From the SYK model to a theory of the strange metal

International Centre for Theoretical Sciences, Bengaluru

Subir Sachdev

December 8, 2017

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS

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Magnetotransport in a model of a disordered strange metal

Aavishkar A. Patel,[1,2] John McGreevy,[3] Daniel P. Arovas,[3] and Subir Sachdev[1,4,5]

[1] *Department of Physics, Harvard University, Cambridge MA 02138, USA*

[2] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara CA 93106-4030, USA*

[3] *Department of Physics, University of California at San Diego, La Jolla, CA 92093, USA*

[4] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada N2L 2Y5*

[5] *Department of Physics, Stanford University, Stanford CA 94305, USA*

Aavishkar Patel

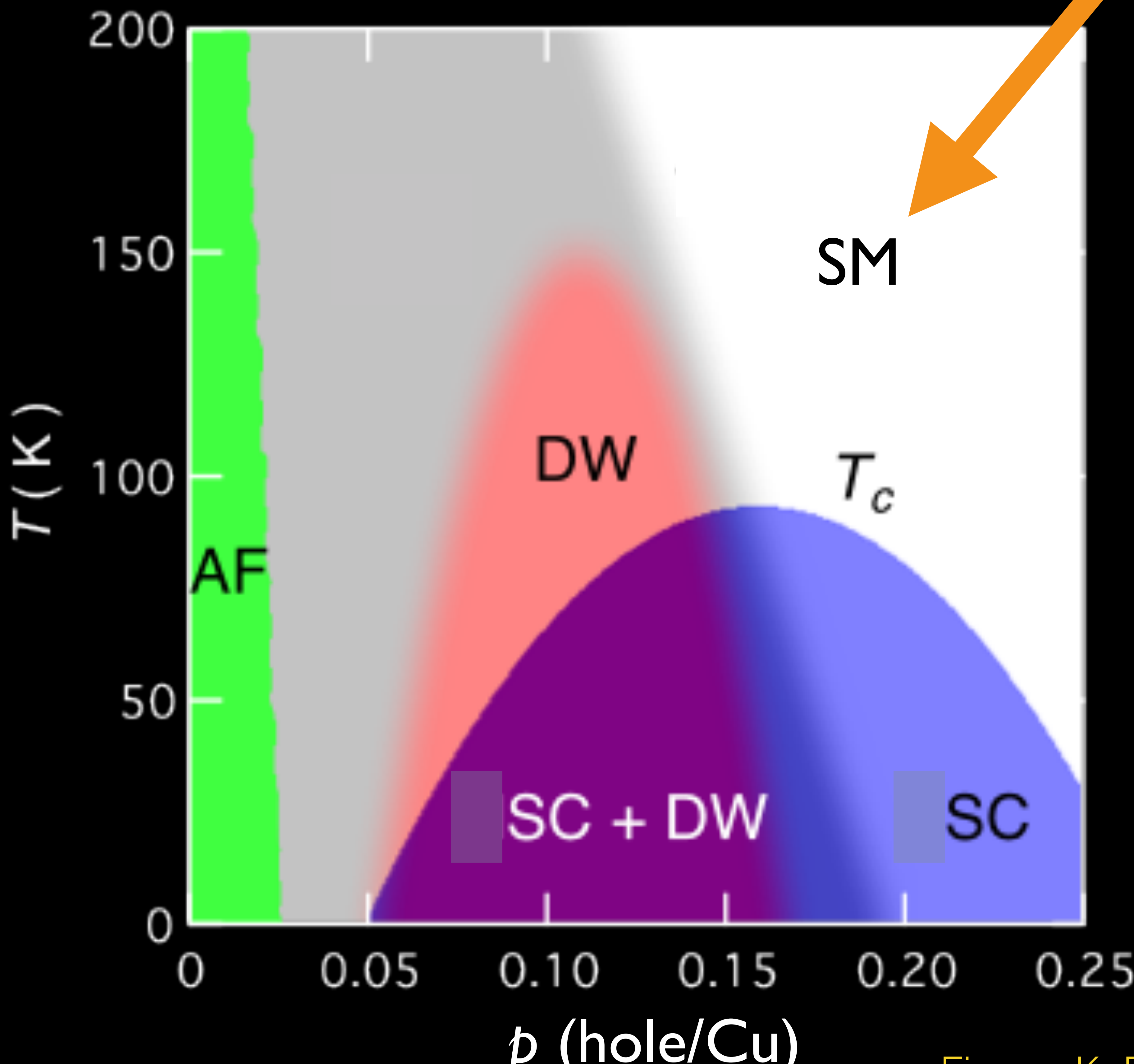
Quantum matter with quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter without quasiparticles

Strange metal



Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

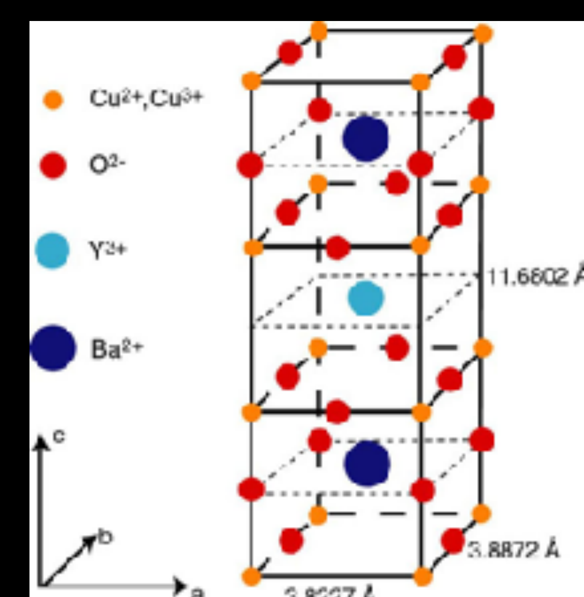
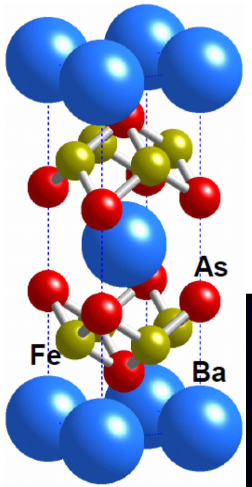
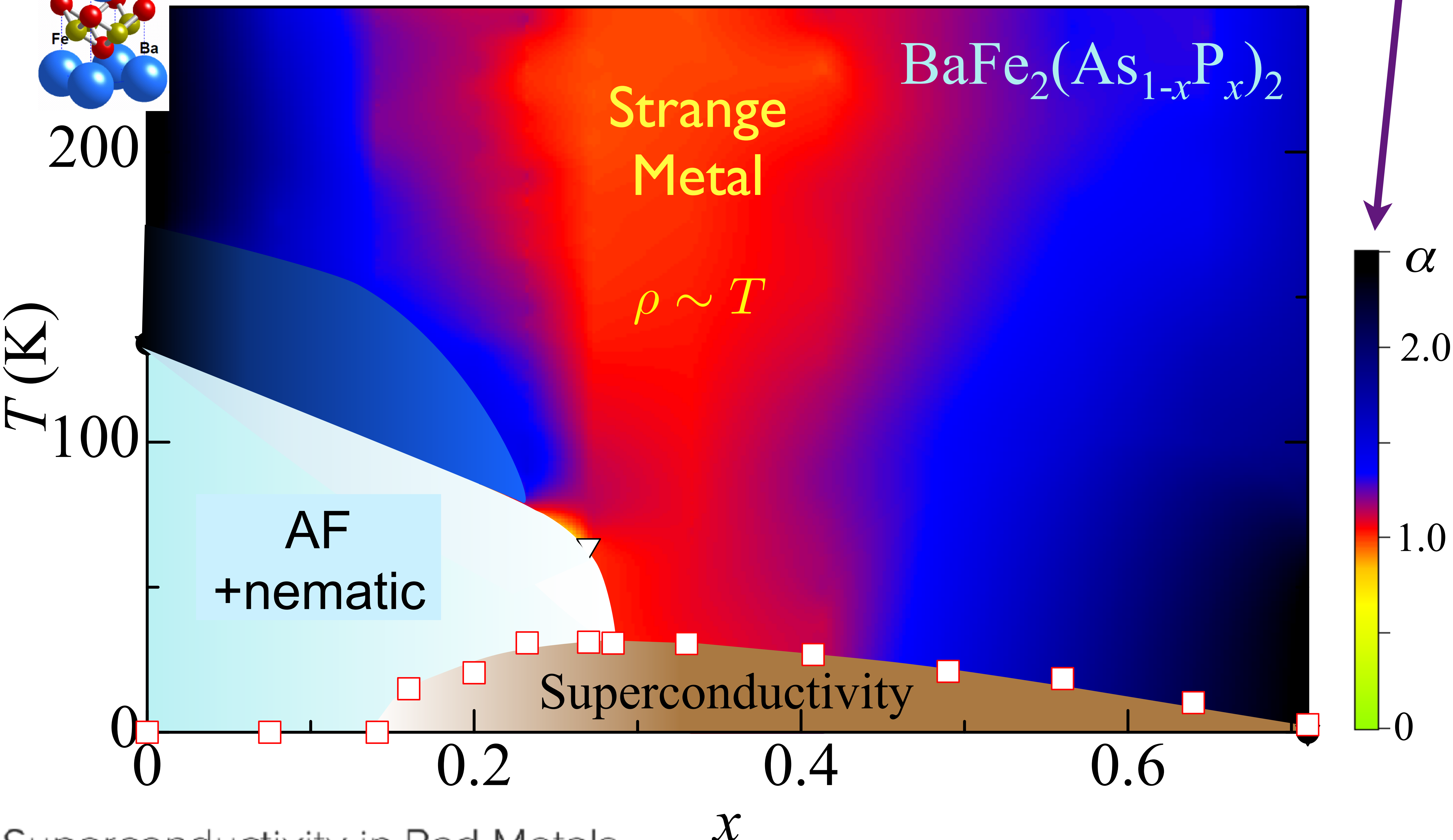


Figure: K. Fujita and J. C. Seamus Davis

Quantum matter without quasiparticles

Resistivity
$\sim \rho_0 + AT^\alpha$





Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

“Strange”,

“Bad”,

or “Incoherent”,

metal has a resistivity, ρ, which obeys

$$\rho \sim T,$$

and

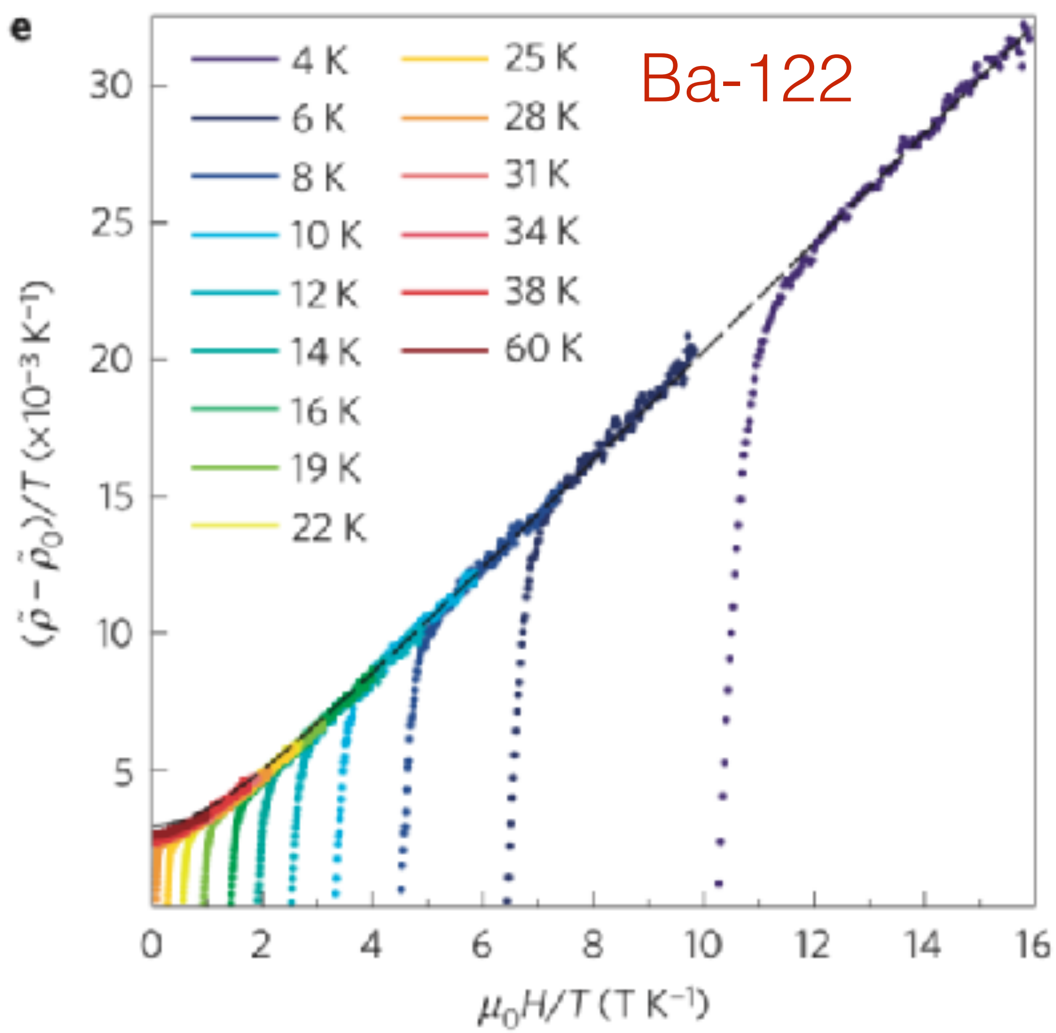
in some cases $\rho \gg h/e^2$

(in two dimensions),

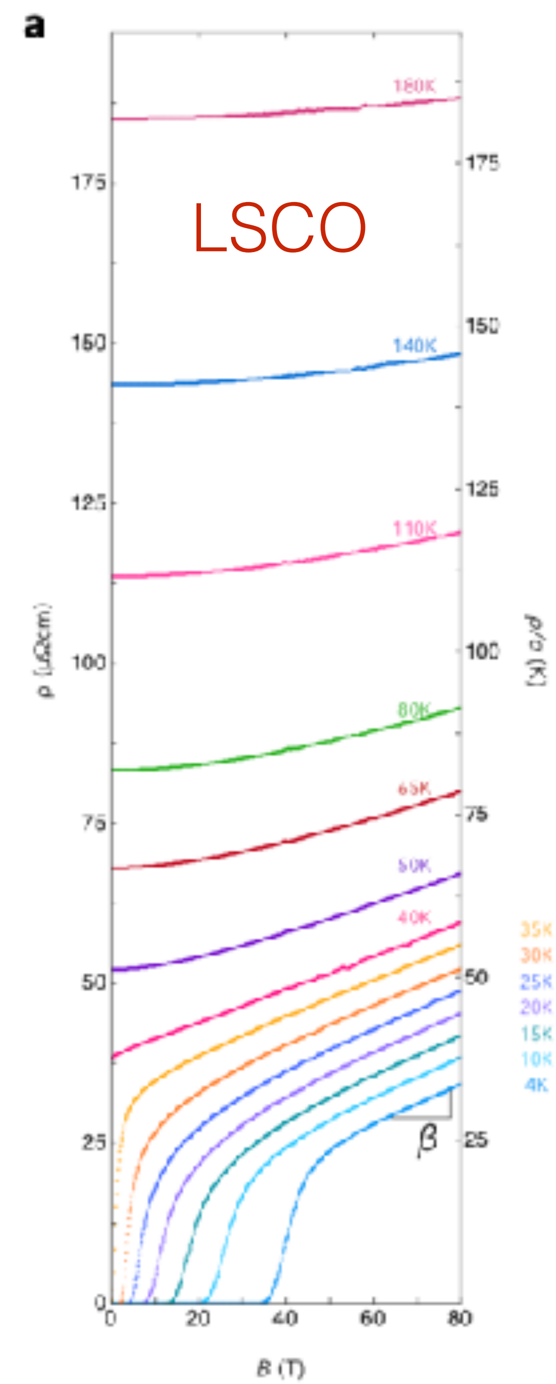
where h/e^2 is the quantum unit of resistance.

Strange metals just got stranger…

B-linear magnetoresistance!?



