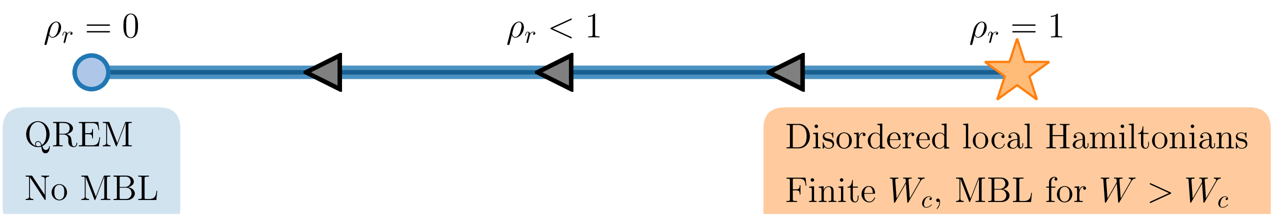
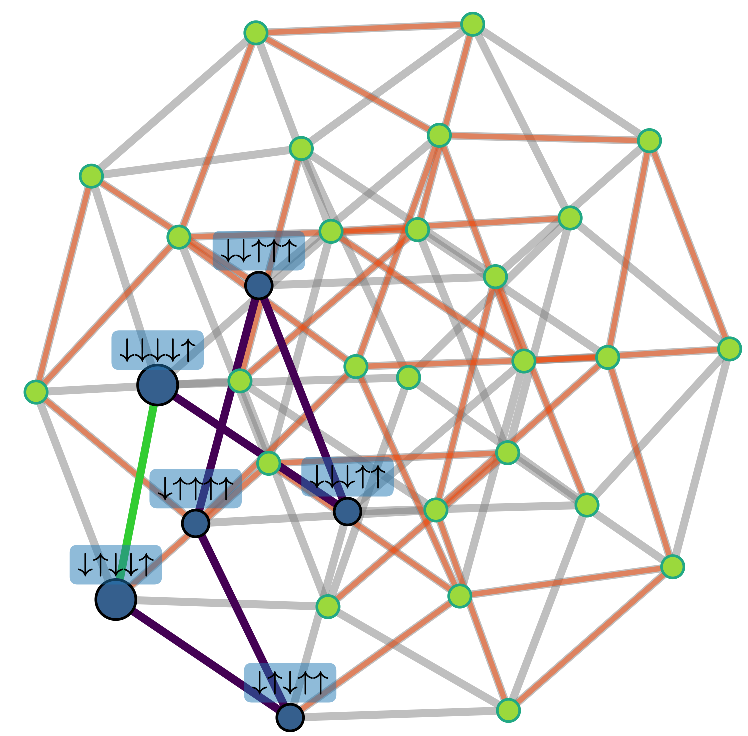
Many-body localisation: a tale of correlations and constraints on Fock space





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Acknowledgements and References

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- **SR**, D. E. Logan, arXiv:1911.12370
- **SR**, A. Lazarides, arXiv:1912.06660
- **SR**, J. T. Chalker, D. E. Logan, Phys. Rev. B 99, 104206 (2019)
- **SR**, D. E. Logan, J. T. Chalker, Phys. Rev. B 99, 220201(R) (2019)

Thermodynamics and Thermalisation

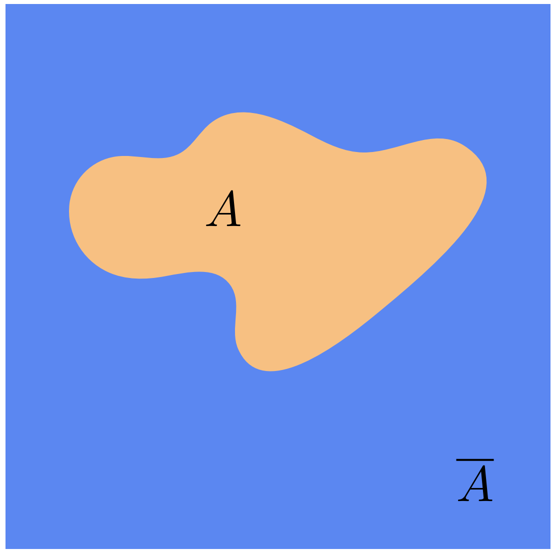
Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

Thermodynamics and Thermalisation

Thermalisation in closed quantum systems

- for a subsystem, the rest of the system acts as a bath
- an out-of-equilibrium state relaxes to a statistical ensemble
- Chaotic (generic) systems (few conserved quantities)

$$\rho_A(t \to \infty) \to \text{Gibbs ensemble}$$



Deutsch (1991), Srednicki (1994), Rigol *et al.* (2008)

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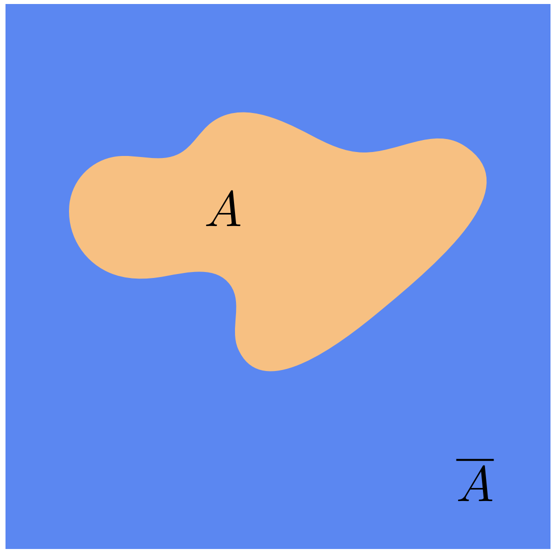

Random Matrix Theory

- Random matrix Hamiltonians as the minimal models
- Enough to capture universal and defining characteristics

Deutsch (1991), Srednicki (1994), Rigol *et al.* (2008)

Can quantum systems fail to thermalise?

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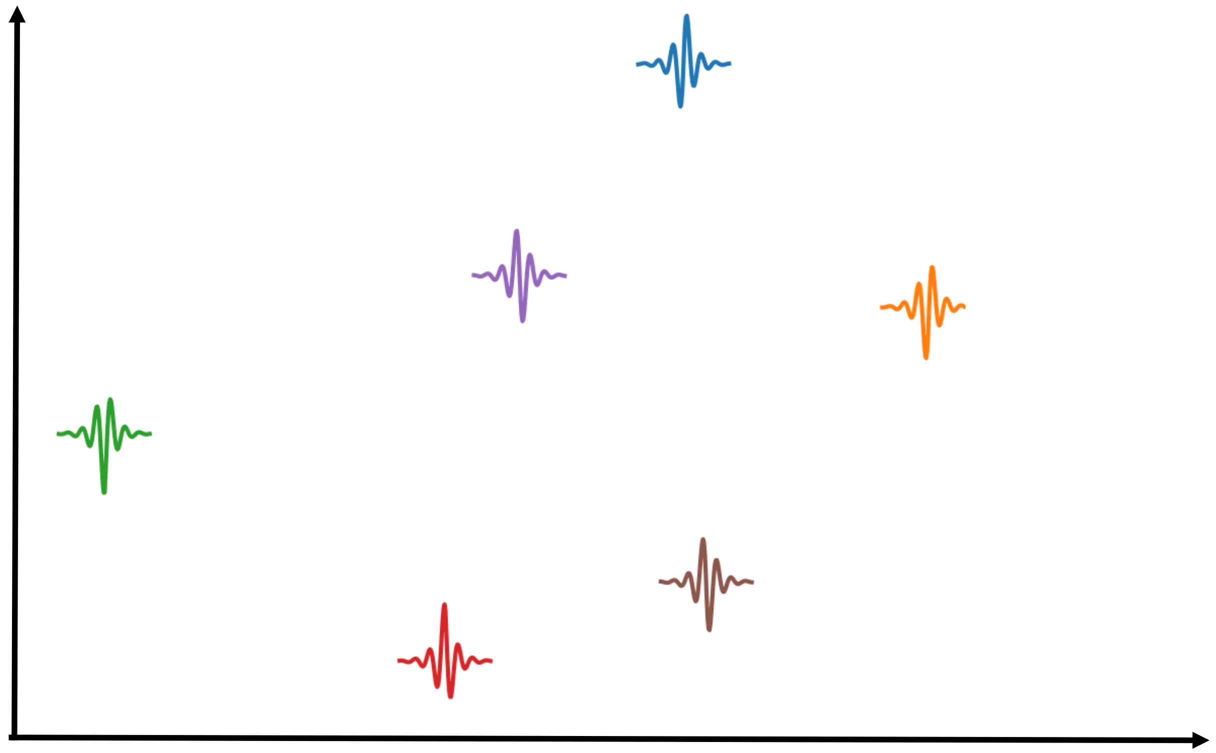
Anderson localisation (1958)

