Convective Momentum Transport and Multiscale Models

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Convective Momentum Transport (CMT):

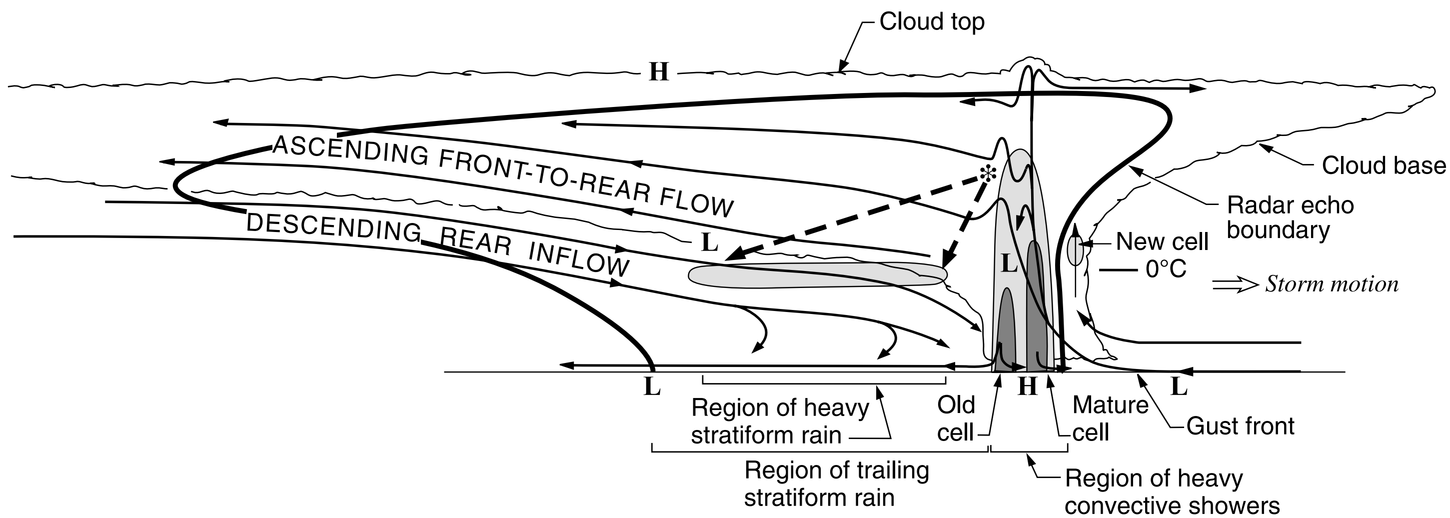
- Energy transfers between scales
  - Upscale vs. Downscale transfer
  - Resolved vs. sub-grid scales

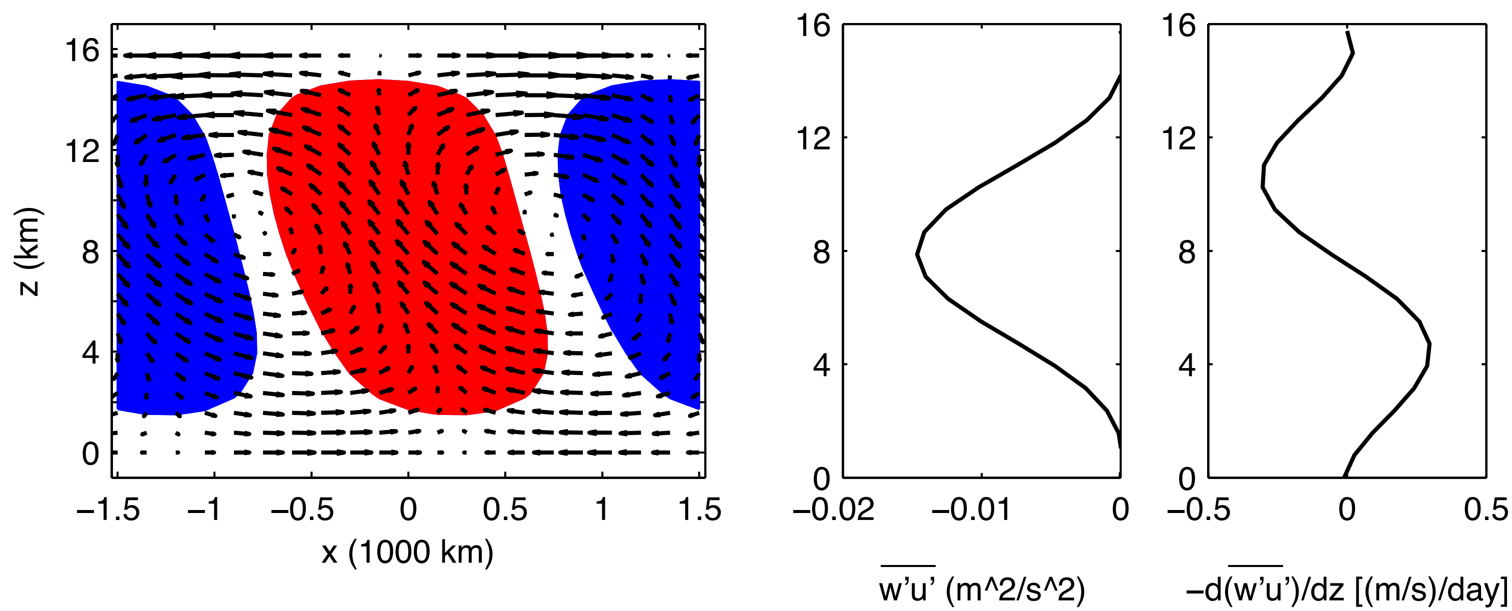
Themes:

