A study of OPE blocks and modular Hamiltonian in AdS_3/CFT_2

Suchetan Das

Ramakrishna Mission Vivekananda Educational and Research Institute

October 30, 2019

References

Bulk of this talk is based on:

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- $\rightarrow$ Connection between OPE and modular algebra?

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- Future direction

Operator Product Expansion

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- In CFT: state operator correspondence $\implies$ convergent OPE at finite separation! $\sim$ linear combination of primaries and descendants. For two scalar operators $\mathcal{O}_i(x), \mathcal{O}_j(0)$ of dimensions (Δ_i, Δ_j):

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k |x|^{\Delta_k-\Delta_i+\Delta_j} C_{ijk}(1 + a x^\mu \partial_\mu + b x^\mu x^\nu \partial_\mu \partial_\nu + \dots)\mathcal{O}_k(0) \quad (2)$$

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- conformal block $G_{\mathcal{O}_k}(x_i) = <\mathcal{O}_1\mathcal{O}_2\mathbb{P}_{\mathcal{O}_k}\mathcal{O}_3\mathcal{O}_4> = <B_k(x_1,x_2)B_k(x_3,x_4)>$

Connection to AdS/CFT(Czech et al 2016, de Boer et al 2016, da Cunha et al 2016, Hijano et al 2015)

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- Using momentum basis representation for scalar OPE block of dim Δ_k from an OPE of two scalars of dim Δ_i, Δ_j in CFT$_d$ $\rightarrow$
  $B_k^{ij} \sim \int_\gamma ds e^{-s\Delta_{ij}} \underbrace{\phi(x(s))}_{\text{scalar field}}$ $\rightarrow$ geodesic operator mass of scalar field ϕ is
  $m^2 = \Delta_k(\Delta_k - d)$ (Ferrara et al 1971,72; da Cunha et al 2016)

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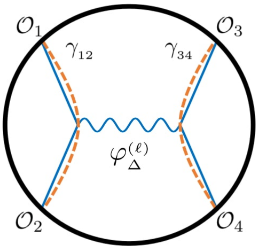
- Conformal block = Geodesic Witten Diagram(GWD) $\sim \int_{\gamma_{12}} \int_{\gamma_{34}} G_{b\partial}G_{b\partial}G_{bb}G_{b\partial}G_{b\partial}$ (Hijano et al 2015)



Spinning OPE block in AdS_3/CFT_2:Motivation

- Closed form expression for CFT_2 spinning conformal block $W_{h_k,\bar{h}_k}(z_i,\bar{z}_i)$:

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- In CFT_2 $\rightarrow$ All spinning primaries can be labelled by two real numbers $h, \bar{h}$.
- Using this feature we generalize the momentum space representation to find an integral expression for spinning OPE block in CFT_2

Spinning OPE block in AdS_3/CFT_2:Result(Das)

- We find an expression for spinning OPE block of dimension $(h_k, \bar{h}_k)$ of two spinning operators $A(z_1, \bar{z_1})$ and $B(0,0)$ of conformal dimension h_i and h_j respectively.

Spinning OPE block in AdS$_3$/CFT$_2$:Result(Das)

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- $A(z_1, \bar{z_1})B(0,0)|_{h_k,\bar{h}_k} = B^{ij}_{h_k,\bar{h}_k}$

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Spinning OPE block in AdS_3/CFT_2:Result(Das)

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- For conserved current OPE blocks, that expression (anti)holomorphically factorized $\rightarrow$ In this case, using HKLL kernal for bulk AdS_2 field and following the lines of da Cunha, Guica$\rightarrow$

$$
B^{ij}_{h_k,\bar{h}_k} \sim \int_{-\infty}^{\infty} d\lambda e^{-\lambda h_{AB}} \underbrace{\phi^{(0)}_{AdS_2}(x(\lambda))}_{\text{mass}=h_k(h_k-1)} \int_{-\infty}^{\infty} d\lambda' e^{-\lambda'\bar{h}_{AB}} \underbrace{\phi^{(0)}_{AdS_2}(x'(\lambda'))}_{\text{mass}=\bar{h}_k(\bar{h}_k-1)} \tag{5}
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Connection to AdS$_3$ and conformal block

- **AdS$_3$ connection**$\rightarrow$ Using HKLL for massless symmetric higher spin fields in radial gauge (Sarkar and Xiao) we get

$$\phi_{zzz...}(z,y) = \mathcal{O}_{zzz...}(z), \phi_{\bar{z}\bar{z}\bar{z}...}(\bar{z},y) = \mathcal{O}_{\bar{z}\bar{z}\bar{z}...}(\bar{z}) \tag{6}$$

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- Two point function of AdS_2 representation of conserved OPE blocks $\underbrace{\Longrightarrow}_{AdS_2 GWD}$ CFT_2 spinning conformal block$\rightarrow$ check!

Modular Theory

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- To understand the emergence of bulk gravity and bulk causality in holographic theories from the entanglement structure of boundary field theory.
- To study universal constraints on QFT and quantum gravity from inequalities associated to modular Hamiltonian, e.g. Average null energy condition from monotonicity of relative entropy.

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- Special symmetric case $\rightarrow$ local expressions for H_{mod}
- e.g: Minkowski vacuum on half space($x^1 > 0$) $H_{mod} = \int_{x^1>0} x_1 T_{00} d^{d-1}x$,
  spherical region in vacuum CFTs,
  locally excited states in CFT$_2$,
  future horizon on a null plane etc.

