Quasi-static probes of the QCD plasma

Asian School on Lattice Field Theory

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Outline of the talk

- Introduction
- Configurations,Measurements,Operators
- Analysis Details: fits and local masses
- Chiral Symmetry restoration
- Tuning explicit chiral symmetry breaking
- Screening masses
- Finite volume
- Continuum Limit
- Statistical Data compression

Motivation

- Nature and composition of quasiparticles in QGP plasma : subject of intense investigation for the past two decades [MILC, RBC-Bielefeld, ILGTI, hot-QCD, BW, CP-PACS, QCD-TARO, W-HOT]
- Above $\sim 2-3T_c$, weak coupling resummation schemes are known to agree with lattice results on Equation of state and susceptibilities. [Laine et al.]
- Around $\sim T_c$, only lattice methods reliable in making quantitative statements
- Distinguishing the hadronic phase from the plasma phase? Important for experiments! Applicability of thermodynamics: $\ell\mu_0 \gg 1$.
- Also important for estimating finite volume corrections for thermodynamics
- Chiral symmetry restoration in the medium

Configuration details

- Configurations used for analysis are reported in Gavai, Gupta PRD 78, 114503 (2008)
- Main features for recap:
  - R-algorithm for hybrid molecular dynamics used : naive staggered fermions + Wilson gauge action
  - Scan in temperature from $0.89T_c$ to $1.92T_c$ on $N_\tau = 6$ lattices, keeping $m_\pi \simeq 230$ MeV
  - For screening mass study, main results quoted for $N_s = 24$
  - For finite volume study, $N_s = 8, 12, 18, 24, 30$
  - For continuum limit, $N_t = 4, 6$
- The screening correlators at zero momentum are:

$$C_z^\gamma = \frac{1}{\mathcal{V}} \left\langle \sum_{\mathbf{x}} \mathrm{Tr} \left[G(\mathbf{x}, z) G^\dagger(\mathbf{x}, z) \right] \phi_\gamma(\mathbf{x}) \right\rangle$$

where **x**=(x,y,t); $\mathcal{V} = N_x N_y N_t$

$G(\mathbf{x}, z) \rightarrow$ inverse of Dirac operator

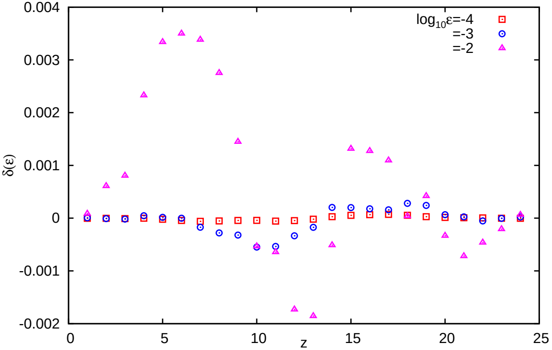
ϕ_γ = phase factor for meson with quantum no γ

Operators & Notations

- Measured all 8 possible local staggered mesons
- At $T = 0$, correspond to PS, S, V and AV mesons
- S $\rightarrow$ the flavour non-singlet scalar meson
- All three polarizations of V/AV identical after averaging

- At $T > 0$ symmetries of (x,y,t) slice different with respect to direction of propagation (S Gupta, 1999)
- S/PS lie in the trivial rep A_1^{++}
- (Vx+Vy): Vs and Vt separately lie in A_1^{++}
- (Vx-Vy): VB lie in another representation B_1^{++}
- Operators do not mix under symmetries of the (x,y,t) slice
- The VB and AVB correlators vanish at all T
- Generic notation: C_z^{γ}

Regarding convergence



- $\beta = 5.42$
- $T = 0.94T_c$
- $am_q = 0.0167$
- valence and sea quark mass identical

Tolerance of the CG algorithm $\epsilon = 10^{-5}$

Check of convergence: $\delta(\epsilon) = 1 - \frac{C^{PS}[\epsilon]}{C^{PS}[\epsilon']}$

Increasing the tolerance by an order of magnitude required ~ 250 more iterations of the CG routine

