Quantum phase transitions in condensed matter

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Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. The holographic view: charged black-branes

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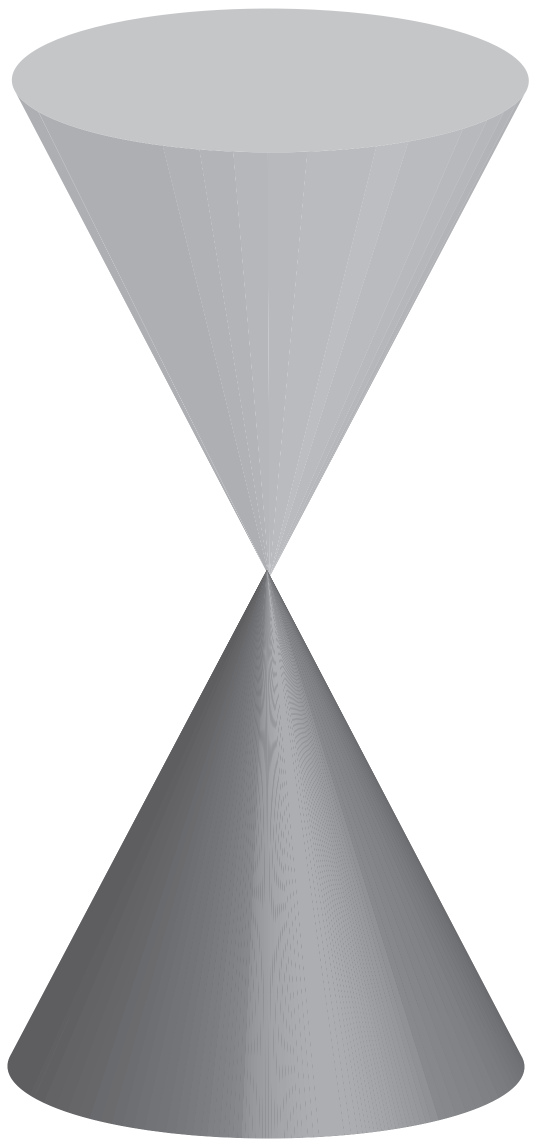
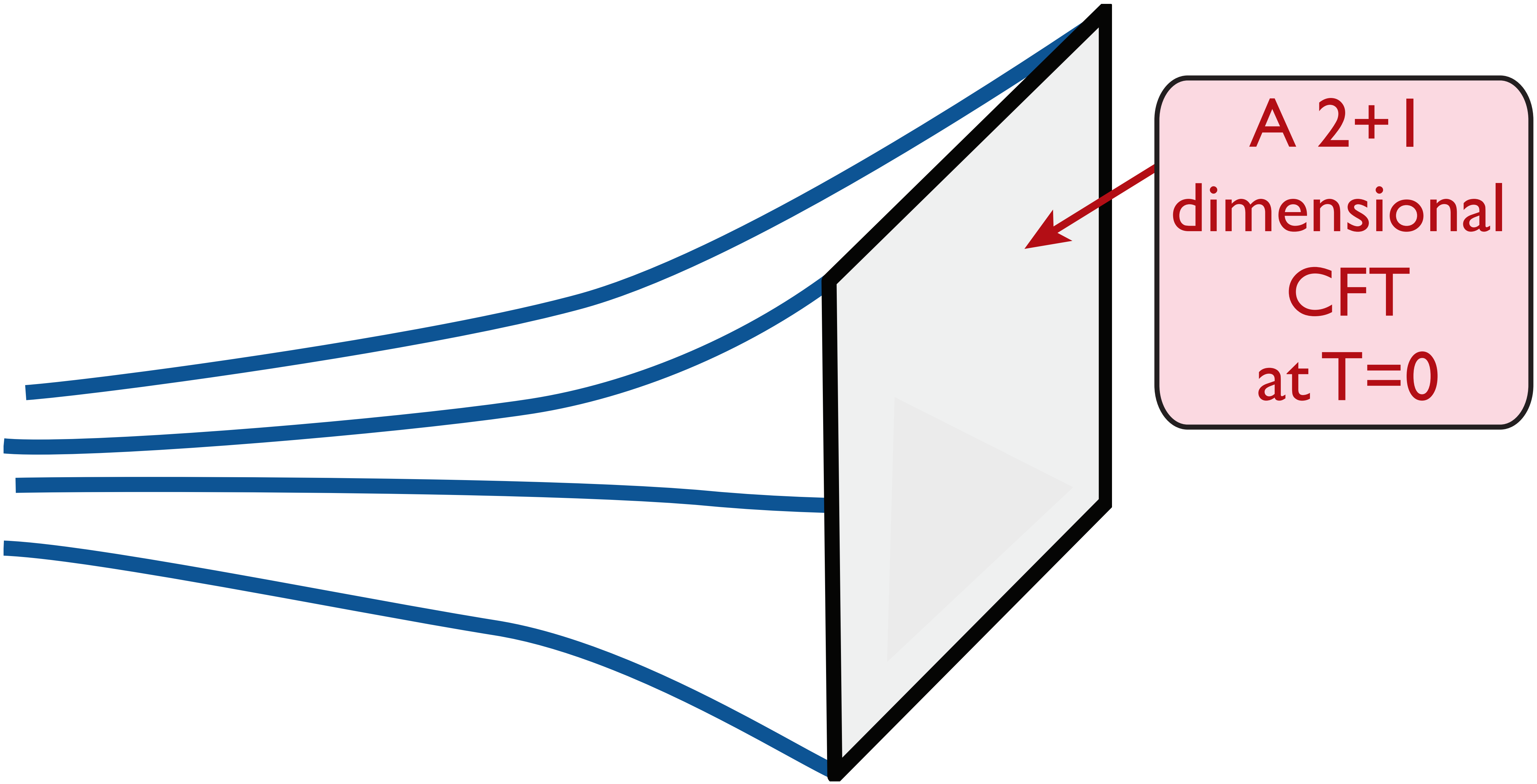
Begin with a CFT