Exponential localisation of non-interacting of fermions on a disordered lattice



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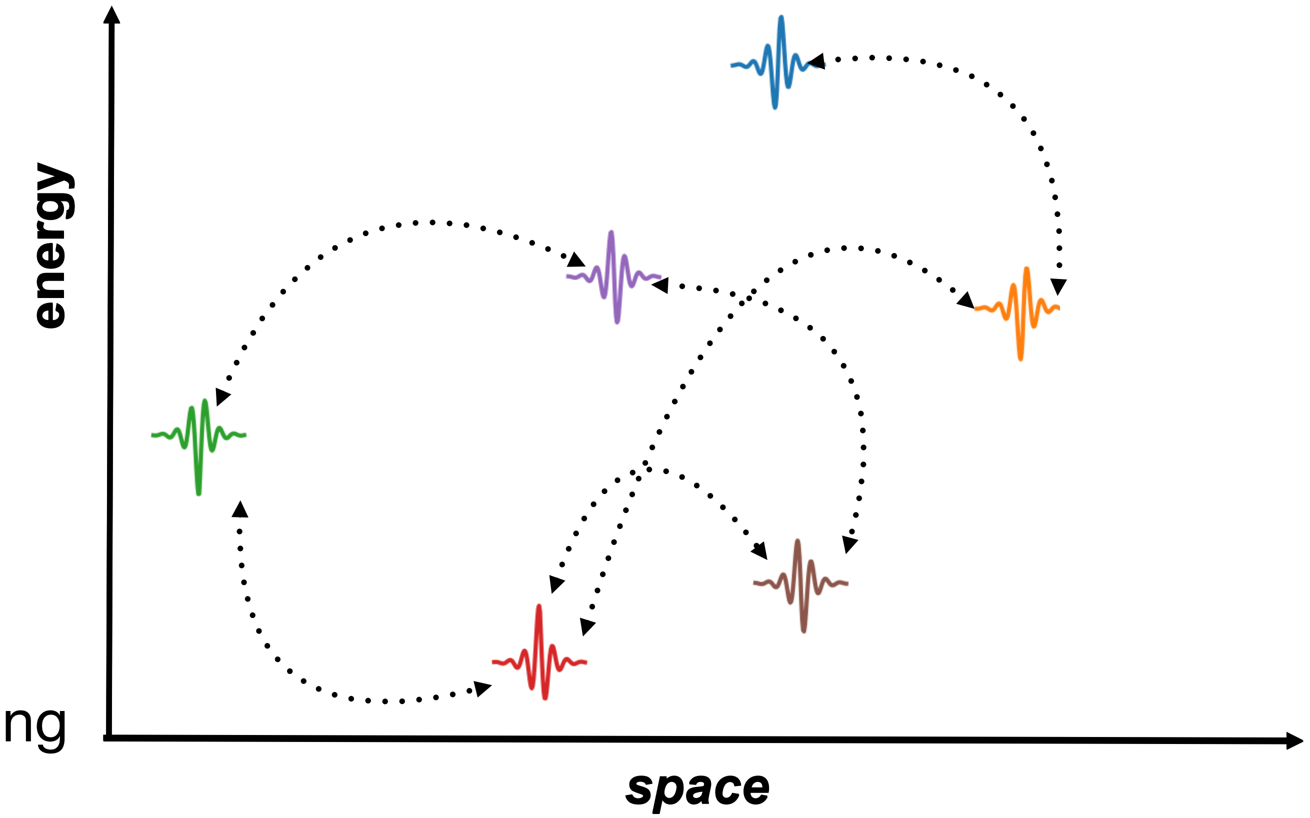
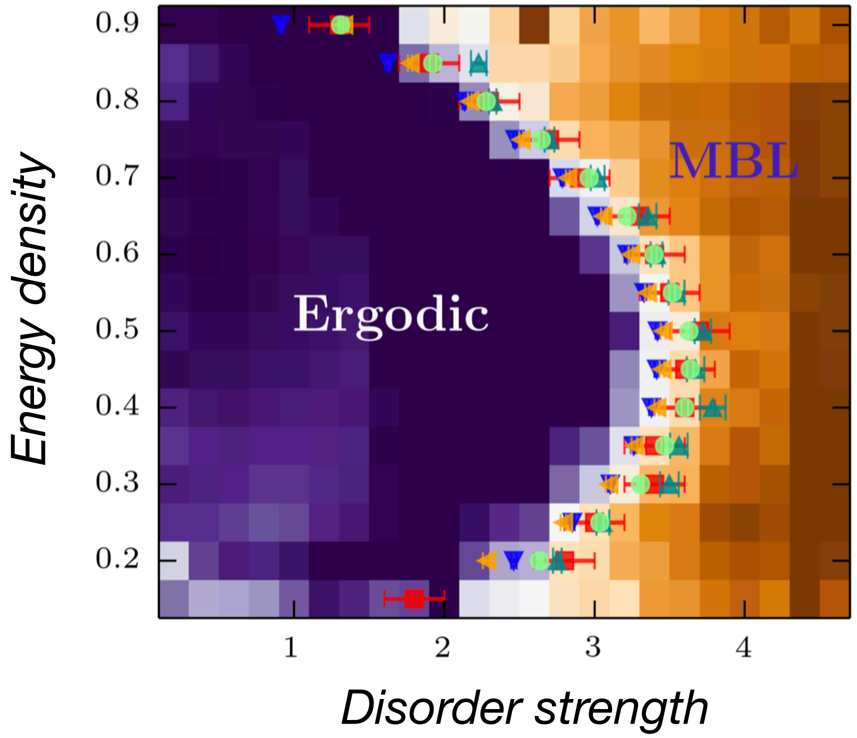


Fate of localisation upon adding interactions: **Many-body localisation**



- Existence of an MBL phase in 1D in local systems
- Finite critical disorder strength
- Higher dimensions still unclear !!

[Basko *et al.*, Znidaric *et al.*, Oganesyan+Huse, Pal+Huse, Kjäll *et al.*, Luitz *et al.*,Nandkishore+Huse, Abanin+Papic, Vasseur+Potter+Parameswaran, Vosk+Huse+Altman, **SR**+Logan+Chalker, ….]

Central question

Random matrix Hamiltonian = minimally structured and simplest non-trivial model

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but generically exhibits ergodicity/ quantum chaotic behaviour/ thermalisation

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Minimal requirements on a random many-body Hamiltonian for a many-body localised phase to be stable ?

MBL and Fock space

- Generic many-body Hamiltonian = tight binding Hamiltonian on the Fock space graph

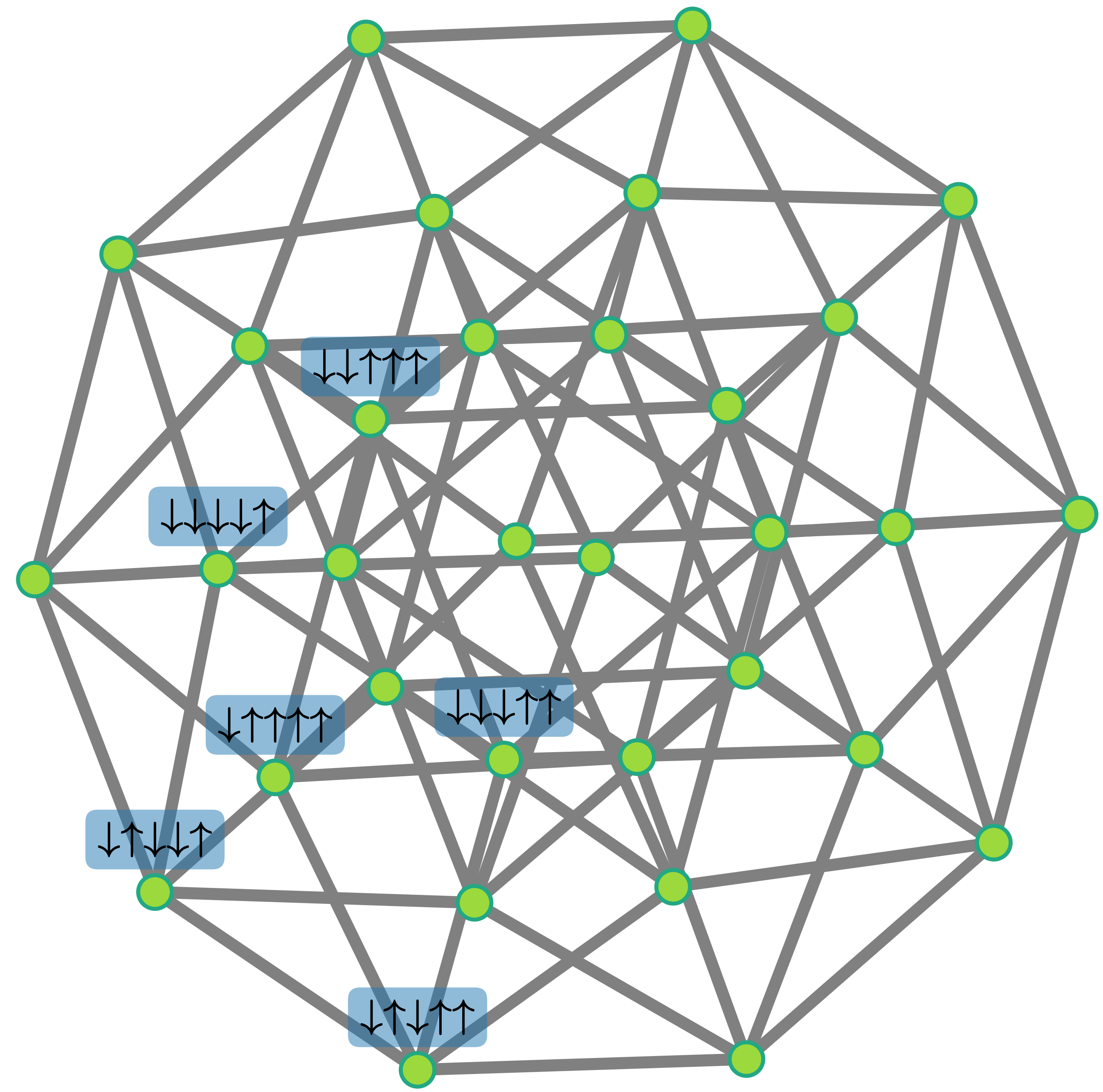
$$H = \sum_{I} \mathscr{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathscr{T}_{IK} |I\rangle\langle K|$$