Analysis Details

Covariance matrix $\mathcal{C}_{zz'}$ was used to fit the correlation functions $C(z)$

$$
\begin{aligned}
C(z) &= A_1(\, \mathrm{e}^{-\mu_1 z} + \mathrm{e}^{-\mu_1 (N_z - z)}\,) \\
&+ (-1)^z A_2(\, \mathrm{e}^{-\mu_2 z} + \mathrm{e}^{-\mu_2 (N_z - z)}\,)
\end{aligned}
$$

μ_1,μ_2: screening masses of the lightest meson and its parity partner
A_1, A_2: the corresponding amplitudes

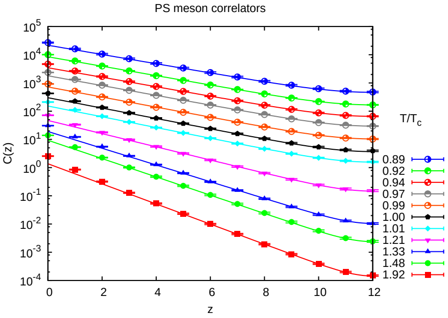
Goldstone pion is the non-oscillating pion with positive A_1

by minimizing the χ^2:

$$
\chi^2 = \sum_{zz'} \frac{C_z - \langle C(z) \rangle}{\sigma(z)}\, \mathcal{C}^{-1}_{zz'}\, \frac{C_{z'} - \langle C(z') \rangle}{\sigma(z')}
$$

Fit details-1

- Inversions done with Mathematica routines
- Inversions much more accurate than statistical errors
- Pion correlators equally good at all temperatures; characterized by single mass fits very well
- Other correlators noisy at small T and large z



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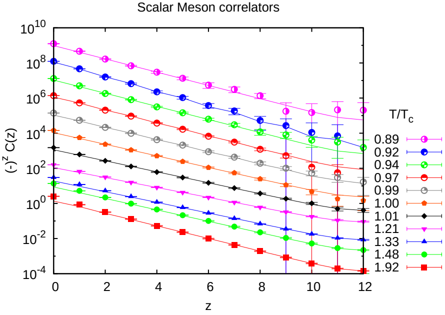
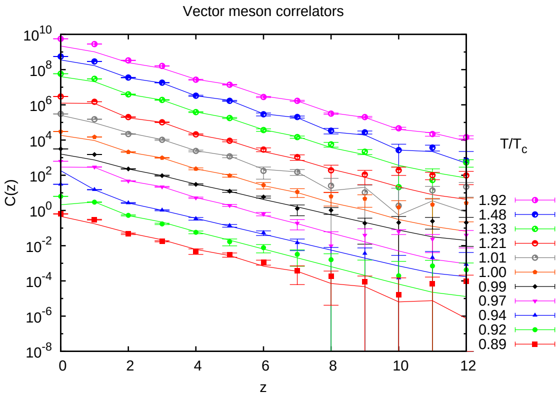


Fit details-2



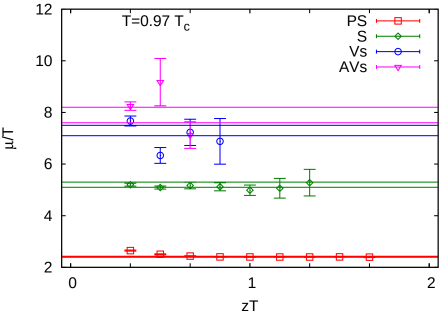
Large contribution from the parity partner for the vector.

- Results indicate considerable correlation entering through $\mathcal{C}_{zz'}$
- Noisy points excluded as much as possible
- Stability of fit checked by varying the fit range
- Most of the fits have $\chi^2/dof \sim 1$

Local Masses

Due to oscillations, local masses using 2-z slices Gavai, Gupta, Majumdar(2002)

$$\frac{C(z+1)}{C(z-1)} = \frac{\cosh[-m(z)(z+1-N_z/2)]}{\cosh[-m(z)(z-1-N_z/2)]}$$



Agree with the fitted values: Plateaus in most cases

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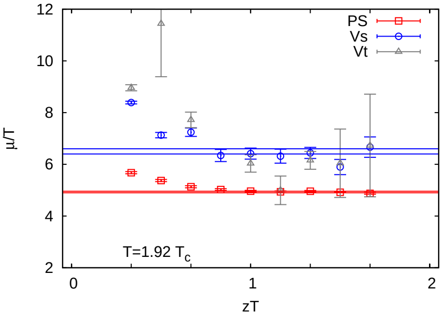

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Chiral symmetry restoration: What to expect?