Holographic representation: AdS$_4$



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

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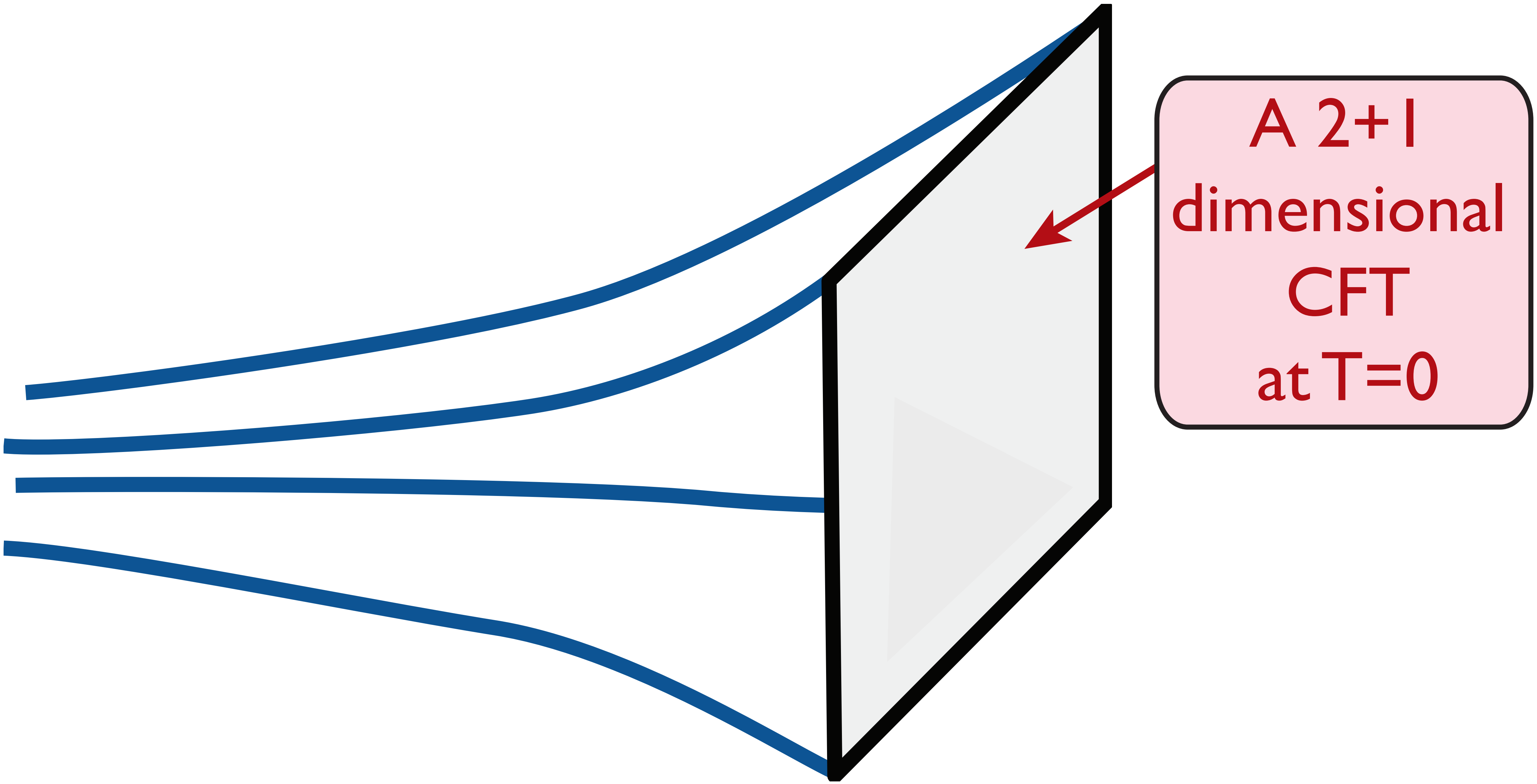
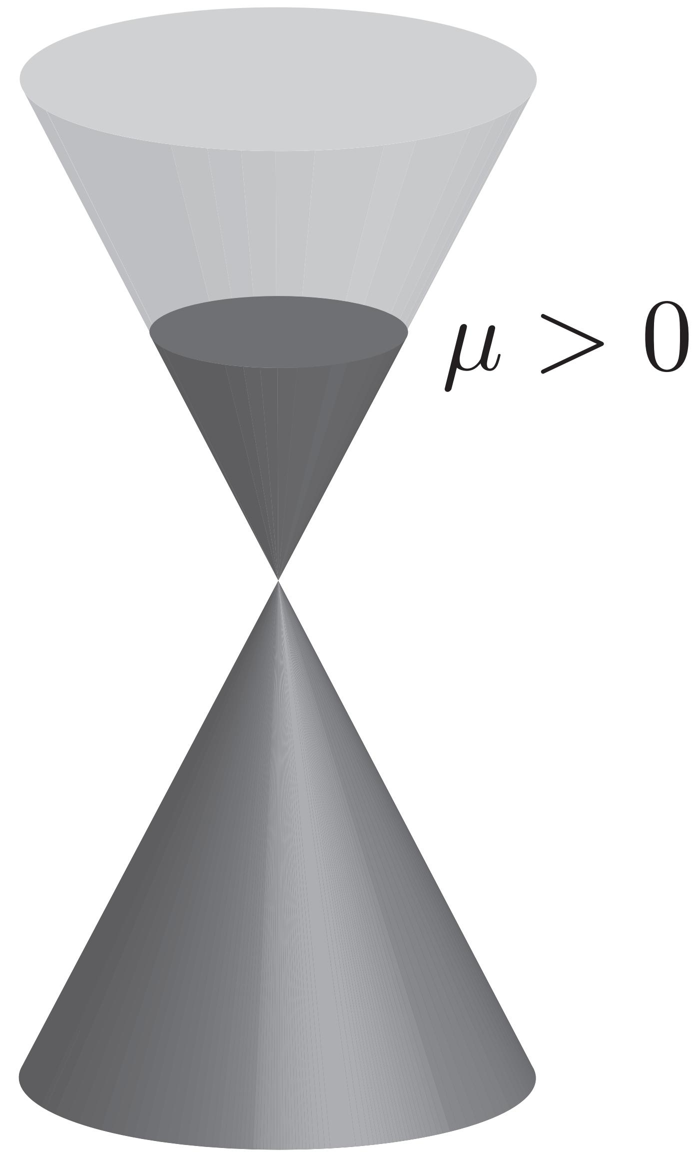