Divergence in EE: motivation to study algebraic QFT

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- We will see in the algebraic formulation of QFT, Tomita Takesaki modular theory provides the notion of UV finite quantities naturally!

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- From now we will consider: A set of bounded operators of a region U forms an algebra $\mathcal{A}_U(\mathcal{O})$. If U' is maximal open set under the condition of being spacelike separated then $[\mathcal{A}_U, \mathcal{A}_{U'}] = 0$.

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- Invertible $S_\psi \implies$ a polar decomposition: $S_\psi = J_\psi \Delta_\psi^{1/2}$ s.t J_ψ is antiunitary $\to$ modular conjugation and Δ_ψ Hermitian and positive definite $\to$ modular operator.

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- ψ is separating and cyclic for both regions U and U'.
- $S_\psi^2 = 1$, $S_\psi|\psi>= |\psi>$, $S_\psi^{-1} = S_\psi$, $S' = S^\dagger$
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- Example(2): Modular operator for right Rindler wedge in Minkowski spacetime $\rightarrow$
  $\ln \Delta_\Omega = \ln \rho_r - \ln \rho_l = \exp(-2\pi K)$, $K = K_r - K_l = \int_{t=0} d^{d-1}x x_1 T_{00}$, $J = CRT$
  $\rightarrow$ coincides with the result of Bisognano and Wichman (1976) using Tomita-Takesaki and no path integral!

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  where ψ is a pure state such that $K|\psi >= 0$.
- $\omega = 0$ corresponds to zero mode(A_0)!$\rightarrow$ does not change under flows!
  $\omega \neq 0$ corresponds to non zero modes(A_ω) $\rightarrow$ changes under flows.

Modular Hamiltonian and eigenmodes in Holography: Why modular modes?

- $S(\rho_R) = \frac{\gamma_R}{4G_N} + S(\rho_{\mathcal{H}_R}) + \mathcal{O}(G_N)$ FLM where $\gamma_R \to$ HRRT surface(bulk codimension-2 surface) such that,

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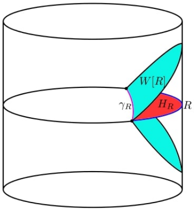

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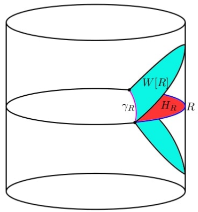

- JLMS $\rightarrow K_R = K_{\mathcal{H}_R} + \mathcal{O}_{G_N}$,
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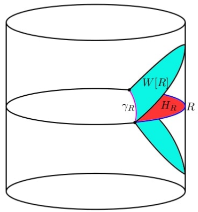
- Entanglement wedge(EW) of R $\to$ bulk domain of dependence $D[\mathcal{H}_R]$ of $\mathcal{H}_R$.



- JLMS $\to K_R = K_{\mathcal{H}_R} + \mathcal{O}_{G_N}$,
  $\implies [\phi(x), K_R] = [\phi(x), K_{\mathcal{H}_R}]$ for bulk x $\to$ spacelike separated to γ_R
- To reconstruct EW generically, one should smear modular evolved boundary operators(JLMS) instead of local boundary operators(HKLL)!

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- modular berry connection: modular zero modes of connected subregions in CFT defines a modular connection in the space of modular Hamiltonians which encodes bulk Riemann curvature in AdS/CFT. (Czech et al. 2017,2019)

Application II: Construction of zero modes in excited states(Das,Ezhuthachan)

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- Similarly we can get the nth order correction perturbatively and get, $A_0^{(n)} = \int_{-\infty}^{\infty} dk \frac{<\left([\Delta^{(n)}, A_0^{(0)}] + [\Delta^{(n-1)}, A_0^{(1)}] + \cdots + [\Delta^{(1)}, A_0^{(n-1)}] + [\Delta^{(0)}, A_0^{(n)}]\right)\Delta^{(0)-1} A_k^{(0)}>}{<B_k^{(0)} \mathcal{O}^i>(1-e^{-k})} B_k^{(0)}$

Connection between OPE blocks and modular algebra

CFT$_2$ modular Hamiltonian:connection to OPE blocks

- An integral expression(shadow formalism) of OPE block of dimension $h_k, \bar{h}_k$ constructed out of OPE of two operators with dimension and spin of $(\Delta_i, l_i), (\Delta_j, l_j)$ is:

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- But positive self adjoint $\Delta \implies$ positive real eigen values!
  $\rightarrow$ ope blocks do not lie in the domain over which the Δ is self-adjoint! $\rightarrow$ do not satisfy orthogonality!(resolution?$\rightarrow$ will see later!)

Construction of the dual of non zero modular modes in CFT$_2$

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- Question: zero modes are localized on the RT surface(geodesic), where are the non-zero modes localized in AdS$_3$?
  answer:$\to$ somewhere inside EW!(JLMS,FL)
  $\to$ more precisely, dual of scalar block $B^{ij}_k(l_{ij}\neq 0)$ is localized inside a Lorentzian cylinder!(DE)

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- $\rightarrow$ an non-trivial example of entanglement wedge observable!

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- Lastly, we commented on the connection of OPE blocks and modular algebra in AdS_3/CFT_2:
  $\rightarrow$ OPE blocks $=$ modular modes(zero and non zero) for vacuum CFT_2
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Thank You!