- In the confined phase, no relations between A_1,A_2 and μ_1,μ_2
- In chiral symmetry restored phase,

$$C_z^{PS} = (-1)^z C_z^S \qquad C_z^{AVs} = (-1)^z C_z^{Vs} \qquad C_z^{AVt} = (-1)^z C_z^{Vt}$$

which implies

$$A_1^{Vs} = A_2^{AVs}, \qquad \mu_1^{Vs} = \mu_2^{AVs} \quad \text{and} \quad (Vt \leftrightarrow AVt,\ PS \leftrightarrow S)$$

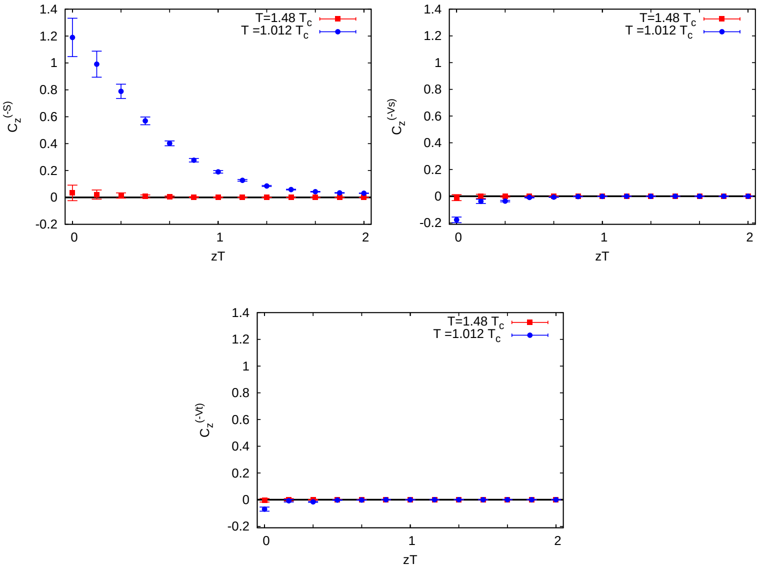
- Easiest to see through chiral projections

$$C_z^{(\pm Vs)} = C_z^{Vs} \pm (-1)^z C_z^{AVs} \qquad C_z^{(\pm S)} = C_z^{PS} \pm (-1)^z C_z^{S}$$

$$C_z^{(\pm Vt)} = C_z^{Vt} \pm (-1)^z C_z^{AVt}$$

- If $C_z^{-\gamma}$ vanishes for all z, then chiral symmetry restored for the full spectrum of excitations

How is chiral symmetry restored?



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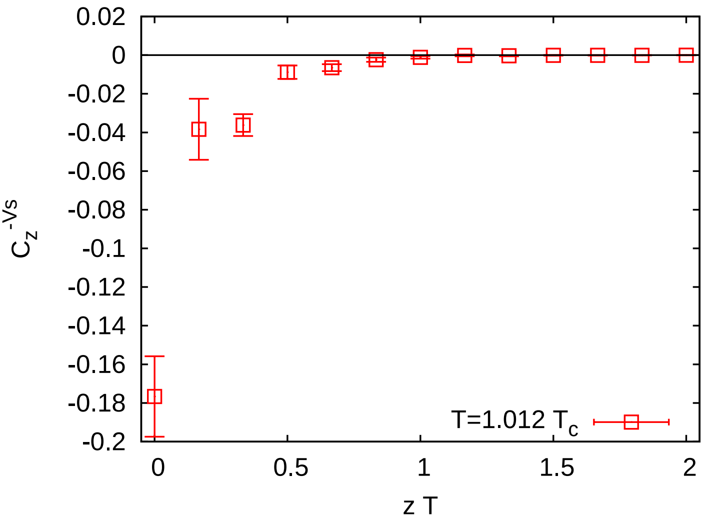


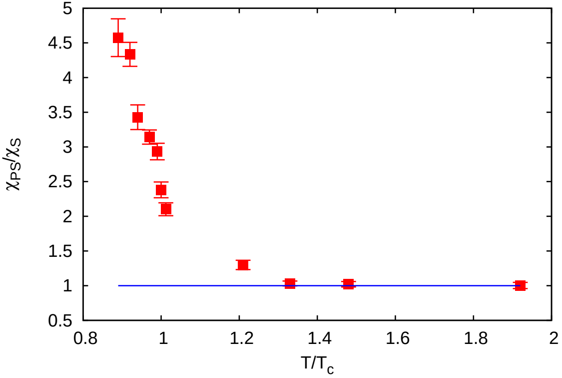
How is chiral symmetry restored?