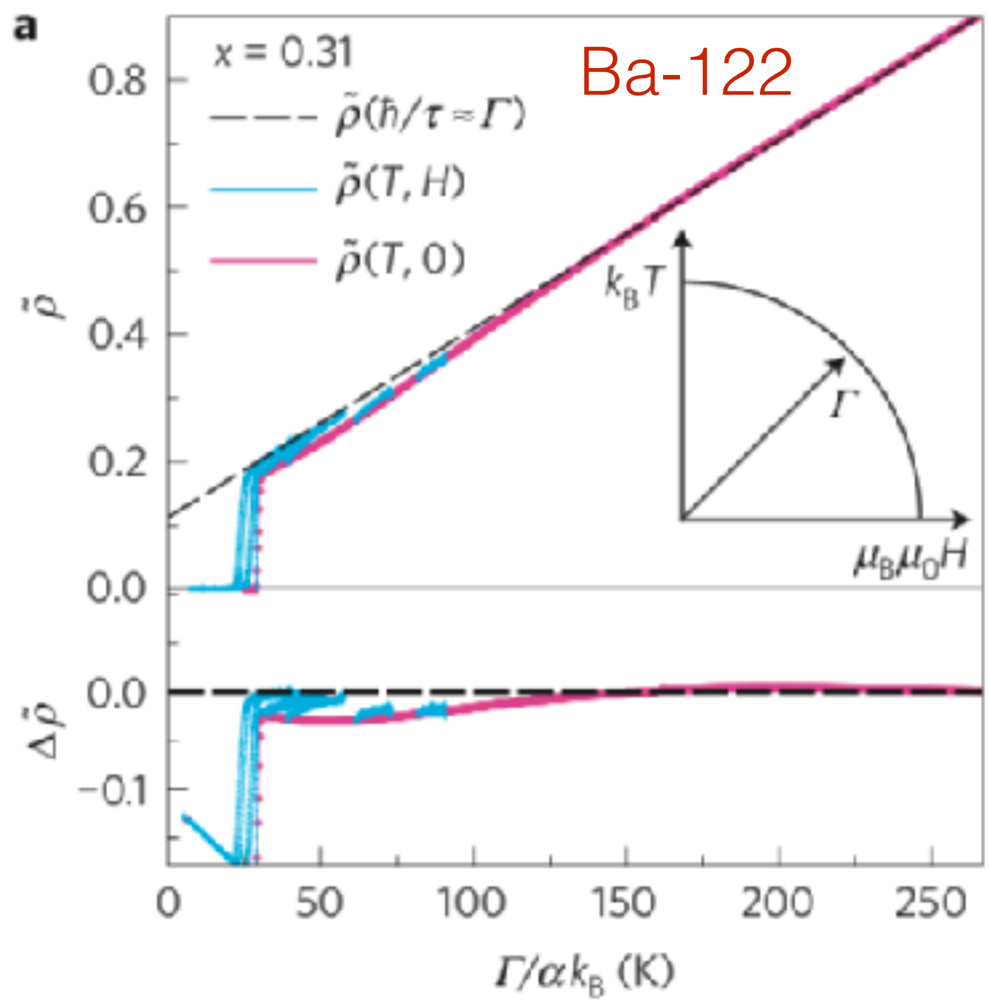
1. M. Hayes et. al., Nat. Phys. 2016



P. Giraldo-Gallo et. al., arXiv:1705.05806

Strange metals just got stranger…

Scaling between B and T !?



$$\rho(H,T)-\rho(0,0)\propto\sqrt{(\alpha k_B T)^2+(\gamma\mu_B\mu_0 H)^2}\equiv\Gamma$$

I. M. Hayes et. al., Nat. Phys. 2016

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:** The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

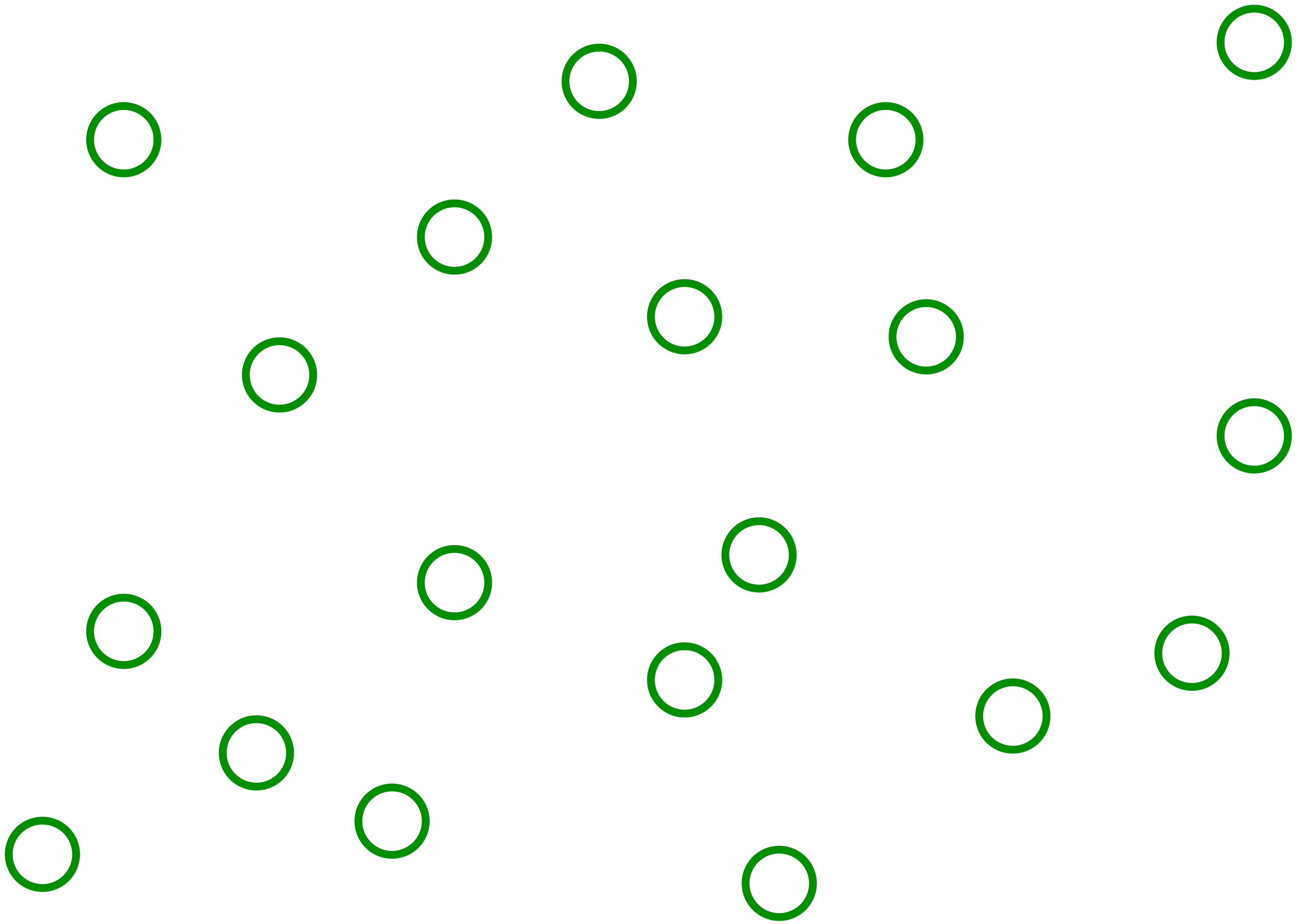
Quantum matter <u>with</u> quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

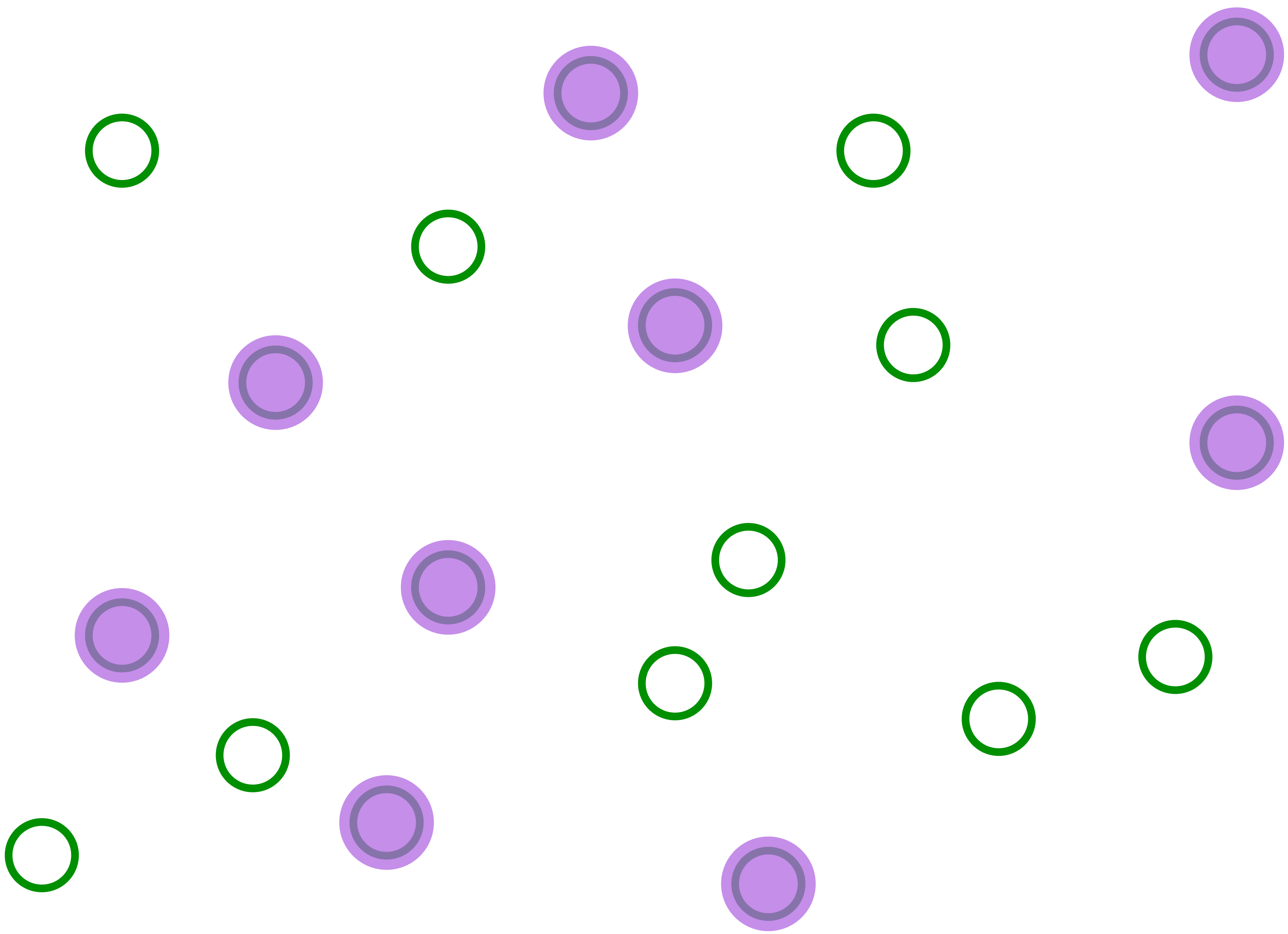
where E_F is the Fermi energy.

A simple model of a metal with quasiparticles



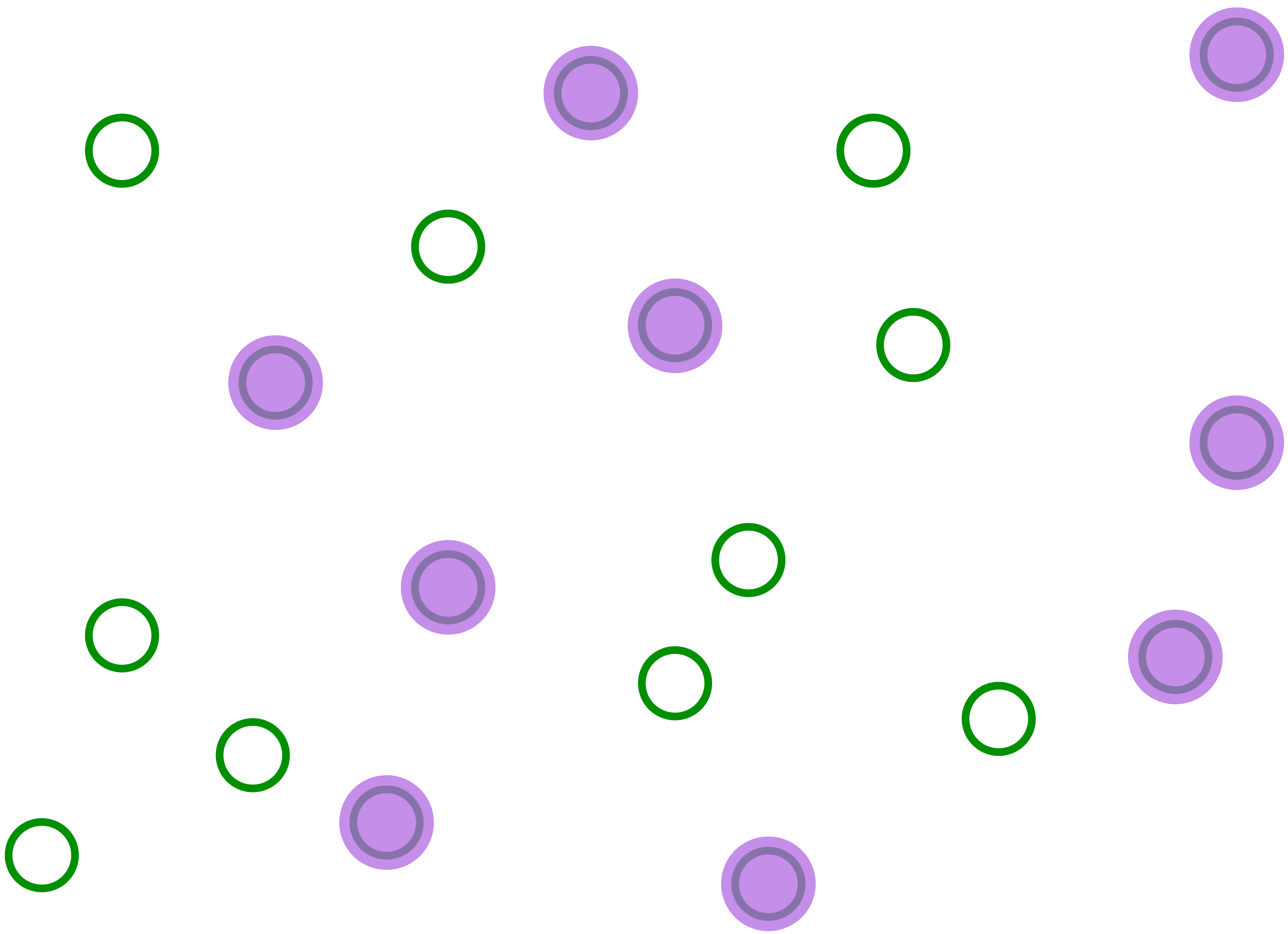
Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

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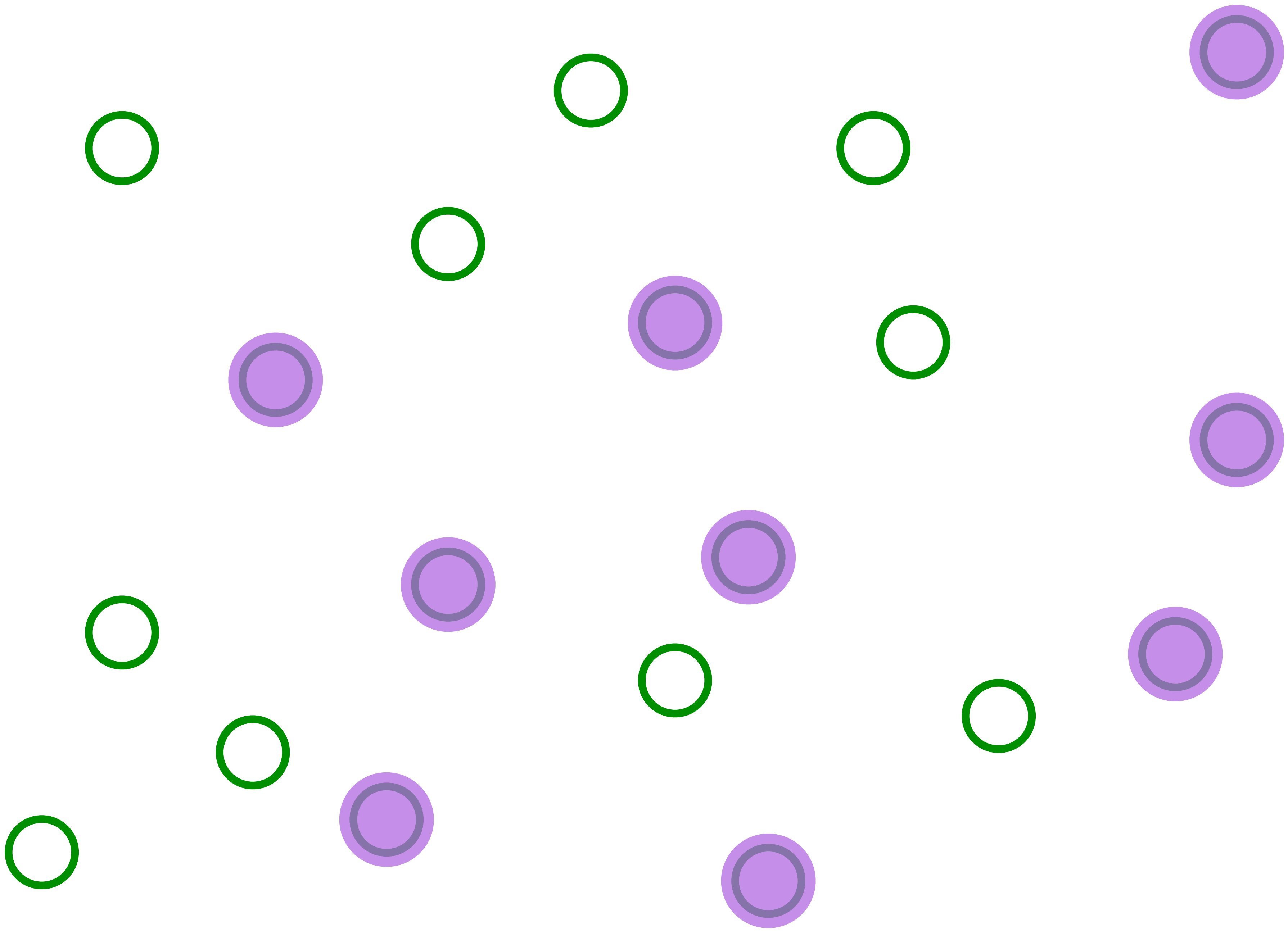

