Strings, Black Holes and Quantum Information

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Thesis Synopsis Seminar

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Part 1

Breakdown of String Perturbation Theory & Implications for Black Hole Information Paradox

References:

- *Breakdown of String Perturbation Theory for Many External Particles*
  Phys. Rev. Lett. 118 (2017); arXiv: 1611.08003 with Suvrat Raju
- *Loss of Locality in Gravitational Correlators with a Large Number of Insertions*
  Phys. Rev. D96 (2017); arXiv: 1706.07424 with Suvrat Raju

Locality in Quantum Gravity

- Notions of locality and causality in any theory of quantum gravity are approximate, generally expected to breakdown close to the Planck scale where quantum fluctuations of the spacetime metric become significant.

- In a local QFT without gravity, locality and causality require

$$[\phi(t_i, x_i), \phi(t_j, x_j)] = 0 \qquad \text{for} \qquad (x_i - x_j)^2 > 0$$

- In gravity, non-local effects can become important over macroscopic scales for a certain class of observables, such as very high point correlation functions.

$$\implies \langle \phi(t_1, x_1)\phi(t_2, x_2) \cdots [\phi(t_i, x_i), \phi(t_j, x_j)] \cdots \phi(t_n, x_n) \rangle \neq 0$$

even for spacelike separations $|x_i - x_j| \gg l_{pl}$ for sufficiently large n.

- The black hole information paradox provides a setting where such large scale non-local effects become important.

Black Holes & Hawking Radiation

- Considering the propagation of quantum field in the background of a black hole formed from gravitational collapse, Hawking derived the remarkable result that black holes can evaporate by emitting thermal radiation at a temperature

[Hawking, 1975]

$$T = \frac{\kappa}{2\pi}$$

κ : surface gravity

- As the black hole evaporates it loses mass at a rate

$$\frac{dM}{dt} = -\sigma T^4 A \implies \frac{dM}{dt} \propto \frac{1}{M^2}$$

using $T \propto 1/M, \quad A \propto M^2$ in d=4 dimensions

- The black hole thus has a finite lifetime $t_{evap} \propto M^3$
- For a solar mass black hole $t_{evap} \sim 10^{67}$ years.

The Information Paradox

- Hawking calculation reveals that the outgoing radiation is only characterised by a few macroscopic parameters and is independent of the details of the matter that collapsed to produce the black hole.

- If the initial matter were in a pure quantum state and the black hole completely evaporates, the outgoing Hawking quanta finally ends up being in a mixed state.

- This situation is inconsistent with the prescribed rules of quantum mechanics which does not allow unitary evolution from a pure state to a mixed state for an isolated quantum system.

- This is usually referred to as the information loss paradox.

Resolution through small corrections

- It is in principle possible to avoid this paradox by considering non-perturbatively small corrections to Hawking's semiclassical calculation.

- In a large quantum system, pure states can mimic thermal states to great deal of accuracy for a large class of coarse-grained observables.

$$\mathrm{Tr}(\rho_{pure} A) = \frac{1}{\mathcal{Z}} \mathrm{Tr}(e^{-\beta H} A) + \mathcal{O}(e^{-S})$$

$$\rho_{pure} = \frac{1}{\mathcal{Z}} \rho_{thermal} + \mathcal{O}(e^{-S}) \rho_{corrections}$$

- The departure from purity can be detected if we measure more complicated fine grained observables such as $\mathcal{O}(S)$-point correlation functions.

- Information about the initial state can thus be imprinted in subtle correlations among a very large number of Hawking quanta.

Modern Information Paradoxes

- The formulation of the information loss paradox has been sharpened over the years since Hawking's result. Modern versions of the paradox make crucial reference to the interior of the black hole.

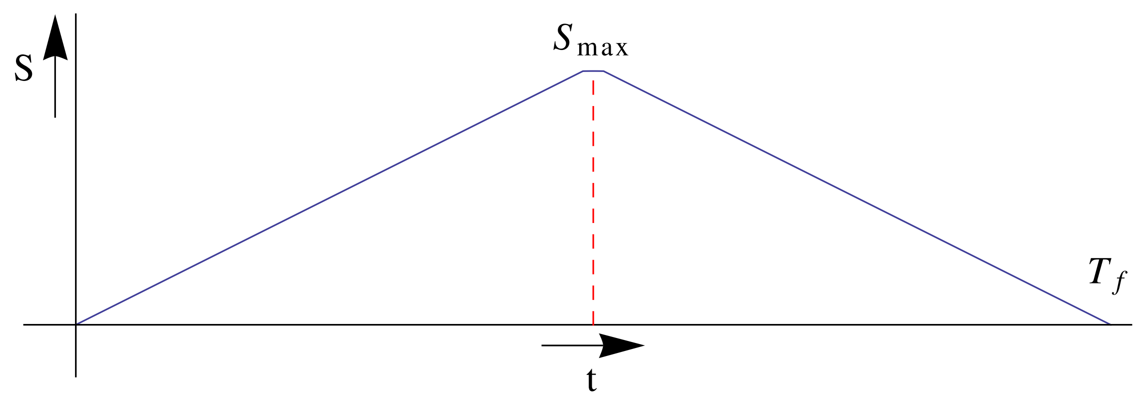
- Recently there has been a lot of activity in this area particularly in the context of the AdS/CFT correspondence which relates quantum gravitational theories in asymptotically anti de-Sitter spacetimes to certain classes of quantum field theories living on the boundary.

- We shall now consider two versions of the modern information loss problem : the cloning and strong subadditivity paradoxes.

Page Curve

- Consider a black hole formed from collapse that gradually evaporates.

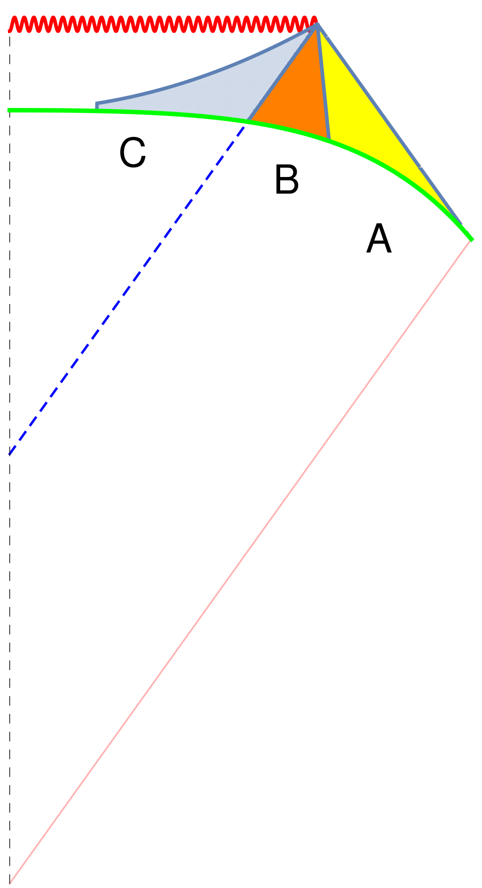
- If black hole evaporation process is unitary, the Von-Neumann entropy of the Hawking radiation, $S = -\mathrm{Tr}(\rho \log \rho)$ should vary with time following the Page-curve.