- Badly broken in S/PS channels, at all distances just above T_c
- For Vs, broken at short distances just above T_c!
- For Vt, nearly restored at all distances just above T_c
- Quantitatively, the following quantity indicates a measure of chiral symmetry breaking

$$\chi^2 = \sum_{zz'} [C_z] \, \Sigma^{-1}_{zz'} \, [C_{z'}] \, .$$

- Large in the hadronic phase, decreases in the deconfined phase signalling a restoration in all channels by $1.33T_c$
- Similar test used to conclude the vanishing of the VB and AVB mesons

More patterns of symmetry restoration



Define: $\chi_{PS} = \sum_z C_z^{PS}$ and $\chi_S = \sum_z (-1)^z C_z^S$
Ratio becomes consistent with unity at $1.33T_c$

Why these patterns?

- At finite quark mass, there is actually a cross-over at T_c
- At high T, $\mu \propto T$
- At low T, just above T_c, could depend on m_π in a non-trivial way: $f(m_\pi/T)$
- Such argument would imply explicit symmetry breaking till 1.35 T_c with the following estimates $m_\pi \approx 230$ MeV and $T_c \approx 175$ MeV

- RBC-Bielefeld results (2010) with 2+1 flavours find similar degeneracy in V/AV but not in PS/S
- Non-degeneracy in PS/S lifted by 1.3 T_c
- Exactly similar restoration seen in the strange quark mesons
- They argue this to be because of $U_A(1)$ restoration

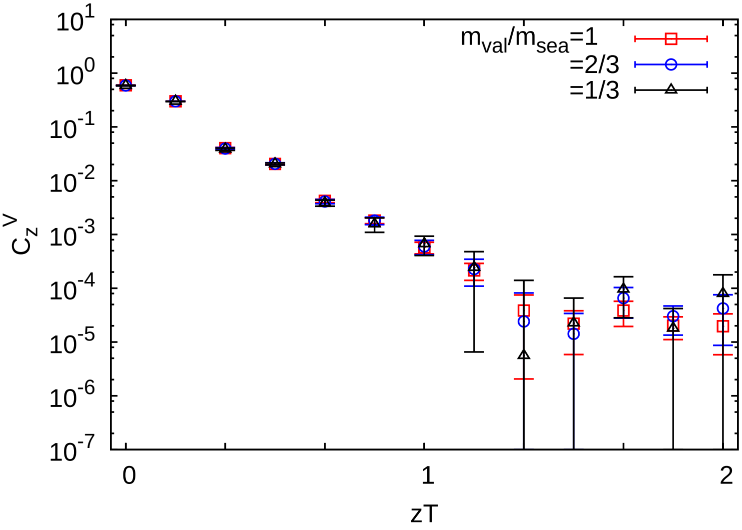
Tuning explicit chiral symmetry breaking

- If the theory was free,

$$\frac{\mu}{T} = 2\sqrt{\pi^2 + \left(\frac{M}{T}\right)^2} \simeq 2\pi\left[1 + \frac{1}{2}\left(\frac{m}{\pi T}\right)^2\right].$$

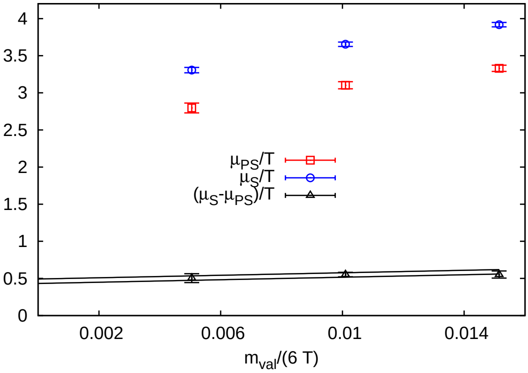
- With the quark mass, the effect is negligible!
- But in the interacting case, need to use the thermal mass of the quarks
- Large effects, should be investigated non-putatively
- In principle, involves a new set of computations
- Do partially quenched theory
- Investigated all temperatures above T_c at two different valence quark mass
  $m_{val} = 2/3 \ \& \ 1/3 \ m_{sea}$

Tuning explicit χ-symmetry breaking: V channel



No effect in the V/AV channel!