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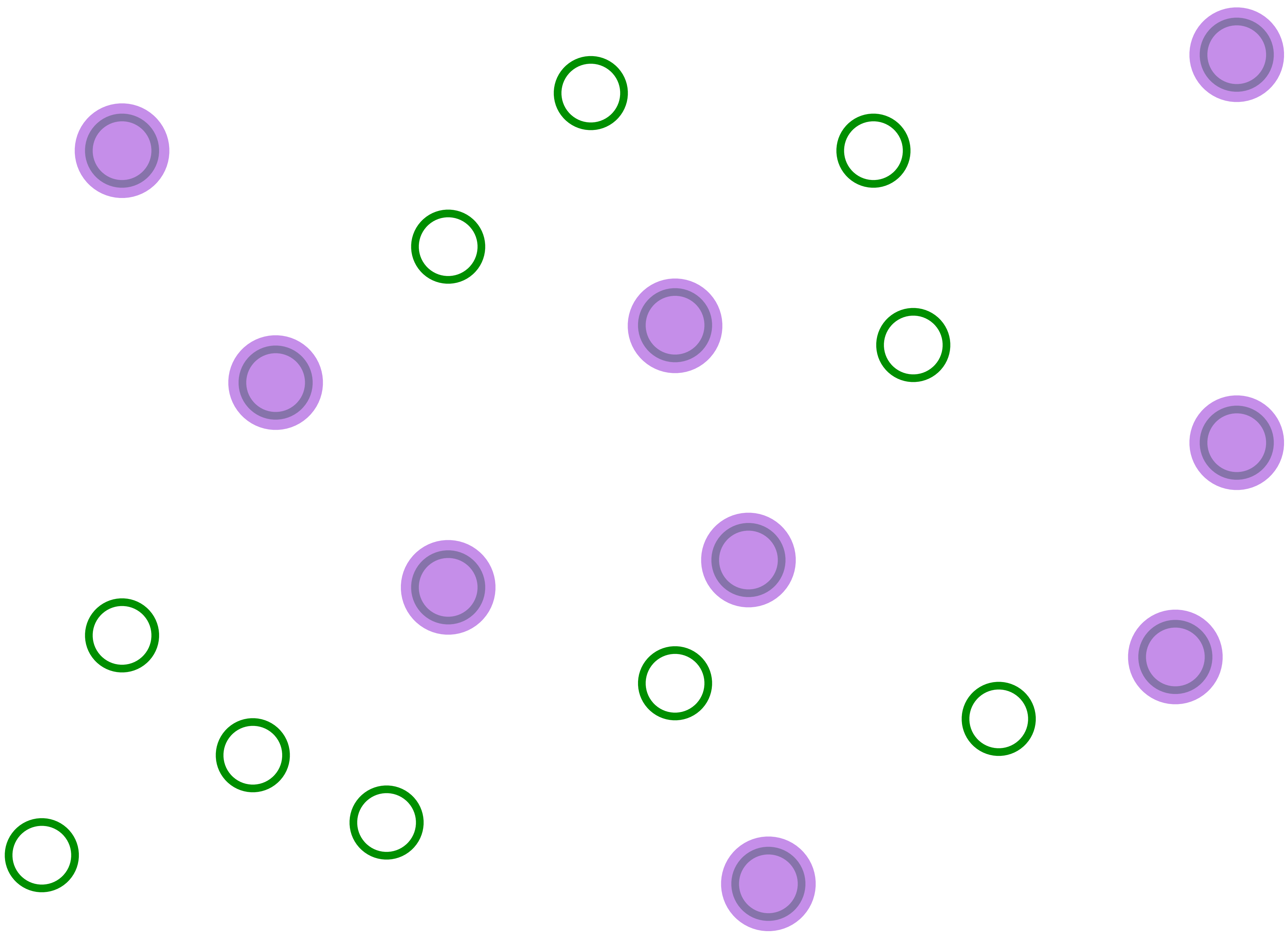

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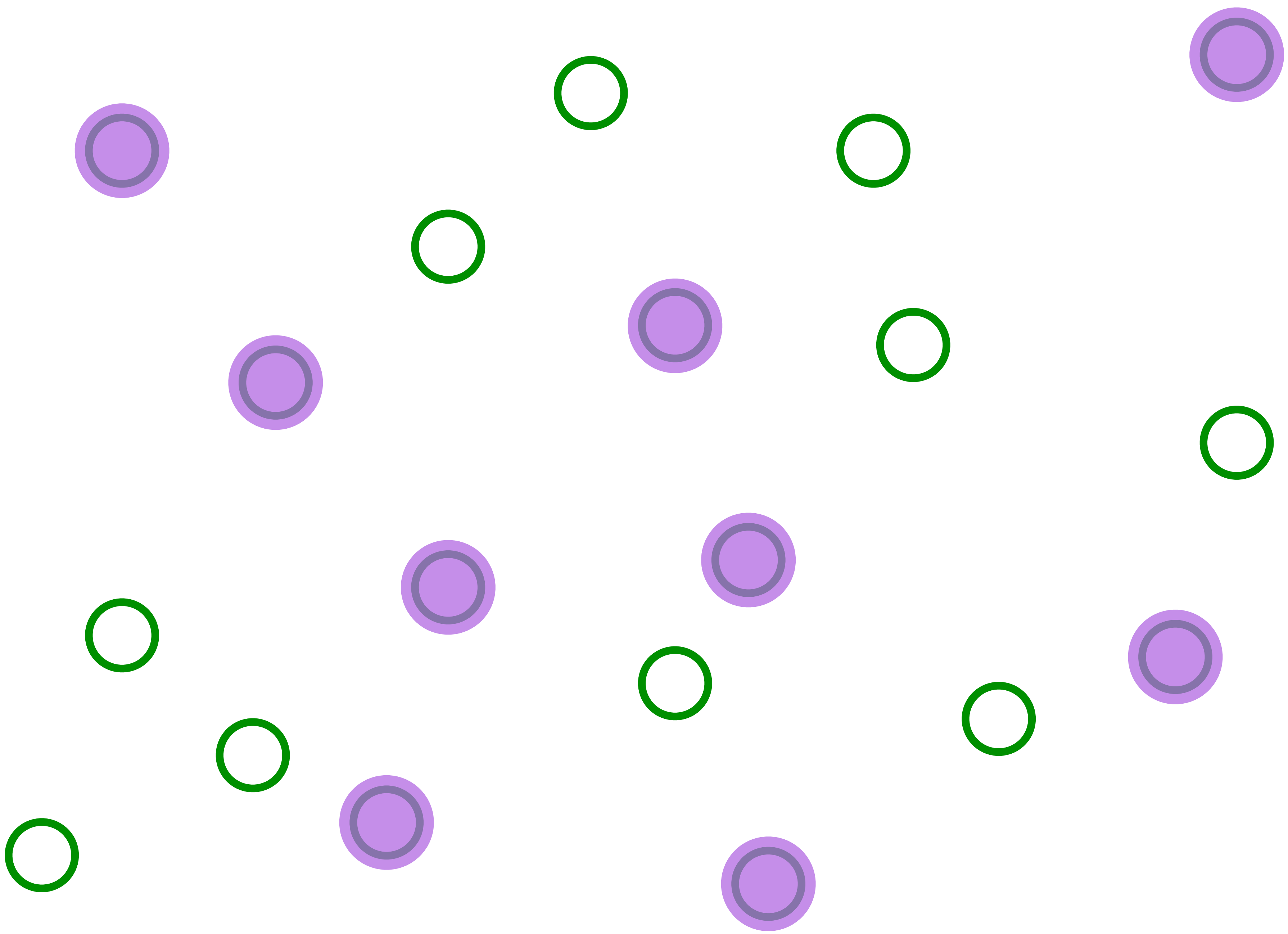

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

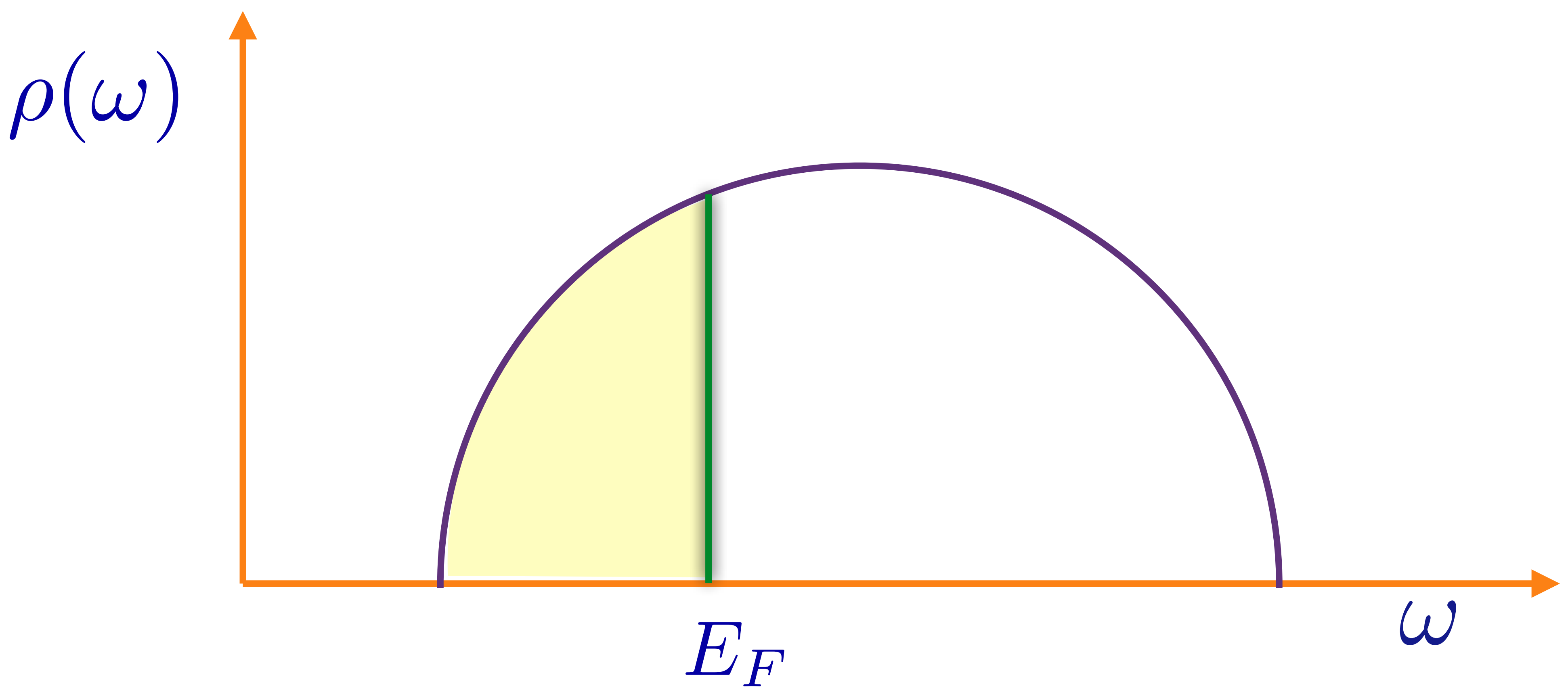
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

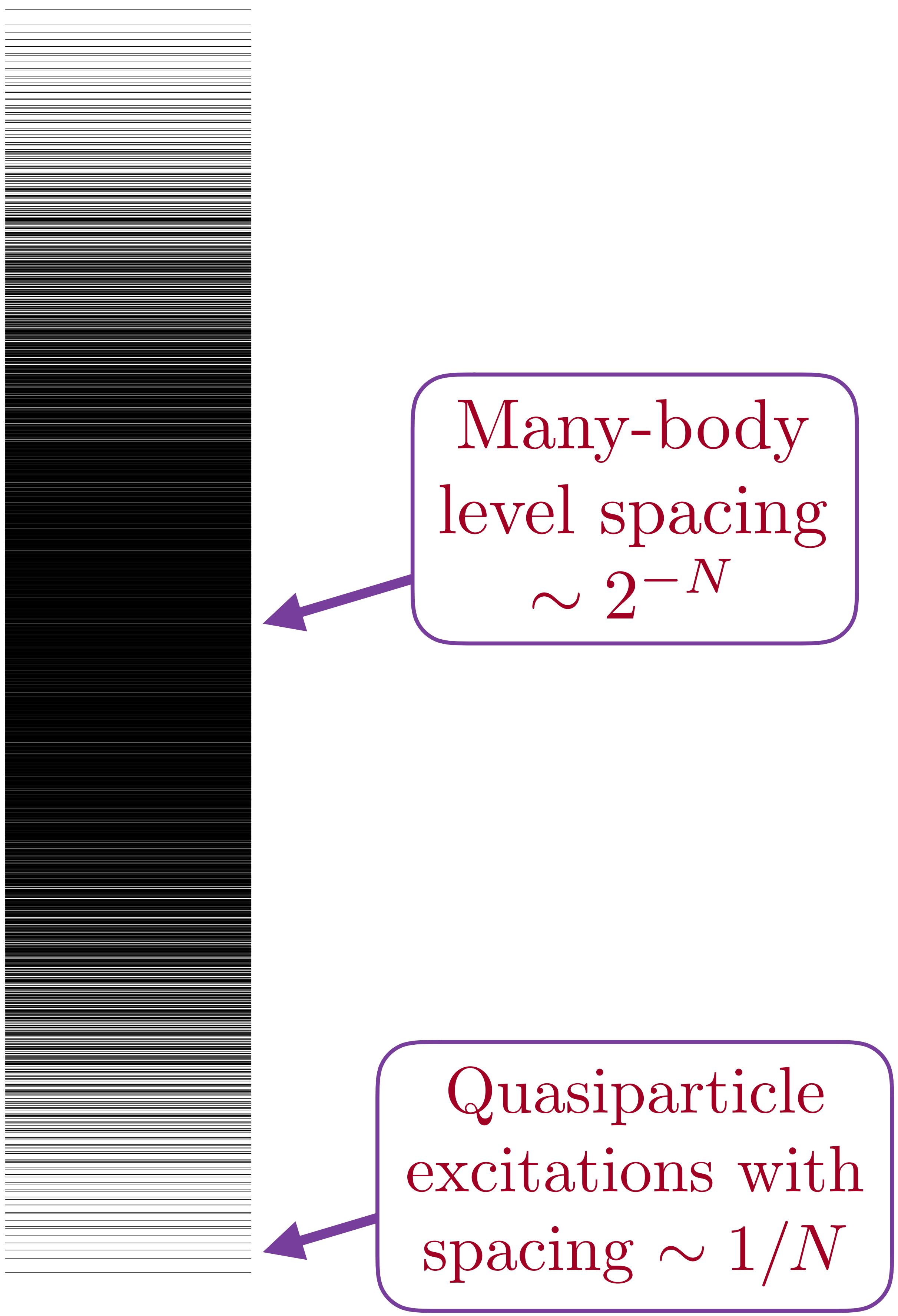
Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



A simple model of a metal with quasiparticles



There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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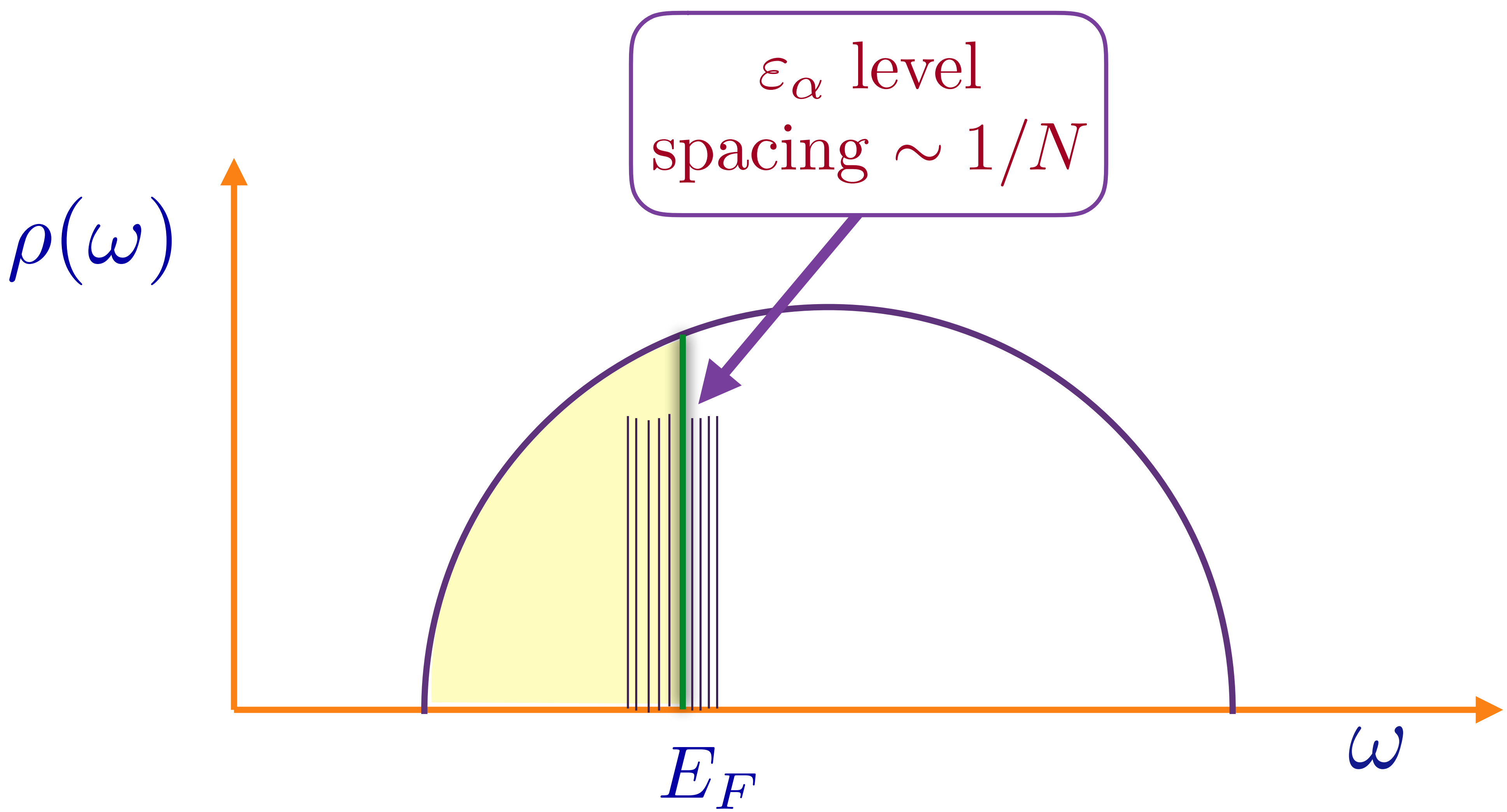