$|I\rangle \equiv$ Fock-basis state

- For example, a disordered spin-1/2 chain

$$H = \sum_{i} [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

$|I\rangle \equiv \sigma^z$- product state



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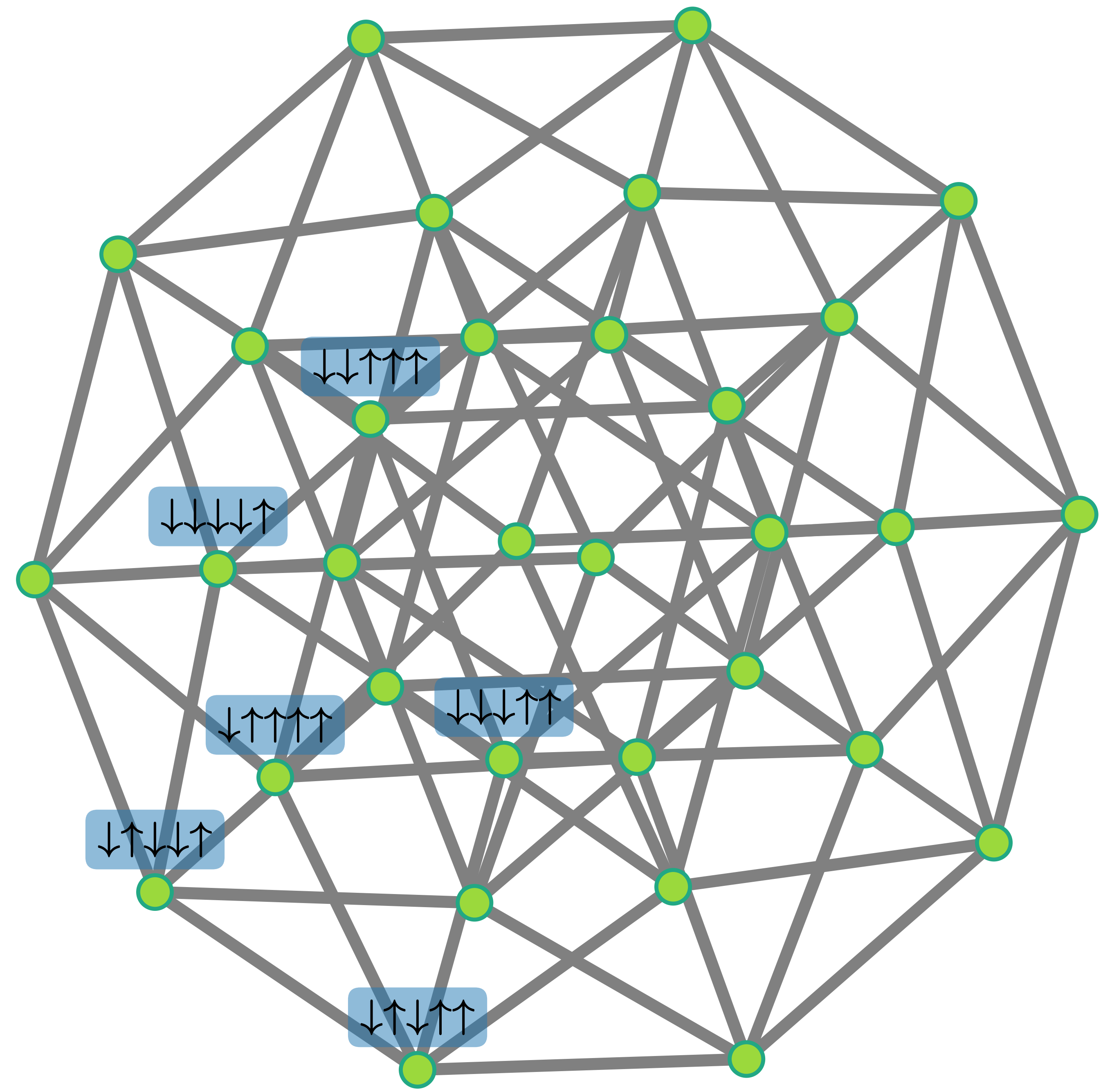

- Dimension of graph exponentially large in system size: $N_{\mathscr{H}} \sim \exp(N)$
- Generically extensive connectivities; distributions can be non-trivial

Specifying the problem

- Generic many-body Hamiltonian = tight binding Hamiltonian on the Fock space graph

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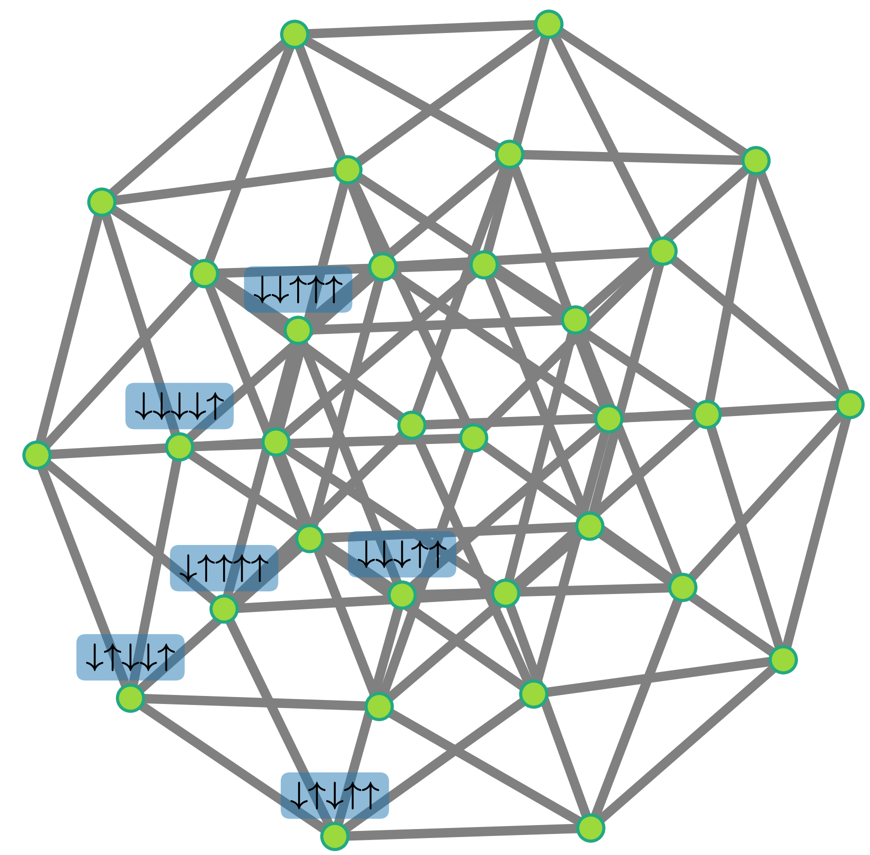

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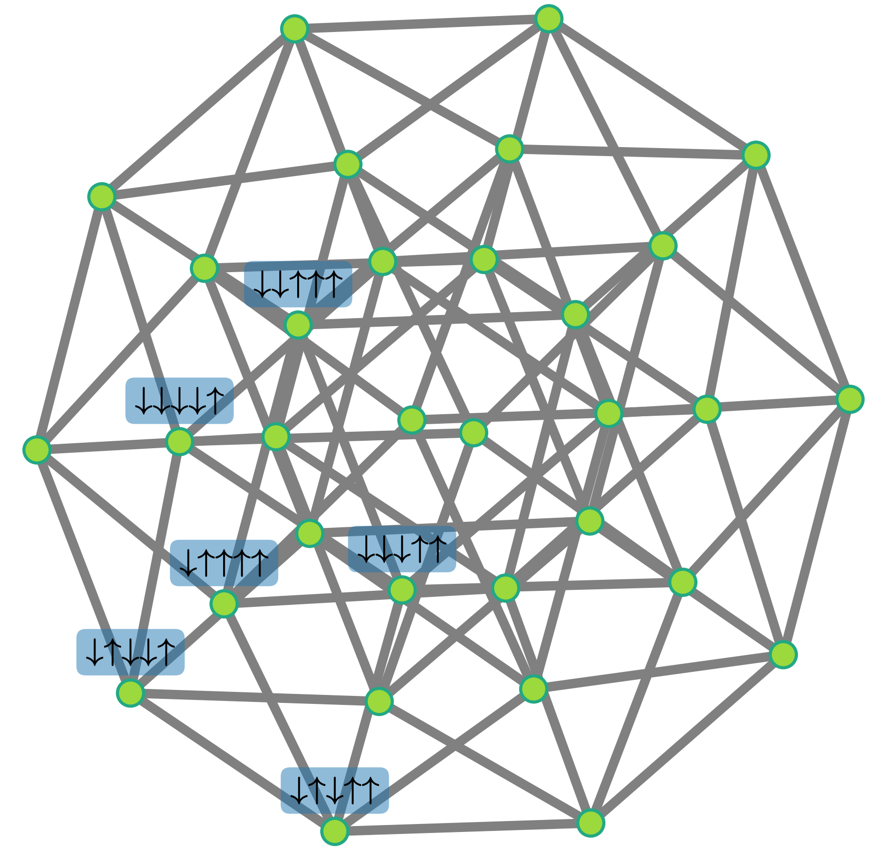

NOT an Anderson localisation problem on a high-*d* graph

- Non-trivially correlated Fock-space site energies
- Effective width of the Fock-space disorder scales with system size
- Connectivities on the graph non-trivially distributed
- Mean/typical connectivity scales with system size

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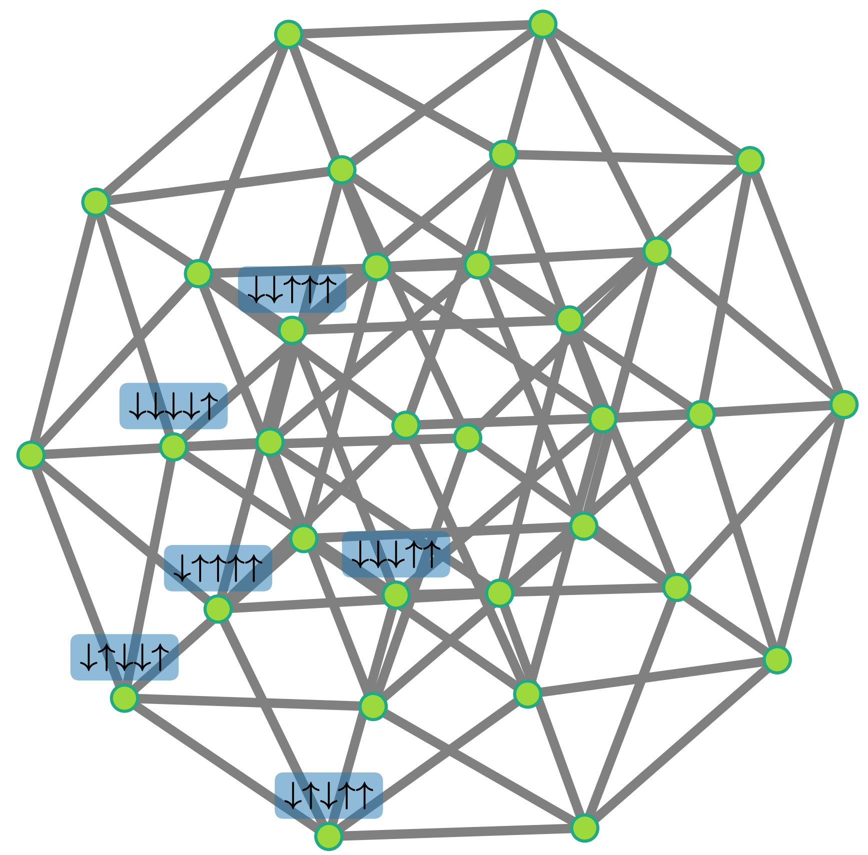