$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$
with $f(r) = 1$

Apply a chemical potential



AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

This is to be solved subject to the constraint

$$A_\mu(r \to 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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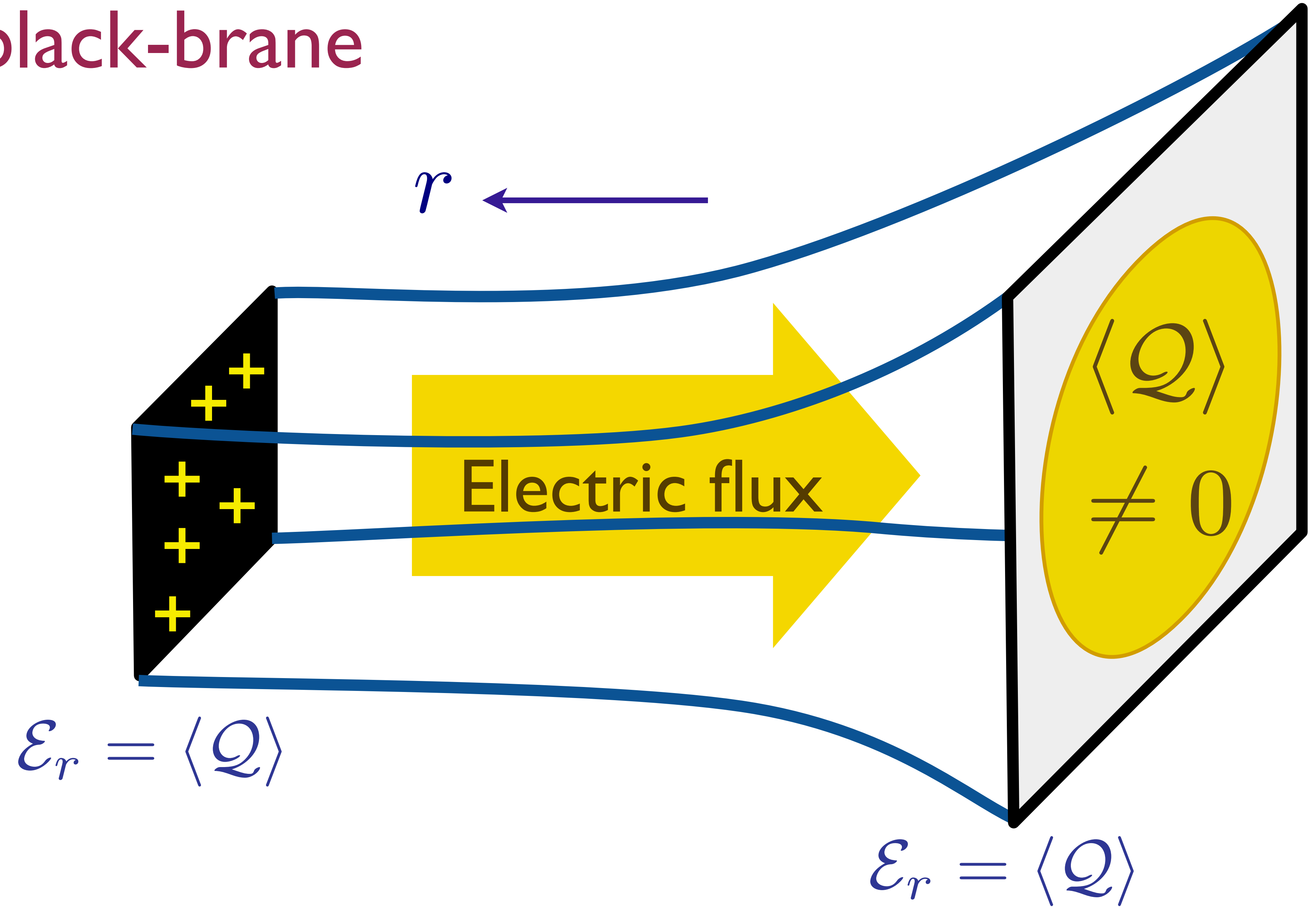
At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

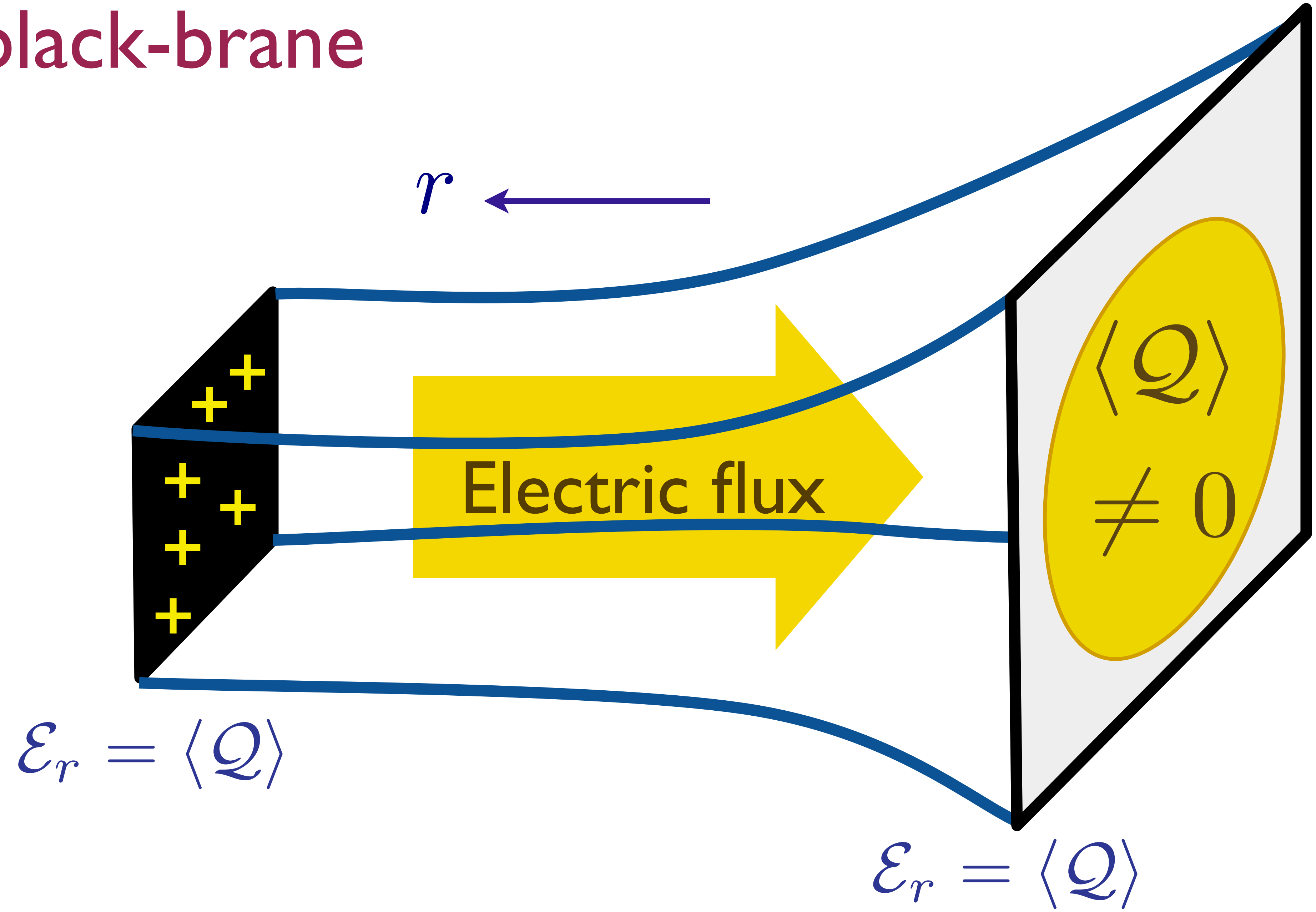
The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4-Reissner-Nordtröm black-brane