The PS/S channel-I

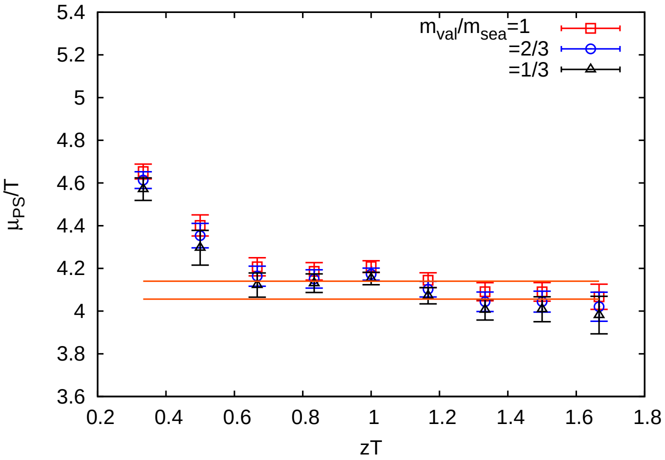


Linear extrapolation gives

$$\left.\frac{\mu_S - \mu_{PS}}{T}\right|_{T/T_c=1.21} = 0.46 \pm 0.03 \qquad (\text{for } m_{sea}/T_c = 0.1,\ m_{val} = 0).$$

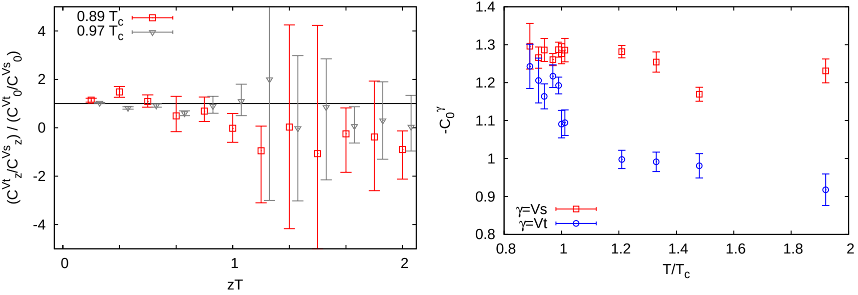
Decreasing the sea quark mass further could however decrease this difference!

The PS/S channel-II



Just restored: no significant change in the local-mass plots
The first signs of a curvature!