- Multiscale modeling
- Stochastic sub-grid-scale parameterization

CMT due to Squall Line

Houze et al 1989





CMT and Convective Scales

Mesoscale convection and CMT

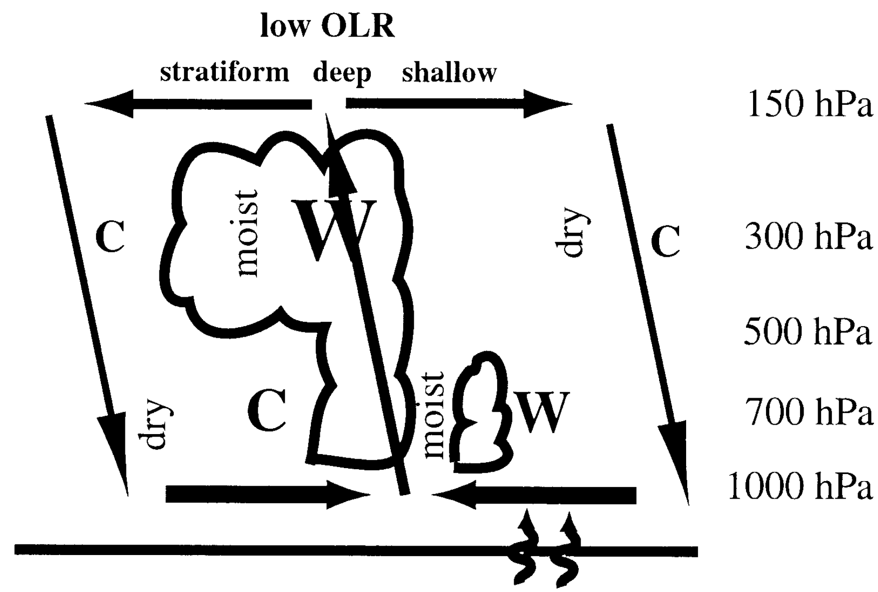
- LeMone 1983, Smull & Houze 1987, Moncrieff 1992, ...