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right)$ and $R = \frac{\sqrt{6}Lg_4}{\kappa\mu}$, and $A_\tau = \mu\left(1 - \frac{r}{R}\right)$

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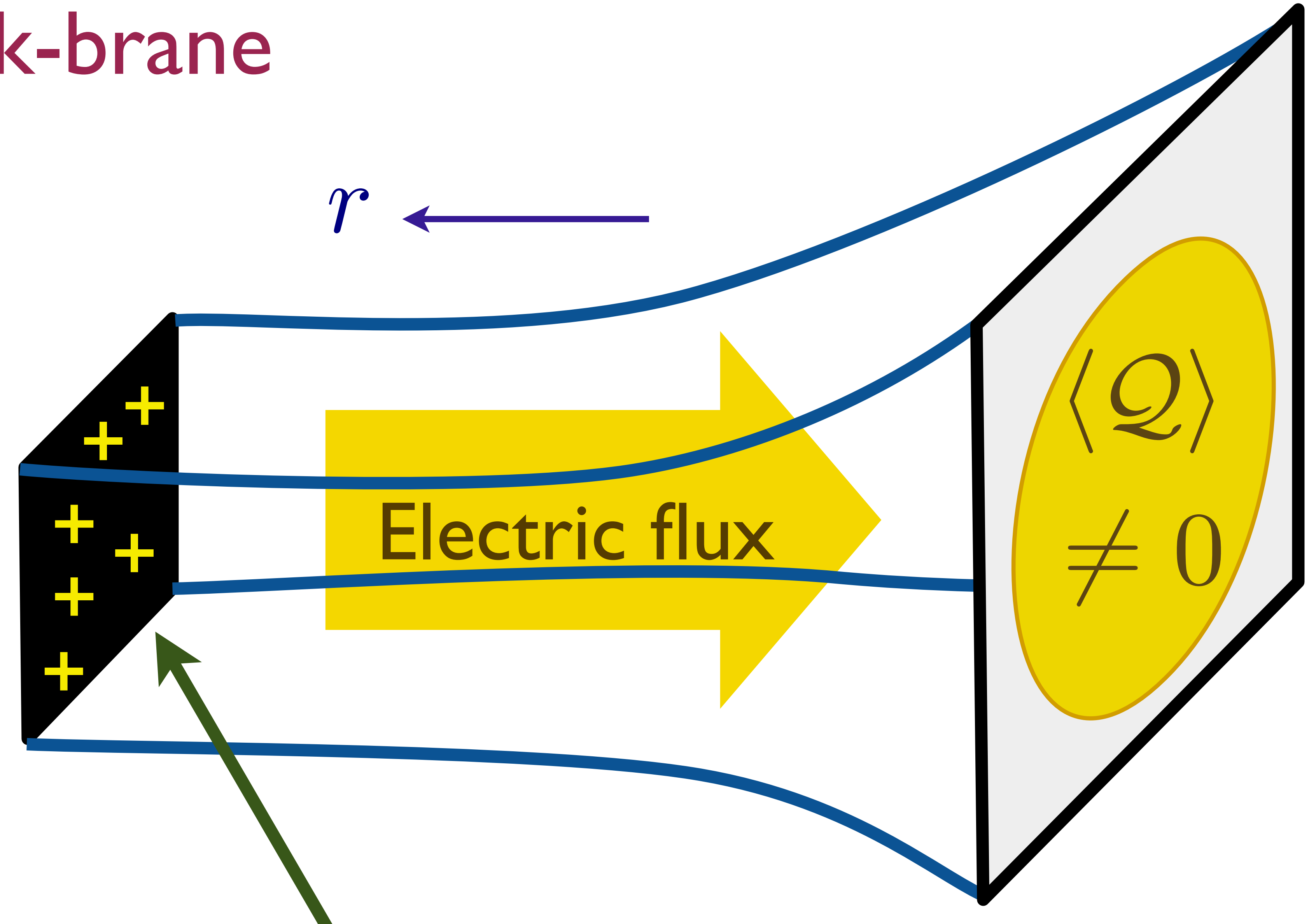