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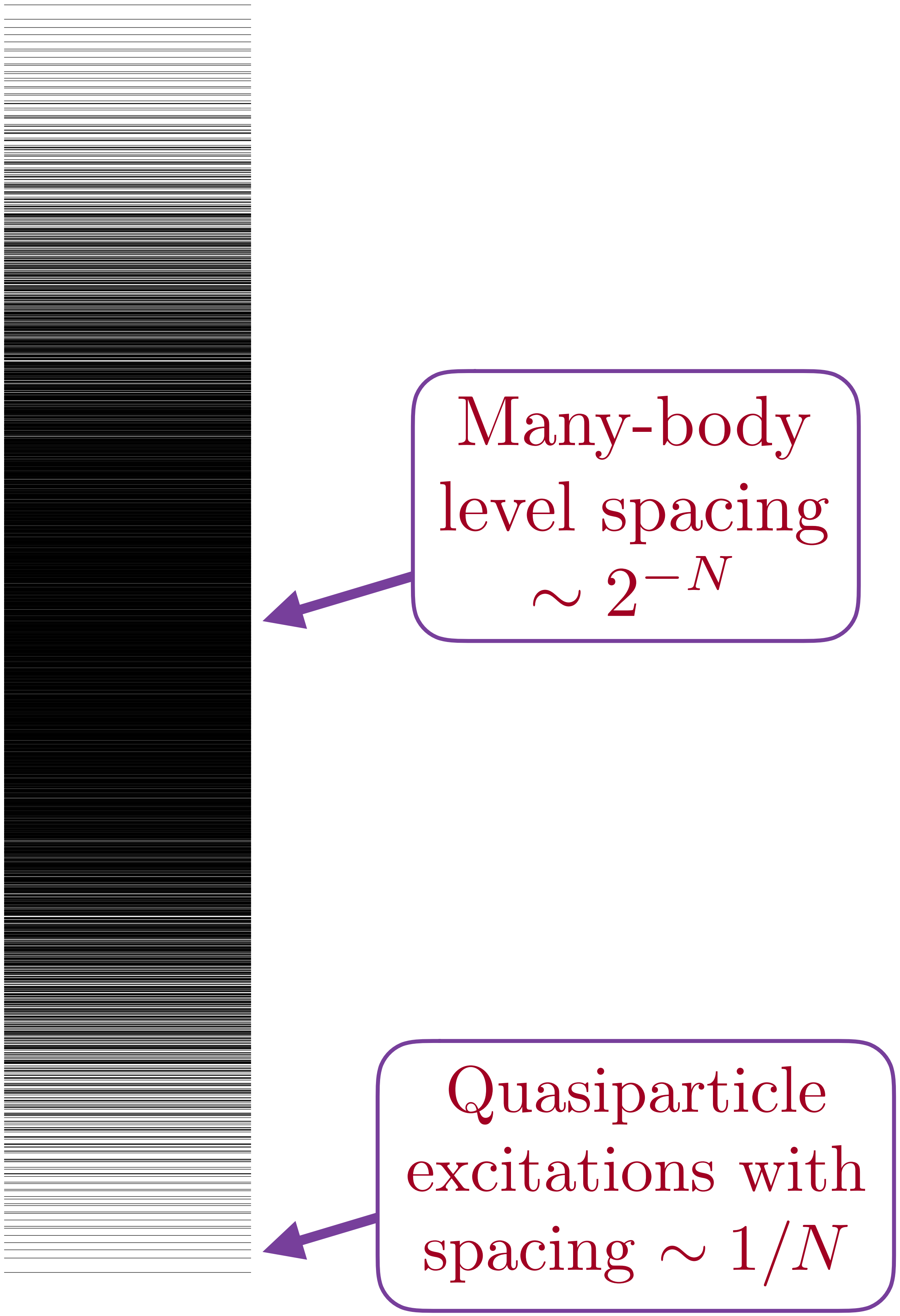

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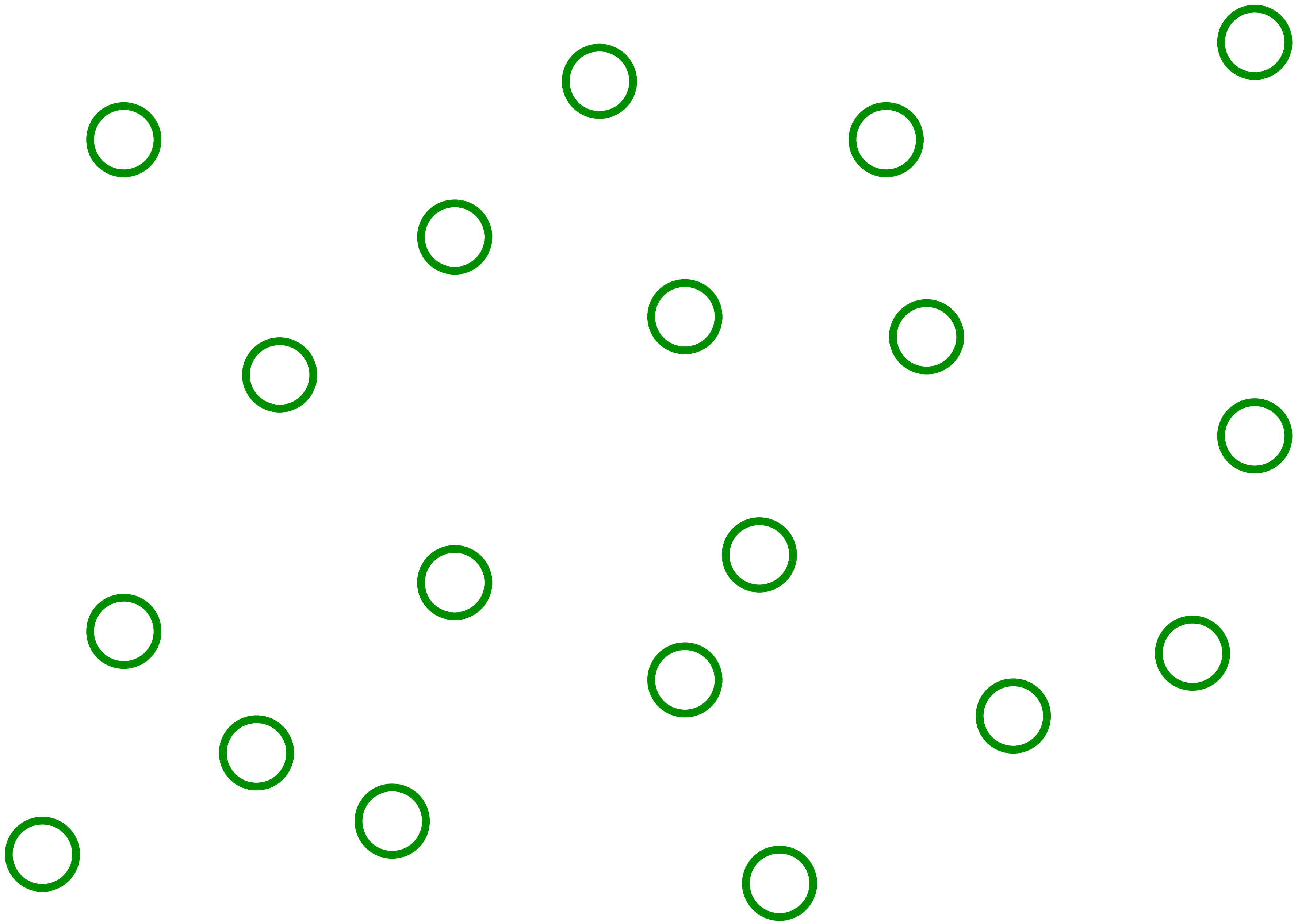
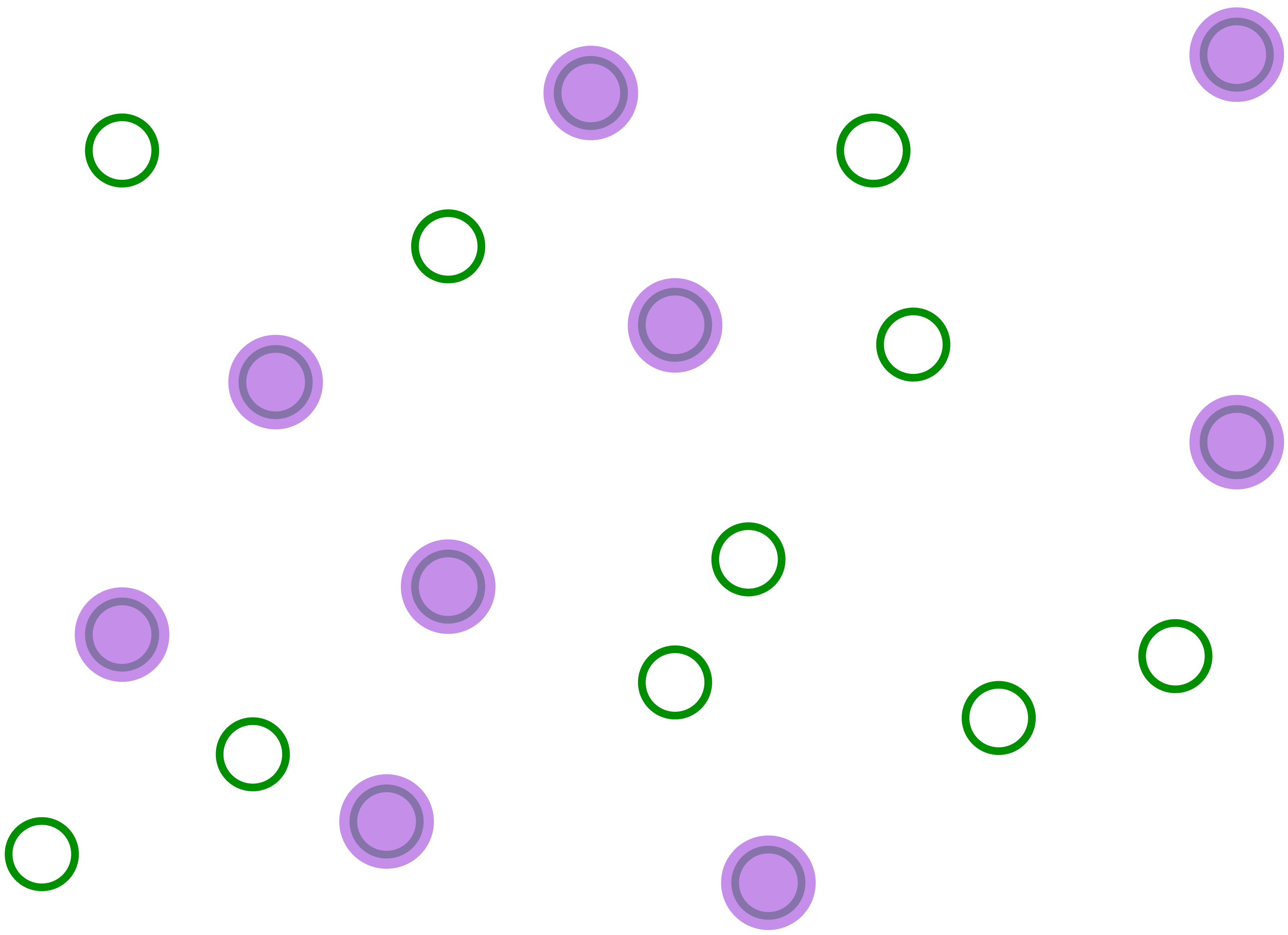
The Sachdev-Ye-Kitaev (SYK) model