Screening Masses-I

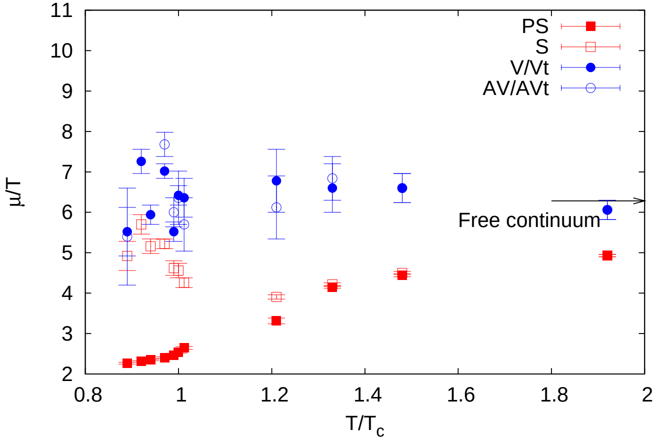


Below T_c, Vs/Vt behave similarly; can be averaged over

Screening mass from the averaged correlate below T_c quoted

Fit ranges uniform for the values plotted in the fig

Screening Masses-II

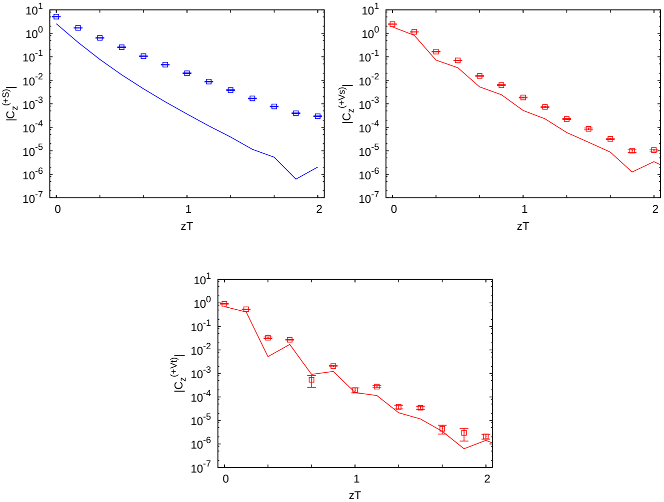


Lowest screening masses as a function of T/T_c

Lowest Vs/Av's slightly larger, but consistent with Vt/ATV to 2-σ

Approach to ideal gas value different for different mesons

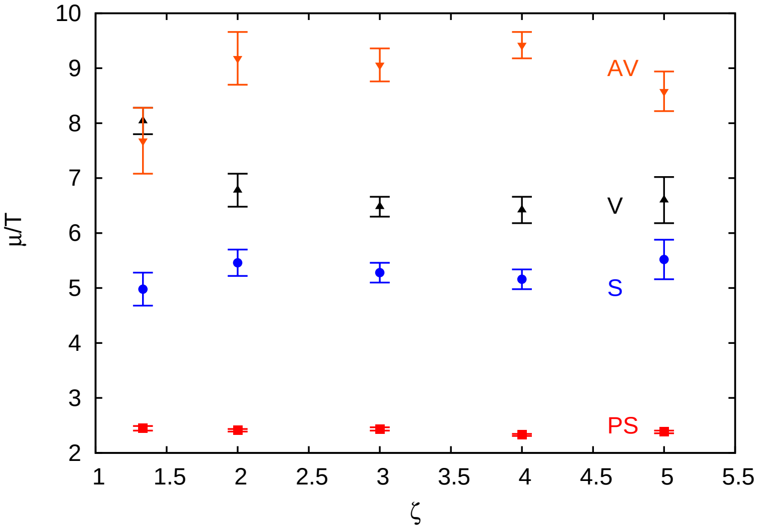
Comparison with FT



$C_z^{(+\gamma)}$ at $T=1.92T_c$ with $m/T=0.1$

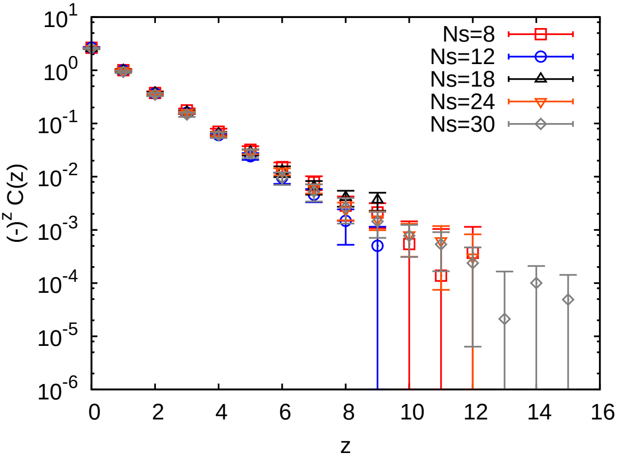
Free field correlates at same value of the bare quark mass.

Finite volume studies-I



$\zeta = LT$. Screening-masses already at the infinite volume limit!

Finite volume studies-II



Correlation functions for the scalar show no volume dependence.

Continuum Limit

T/T_c	$a=1/(4T)$		$a=1/(6T)$	
	S/PS	Vt/ATV	S/PS	Vt/ATV
1.5	3.67 ± 0.02	5.44 ± 0.08	4.44 ± 0.04	6.6 ± 0.4
2.0	4.08 ± 0.01	5.72 ± 0.04	4.93 ± 0.03	6.1 ± 0.2