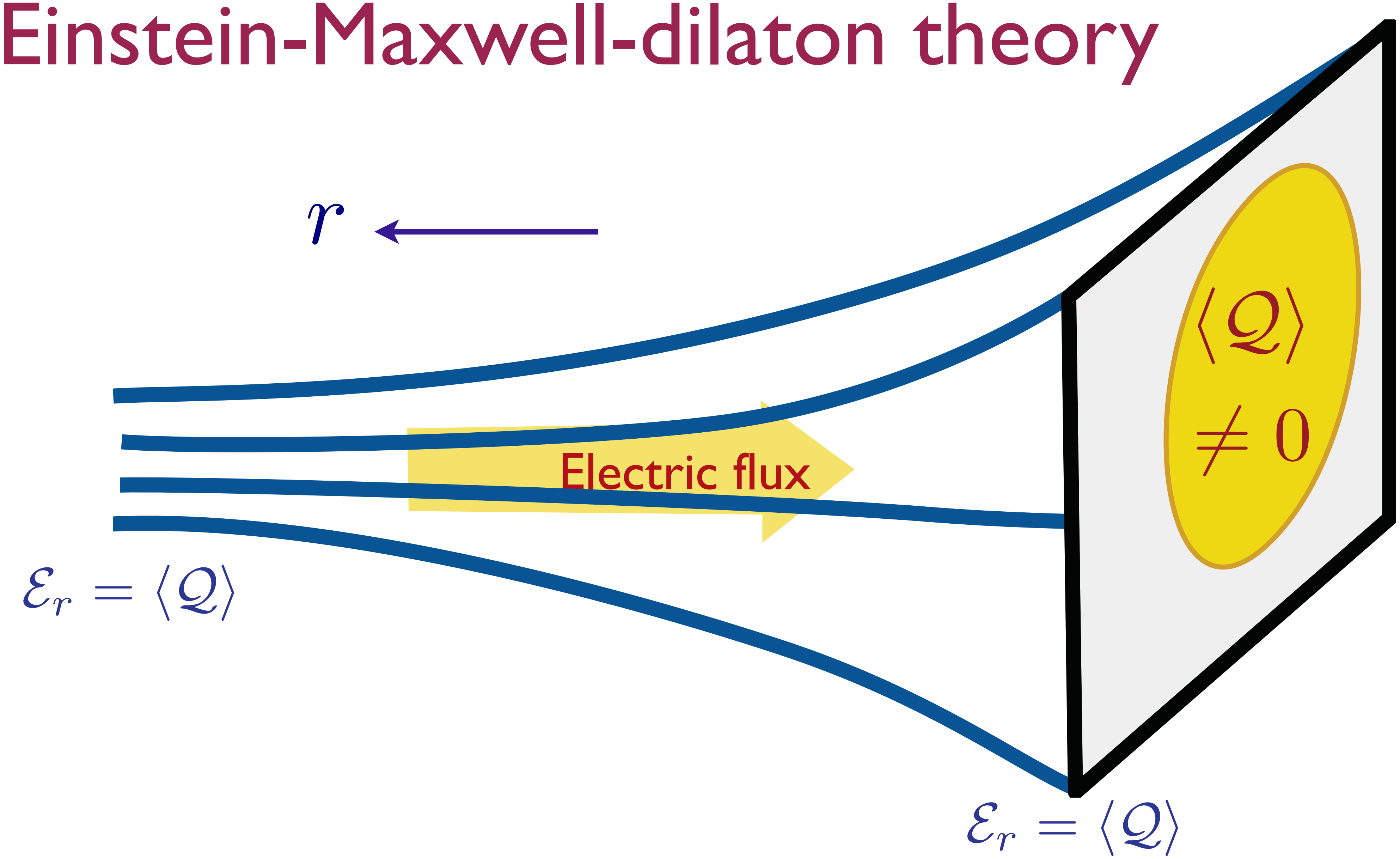
At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $AdS_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the above form with

$$\begin{aligned} \theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}. \end{aligned}$$