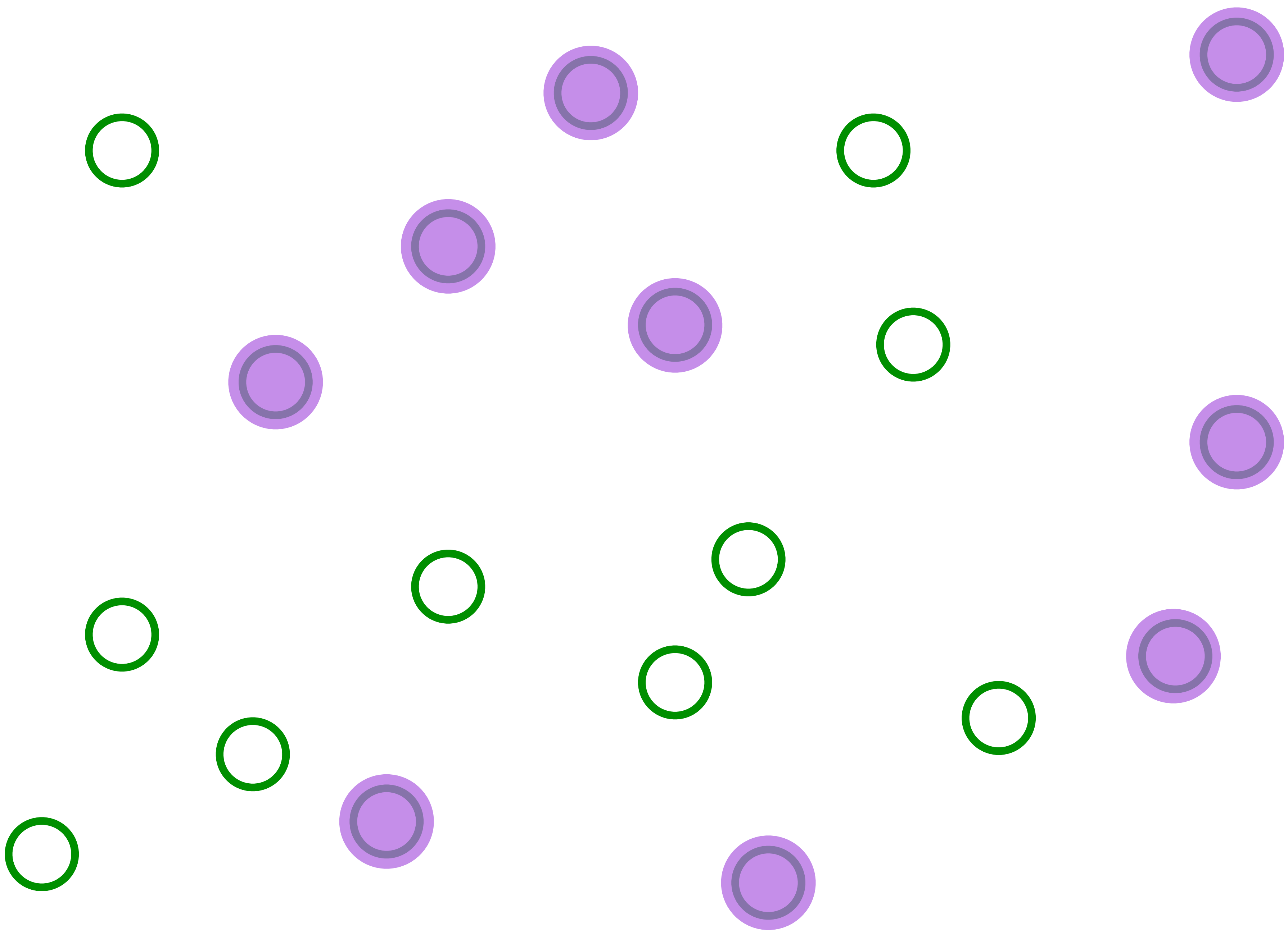
Pick a set of random positions

The SYK model



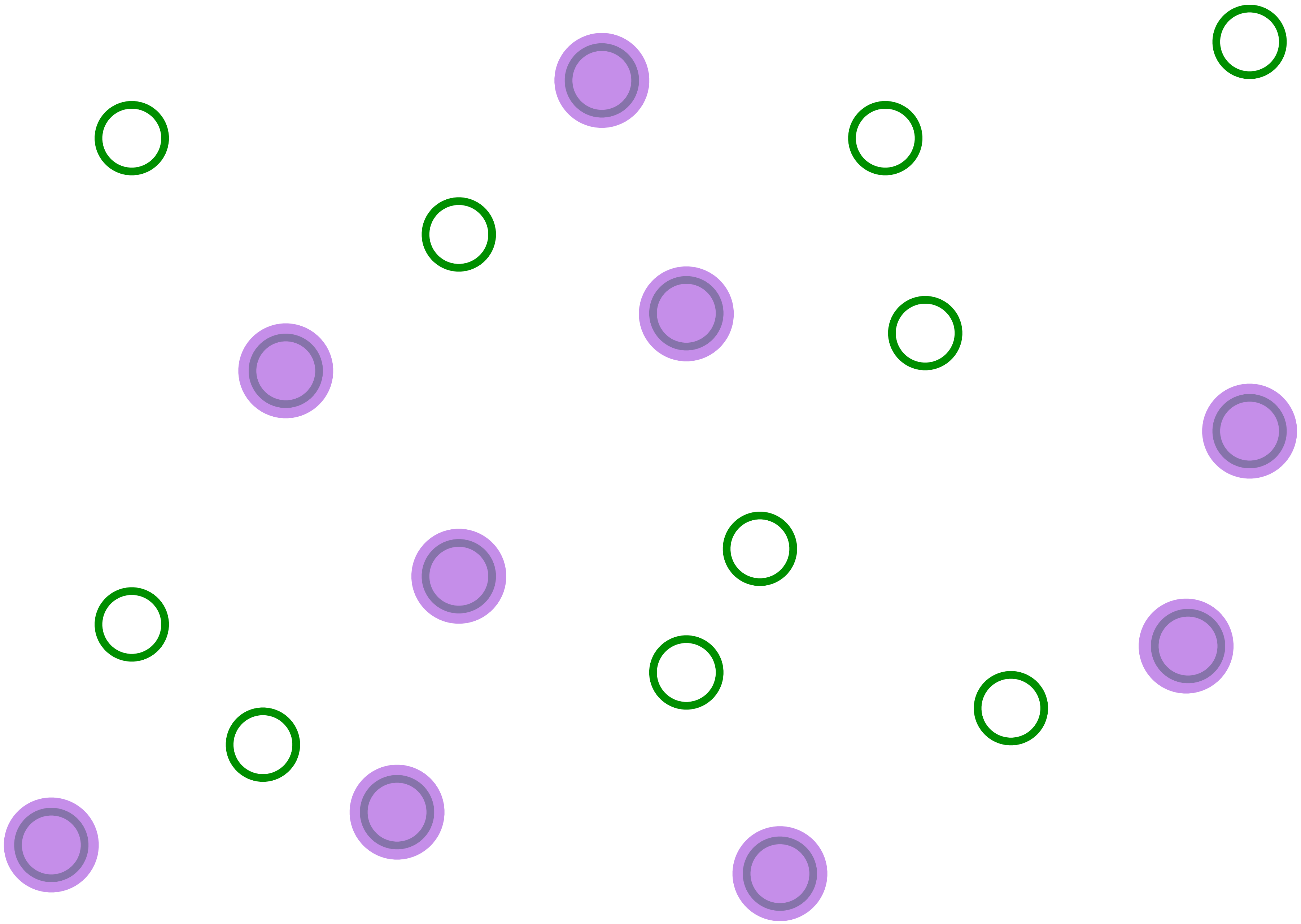
Place electrons randomly on some sites

The SYK model



Entangle electrons pairwise randomly

The SYK model



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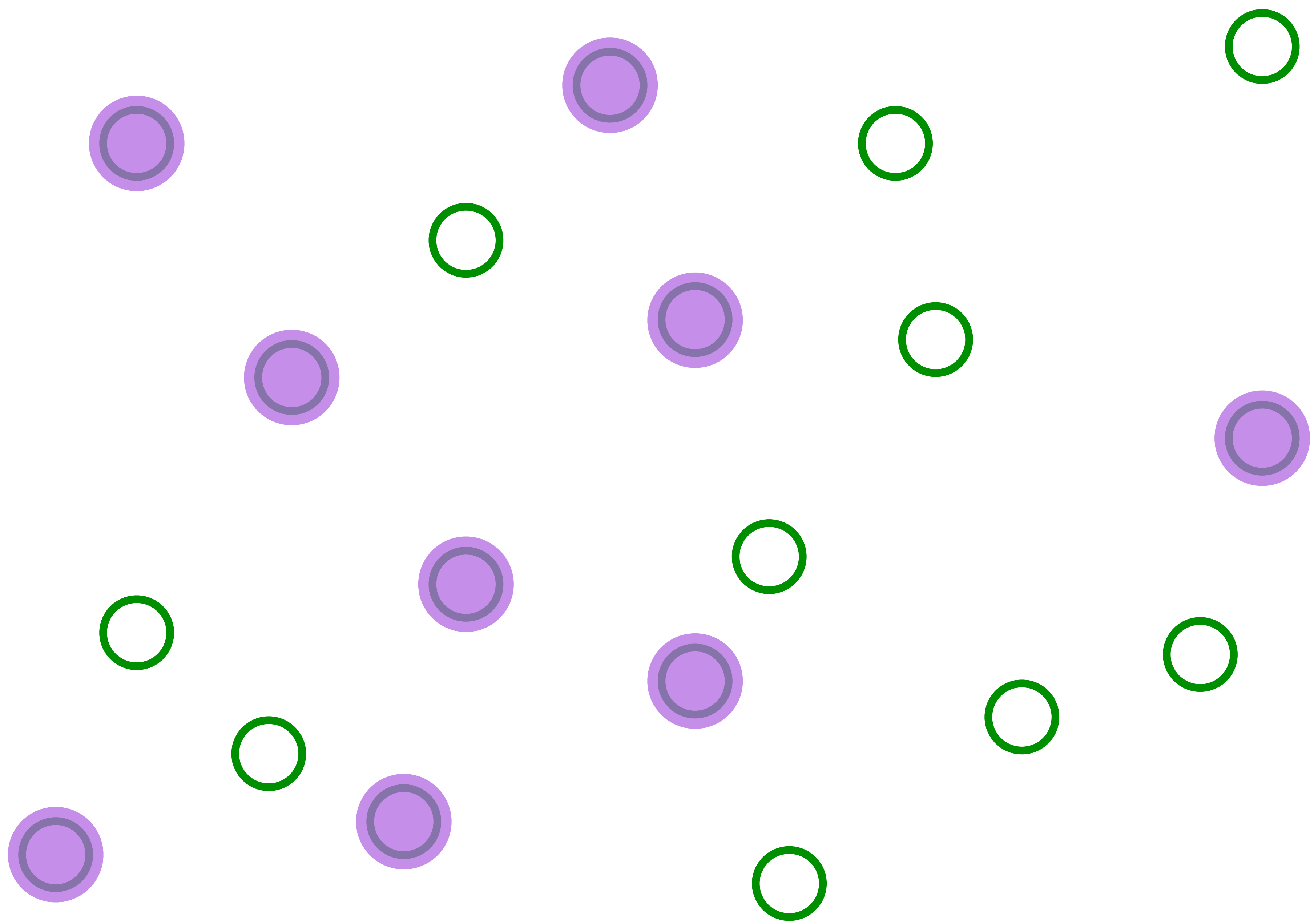

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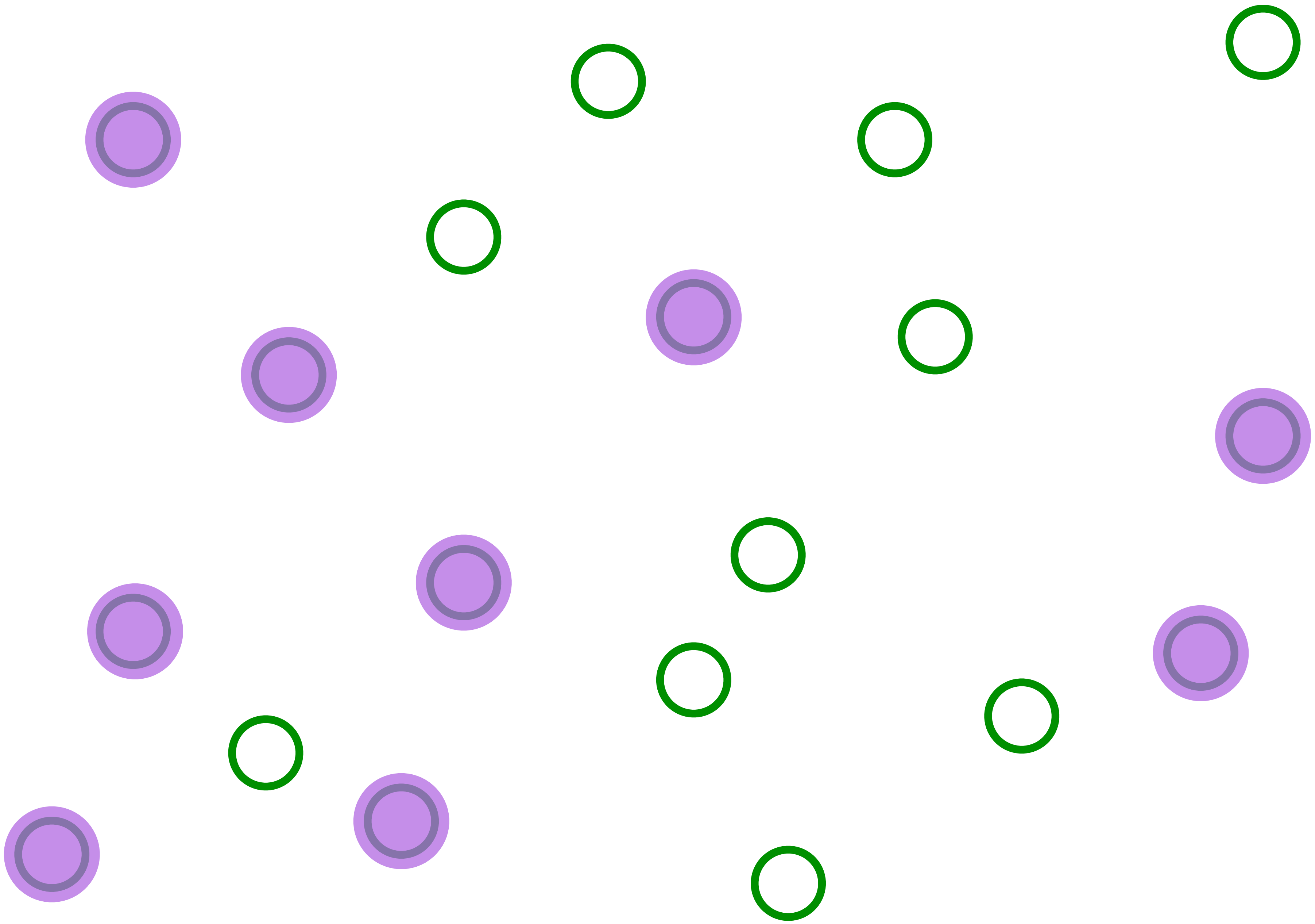

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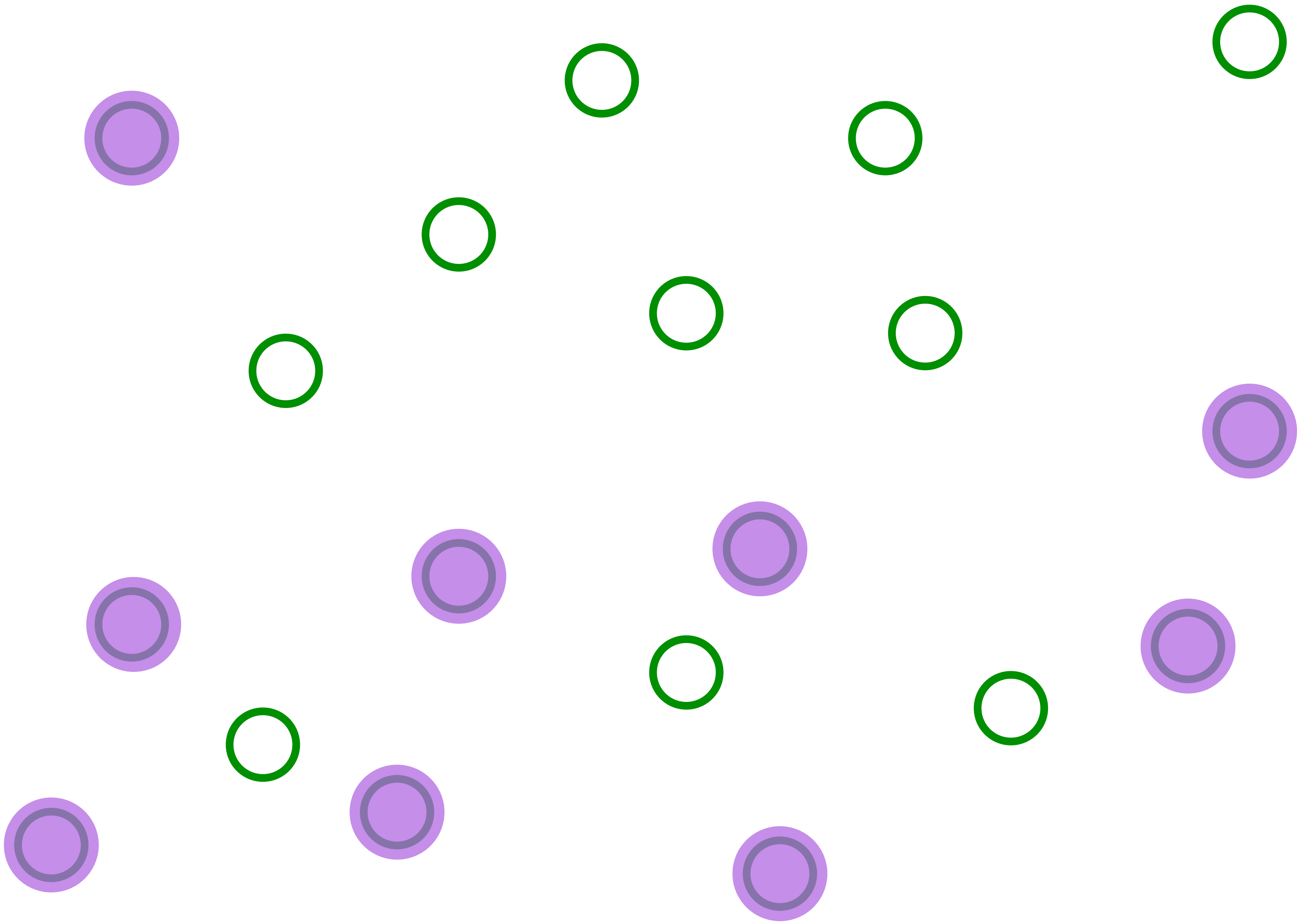

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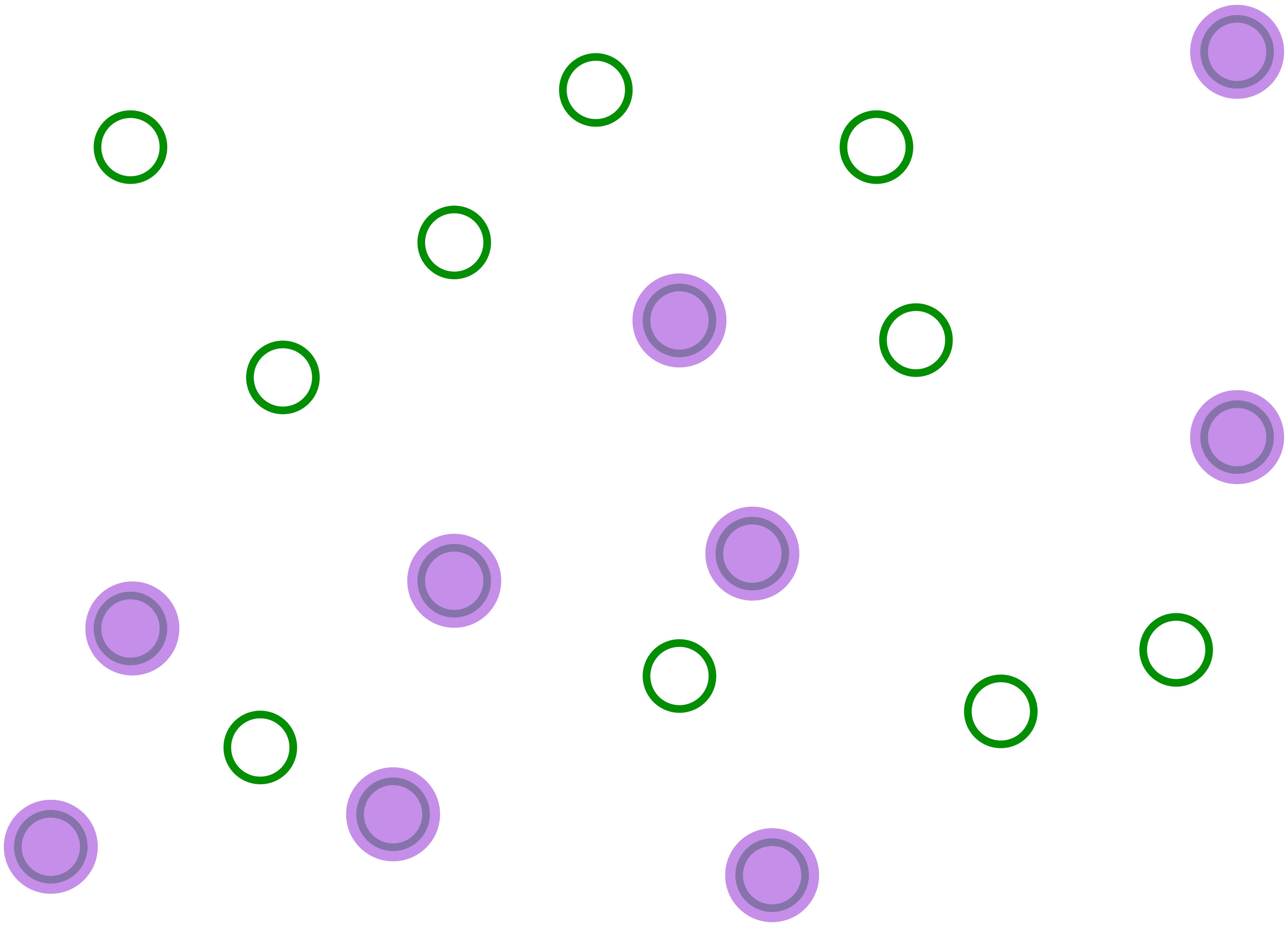