[D.Page, 1993]

- The decrease of the Von-Neumann entropy after the Page time indicates that the outgoing Hawking radiation starts to become pure $\Longrightarrow$ information exists the black hole after the Page time.

Strong Subadditivity Paradox



- Consider a stage in the evaporation process after the Page time. Then we must have

[Mathur, AMPS, 2009-2012]

$$S_{AB} < S_A$$

- But smoothness of the horizon requires the near horizon modes to be almost maximally entangled.

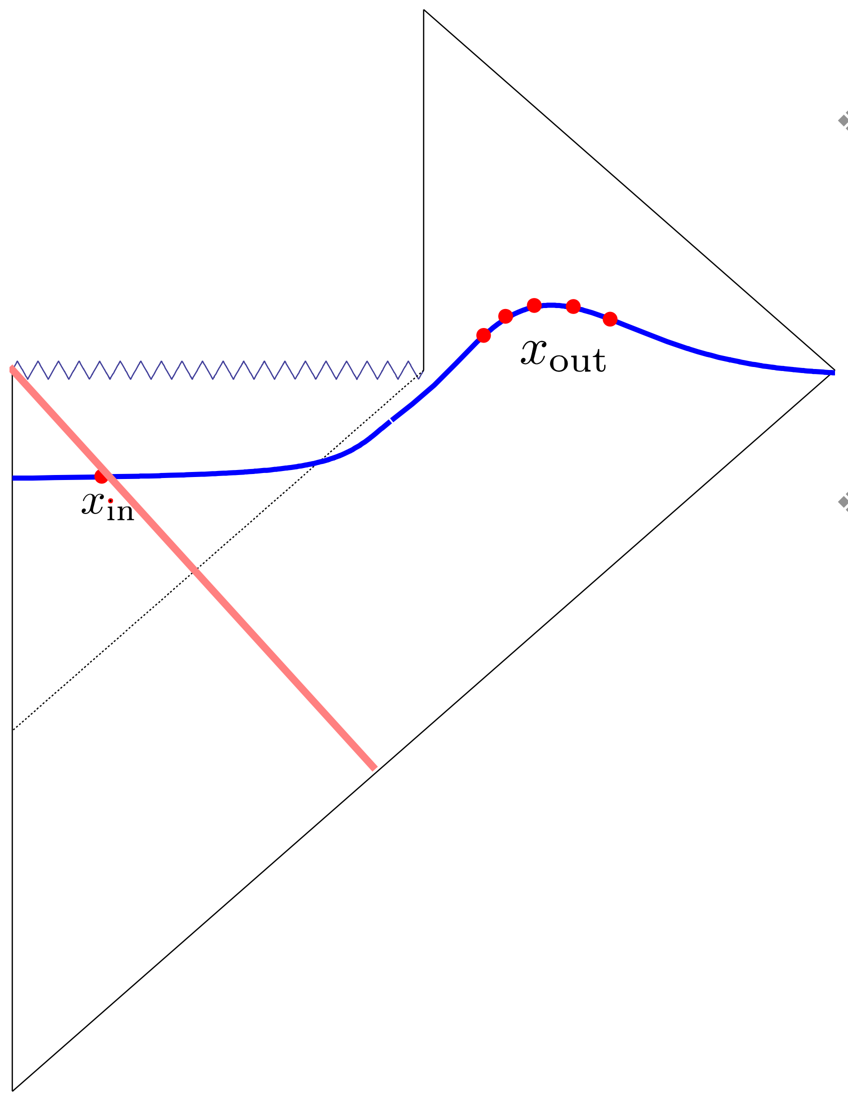
$$S_{BC} < S_C$$

- But this contradicts strong subadditivity of von-Neumann entropy

$$S_{AB} + S_{BC} > S_A + S_C$$

- This presents an $\mathcal{O}(1)$ contradiction which cannot be resolved by including small corrections.

Cloning Paradox



- The black hole spacetime can be foliated by a set of *nice slices* that captures significant fraction of the outgoing radiation.
- Information present in infalling matter seems to have been duplicated outside on the same spatial slice beyond the Page time.
- This is contradiction with linearity of quantum mechanics.

[Susskind, Thorlacius, Uglum, Preskill, Hayden]

Complementarity

- A possible way to avoid the cloning paradox is to posit that degrees of freedom in the interior of the black are not independent of the degrees of freedom outside. This suggests

$$\phi(x_{in}) \approx P\left(\phi(x_{1,out})\phi(x_{2,out})\ldots\phi(x_{S,out})\right)$$

[t'Hooft, Susskind, Uglum, Thorlacius, Polchinski,…]

- This implies a subtle breakdown of locality over macroscopic scales.

- The recent prescription of Papadodimas-Raju for describing the interior of large black holes in AdS implements this idea by representing CFT operators dual to local bulk fields in the black hole interior as *complicated* $\mathcal{O}(N^2)$ combinations of CFT operators dual to exterior bulk fields.

[K. Papadodimas, S. Raju, 2013-2015]

- It is possible to realise such polynomial constraints for the algebra of local observables in a precise way even in empty AdS.

[S. Banerjee, JW Bryan, K. Papadodimas, S. Raju, 2016]

Non-Localities from Gauss Law

- Since the Hamiltonian in a theory of gravity is a boundary term, it leads to non-local Gauss law commutators.

$$i[H, \phi(t,x)] = \dot{\phi}(t,x)$$

- In the case of large N QFTs with AdS holographic duals, these are suppressed by factors of $1/N$.

- Breakdown of $1/N$ perturbation theory enhances this to $\mathcal{O}(1)$ effect.

- In asymptotically flat spacetimes, such commutators are suppressed by factors of E/M_{pl} where E is a measure of the energy of the configuration of operators under consideration.

[W. Donnelly, S. Giddings, 2016]

- The breakdown of gravitational perturbation theory can again enhance these effects.

A Path Integral Perspective

- Consider defining a theory of quantum gravity through a path integral

$$\mathcal{Z} = \int \mathcal{D}g\mathcal{D}\phi \; e^{iS[g,\phi]}$$

- A semiclassical spacetime corresponds to a saddle point of this path integral.

- Breakdown of gravitational perturbation theory indicates a breakdown of the saddle point approximation.

$\Longrightarrow$ Modification of any notion of locality or causality defined with respect to the original saddle.

Large n String Amplitudes

- The suggestive relation between breakdown of locality and gravitational perturbation theory motivates us to study string scattering amplitudes with many external particles.

- The large n limit of string amplitudes is in itself an interesting and novel regime.

- Exploring the limits of validity of perturbation theory often yields valuable insight into non-perturbative aspects. In the context of string theory the analysis of the breakdown of string perturbation theory at large loop orders played an important role in the discovery of D-branes.