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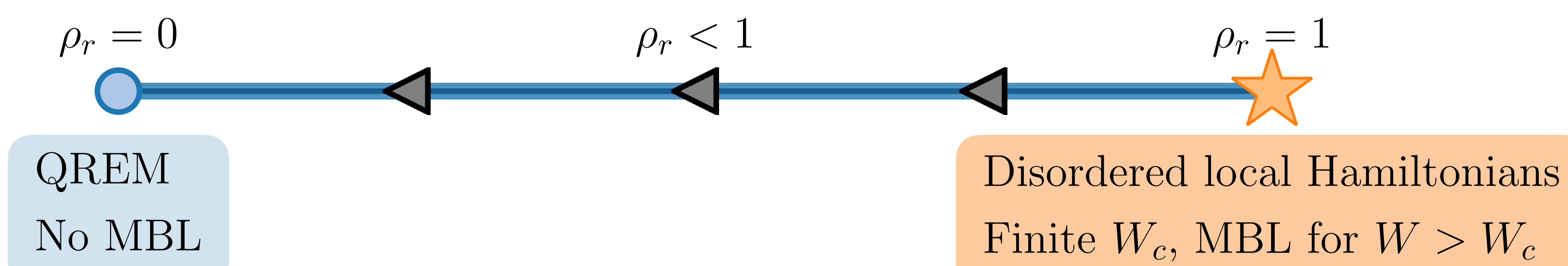
Minimal specification

- Joint $N_{\mathscr{H}}$-dimensional distribution of the $\mathscr{E}_I$'s $\equiv \mathscr{P}_{N_{\mathscr{H}}}(\{\mathscr{E}_I\})$
- Distribution of the connectivities on the Fock-space graph

Pathways to MBL

(I) Correlations in Fock-space site energies

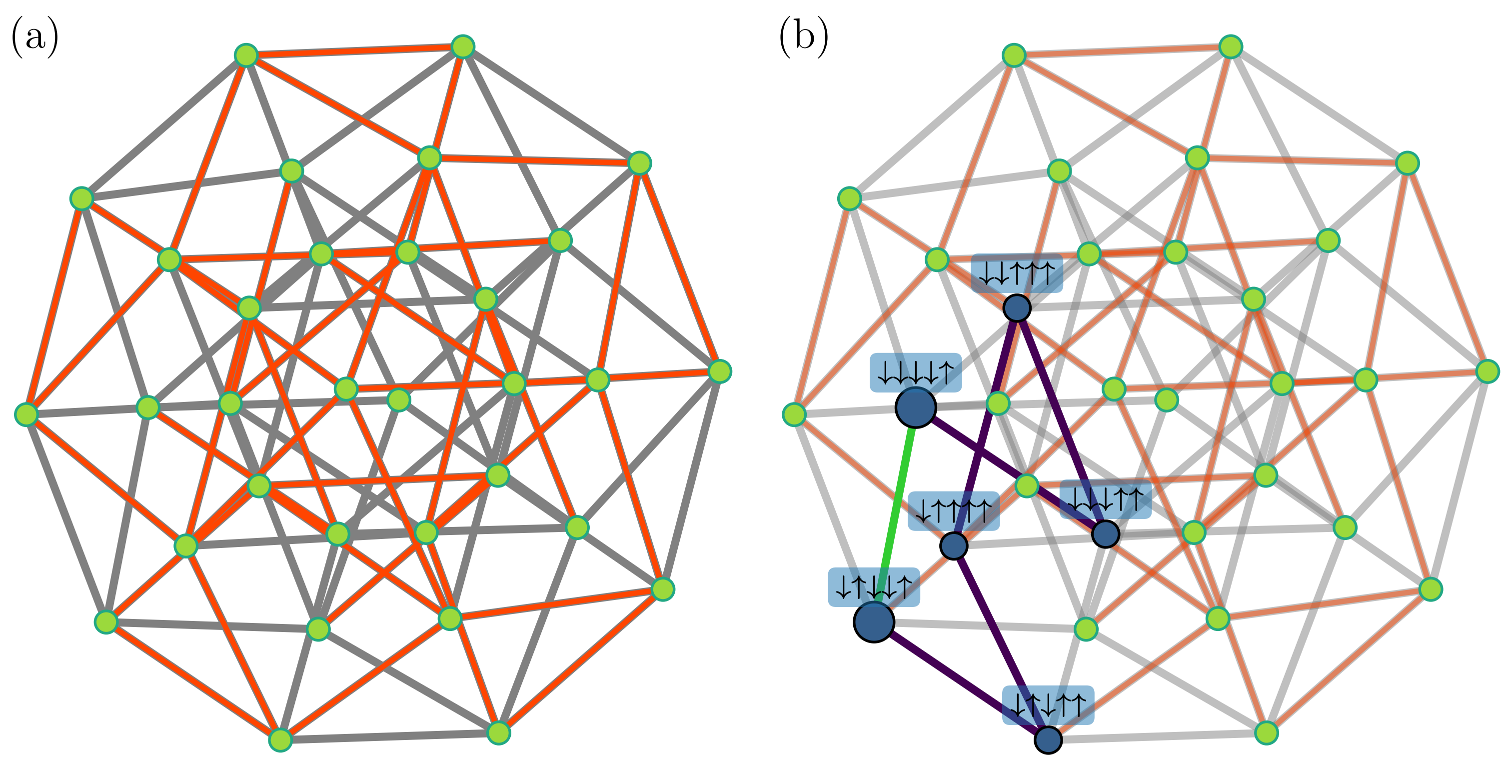
- Strong correlations in the Fock-space site energies present.
- An MBL phase is stable only if the correlations are ***maximal*** at finite distances on the graph



SR and D. E. Logan, arXiv:1911.12370

(II) Constrained dynamics in Fock space

- Constraints can lead to bottlenecks on the Fock space
- Bottlenecks becoming potentially off-resonant can drive a stable MBL phase



SR and A. Lazarides, arXiv:1912.06660

(I) Correlations in Fock-space site energies and MBL

Correlations in Fock-space site energies: Why?

Diagonal part of the Hamiltonian for a system with local interactions

$$H_{\rm diag} = \sum_{i=1}^{N} \mathcal{J}_i^{(1)} O_1(\sigma_i^z, \cdots, \sigma_{i+m_1}^z) + \sum_{i=1}^{N} \mathcal{J}_i^{(2)} O_2(\sigma_i^z, \cdots, \sigma_{i+m_2}^z) + \cdots$$

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– number of random numbers required is polynomially large in N
– they constitute the $\sim \exp(N)$ Fock-space site energies $\{\mathcal{E}_I\}$

⇓

Results in correlations in the set $\{\mathcal{E}_I\}$

⇓

Encoded in the joint distribution
$\mathcal{P}_{N_{\mathcal{H}}}(\{\mathcal{E}_I\})$

Correlations in Fock-space site energies: What?

- For a large class of local Hamiltonians, the joint distribution is a multivariate Gaussian

$$\mathscr{P}_{N_{\mathscr{H}}}(\{\mathscr{E}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|\mathbf{C}|}} \exp\left[-\frac{1}{2}\vec{\mathscr{E}}^T \cdot \mathbf{C}^{-1} \cdot \vec{\mathscr{E}}\right]$$

- Covariance matrix $\mathbf{C}$ completely specifies the distribution

The matrix elements of $\mathbf{C}$ depend only on the *Hamming distance*

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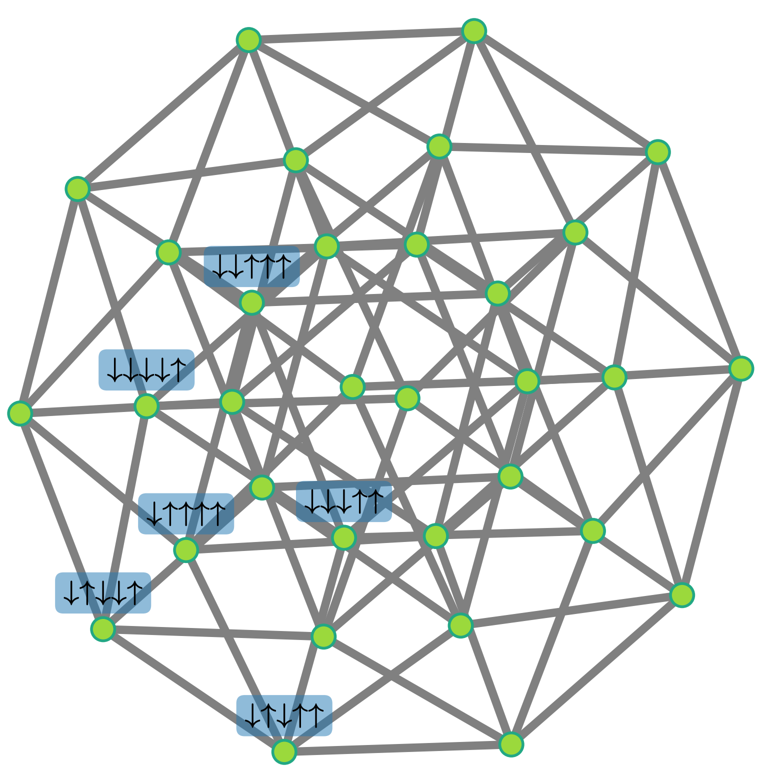

- Distance defined via spin-flips = hopping on the Fock space
- Hamming distance =
  - Shortest path length between two nodes
  - Number of spins different between two product states

Correlations in Fock-space site energies: What?

General form of the covariance that depends only on Hamming distance

$$C(r) = W^2 N \rho(r, N); \quad \rho(r=0, N) = 1$$

Effective disorder strength

Scaling with system size

All dependence on Hamming distance

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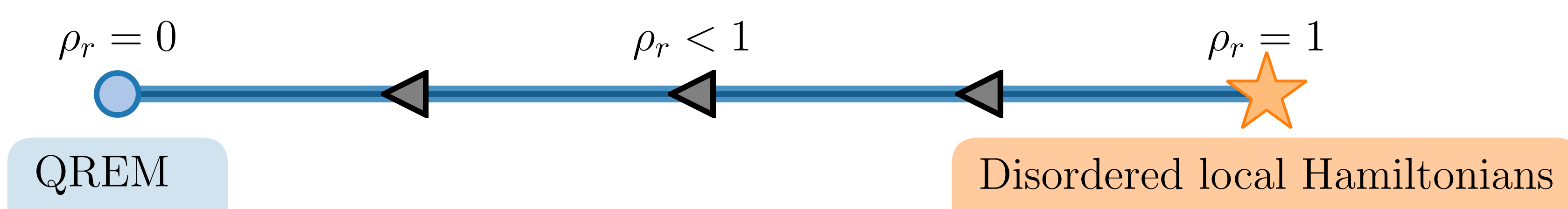
For finite Hamming distances