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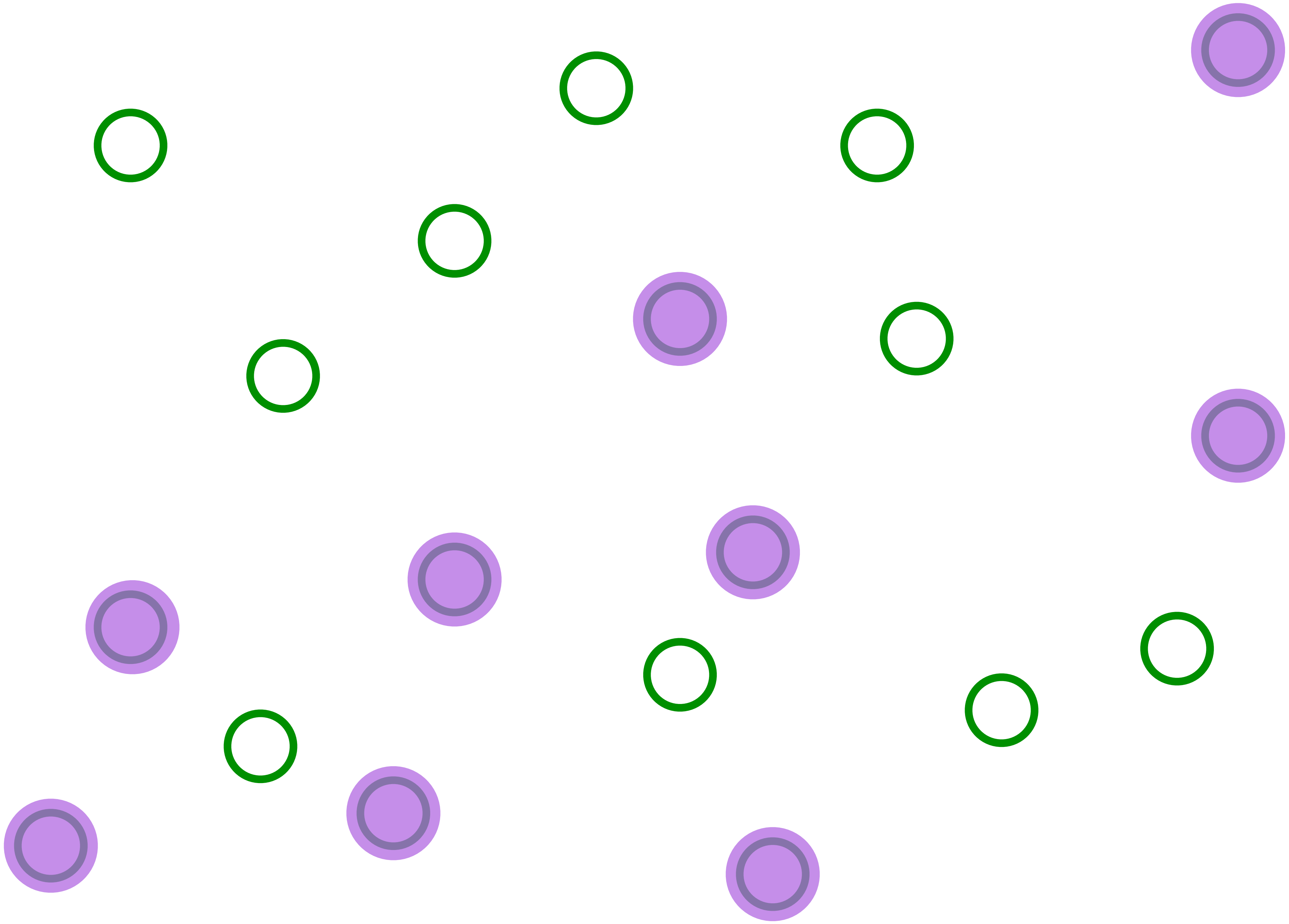


This describes both a strange metal and a black hole!

The SYK model

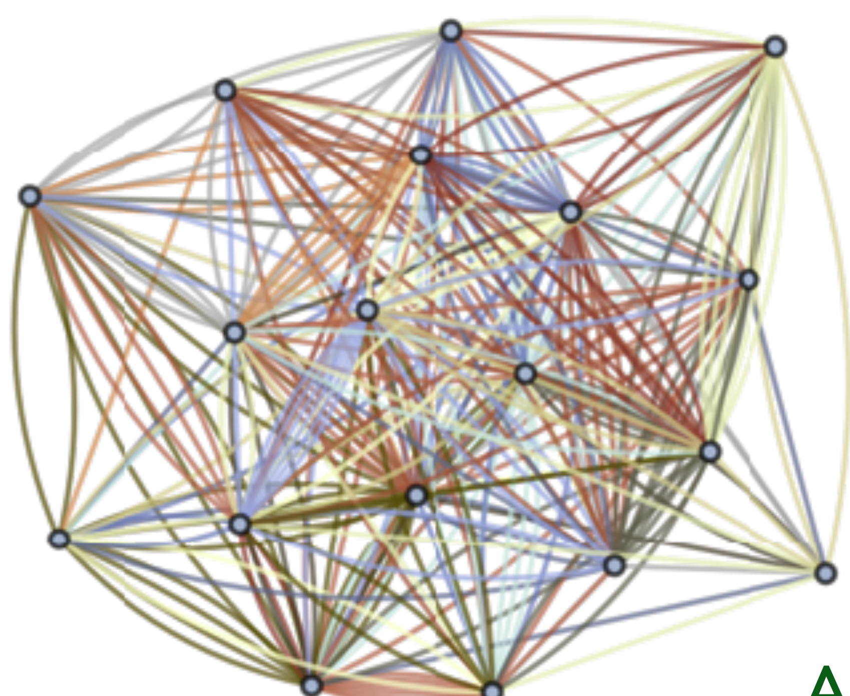
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

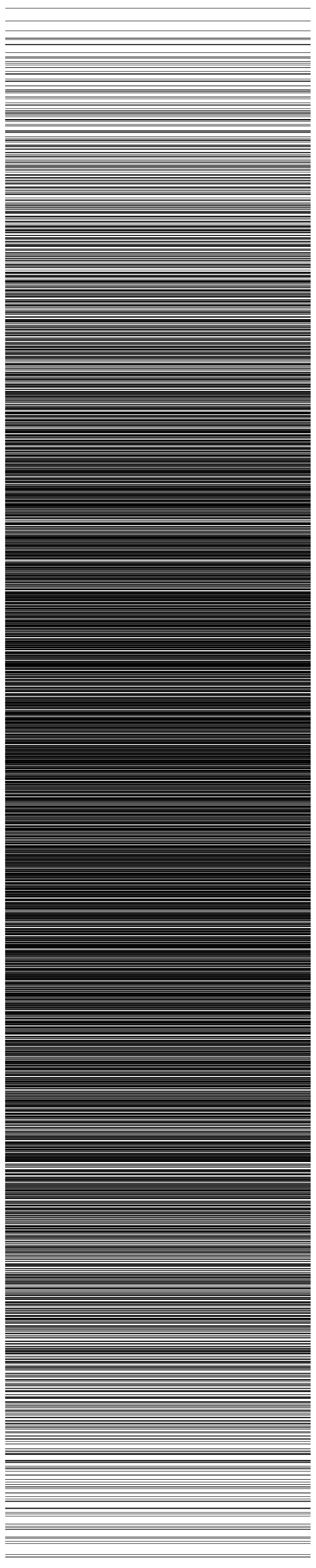
$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

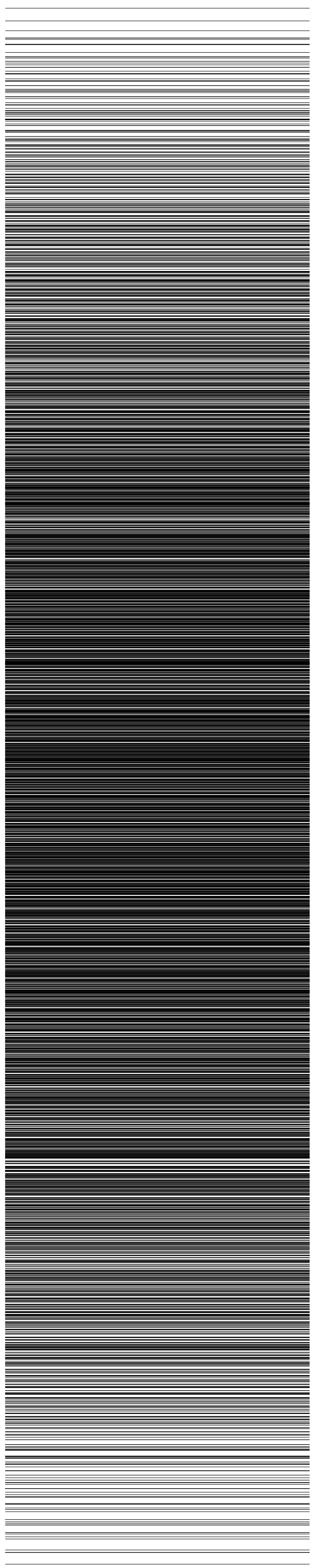
$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

No quasiparticles !
$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha$
$+ \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

…ev,
PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

- Low energy, many-body density of states
  $\rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

(for Majorana model)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

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- Low temperature entropy $S = Ns_0 + N\gamma T + \ldots$.

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

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- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX, **5**, 041025 (2015)

The SYK model

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- $T > 0$ Green's function has conformal invariance $G \sim e^{-2\pi \mathcal{E} T \tau} (T / \sin(\pi k_B T \tau / \hbar))^{1/2}$; $\mathcal{E}$ measures particle-hole asymmetry.
- The last property indicates $\tau_{\text{eq}} \sim \hbar / (k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Quantum matter without quasiparticles:

- If there are no quasiparticles, then

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S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

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- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \,.$$

S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

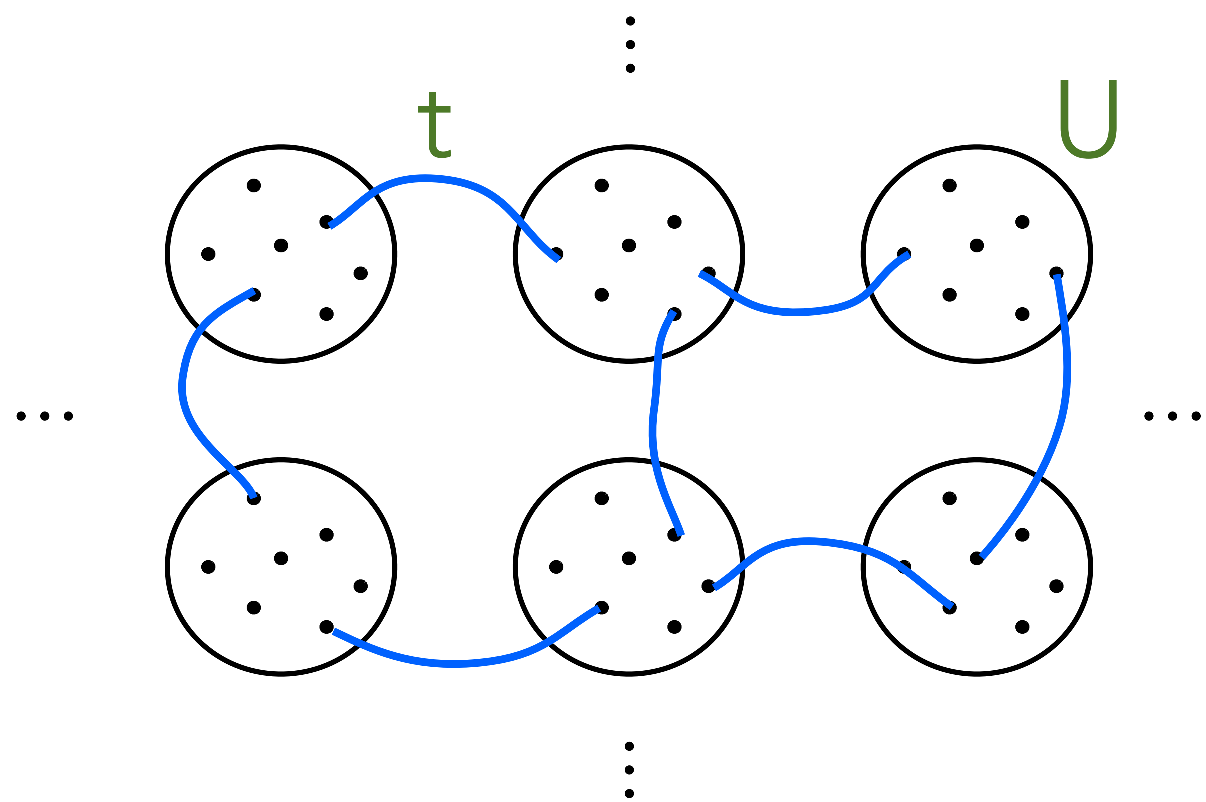
  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

arXiv:1705.00117

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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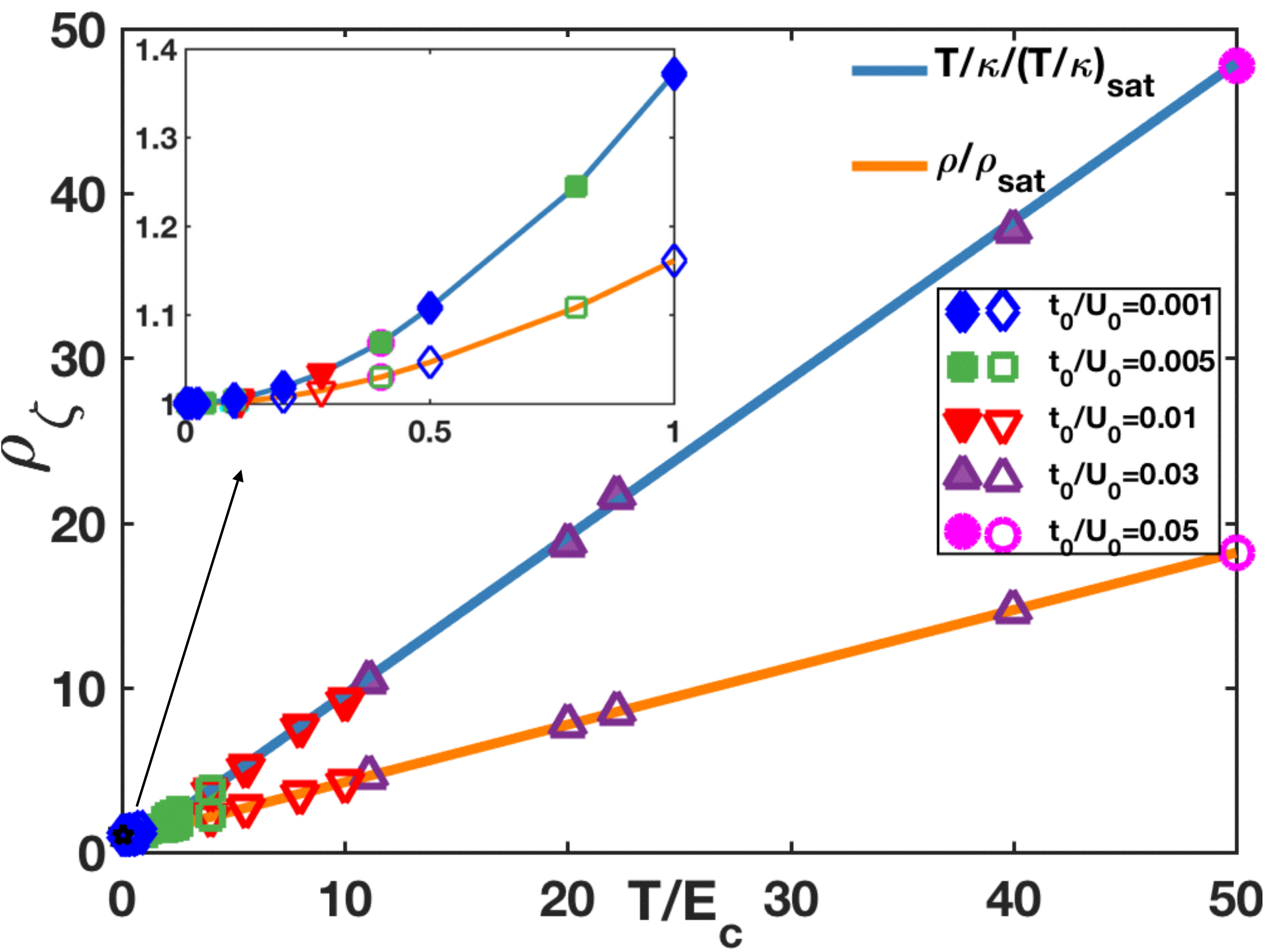
Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