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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

This is the most general metric which is invariant under the scale transformation

$$\begin{aligned} x_i &\to \zeta\, x_i \\ t &\to \zeta^z\, t \\ ds &\to \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.
We have used reparametrization invariance in r to define it so that it scales as

$$r \to \zeta^{(d-\theta)/d} r\,.$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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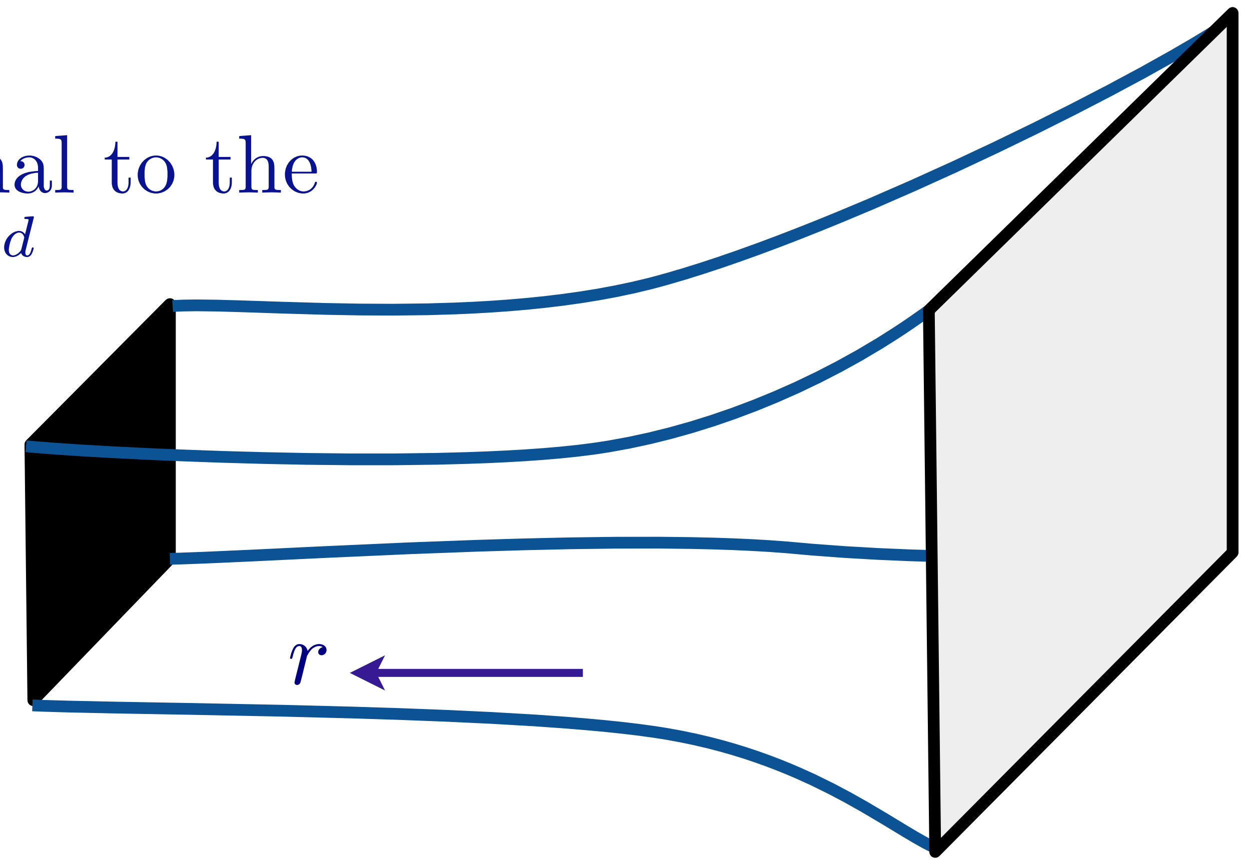
At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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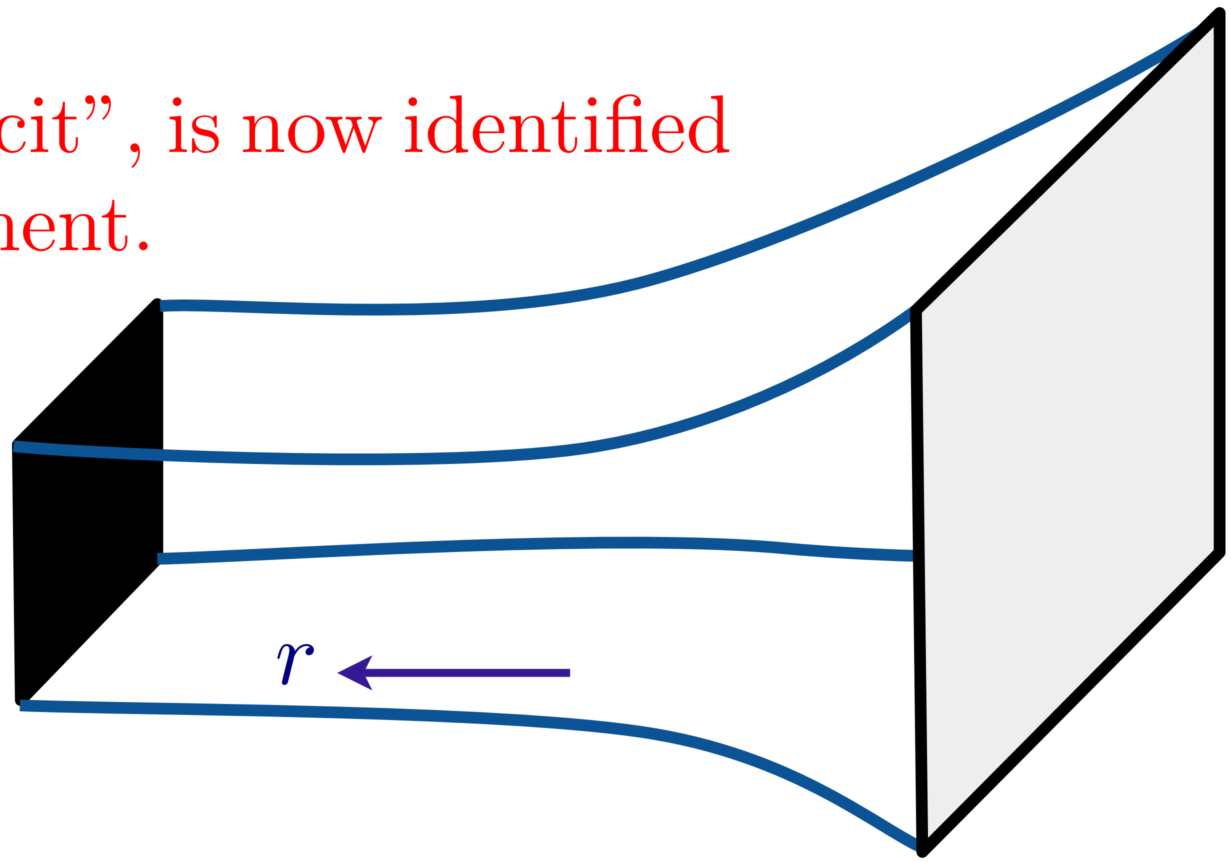
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$, the "dimension deficit", is now identified as the violation of hyperscaling exponent.