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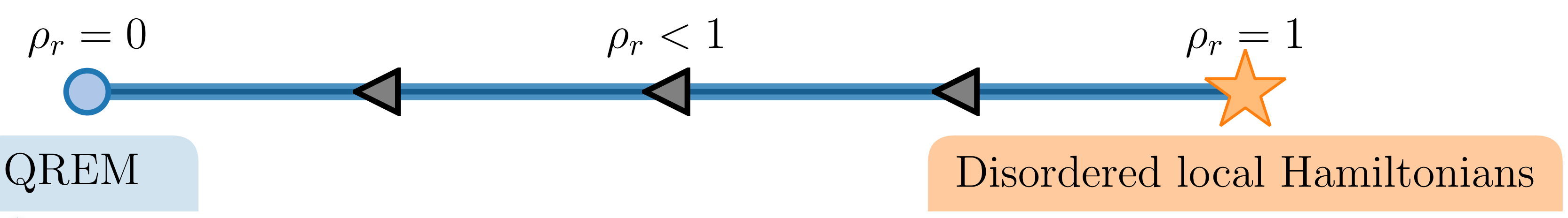
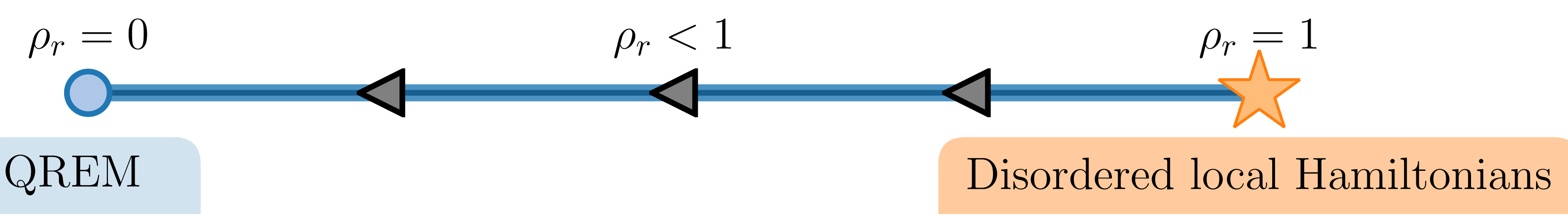


- Quantum random energy model

$$H_{\mathrm{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

- Fock-space site energies: Gaussian IRVs
- **Uncorrelated limit**

Correlations in Fock-space site energies: What?



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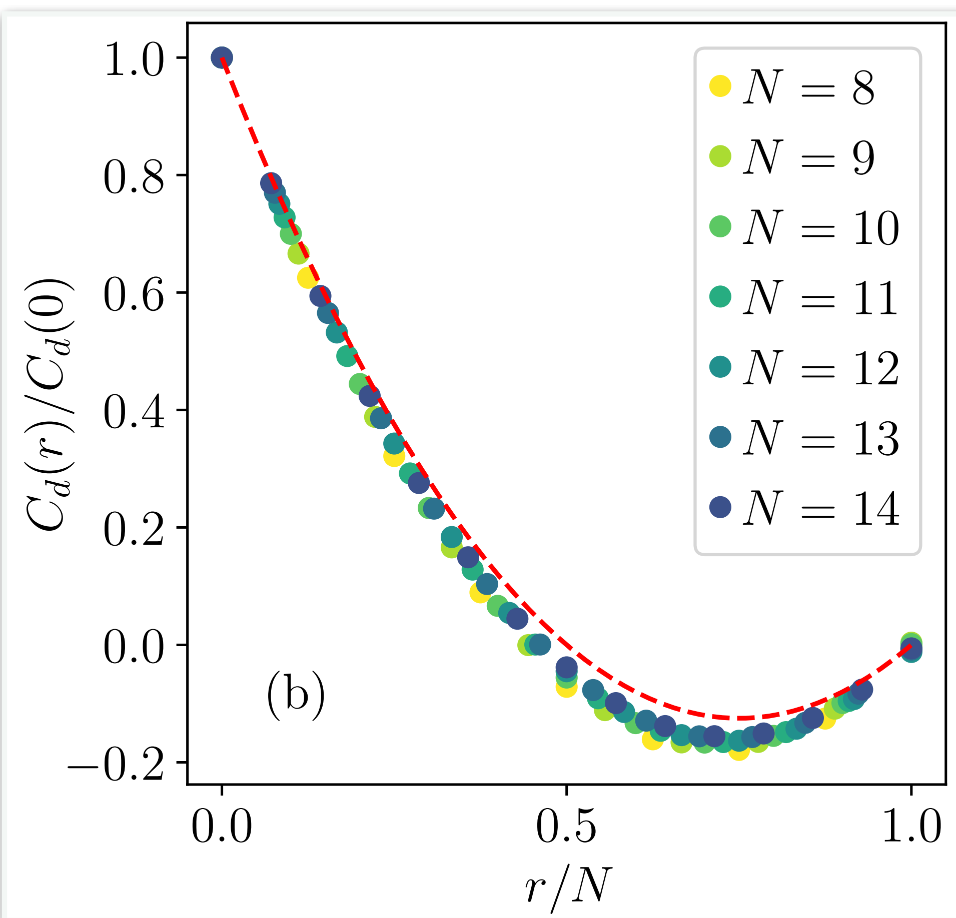

- Disordered spin-1/2 chain

  $H_{\mathrm{TFI}} = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$
- $\rho_r = a\left(1 - \frac{2r}{N}\right)^2 + (1-a)\left(1 - \frac{2r}{N}\right)$
- More generally a polynomial of r/N
- At finite distances, site-energies completely slaved
- **Maximally correlated limit $\rho(r) \to 1$ as $N \to \infty$**

Correlations: self-consistent theory for MBL

Theoretical diagnostic for stability of MBL?

— local propagator in Fock space

$$G_I(\omega) = \langle I | (\omega + i\eta - H)^{-1} | I \rangle$$
$$= [\omega^+ - \mathscr{E}_I - \Sigma_I(\omega)]^{-1}$$

Self-energy: $\Sigma_I(\omega) = X_I(\omega) + i\Delta_I(\omega)$

Why are we interested in the self-energy?

— probabilistic order parameter for localisation-delocalisation transition

— delocalised phase: $\Delta_I(\omega)$ is non-vanishing with unit probability

— localised phase: $\Delta_I(\omega)$ is vanishing ($\sim \eta$) with unit probability

- — $\Delta_I(\omega)/\eta$ has a distribution (Lévy) in the thermodynamic limit

[Abou-Chacra+Anderson+Thouless (1973), Economou (Springer, 2006)]

Correlations: self-consistent theory for MBL

Self-consistent probabilistic mean-field theory — sketch

1
$$\Sigma_I(\omega) = \sum_K \frac{\mathcal{T}_{IK}^2}{\omega^+ - \mathscr{E}_K - \Sigma_K(\omega)} + \cdots$$
Renormalised perturbation series

[Feenberg (1948)]

Replace by its typical value

2
$$\Sigma_I(\omega) = \sum_K \frac{\mathcal{T}_{IK}^2}{\omega^+ - \mathscr{E}_K - \Sigma_{\rm typ}(\omega)} + \cdots$$

3 Obtain distribution of $\Sigma_I(\omega)$ from the joint distributions of $\{\mathscr{E}_K\}$; denoted by $\mathscr{P}_\Sigma(\Sigma, \Sigma_{\rm typ})$

Information about correlations gets fed in!

4 Impose self-consistency: $\Sigma_{\rm typ}$ arising from $\mathscr{P}_\Sigma(\Sigma, \Sigma_{\rm typ})$ must coincide with *input* $\Sigma_{\rm typ}$

Logan+Welsh PRB 2019, **SR**+Logan SciPost 2019, **SR**+Logan arXiv:1911.12370

Correlations: how do they enter the self-energy calculation

$$\Sigma_I(\omega) = \sum_K \frac{\mathscr{T}_{IK}^2}{\omega^+ - \mathscr{E}_K - \Sigma_{\text{typ}}(\omega)} + \cdots$$

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sum over neighbours of $|I\rangle$ on Fock space

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- Maximally correlated limit

  $\Rightarrow \mathcal{E}_K = \mathcal{E}_I + \mathcal{O}(1)$

- But recall, distribution of $\mathcal{E}_I$ has a width which scales as $\sqrt{N}$
- On this scale, $\mathcal{E}_K \approx \mathcal{E}_I$

$$\Sigma_I(\omega) = \frac{N\Gamma^2}{\omega^+ - \mathcal{E}_I - \Sigma_{\text{typ}}(\omega)} + \cdots$$

Upshot: the self-energy is just a single term

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Upshot: the self-energy is just a single term

- Completely uncorrelated limit

  $\Rightarrow \mathscr{E}_K = \mathscr{E}_I + \mathcal{O}(\sqrt{N})$
- Width of distribution of $\mathscr{E}_I$ still scales as $\sqrt{N}$
- On this scale, all the site energies are independent random variables

Self-energy continues to be a sum of extensive number of terms

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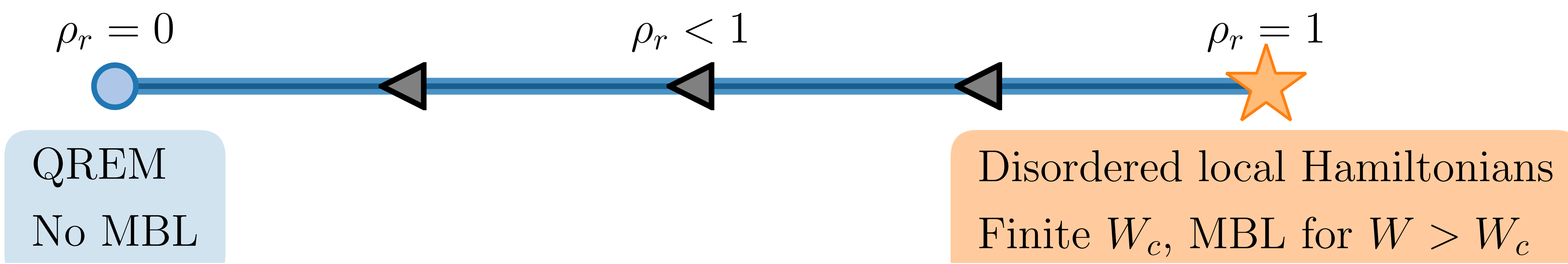
Correlations: self-consistent stability of MBL