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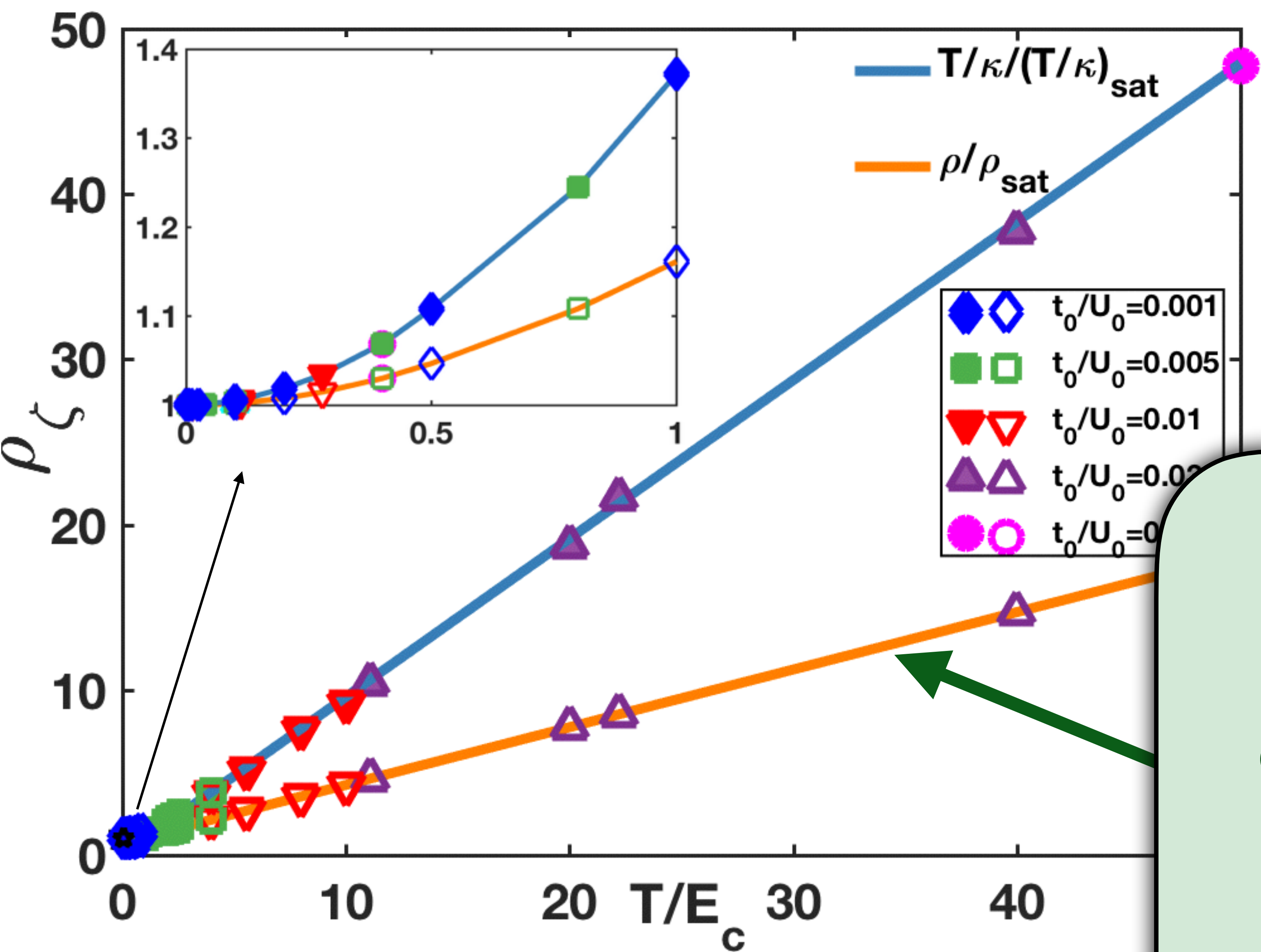

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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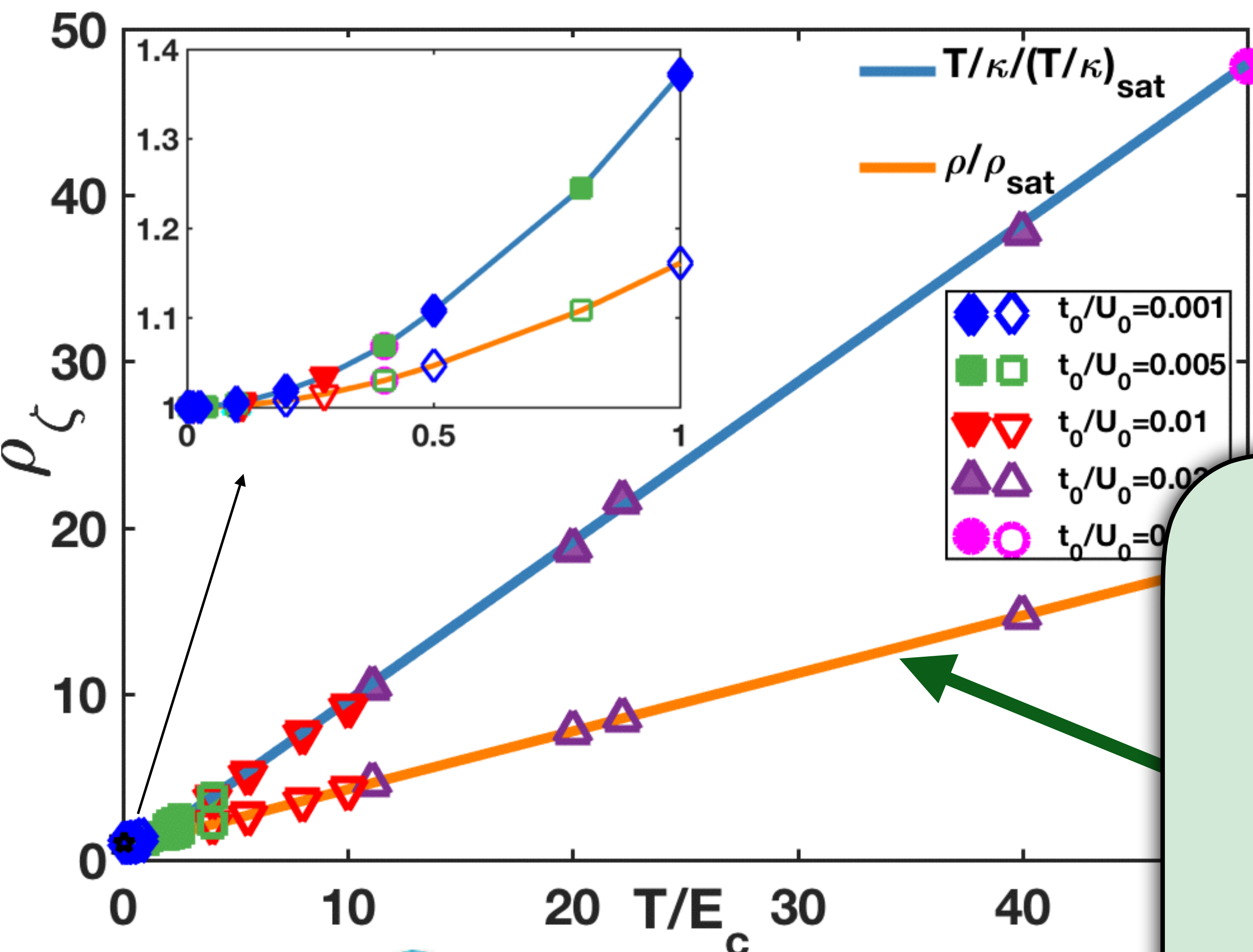
Low ‘coherence’ scale

$$E_c \sim \frac{t_0^2}{U}$$



For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right)\,, \qquad s = s_0$$

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

arXiv:1705.00117

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

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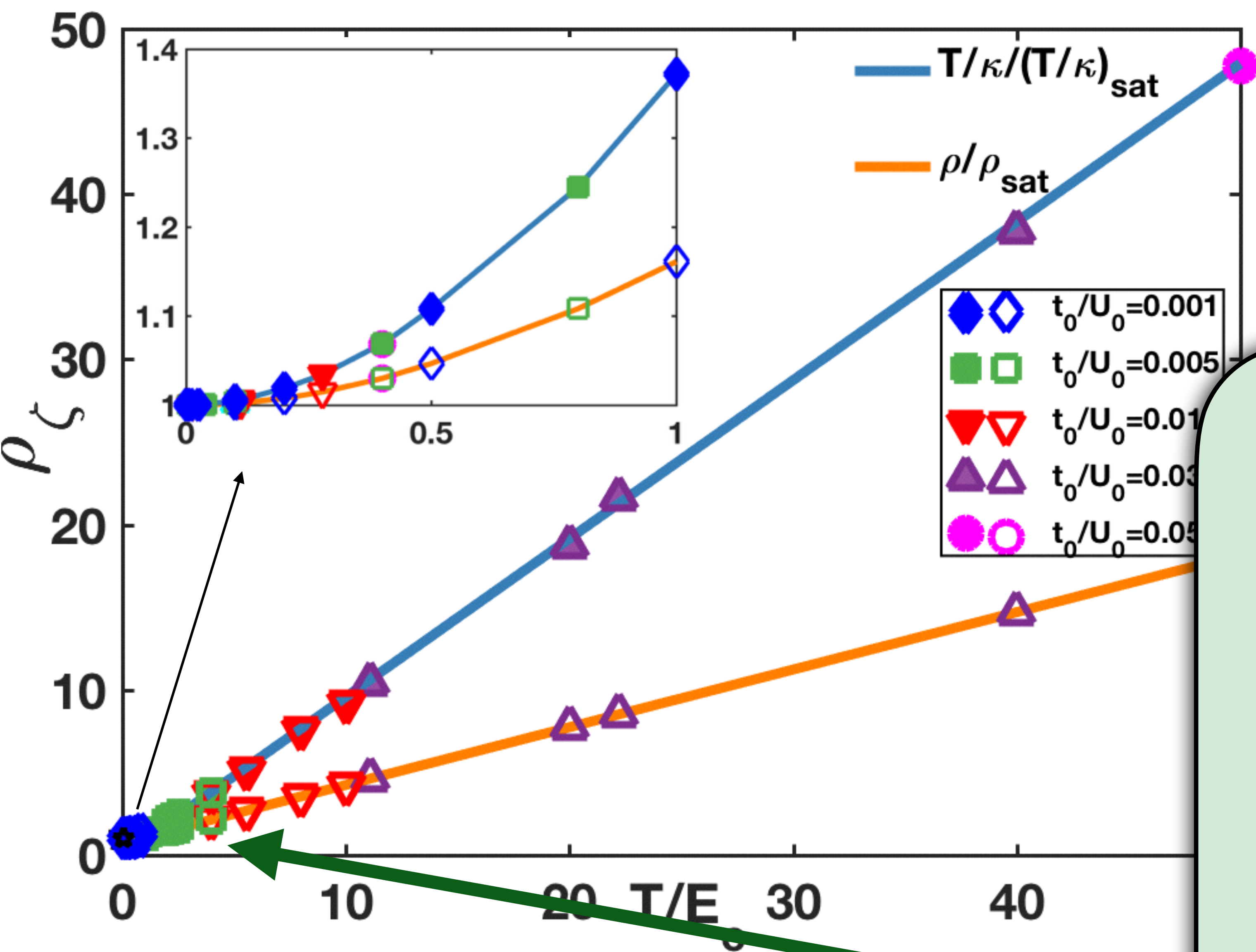


For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

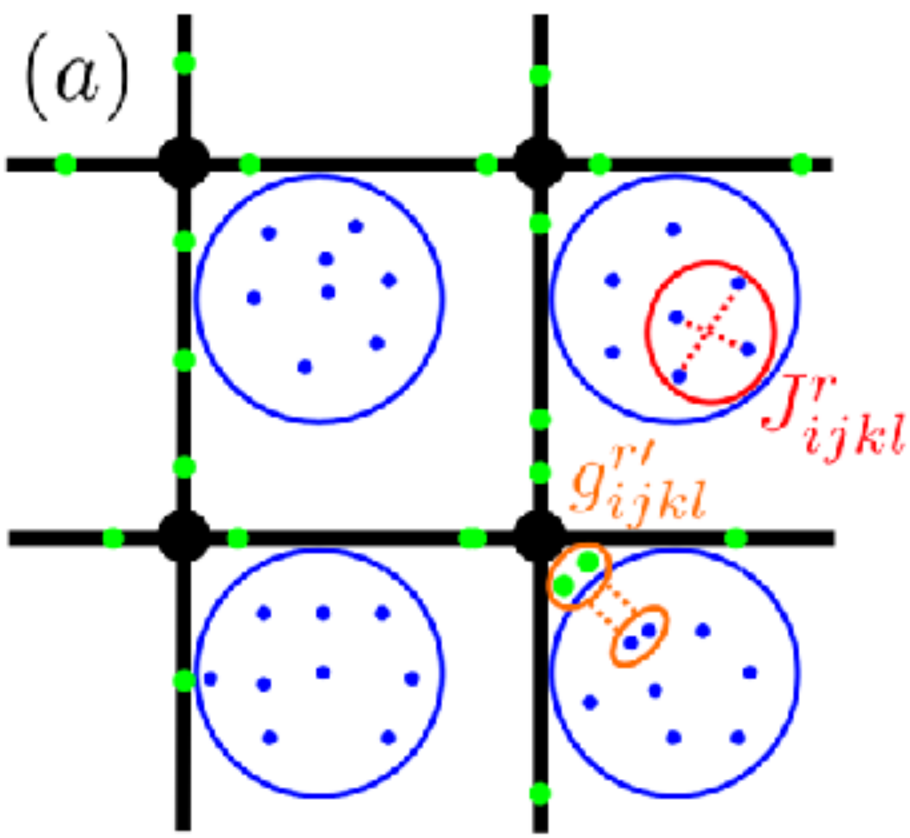
$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Infecting a Fermi liquid and making it SYK

- Can we build a bridge between the 0-dimensional SYK model and a more conventional FS-based system?

$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^\dagger f_{ri}$$

$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{r} f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl}^{r} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$



A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, **to appear…**

See also: D. Ben-Zion and J. McGreevy, arXiv: 1711.02686

Infecting a Fermi liquid and making it SYK

$$\Sigma(\tau-\tau') = -J^2 G^2(\tau-\tau')G(\tau'-\tau) - \frac{M}{N} g^2 G(\tau-\tau')G^c(\tau-\tau')G^c(\tau'-\tau),$$

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)},$$ (*f* electrons)

$$\Sigma^c(\tau-\tau') = -g^2 G^c(\tau-\tau')G(\tau-\tau')G(\tau'-\tau),$$

$$G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)}.$$ (*c* electrons)

Exactly solvable in the large *N*,*M* limits!

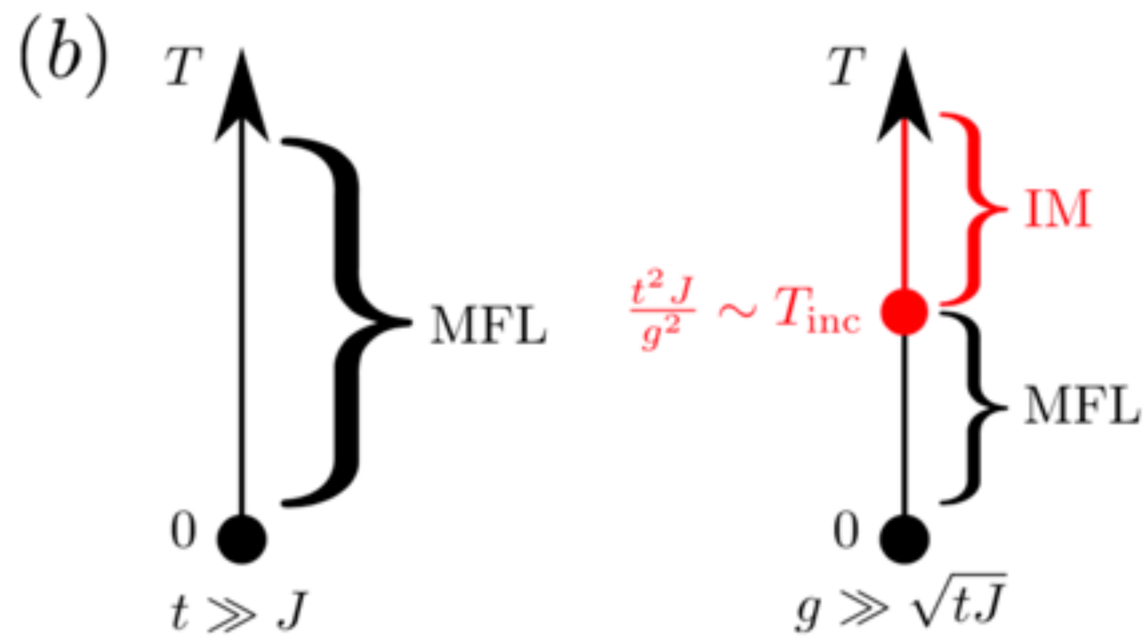
- Low-*T* phase: *c* electrons form a Marginal Fermi-liquid (MFL), *f* electrons are local SYK models

$$\Sigma^c(i\omega_n) = \frac{ig^2\nu(0)T}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\left(\frac{\omega_n}{T}\ln\left(\frac{2\pi T e^{\gamma_E-1}}{J}\right) + \frac{\omega_n}{T}\psi\left(\frac{\omega_n}{2\pi T}\right) + \pi\right),$$

$$\Sigma^c(i\omega_n) \to \frac{ig^2\nu(0)}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\omega_n \ln\left(\frac{|\omega_n|e^{\gamma_E-1}}{J}\right), \quad |\omega_n| \gg T \quad (\nu(0)\sim 1/t)$$