Data for $a = 1/(4T)$ from Gavai, Gupta, Majumdar; 2004
Check consistency with $2\pi T$ with the following form

$$\left.\frac{\mu}{T}\right|_{N_t} = 2\pi + \frac{s}{N_t^2}.$$

Gives reasonable fits in the V/AV channel
Fails in the S/PS channel
Weak-coupling results emerge at still smaller lattice spacings?
Note: In the weak coupling theory, the screening of meson-like correlations occurs via multi-particle exchange $\Rightarrow$ curvature of correlation functions and curvature in the local mass plots

Statistical Data compression-I

- Estimates of correlation functions at different z not independent
- The covariance matrix gives a measure of this

$$\Sigma_{zz'} = \left\langle \left(C_z^\gamma - \langle C_z^\gamma \rangle\right)\left(C_{z'}^\gamma - \langle C_{z'}^\gamma \rangle\right)\right\rangle ,$$

- Fits made by minimizing the χ^2 function

$$\chi^2 = D^t \Sigma^{-1} D, \qquad \text{where} \qquad D = C^\gamma - f,$$

$C^\gamma \to$ vector of measured correlation functions, C_z^γ
$f \to$ vector made from the function to be fitted

- Highly correlated data $\Rightarrow$ singular covariance matrix $\Rightarrow$ large condition number
- Usual procedure SVD, to protect rounding off errors from dominating in the inverse
- Use *Mathematica* to control the precision of the computation

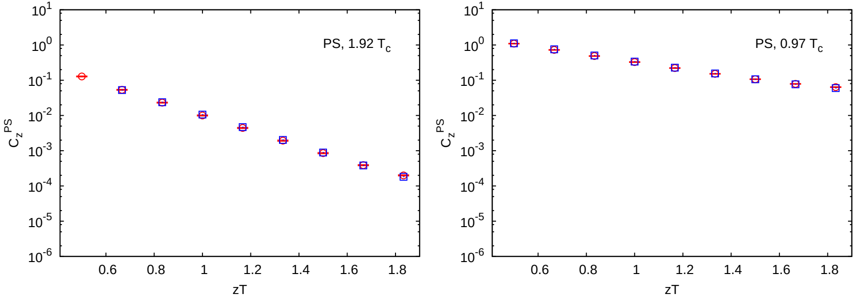
Statistical Data compression-II

- Expect that properties of $\Sigma_{zz'}$ and its eigenvectors completely determined by statistical properties of measurements
- Measurement at each z on different decorrelated configurations would give a diagonal $\Sigma_{zz'}$
- λ_α and v_α be the eigenvalues and the eigenvectors of Σ^{-1}
- Look for the projection of the C_z^γ on the eigenvectors

$$C^\gamma = \sum_\alpha c_\alpha^\gamma v_\alpha$$

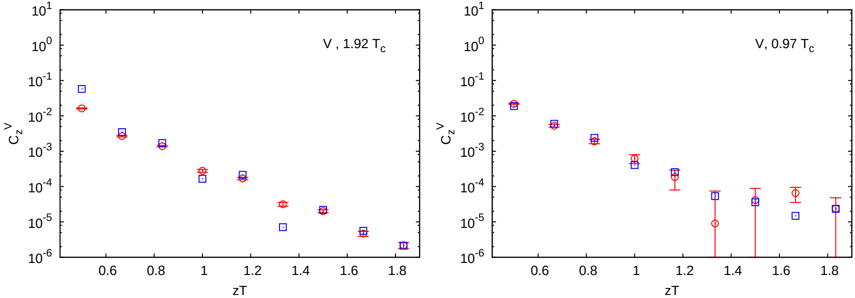
- If some c_α^γ is zero within errors, then drop them!
- Notion of “data compression”
- For the case above, *none* c_α^γ are zero!
  Correlator not compressible
- What happens with our data?

Statistical Data compression-III



All the information is contained in a single eigenvector (for PS)!
High degree of data compression

Statistical Data compression-IV



Procedure also works reasonably well for the V correlator

Origin of strong covariances in correlation function?

Conclusions

- Systematic study of the correlation functions of different meson-like operators both in the hadronic and the plasma phase
- Staggered fermions with Wilson action used in the study
- Systematic study of chiral symmetry restoration in various channels
- Examined the temperature dependence of the screening masses, their finite volume behaviour (in the confined phase), dependence on valence quark mass and continuum limit

Discussions with Saumen Datta, Nilmani Mathur and Jyotirmoy Maiti were of great help!

THANK YOU