We now know how to handle the correlations in the Fock-space site energies

After some algebra …

Correlations: a central result



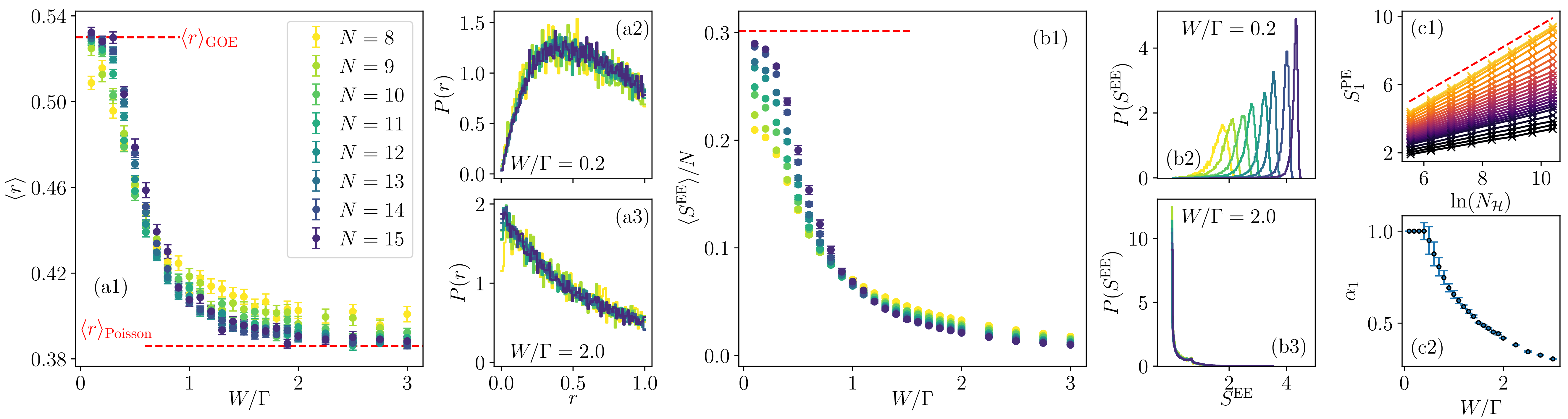
- MBL possible only if Fock-space site energies at finite distances maximally correlated; minimum requirement for MBL to be stable
- Any randomness/independence in them leads to delocalisation

SR+Logan arXiv:1911.12370

Correlations: Numerical checks

$$H_{\mathrm{TFI}} = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

Example of **maximally correlated** model



Presence of an MBL phase and an MBL transition clear from standard numerical diagnostics

- Statistics of level-spacing ratio (GOE statistics vs. Poisson statistics)
- Entanglement entropy (volume law vs. area law)
- Scaling of participation entropies of many-body wave functions

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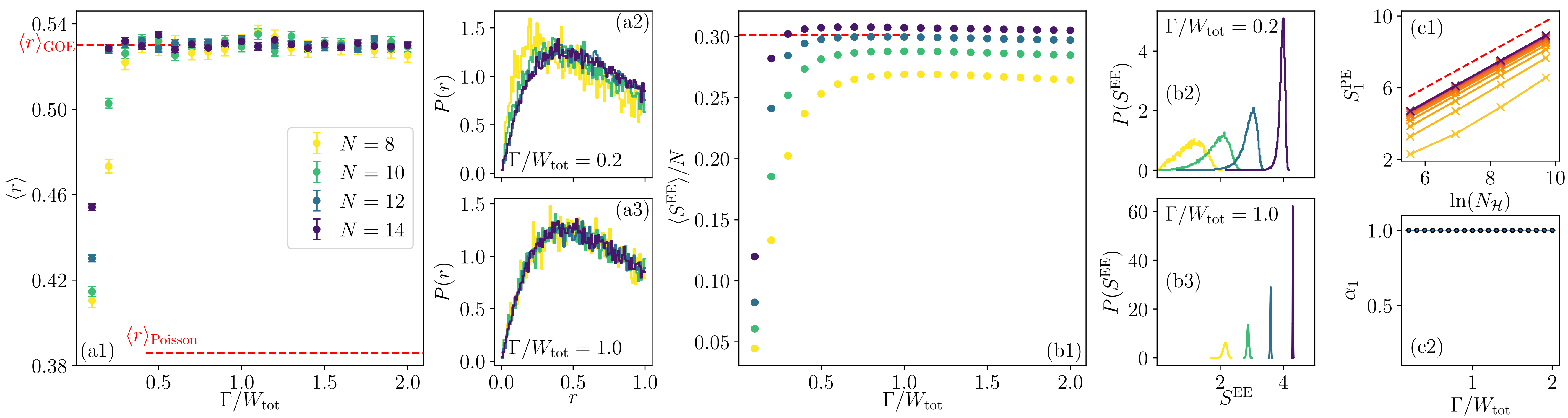
Correlations: Numerical checks

$$H_{\text{ExpREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

$$C(r) = W^2 N \exp(-r/N)$$

Example of **partially correlated** model



Numerics consistent with the absence of an MBL phase

SR+Logan arXiv:1911.12370

(II) Constrained dynamics in Fock space and MBL

Constrained dynamics

— switch off some links on the Fock space thereby creating bottlenecks

Example

— start with the QREM as our reference *unconstrained* model (no MBL in the model)

Baldwin+Pal+Laumann+Scardichhio
PRB (2016)

$$H_{\mathrm{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Gaussian IRVs

spins free to flip

SR+Lazarides arXiv:1912.06660

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Gaussian IRVs

spins free to flip

— introduce *East-glass* type constraints Ritort+Sollich (2003), Garrahan (2018)

$$H_{\mathrm{EastREM}} = \sum_I \mathcal{E}_I |I\rangle\langle I| + \frac{\Gamma}{2} \sum_i \sigma_i^x(1 + \sigma_{i+1}^z)$$

Gaussian IRVs

a spin can flip only if the one to its right is up

SR+Lazarides arXiv:1912.06660

Constraints: how do they cause bottlenecks?

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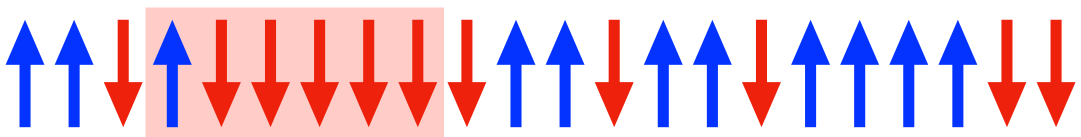

Frozen block of spins; can melt only from the right; arrested dynamics !!

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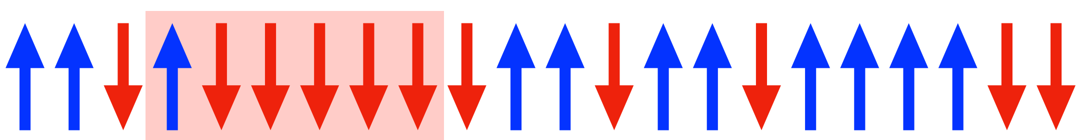

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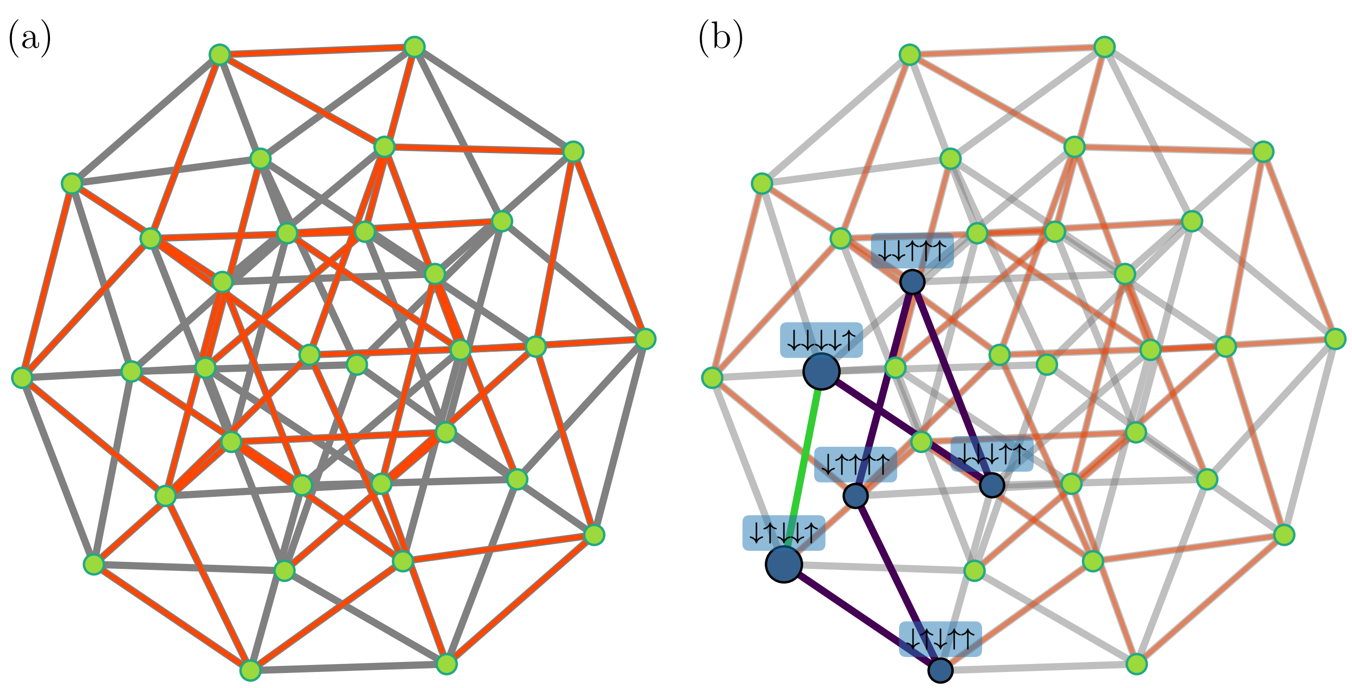
On Fock space