Infecting a Fermi liquid and making it SYK

- High-T phase: c electrons form an "incoherent metal" (IM), with local Green's function, and no notion of momentum; f electrons remain local SYK models



$$G^c(\tau) = -\frac{C_c}{\sqrt{1+e^{-4\pi\mathcal{E}_c}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}_c T\tau}, \quad G(\tau) = -\frac{C}{\sqrt{1+e^{-4\pi\mathcal{E}}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}T\tau}, \quad 0 \le \tau < \beta$$

$$C = \cosh^{1/4}(2\pi\mathcal{E})\frac{\pi^{1/4}}{J^{1/2}}\left(1 - \frac{M}{N}\frac{\Lambda\nu(0)}{2\pi}\frac{\cosh(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)}\right)^{1/4}, \quad C_c = \frac{\cosh^{1/2}(2\pi\mathcal{E})\Lambda^{1/2}\nu^{1/2}(0)}{2^{1/2}Cg},$$

$$(\Lambda \sim t, \quad \nu(0) \sim 1/t)$$

Linear-in-*T* resistivity

Both the MFL and the IM are not translationally-invariant and have linear-in-*T* resistivities!

$$\sigma_0^{\mathrm{MFL}} = 0.120251 \times M T^{-1} J \times \left(\frac{v_F^2}{g^2} \right) \cosh^{1/2}(2\pi \mathcal{E}). \;\; (v_F \sim t)$$

$$\sigma_0^{\mathrm{IM}} = (\pi^{1/2}/8) \times M T^{-1} J \times \left(\frac{\Lambda}{\nu(0) g^2} \right) \frac{\cosh^{1/2}(2\pi \mathcal{E})}{\cosh(2\pi \mathcal{E}_c)}.$$

[Can be obtained straightforwardly from Kubo formula in the large-*N*,*M* limits]

The IM is also a “Bad metal” with $\sigma_0^{\mathrm{IM}} \ll 1$

Magnetotransport: Marginal-Fermi liquid

- Thanks to large *N*,*M*, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields…

$$(1-\partial_\omega \mathrm{Re}[\Sigma_R^c(\omega)])\partial_t \delta n(t,k,\omega) + v_F \hat{k}\cdot \mathbf{E}(t)\; n_f'(\omega) + v_F(\hat{k}\times \mathcal{B}\hat{z})\cdot \nabla_k \delta n(t,k,\omega) = 2\delta n(t,k,\omega)\mathrm{Im}[\Sigma_R^c(\omega)],$$

$(\mathcal{B} = eBa^2/\hbar)$ (i.e. flux per unit cell)

$$\sigma_L^{\mathrm{MFL}} = M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{-\mathrm{Im}[\Sigma_R^c(E_1)]}{\mathrm{Im}[\Sigma_R^c(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2},$$

$$\sigma_H^{\mathrm{MFL}} = -M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{(v_F/(2k_F))\mathcal{B}}{\mathrm{Im}[\Sigma_R^c(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2}.$$

$$\sigma_L^{\mathrm{MFL}} \sim T^{-1}s_L((v_F/k_F)(\mathcal{B}/T)), \quad \sigma_H^{\mathrm{MFL}} \sim -\mathcal{B}T^{-2}s_H((v_F/k_F)(\mathcal{B}/T)).$$

$$s_{L,H}(x\to\infty)\propto 1/x^2, \quad s_{L,H}(x\to 0)\propto x^0.$$

Scaling between magnetic field and temperature in **orbital** magnetotransport!

Macroscopic magnetotransport in the MFL

- Let us consider the MFL with additional **macroscopic** disorder (charge puddles etc.)

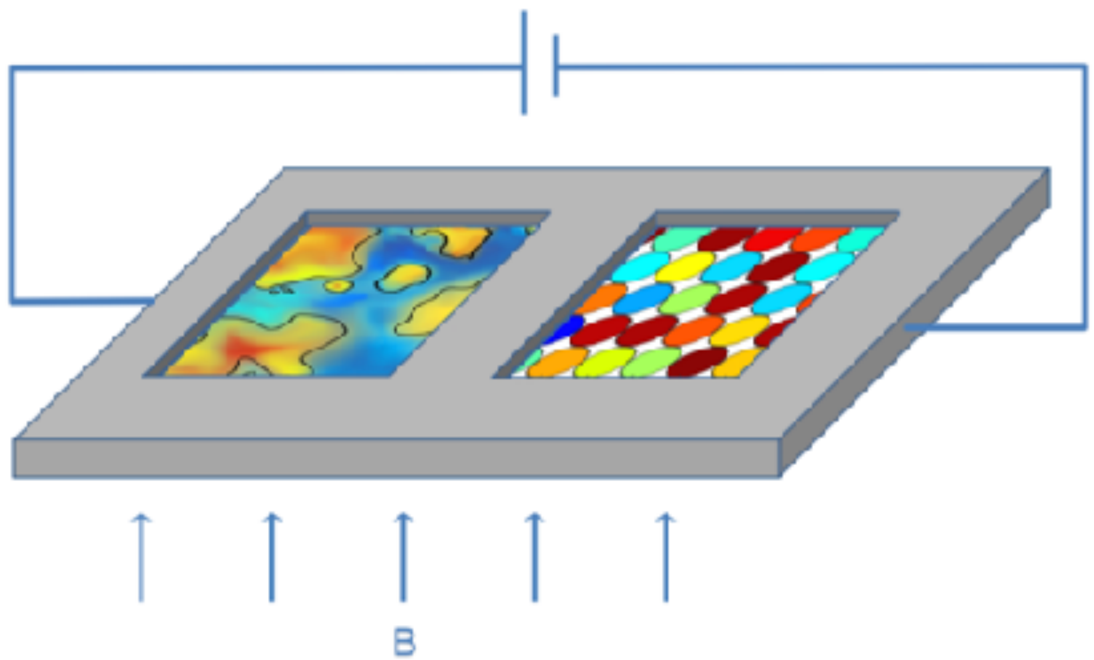


Figure: N. Ramakrishnan et. al., arXiv: 1703.05478

- No macroscopic momentum, so equations describing charge transport are just

$$\nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla\Phi(\mathbf{x}).$$

- Very weak thermoelectricity for large FS, so charge effectively decoupled from heat transport.

Physical picture

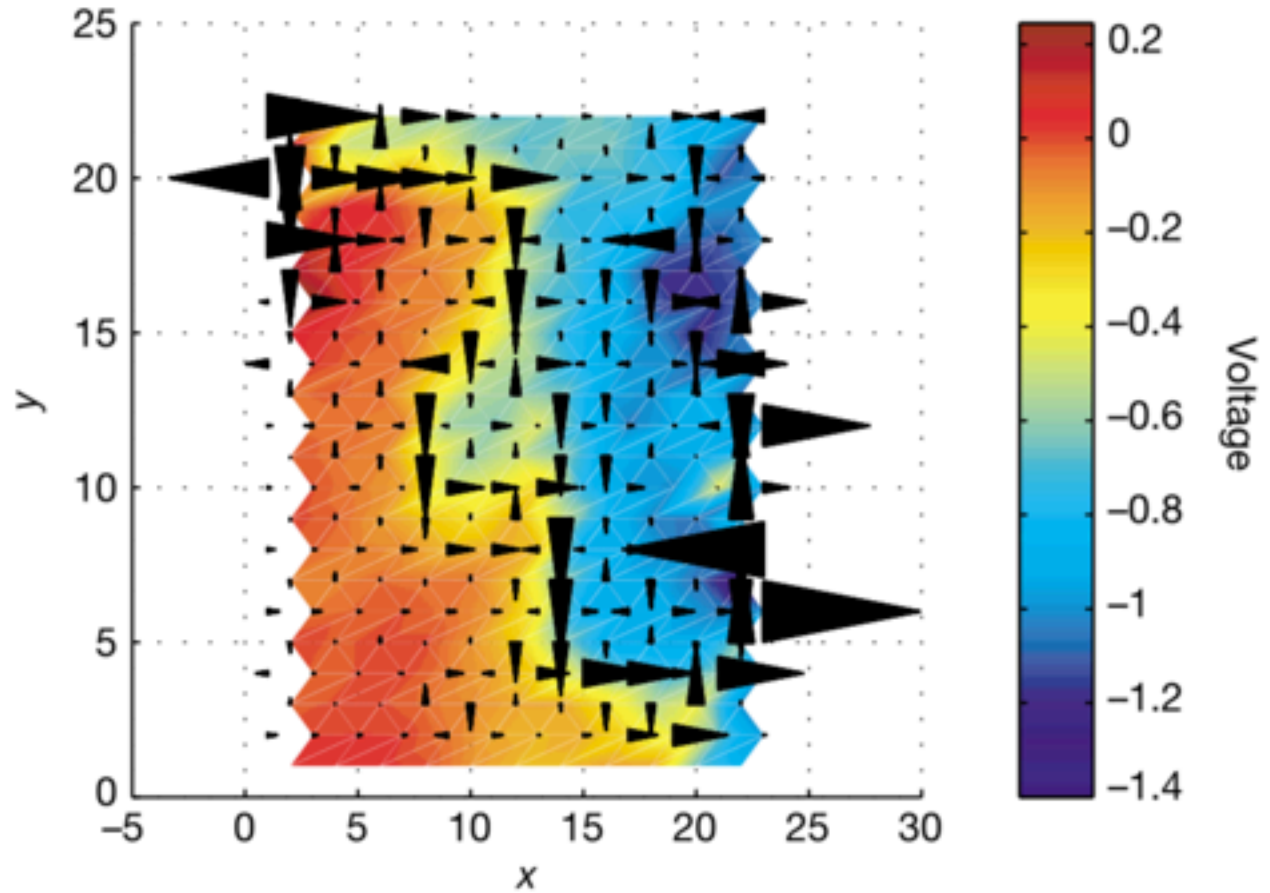
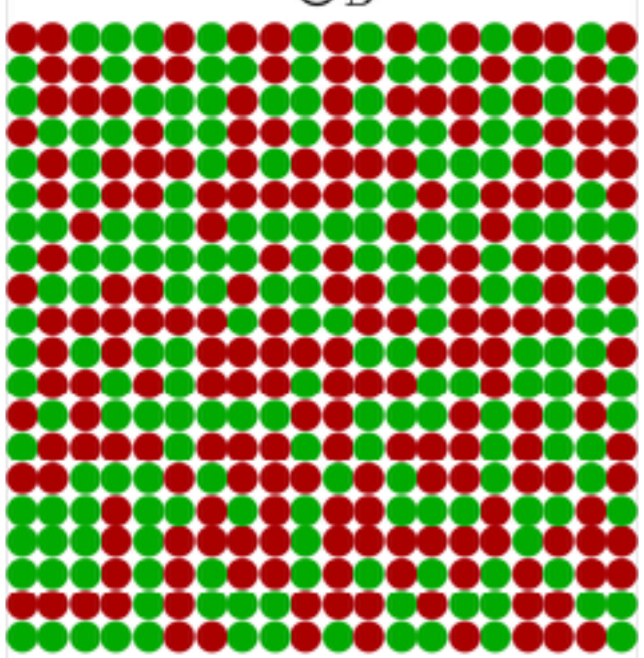


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in H.

- Current path length increases linearly with B at large B due to local Hall effect, which causes the global resistance to increase linearly with B at large B.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Solvable toy model: two-component disorder



- Two types of domains *a,b* with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly

$$\left(\mathbb{I}+\frac{\sigma^a-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^a-\sigma^e)+\left(\mathbb{I}+\frac{\sigma^b-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^b-\sigma^e)=0.$$

$$\rho^e_L \equiv \frac{\sigma^e_L}{\sigma^{e2}_L+\sigma^{e2}_H} = \frac{\sqrt{(\mathcal{B}/m)^2\left(\gamma_a\sigma^{\rm MFL}_{0a}-\gamma_b\sigma^{\rm MFL}_{0b}\right)^2+\gamma_a^2\gamma_b^2\left(\sigma^{\rm MFL}_{0a}+\sigma^{\rm MFL}_{0b}\right)^2}}{\gamma_a\gamma_b(\sigma^{\rm MFL}_{0a}\sigma^{\rm MFL}_{0b})^{1/2}\left(\sigma^{\rm MFL}_{0a}+\sigma^{\rm MFL}_{0b}\right)},$$

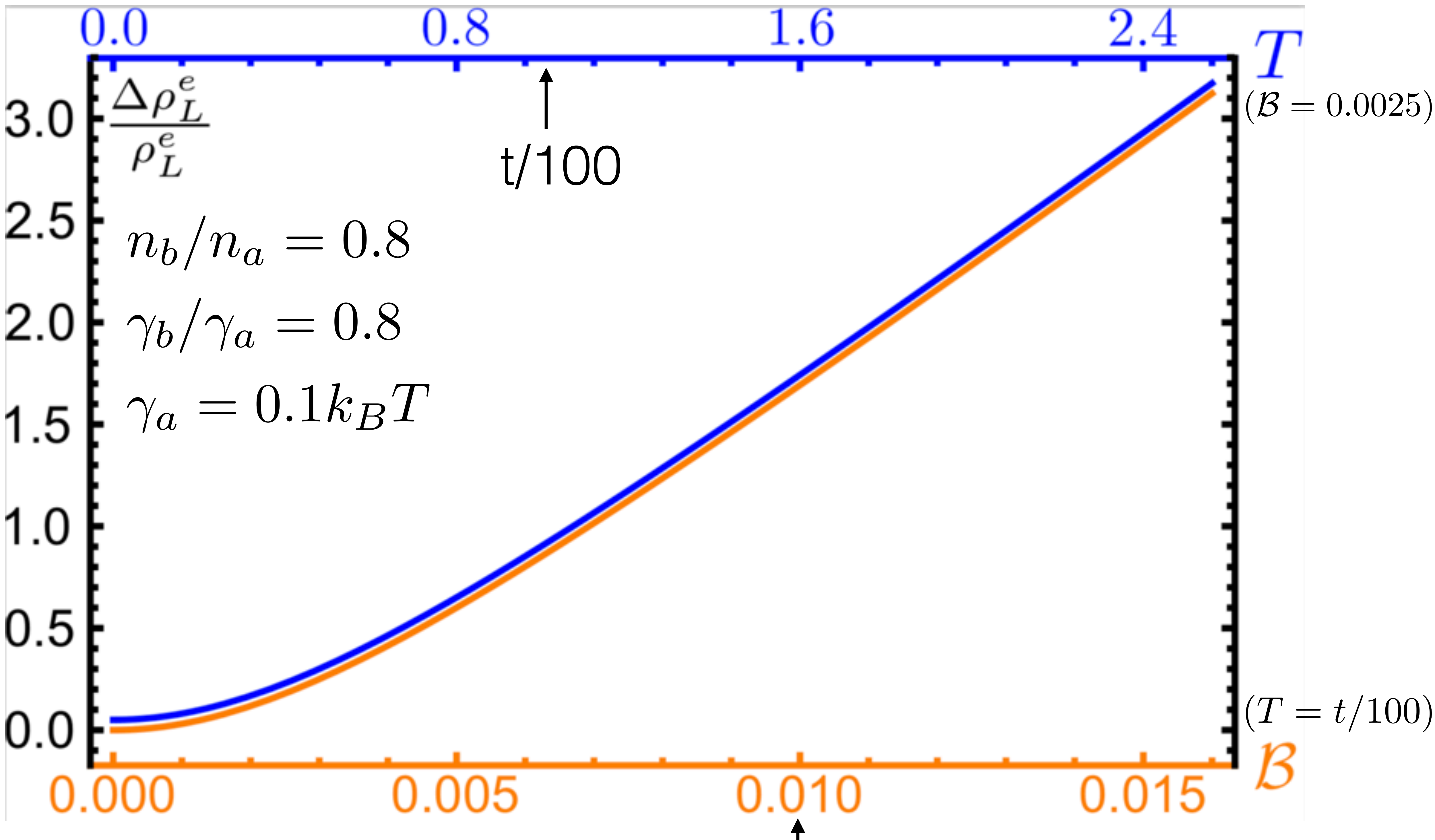
$$\rho^e_H \equiv -\frac{\sigma^e_H/\mathcal{B}}{\sigma^{e2}_L+\sigma^{e2}_H} = \frac{\gamma_a+\gamma_b}{m\gamma_a\gamma_b\left(\sigma^{\rm MFL}_{0a}+\sigma^{\rm MFL}_{0b}\right)}. \quad (m = k_F/v_F \sim 1/t)$$

$\gamma_{a,b} \sim T$ (i.e. effective transport scattering rates)

$$\rho^e_L \sim \sqrt{c_1T^2+c_2B^2}$$

Scaling between *B* and *T* at microscopic orbital level has been transferred to global MR!

Scaling between *B* and *T*



~ 50 T (a = 3.82 A)

Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states: $E \neq \sum_{\alpha} n_{\alpha}\varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \ldots$
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These are also characteristics of black holes in quantum gravity.

Magnetotransport in strange metals

- Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-T resistance, with a magnetoresistance which scales as $B \sim T$.

- Macroscopic disorder then leads to linear-in-B magnetoresistance, and a combined dependence which scales as $\sim \sqrt{B^2 + T^2}$

- Higher temperatures lead to an incoherent metal with a local Green's function and a linear-in-T resistance, but negligible magnetoresistance.