[Shenker, 1990, Polchinski, 1995]

Breakdown of String Perturbation Theory

- Consider n-point scattering amplitudes of massless closed string states in the regime where string and Planck scales are widely separated, n is large & energy per-particle, E is small.

$$g_s^2 = \frac{2\pi l_{pl}^{d-2}}{(2\pi\sqrt{\alpha'})^{d-2}} \to 0, \quad n \to \infty$$

$$\frac{\log(E\sqrt{\alpha'})}{\log(n)} \to -\gamma, \quad 0 < \gamma < \frac{1}{(d-2)}$$

- String Perturbation theory breaks down when

$$\frac{\log(g_s)}{\log(n)} = \frac{(d-2)\gamma - 1}{2} + \mathcal{O}\left(\frac{1}{\log(n)}\right)$$

$$\boxed{\implies \quad n \propto g_s^{\frac{2}{(d-2)\gamma-1}} \quad \text{or} \quad n \propto \left(\frac{M_{pl}}{E}\right)^{d-2}}$$

Factorial Growth of Amplitudes

- To obtain this result we show that tree level scattering amplitudes for massless particles in closed bosonic and superstring theories grow at least as fast as

$$M^{\text{tree}}(k_1 \ldots k_{\frac{n}{2}} \to k_{\frac{n}{2}+1} \ldots k_n) \sim \frac{n!}{M_{pl}^{\frac{(d-2)n}{2}-d}}.$$

for large n.

- The threshold for breakdown of string perturbation theory can then be derived from bounds on growth of tree level amplitudes imposed by unitarity.

$$\int d\Pi_{\frac{n}{2}} \left|M^{\text{tree}}(\{k_1 \ldots k_{\frac{n}{2}} \to \{k_{\frac{n}{2}+1} \ldots k_n\})\right|^2 \leq 2\left|M^{\text{tree}}(\{k_i\} \to \{k_i\})\right|$$

Analytic Results: Overview

- String amplitudes can be expressed as integrals over moduli space of punctured Riemann surfaces. [Giddings, D'Hoker, Phong, Sonoda, 1987]
- For bosonic strings at tree level

$$M^{\text{tree}}(k_1 \ldots k_n) = g_s^{n-2} \int_{\mathcal{M}_{0,n}} d\mu_{WP} (\det{}' P_1^\dagger P_1)^{\frac{1}{2}} (\det{}' \Delta)^{\frac{-d}{2}} \langle \mathcal{V}_1(k_1; z_1, \bar{z}_1) \ldots \mathcal{V}_n(k_n; z_n, \bar{z}_n) \rangle$$

- In the large *n* limit, the major contribution to the integral can be shown to come from the Weil-Petersson volume of the moduli space.

$$V_{0,n} = \int_{\mathcal{M}_{0,n}} d\mu_{WP} \qquad\qquad \lim_{n\to\infty} V_{0,n} \propto n!$$

[M. Mirzakhani, P. Zograf, 2008-2013]

- The contributions from the functional determinants and worldsheet correlation functions do not suppress this factorial growth.

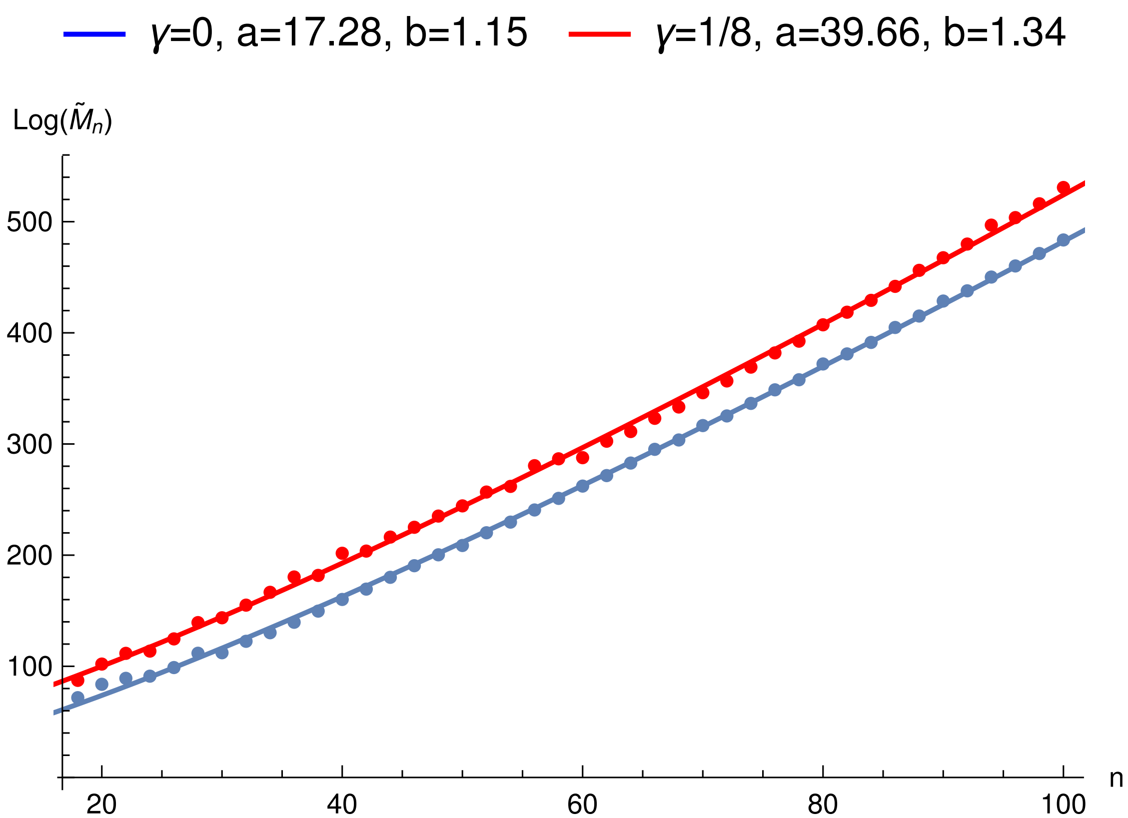
Numerical Results

- The $n!$ growth can be independently verified by a numerical analysis.
- At large n the string amplitude can be evaluated by a saddle point approximation. The moduli space integral localises onto the solutions of the scattering equations