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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, with $\theta = d - 1$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid

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Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \begin{cases} P & , \quad \text{for } \theta < d-1 \\ P \ln P & , \quad \text{for } \theta = d-1 \\ P^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases} .$$

The log-violation of "area law" again prefers the value $\theta = d - 1$. Moreover, the co-efficient of $P \ln P$ is
(i) independent of the shape of the entangling region just as expected for a circular Fermi surface, and
(ii) has the same universal dependence on $\mathcal{Q}$ expected from the Luttinger theorem.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
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- Are there N^2 Fermi surfaces of 'quarks' with $k_F \sim 1$, or 1 Fermi surface of a 'baryon' with $k_F \sim N^2$?
- Why is k_F not observable as Friedel oscillations in correlators of the density, and other gauge-neutral operators ?

Answer: need non-perturbative effects of monopole operators
T. Faulkner and N. Iqbal, arXiv:1207.4208;
S. Sachdev, Phys. Rev. D **86**, 126003 (2012)

R. A. Davison, M. Goykhman, and A. Parnachev, arXiv:1312.0463.

Consider the truncation of IIB supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$ where two of the three U(1) charges are equal and non-vanishing while the third one is zero (S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010)). In the IR limit, the bulk metric is of the general scaling form, but with $\theta = -z$ and $z \to \infty$:

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^6} + \frac{dr^2}{r^2} + dx_i^2\right).$$

This metric is conformal to $\mathrm{AdS}_2 \times \mathrm{R}^3$. From our results before, we have

the entropy $S \sim T$ and the entanglement entropy $S_E \sim P$.

Key difference from a Landau Fermi liquid are the equality of hydrodynamic and zero sound velocities, and a viscosity $\eta = S/(4\pi)$.

R. A. Davison, M. Goykhman, and A. Parnachev, arXiv:1312.0463.

Claim: all physical properties of the holographic theory can be interpreted in a theory of a Fermi surface of 'baryons' with $k_F \sim N^{2/3}$, $v_F \sim N^{-2/3}$, and interactions such that the system is precisely at the nematic quantum critical point in $d = 3$!

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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography