $\mathcal{R} = 0.8$.

Equatorial β-plane: Case I

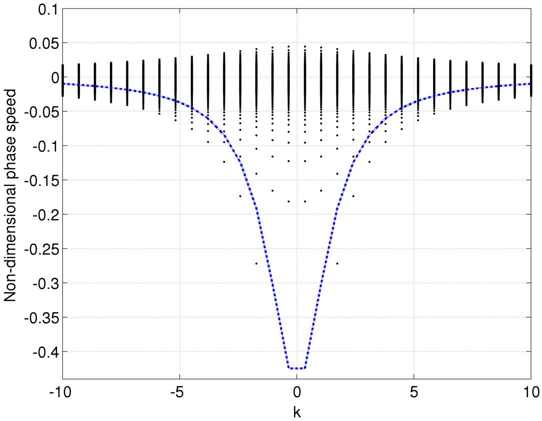


Figure: Phase speed vs k for $\mathcal{R} = 0.8, R_{m_\beta} = 2, R_{m_\alpha} = \infty$. The superposed dashed curves are analytical estimates of the moist Rossby wave phase speed in the zero gradient case with $\mathcal{R} = 0.8$.

Equatorial β-plane: Case I

- Only neutral modes. Rotation stabilizes the problem.
- High frequency response is similar to the "traditional" moist system.
- Main difference is in the low frequency modes. In addition to Rossby wave like westward moving modes, a new eastward propagating system.
- Further, the east-west response is asymmetric! In particular, east $\rightarrow$ slow, west $\rightarrow$ (relatively) fast.

Equatorial β-plane: Case I (largest eastward mode)

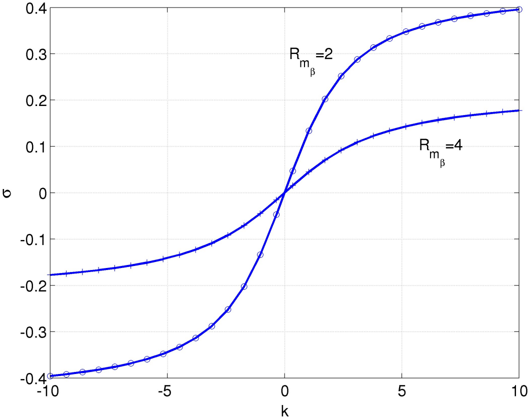


Figure: $\sigma(k)$ vs k for the first or largest scale eastward moving mode.

Equatorial β-plane: Case I (largest eastward mode)

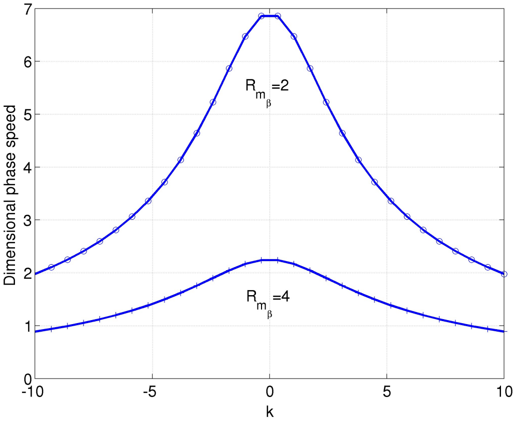


Figure: Phase speed vs k for the first or largest scale eastward moving mode.

Equatorial β-plane: Case I (eastward mode)

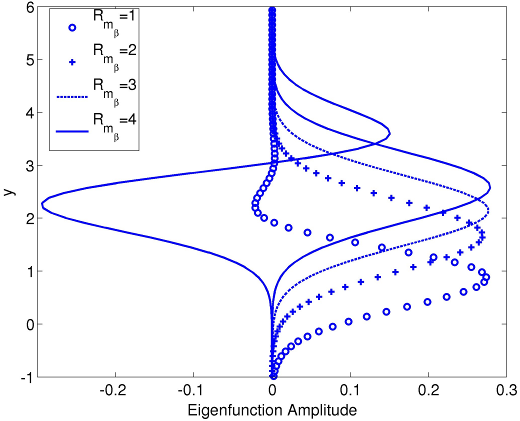


Figure: The first (symmetric) and second (antisymmetric) eigenfunctions of the largest eastward propagating mode for $R_{m_\beta} = 4$ (solid). The first eigenfunctions of the eastward propagating modes for $R_{m_\beta} = 3, 2, 1$ (symbols).