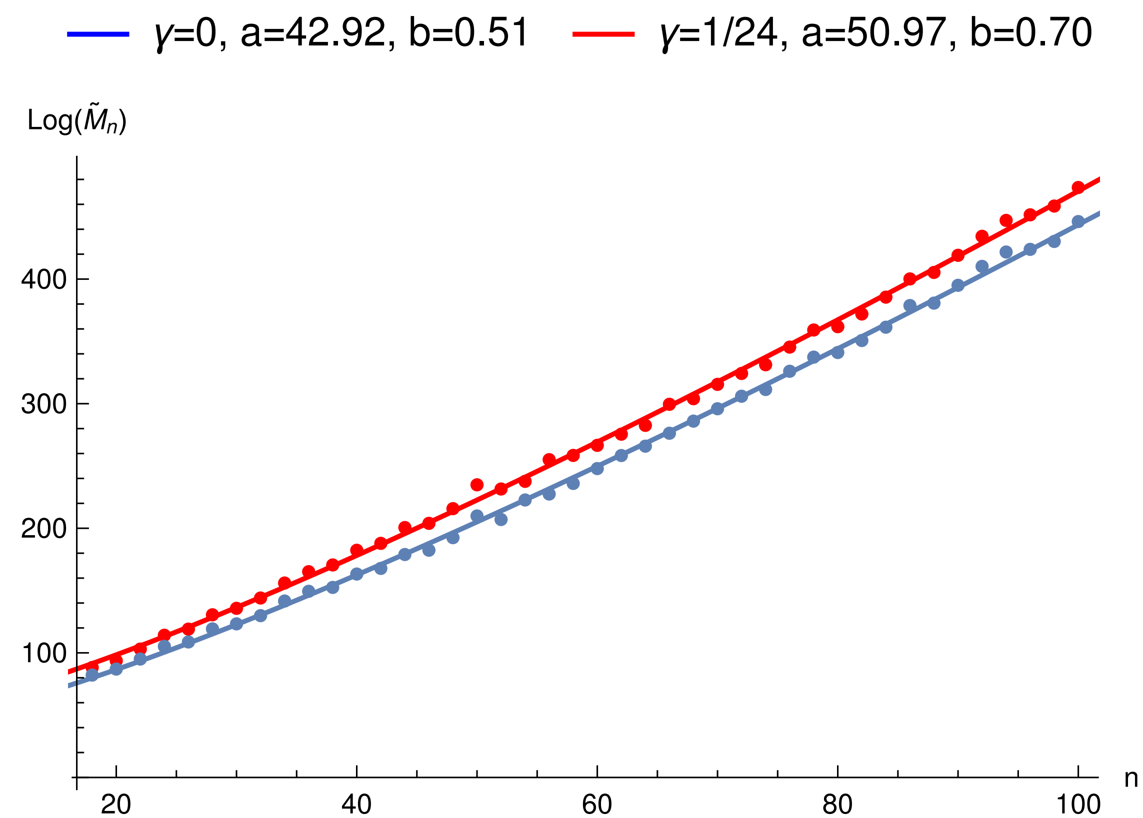
$$E_i = \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j} = 0, \quad \forall\, i \in (1, \ldots, n)$$

- These equations have precisely $(n-3)!$ solutions.
- The string amplitude is then evaluated by sampling over a large set of random solutions to these equations.

Numerical Results



Type II Superstring amplitude



Bosonic string amplitude

$$\log(\tilde{M}_n) = \log(M^{\text{tree}}(k_1 \ldots k_n)) - (n-2)\log(4\pi g_s) + n\log(d-2)$$

$$= \; a + bn + \log((n-3)!)$$

Extracting Information from Hawking Radiation

- To detect cloning/strong subadditivity paradoxes, we need to determine the density matrix ρ_A of the outgoing Hawking radiation.

- For a black hole with entropy S, $\dim(\rho_A) \sim e^S \times e^S$

- Need to measure order S-point connected correlation functions of the Hawking quanta to detect cloning.

- These correlation functions can be related to the scattering matrix elements with S insertions of the Hawking quanta with typical energy per quanta being T.

Implications for Information Paradox

- For a d -dimensional Schwarzschild black hole

$$S = \frac{\Omega_{d-2} r_h^{d-2}}{4G} \qquad T = \frac{d-3}{4\pi r_h}$$

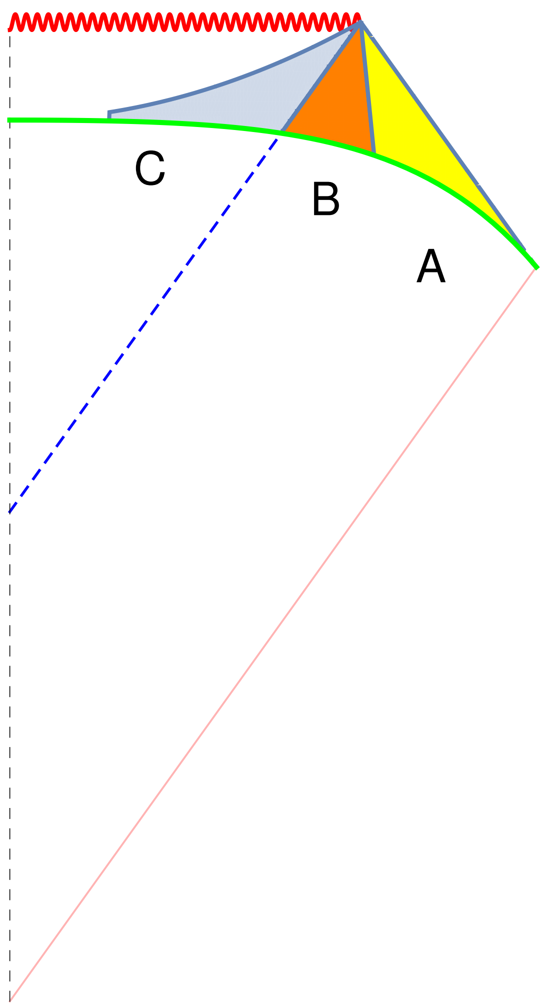
$$r_h = \left(\frac{16\pi G M}{(d-2)\Omega_{d-2}}\right)^{\frac{1}{d-3}}$$

- For a black hole with $M/M_{pl} \gg 1$

$$\frac{\log(S)}{(2-d)\log(T/M_{pl})} \to 1$$

- With $n = S$ and $E = T$, this is precisely the same threshold for breakdown of perturbation theory shown before. Such observables are then likely to be sensitive to *non-perturbative* as well as *non-local* effects.

Complementarity for Flat Space Black Holes



- The breakdown of locality for observables with $\mathcal{O}(S)$ insertions provides strong support towards the possibility of realising complementarity for black holes in asymptotically flat space-times.

$$\phi(x_C) \cong P(\phi(x_{A_1}), \phi(x_{A_2}), \cdots, \phi(x_{A_S}))$$

- This idea of complementarity will then imply that the algebra of local observables does not factorize as

$$\mathcal{A} \neq \mathcal{A}_A \otimes \mathcal{A}_B \otimes \mathcal{A}_C$$

- The degrees of freedom in regions *A*, *B*, *C* do not constitute independent subsystems.

$\implies$ Strong Subadditivity is not applicable.

Summary

- *Complicated* observables in a theory of quantum gravity are potentially sensitive to non-local and non-perturbative effects. This motivated the study of string amplitudes for large number of particles.

- Analytic and numerical evidence shows that closed-string perturbation theory breaks down when $n \sim (E/M_{pl})^{d-2}$.

- The consequent breakdown of locality is sufficient to resolve the cloning and strong subadditivity paradoxes for evaporating black holes in asymptotically flat space-times.

- It will be interesting to explore the nature of non-perturbative effects in string theory that restore unitarity when this threshold for breakdown of perturbation theory is reached.