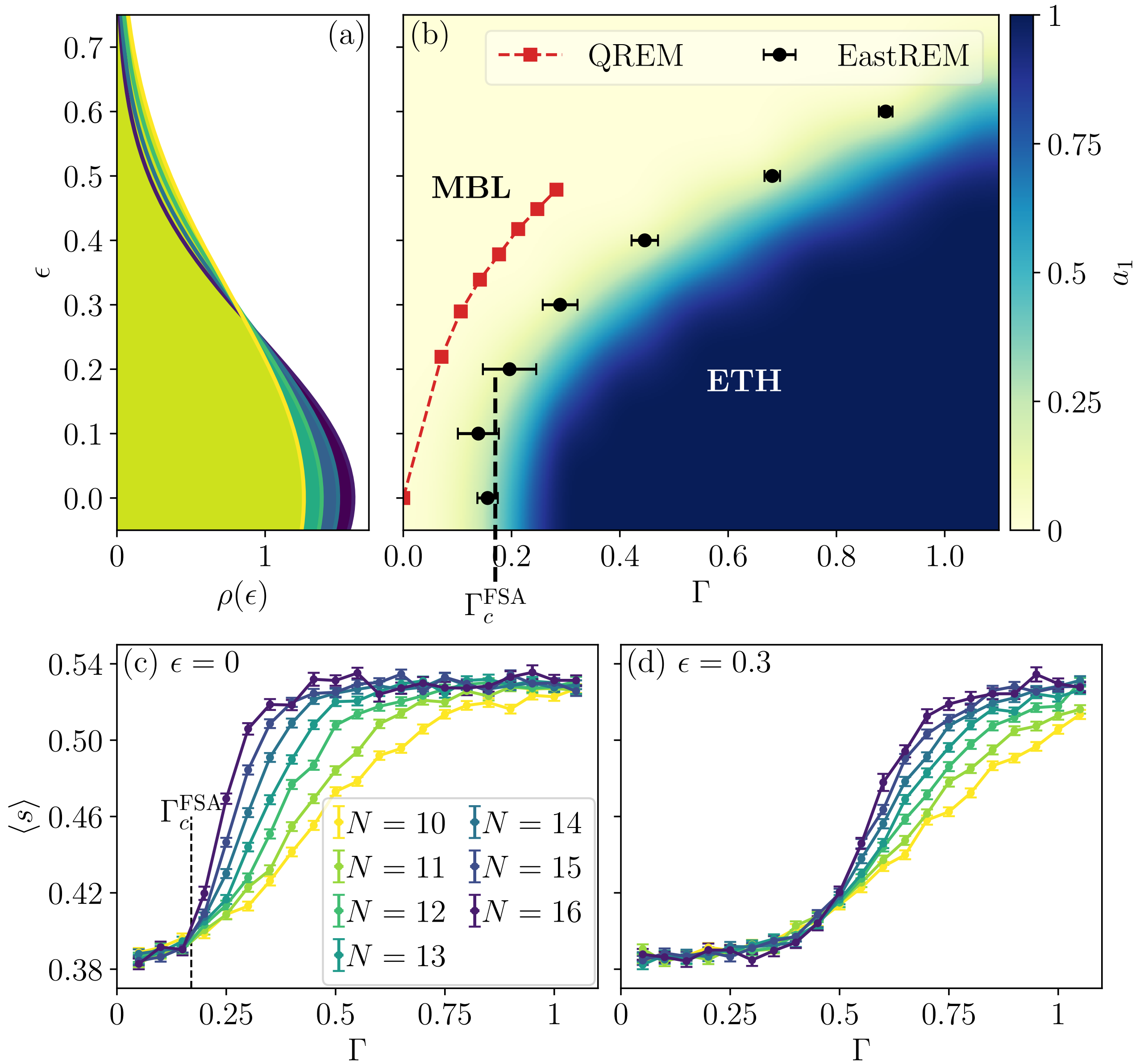
- Constraints switch off some of the links
- Increase the typical distance between two nodes
- Decrease the number of paths between two nodes

SR+Lazarides arXiv:1912.06660

Constraints: can they stabilise a full MBL phase?

Constraints clearly disfavour delocalisation but can they stabilise MBL?

— spectral properties of the EastREM suggest an affirmative answer



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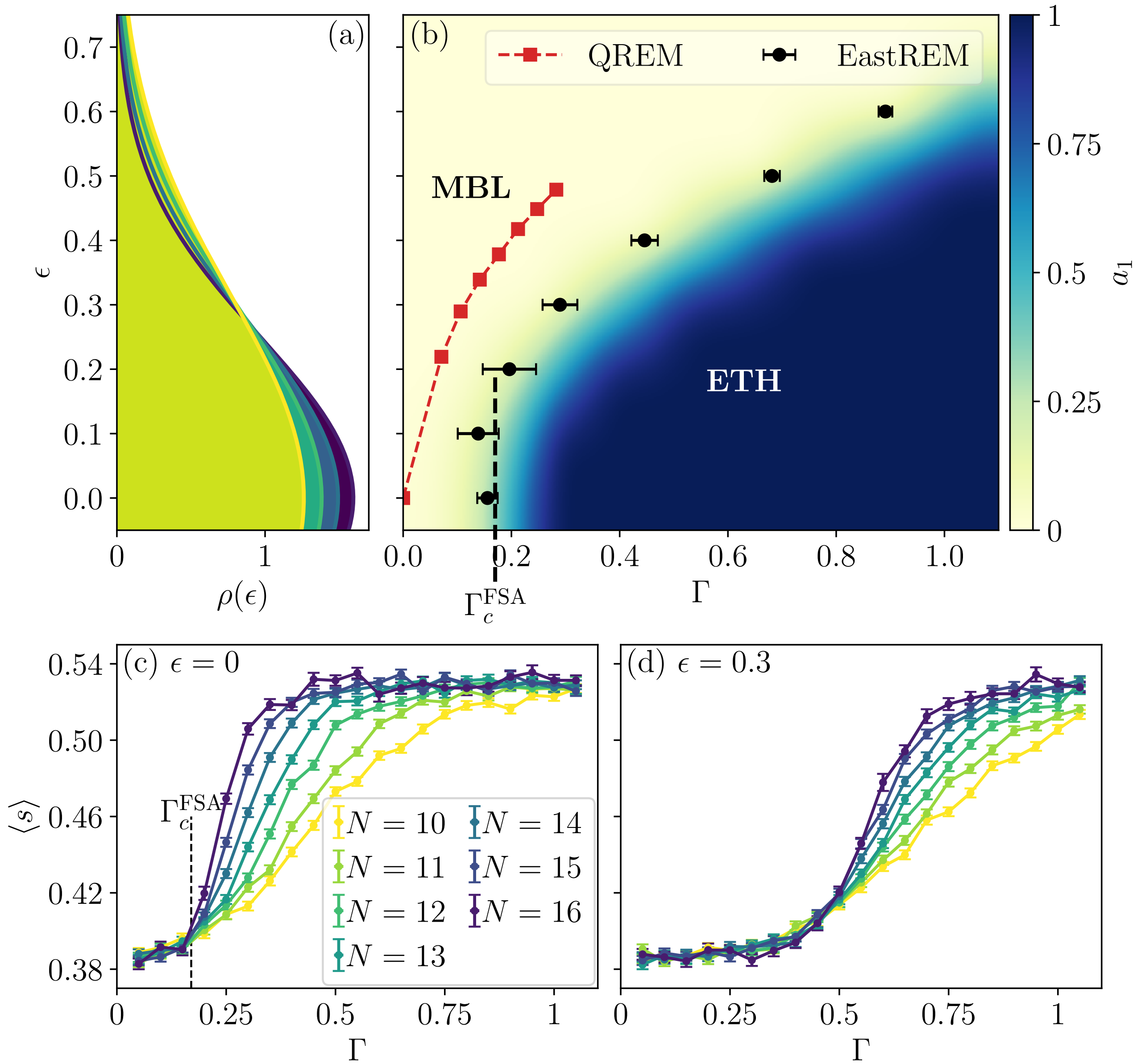
Constraints: can they stabilise a full MBL phase?

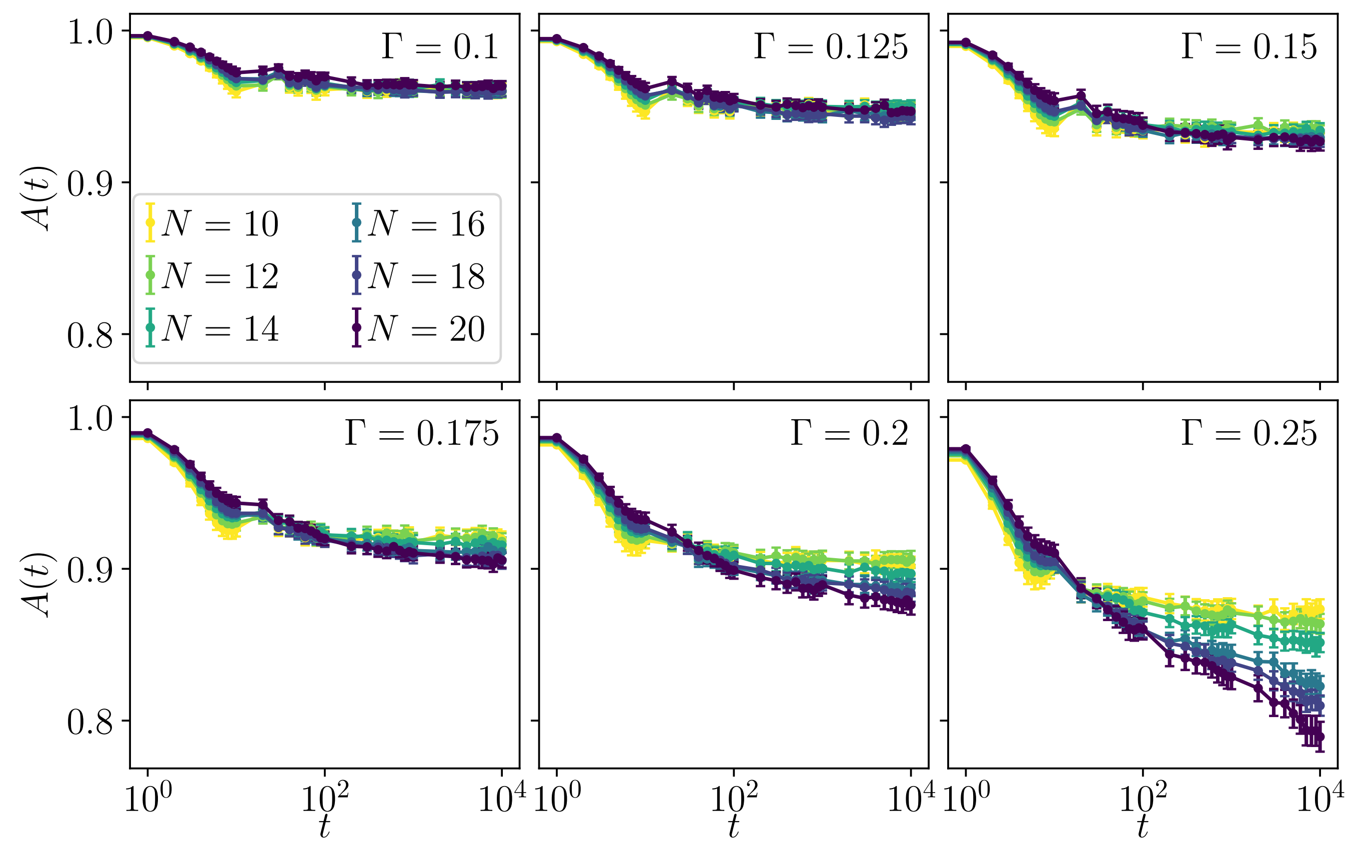
Constraints clearly disfavour delocalisation but can they stabilise MBL?