Equatorial β-plane: Case I (Summary)

- Eastward mode: New feature as compared to the "traditional system", slow phase speed ($\approx 2 - 7$ m/s), first eigenfunction is Gaussian, second eigenfunction gives a quadrupole horizontal field, phase speed levels off for moderate wavenumbers.
- Westward mode: Akin to the "traditional" moist Rossby wave, slightly faster, quasi-biweekly timescale.
- Aquaplanet results.
- Larger gradient (smaller R_{m_β}) $\Rightarrow$ localized near equator and faster. [varying ITCZ width experiments by Dias & Paulius, JAS, 2009]

Equatorial β-plane: Case II (West to East increasing gradient)

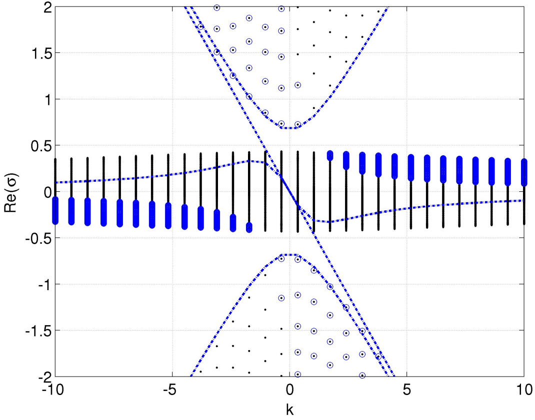


Figure: $\mathcal{R} = 0.8, R_{m_\alpha} = -2, R_{m_\beta} = \infty$. The open circles and thick segments denote unstable modes.

Equatorial β-plane: Case II (East to West increasing gradient)

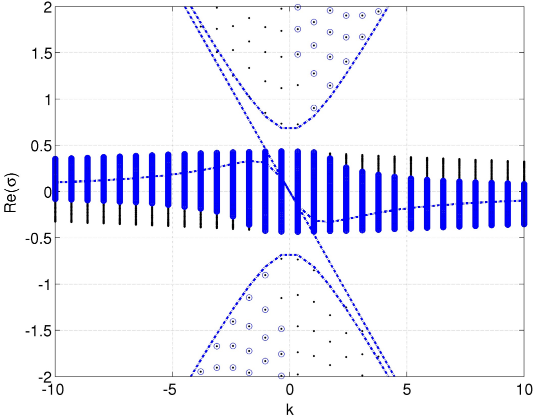


Figure: $\mathcal{R} = 0.8, R_{m_\alpha} = 2, R_{m_\beta} = \infty$. The open circles and thick segments denote unstable modes.

Equatorial β-plane: Case II (Largest unstable modes)

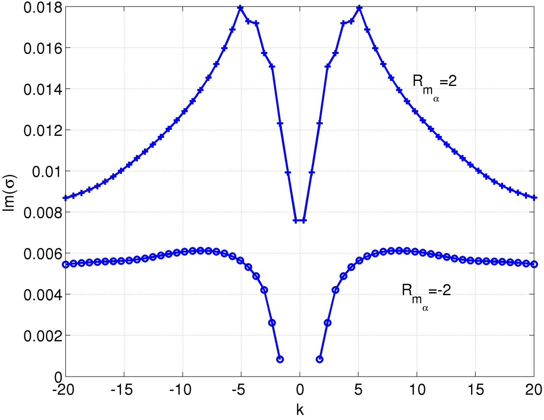


Figure: $\mathcal{R} = 0.8$, $R_{m_\alpha} = \pm 2$, $R_{m_\beta} = \infty$.

Equatorial β-plane: Case II (Summary)

- West to east increasing PW. Yields unstable eastward moving modes. Similar space/time scales as the neutral eastward mode in Case I.
- East to west increasing PW. Yields unstable westward moving large-scale modes. Also, a presence of unstable eastward propagating systems. $\therefore$ not exclusive.
- Maximum growth rate of eastward unstable modes at synoptic scales. "Muscle" & "Skeleton" (Majda & Stechmann, PNAS, 2009; Moncrieff, JAS, 2004).
- Maximum growth rate of westward unstable modes at $\approx$ wavenumber 5. As observed for moist equatorial Rossby waves (Wheeler, Kiladis & Webster, JAS, 2000).

Conclusions

- Observational correspondence between longitudinal gradient of PW and low-frequency moist convective activity.
- A schematic of a PW envelope with eastward and westward moving moist systems.
- Single baroclinic mode model with horizontal moisture gradients (simple generalization of the "traditional" moist setup).
- Non-rotating system. Symmetry and broken symmetry of low-frequency unstable modes.
- Equatorial β-plane. Neutral and/or unstable large-scale, low-frequency, asymmetric, eastward (slow) and westward (relatively fast) propagating modes.

For details...

- J. Sukhatme "Longitudinal Localization of Tropical Intraseasonal Variability" Quarterly Journal of the Royal Meteorological Society, DOI: 10.1002/qj.1984, 2012.
- J. Sukhatme "Large-Scale Moist Modes in the Tropics" submitted to the Quarterly Journal of the Royal Meteorological Society, 2012.