Part 2

Entanglement in Gauge Theories and Gravity

References:

- *On the Entanglement Entropy for Gauge Theories*
  JHEP 09 (2015); arXiv: 1501.102593 with Ronak M Soni, Sandip P. Trivedi
- *Quantum Information Measures for Restricted Sets of Observables*
  Phys. Rev. D98 (2018); arXiv: 1706.07424 with Suvrat Raju

Introduction

- Entanglement is a ubiquitous feature of quantum systems.

- For bipartite quantum systems, a commonly used measure is the von Neumann entropy

$$S = -\mathrm{Tr}(\rho \log \rho)$$

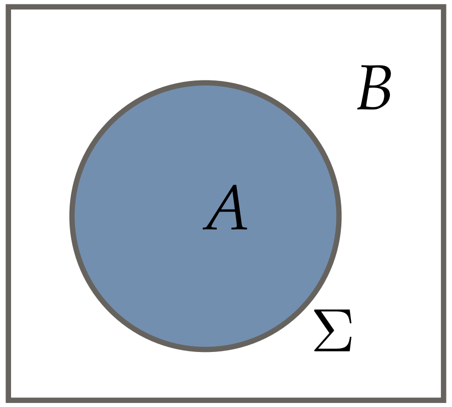
- The study of measures of quantum entanglement have recently found several applications in understanding various features of quantum field theories, such as renormalization group flows.

- Ideas and tools of quantum information theory are also becoming increasingly useful in exploring aspects of quantum gravity, such as the emergence of space-time.

Entanglement in Gauge Theory

- In QFT, one is often interested in the entanglement between degrees of freedom across spatial subregions. Defining the standard measures of entanglement in terms of reduced density matrices requires

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



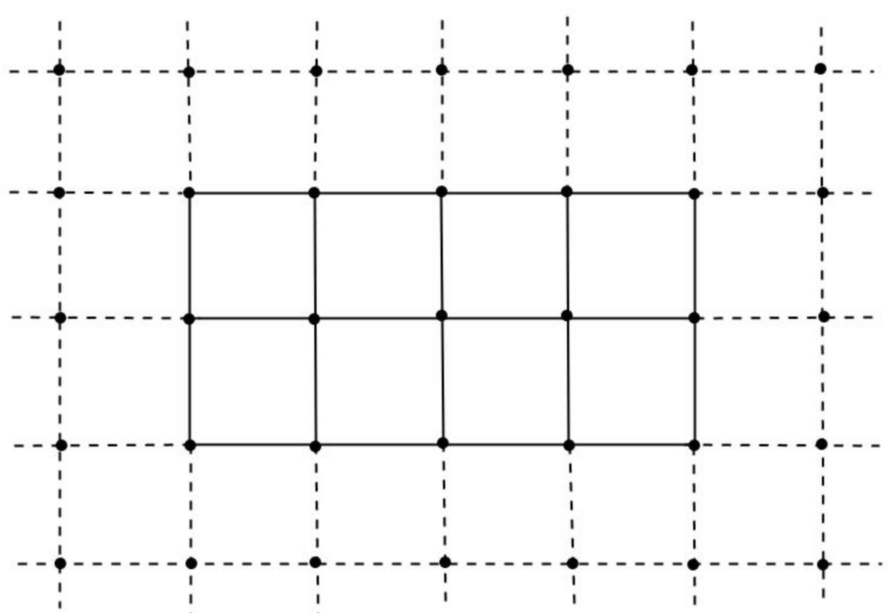
- But this does not strictly hold in QFT due to divergent short range correlations across the entangling surface. In a local QFT, this can be regulated by formulating the theory on a lattice.

- In gauge theories, there exist non-local gauge invariant degrees of freedom such as Wilson loops. The Gauss law constraint then hinders a straightforward tensor product factorisation of the Hilbert space of physical states.

Entanglement in Lattice Gauge Theory

- Lattice gauge theories provide an interesting setup for studying aspects of entanglement for gauge theories.

- Here the basic degrees of freedom live on the links of the lattice. We shall consider the the entanglement between a subset of links and its complement.



Extended Hilbert Space Definition

- Let $|\psi\rangle$ be a gauge-invariant state of the full system. This can embedded in a larger Hilbert space.

$$|\psi\rangle \in \mathcal{H}_{ginv} \subset \mathcal{H} = \bigotimes_{(ij)} \mathcal{H}_{ij}$$

$\mathcal{H}_{ij}$: Hilbert space of all states on the link (ij)

- This extended Hilbert space has a tensor product decomposition $\mathcal{H} = \mathcal{H}_{in} \otimes \mathcal{H}_{out}$

- The entanglement entropy for the set of "inside" links is then defined by

$$S_{EE} = -\mathrm{Tr}_{\mathcal{H}_{in}} \left(\rho_{in} \log \rho_{in} \right)$$

where $\rho_{in} = \mathrm{Tr}_{\mathcal{H}_{out}} |\psi\rangle\langle\psi|$

Gauge Invariant Representation

- The extended Hilbert space can be decomposed as

$$\mathcal{H} = \bigoplus_{\mathbf{k}} \mathcal{H}^{\mathbf{k}}_{in} \otimes \mathcal{H}^{\mathbf{k}}_{out}$$

$\mathbf{k} = \{k_1, k_2, \ldots .k_N\}$: eigenvalues of electric flux operators emanating from boundary vertices.

- Since $|\psi\rangle$ is gauge invariant, in a definite flux/superselection sector

$$\rho^{\mathbf{k}}_{in} = \mathrm{Tr}_{\mathcal{H}^{\mathbf{k}}_{out}} |\psi\rangle\langle\psi| = \mathrm{Tr}_{\mathcal{H}^{\mathbf{k}}_{ginv,out}} |\psi\rangle\langle\psi| = p_{\mathbf{k}} \tilde{\rho}^{\mathbf{k}}_{in} \qquad \sum_{\mathbf{k}} p_{\mathbf{k}} = 1$$

$$\Longrightarrow \quad \boxed{S_{EE} = -\sum_{\mathbf{k}} \mathrm{Tr}_{\mathcal{H}^{\mathbf{k}}_{in}} \rho^{\mathbf{k}}_{in} \log(\rho^{\mathbf{k}}_{in}) = -\sum_{\mathbf{k}} p_{\mathbf{k}} \log(p_{\mathbf{k}}) + \sum_{\mathbf{k}} p_{\mathbf{k}} S(\tilde{\rho}^{\mathbf{k}}_{in})}$$

$$S(\tilde{\rho}^{\mathbf{k}}_{in}) = -\mathrm{Tr}_{\mathcal{H}^{\mathbf{k}}_{ginv,in}} \tilde{\rho}^{\mathbf{k}}_{in} \log(\tilde{\rho}^{\mathbf{k}}_{in})$$

Properties of the Definition

- This definition applies to all lattice gauge theories without additional matter, including discrete, abelian and non-abelian gauge groups.

- It is unambiguous and manifestly gauge invariant.

- Satisfies the strong subadditivity property. For any three set of links *A*, *B*, *C* on the lattice

$$S_{A\cup B} + S_{B\cup C} \geq S_B + S_{A\cup B\cup C}$$

- Agrees with replica trick path integral definition.

Extracting the Entanglement

- In quantum information theory, the amount of entanglement in a bipartite system can be operationally quantified through protocols such as entanglement distillation and dilution.