— spectral properties of the EastREM suggest an affirmative answer

— dynamical autocorrelations also do so



$$A(t) = \frac{1}{N} \sum_i \langle \psi_0 | \sigma_i^z(t) \sigma_i^z | \psi_0 \rangle$$



SR+Lazarides arXiv:1912.06660

Constraint induced MBL: theory?

Forward scattering approximation on Fock space

—leading order approximation to the non-local propagator in Fock space

$$\psi_I = \sum_{p \in \text{paths}^*(0,I)} \prod_{K \in p} \frac{\Gamma}{\mathcal{E}_0 - \mathcal{E}_K}$$

sum over the shortest paths

— for delocalisation to happen $\lim_{r\to\infty} \mathcal{P}\left(\frac{\ln|\psi_r|^2}{2r} > -\xi^{-1}\right) \to 1$

- Constraints increase the shortest path lengths, so the propagators naturally suppressed due to powers of Γ
- Decrease the number of paths between two nodes and hence decreases probabilities of obtaining resonant paths

After some algebra …

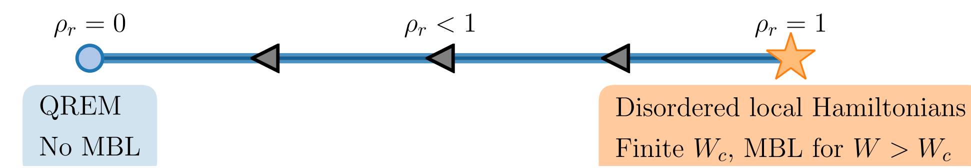
A stable MBL phase at finite Γ is indeed possible

SR+Lazarides arXiv:1912.06660

Summary

(I) Correlations in Fock-space site energies at the origins of MBL

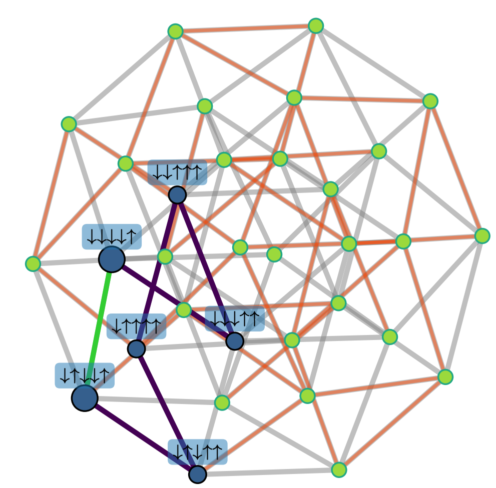
- MBL possible only when the correlations are maximal at finite distances on the Fock-space graph
- Generally the situation for local disordered Hamiltonians
- Any randomness/independence enough to delocalise the system



SR and D. E. Logan, arXiv:1911.12370

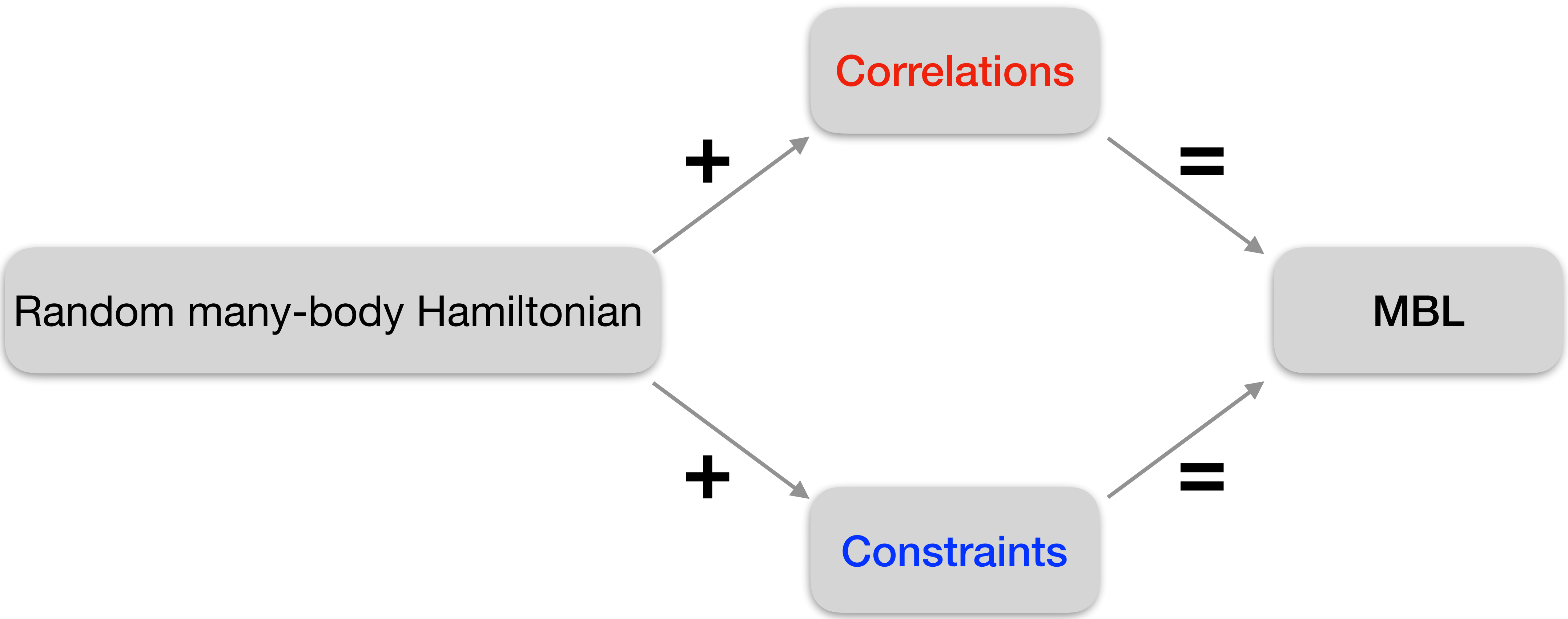
(II) Constrained dynamics on Fock space can lead to full MBL

- Constraints lead to bottlenecks in Fock space
- Potential off-resonance of these bottlenecks can stabilise a full MBL phase
- Stable MBL despite fully uncorrelated Fock-space site energies

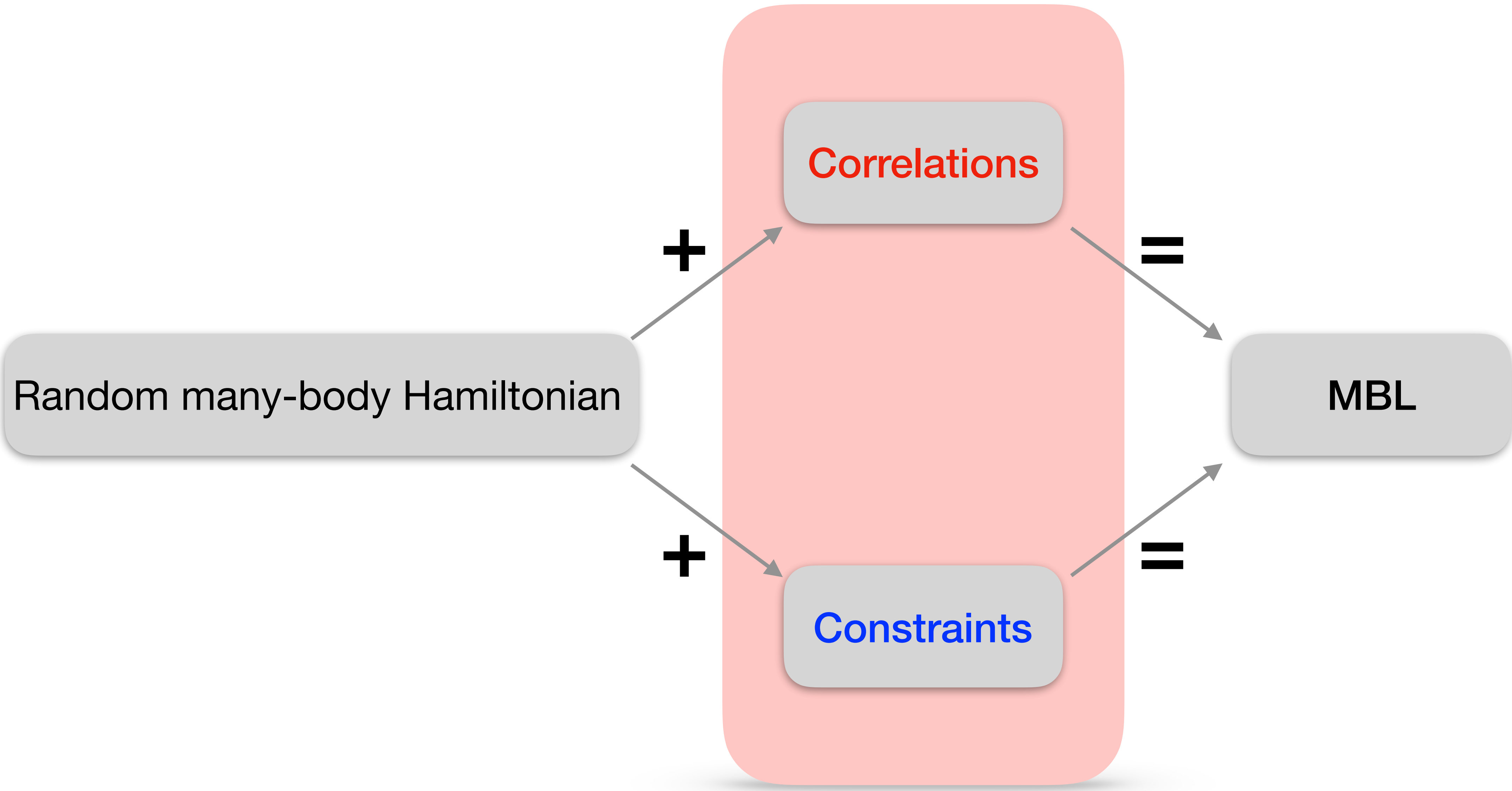


SR and A. Lazarides, arXiv:1912.06660

Outlook


Correlations
+
=
Random many-body Hamiltonian
MBL
+
=
Constraints

Outlook



— Can I think of the correlations as leading to effective constraints on the Fock space ?
— Equivalently, can I think of the constraints as generating a non-trivially correlated model?

If yes, what is the unifying principle which stabilises MBL?