- These protocols are implemented through local operations and classical communications (LOCC).

- Since gauge invariant local operations cannot change the flux values characterising a particular superselection sector, the full entanglement entropy in gauge theories cannot be extracted through such protocols.

$$S_{EE} = -\sum_{\mathbf{k}} p_{\mathbf{k}} \log(p_{\mathbf{k}}) + \sum_{\mathbf{k}} p_{\mathbf{k}} S(\tilde{\rho}_{in}^{\mathbf{k}})$$

non-extractable "classical" piece

extractable "quantum" part

Algebraic Approach to Entanglement

- A more algebraic approach towards studying entanglement can also adopted in settings where a bipartite decomposition of the Hilbert space is not assumed a priori.
- In this case, one considers having access to only a subalgebra of gauge invariant operators for probing a state, for e.g., the subalgebra of operators localised in a given spatial region.
- It is possible to assign a density matrix to this algebra by choosing an element ρ of the algebra such that

$$\mathrm{Tr}(\rho \mathcal{O}_i) = \langle \psi | \mathcal{O}_i | \psi \rangle \quad \forall \, \mathcal{O}_i \in \mathcal{A}$$

- The von Neumann entropy of ρ then serves a definition of the entanglement entropy.
- In case of gauge theories, the lack of tensor product factorisation of the Hilbert space can be associated to the presence of a non-trivial centre in the algebra of gauge-invariant operators.

[Casini, Huerta, Rosabal, 2014]

Entanglement in Gravity

- In a theory of quantum gravity, the problem of defining a notion of entanglement for spatially localised degrees of freedom is even more subtle.
- The Hilbert space does not factorise due to constraints of diffeomorphism invariance.
- It is also difficult to assign closed algebras of approximately local operators to spacetime regions in a theory of gravity. Complementarity suggests that complicated products operators cannot be meaningfully localised within a given region.
- In this case both the usual approach of defining quantum information measures in terms of reduced density matrices as well as the algebraic approaches are not suitable.
- This motivates the need to consider new formulations of information theoretic measures where the set of accessible observables does not form an algebra.

General Setup

- Consider a general quantum system in a state $|\psi\rangle$. Suppose an observer can probe this state using a set of operators

$$\mathcal{A} = \mathrm{span}\{\mathcal{O}_1, \cdots, \mathcal{O}_n\}$$

such that $\mathcal{A}$ does not close to form an algebra.

- The relevant Hilbert space accessible for performing measurements is then

$$\mathcal{H}_\psi = \mathrm{span}\{\mathcal{O}_1|\psi\rangle, \cdots, \mathcal{O}_n|\psi\rangle\}$$

and the set of accessible observables is spanned by the set of 2-point functions

$$g_{ij} = \langle\psi|\mathcal{O}_i\mathcal{O}_j|\psi\rangle$$

- Now given another state $|\phi\rangle$ of the full system, how well can the observer distinguish between $|\psi\rangle$ & $|\phi\rangle$?

Properties of Distance Measures

Any measure $\mathcal{D}_{\mathcal{A}}(\psi, \phi)$ of distance between quantum states must meet the following requirements:

- *Basis Independence* : $\mathcal{D}_{\mathcal{A}}(\psi, \phi)$ must be invariant under unitary transformations of orthogonal basis chosen for the set of observables.
- *Specificity* : $\mathcal{D}_{\mathcal{A}}(\psi, \phi) \geq 0$ & $\mathcal{D}_{\mathcal{A}}(\psi, \phi) = 0 \iff \psi = \phi$
- *Monotonicity* : $\mathcal{D}_{\mathcal{B}}(\psi, \phi) \leq \mathcal{D}_{\mathcal{A}}(\psi, \phi) \quad \text{if } \mathcal{B} \subset \mathcal{A}$
- *Insularity* : Coupling to an auxiliary system should not change the distance measure

$$\mathcal{D}_{\mathcal{A}_1 \otimes \mathcal{A}_{aux}}(\psi_1 \otimes \psi_{aux}, \phi_1 \otimes \psi_{aux}) = \mathcal{D}_{\mathcal{A}_1}(\psi_1, \phi_1)$$

Modular & Relative Modular Operators

- The modular and relative modular operators are useful for constructing distance measures in this case.

[Tomita, Takesaki]

- The *modular operator* on $\mathcal{H}_\psi$ is defined as

$$\Delta_\psi = S_\psi^\dagger S_\psi \qquad \text{with} \quad S_\psi(\mathcal{O}_i|\psi\rangle) = \mathcal{O}_i^\dagger|\psi\rangle$$

$$\implies (\Delta_\psi)_{ij} = \langle\psi|\mathcal{O}_i^\dagger \Delta_\psi \mathcal{O}_j|\psi\rangle = \langle\psi|\mathcal{O}_j\mathcal{O}_i^\dagger|\psi\rangle$$

- The *relative modular operator* on $\mathcal{H}_\psi$ is defined as

$$\Delta(\psi|\phi) = S_{(\psi|\phi)}^\dagger S_{(\psi|\phi)} \qquad \text{with} \quad S_{(\psi|\phi)}(\mathcal{O}_i|\psi\rangle) = \mathcal{O}_i^\dagger|\phi\rangle$$

$$\implies (\Delta(\psi|\phi))_{ij} = \langle\psi|\mathcal{O}_i^\dagger \Delta(\psi|\phi)\mathcal{O}_j|\psi\rangle = \langle\phi|\mathcal{O}_j\mathcal{O}_i^\dagger|\phi\rangle$$

Araki Relative Entropy

- The relative entropy is a commonly used distance measure. A definition of this in terms of the relative modular operator was given by H. Araki

[Araki; Narnhoffer, Thirring]

$$S^{ar}_{\mathcal{A}}(\psi|\phi) = -\langle\psi|\log(\Delta(\psi|\phi))|\psi\rangle$$

- This satisfies all the previous properties when $\mathcal{A}$ is an algebra.

- But if $\mathcal{A}$ is not an algebra, the Araki relative entropy can sometimes fail to distinguish between states.

$$S^{ar}_{\mathcal{A}}(\psi,\phi) = 0 \not\Longrightarrow \langle\psi|\mathcal{O}_i\mathcal{O}_j|\psi\rangle = \langle\phi|\mathcal{O}_i\mathcal{O}_j|\phi\rangle$$

Measures of Distance & Entanglement

- We can define a new class of distance measures that satisfies all our stipulated properties

$$\mathcal{D}_{\mathcal{A}}(\psi,\phi): \boxed{\begin{aligned} S_{\mathcal{A}}(\psi,\phi) &= \log ||X|| + \log ||X^{-1}|| \\ \mathcal{X}_{\mathcal{A}}(\psi,\phi) &= 1 - \frac{2||X||}{(1+||X||)^2} - \frac{2||X^{-1}||}{(1+||X^{-1}||)^2} \end{aligned}}$$

with $X = \Delta_{\psi}^{-1/2}\Delta(\psi|\phi)\Delta_{\psi}^{-1/2}$

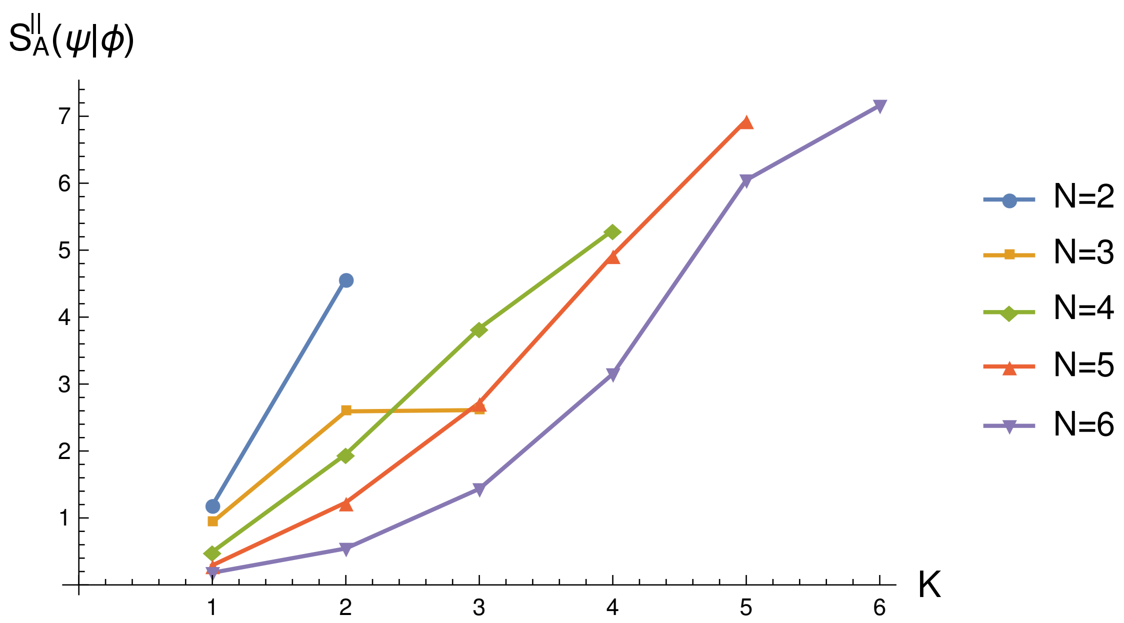
$||X|| \longrightarrow$ *operator norm of* X

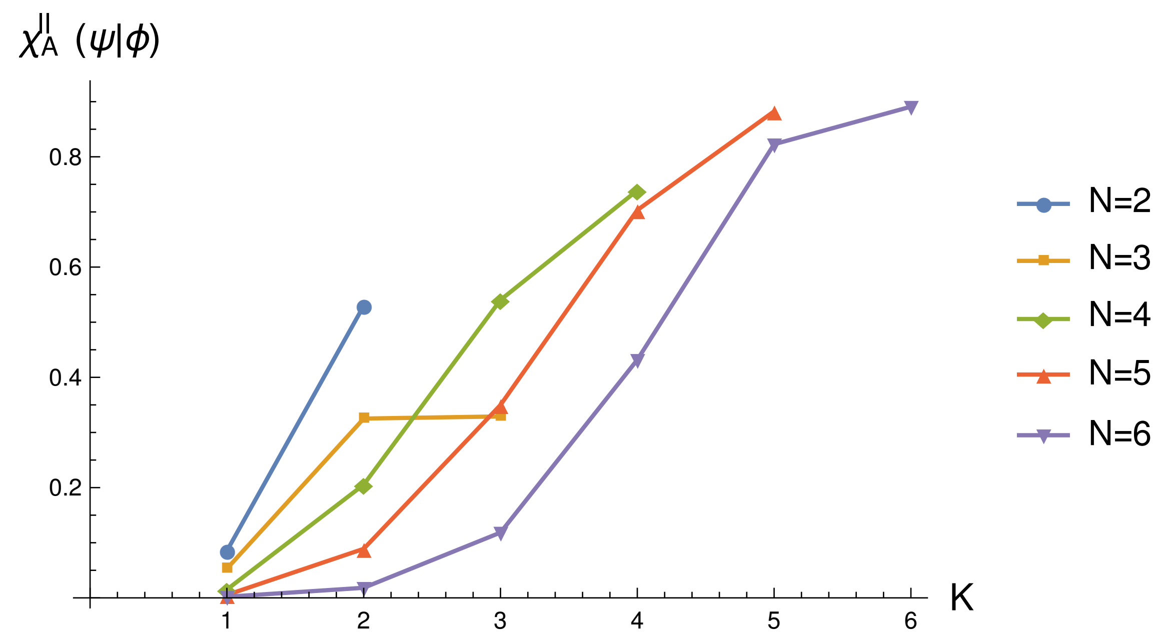
- From these, measures of entanglement can be defined as

$$E_{\mathcal{A}}(\psi) = \inf_{\phi \in sep} \mathcal{D}_{\mathcal{A}}(\psi,\phi)$$

such that this measures only quantum correlations, remains invariant under unitary transformations and does not increase under LOCC operations.

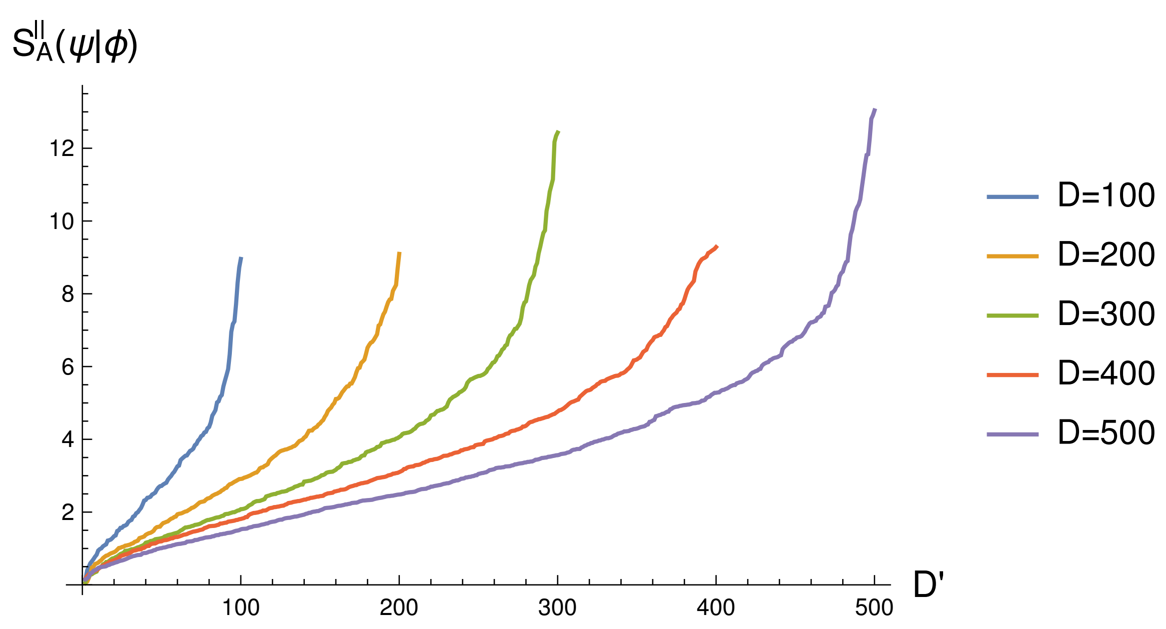
Numerical Examples: Spin chains

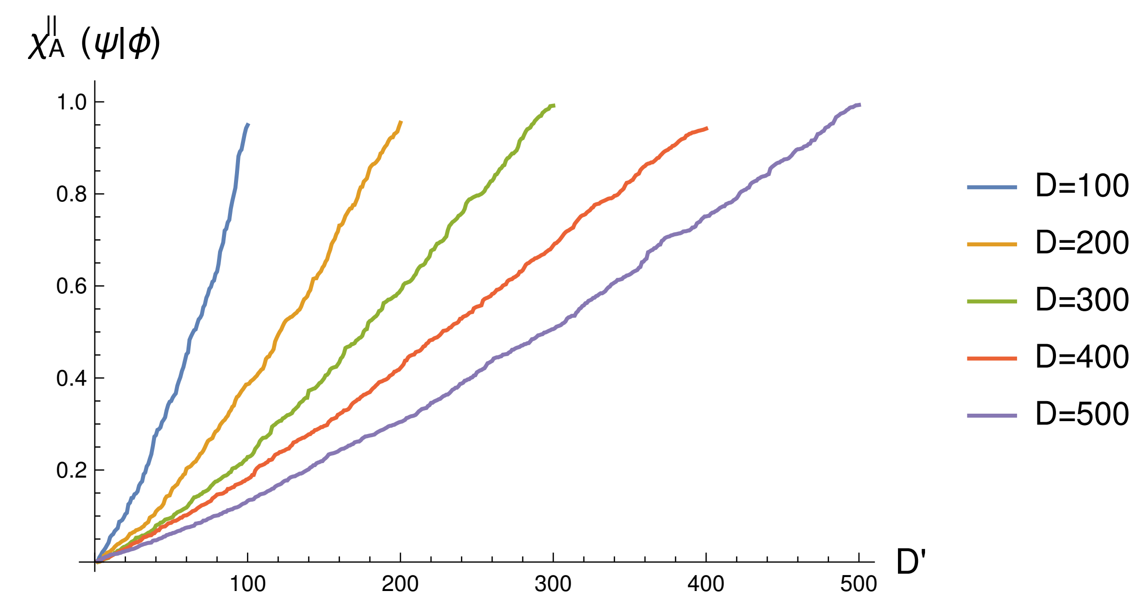




Plot of distance measures for a spin chain of length N that is probed with polynomials of order K in individual spin operators

Numerical Examples: Random Matrices

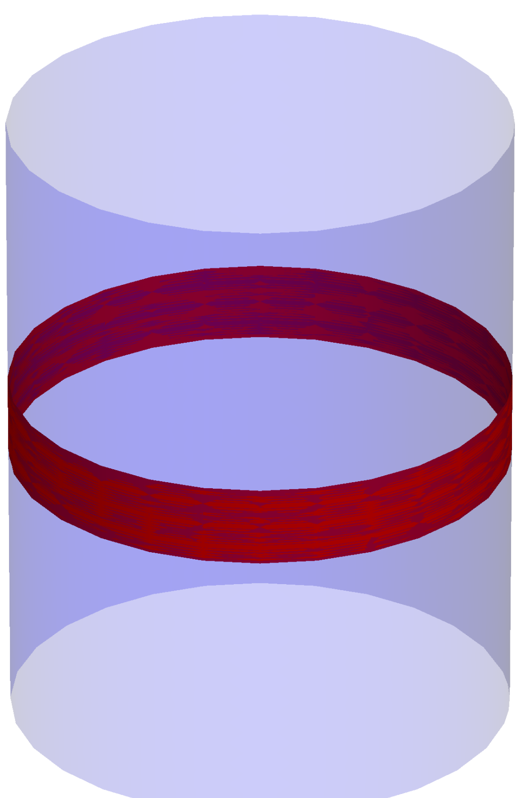




Plot of distance measures for a system of D random matrices probed with a D′-dimensional subset of these operators.

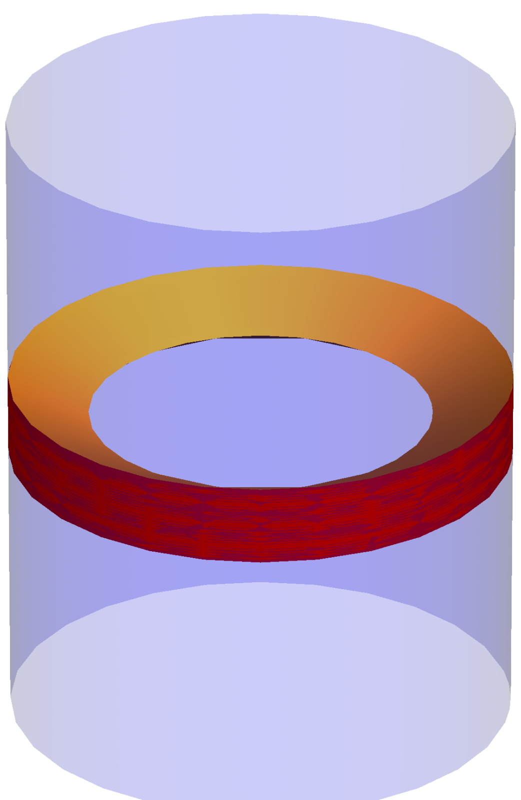
Subregion Duality in AdS/CFT

- Our information measures can be used to study subregion dualities in AdS/CFT.
- Given a set of boundary observables in a region $\mathcal{R}$, what is the bulk region whose information is encoded within these set of observables



$$S_{\mathcal{A}_{\mathcal{R}_C}}(\psi,\phi) = S_{\mathcal{A}_{\mathcal{B}_C}}(\psi,\phi)$$

$$\mathcal{X}_{\mathcal{A}_{\mathcal{R}_C}}(\psi,\phi) = \mathcal{X}_{\mathcal{A}_{\mathcal{B}_C}}(\psi,\phi)$$



$\mathcal{R}$: Time-band on the boundary of width $T < \pi$

$\mathcal{B}_C$: Coarse grained dual bulk region

Summary

- In gauge theories the non-factorisability of the physical Hilbert space makes defining entanglement entropy a subtle issue.
- We considered a particular definition for lattice gauge theories using the extended Hilbert space approach.
- The full entanglement entropy cannot however be made available for extraction in operational quantum information theoretic protocols.
- Motivated by the problem of defining notions of entanglement in gravity, we considered a setup where the set of accessible observables do not form closed algebras.
- We defined a new class of distance between quantum states using the modular and relative modular operators.
- These distance measure have interesting applications in studying aspects of bulk locality in AdS/CFT.

THANK YOU