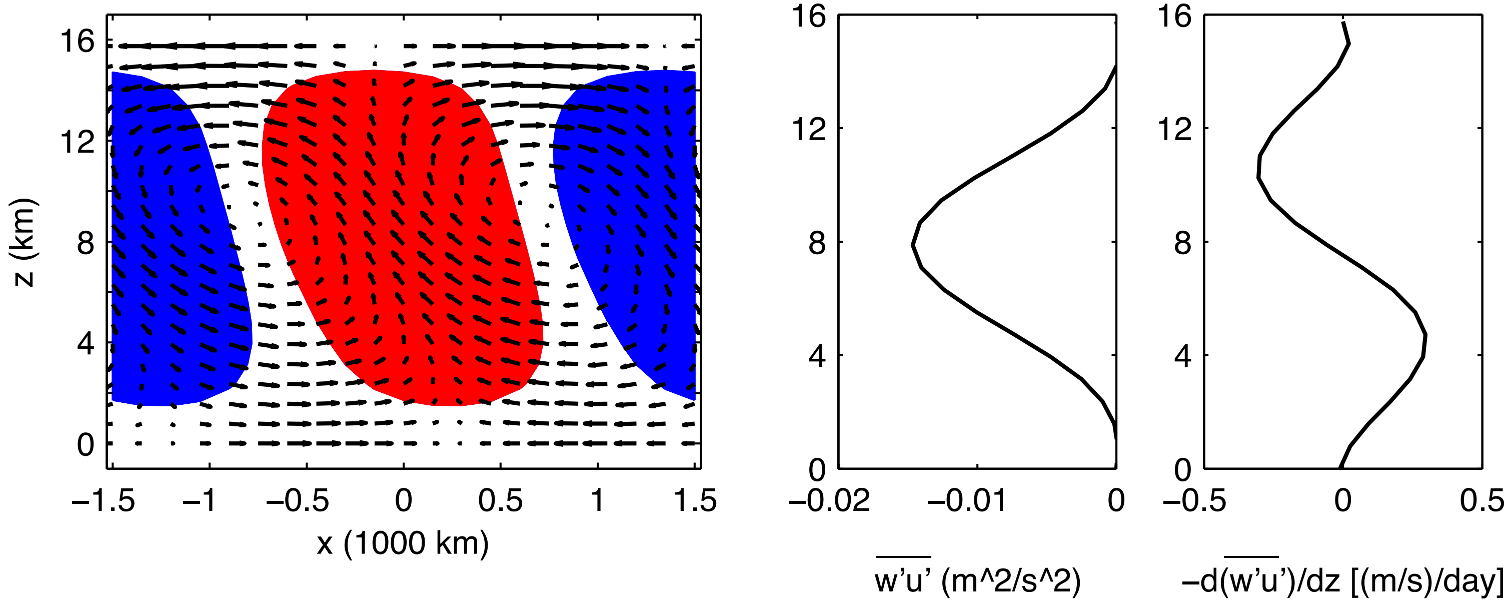
Synoptic scale convection and CMT

- Moncrieff & Klinker 1997, Grabowski & Moncrieff 2001, Majda & Biello 2004, ...
- Less understood than Mesoscale case?

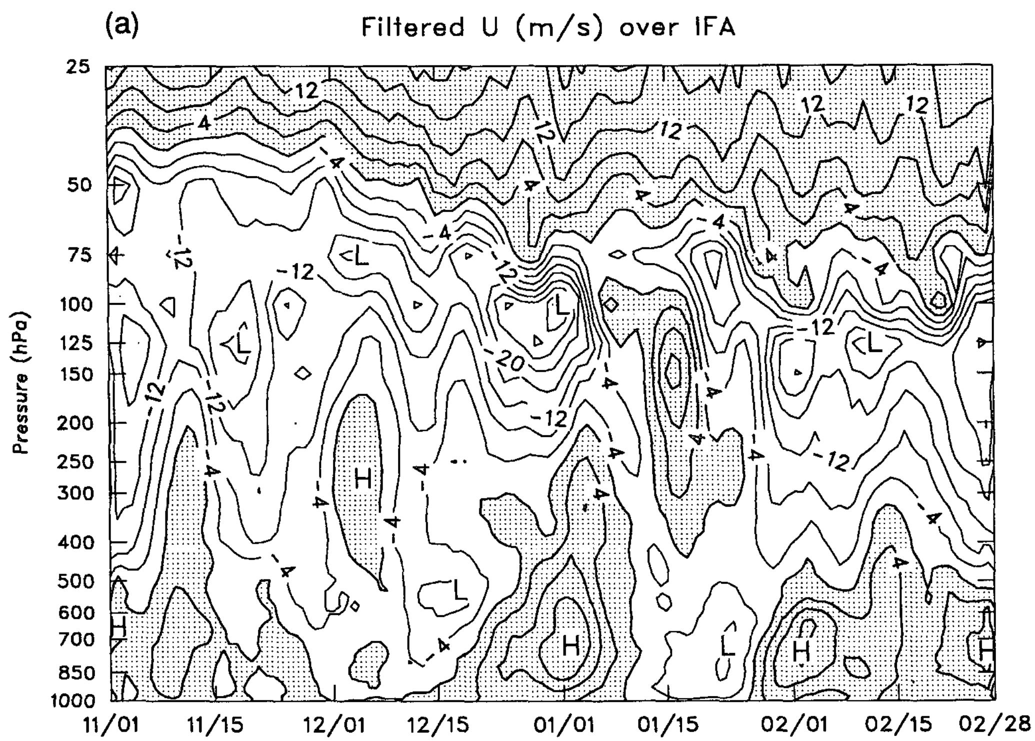
Convectively coupled Kelvin waves have vertical tilts ⇒ CMT

Straub & Kiladis 2003





Westerly wind bursts during TOGA–COARE



Lin & Johnson (1996)

Theory: Majda & Biello 2004, Biello & Majda 2005:

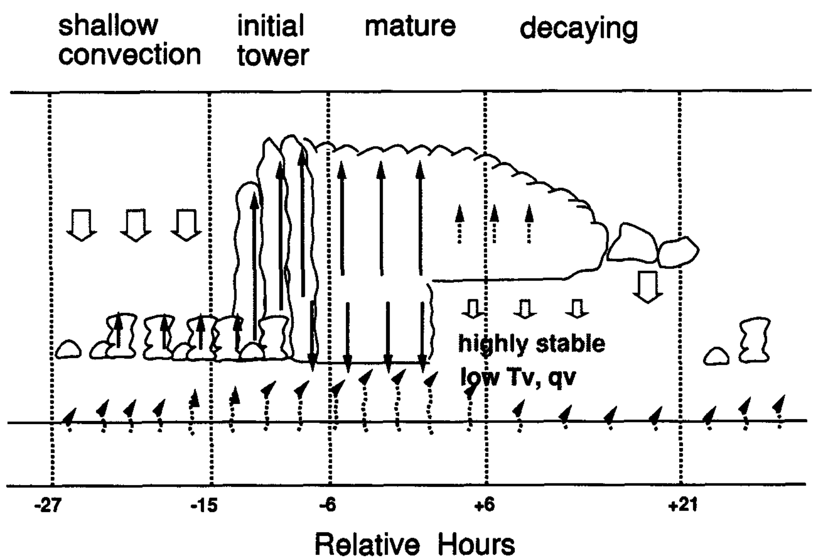
Convective Momentum Transport (CMT)

due to Convectively Coupled Waves (CCW) could accelerate strong WWB aloft:

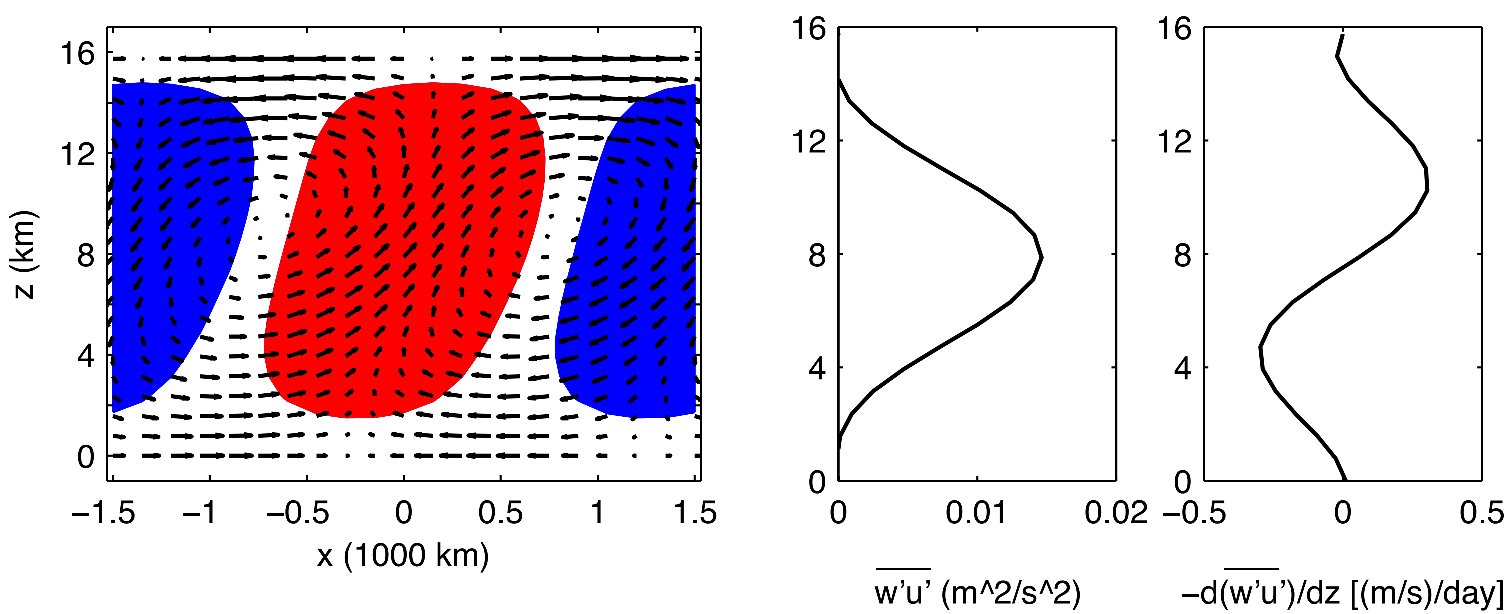
$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

2-day waves have *opposite* vertical tilts and *opposite* CMT

Takayabu, Lau & Sui 1996



Schematics for the quasi 2-day variation in TOGA COARE



How is the mean wind affected by CMT due to CCW?

Acceleration/deceleration of mean wind depends on sign of CMT

Sign of CMT depends on propagation direction of CCW

Propagation direction of CCW depends on mean wind

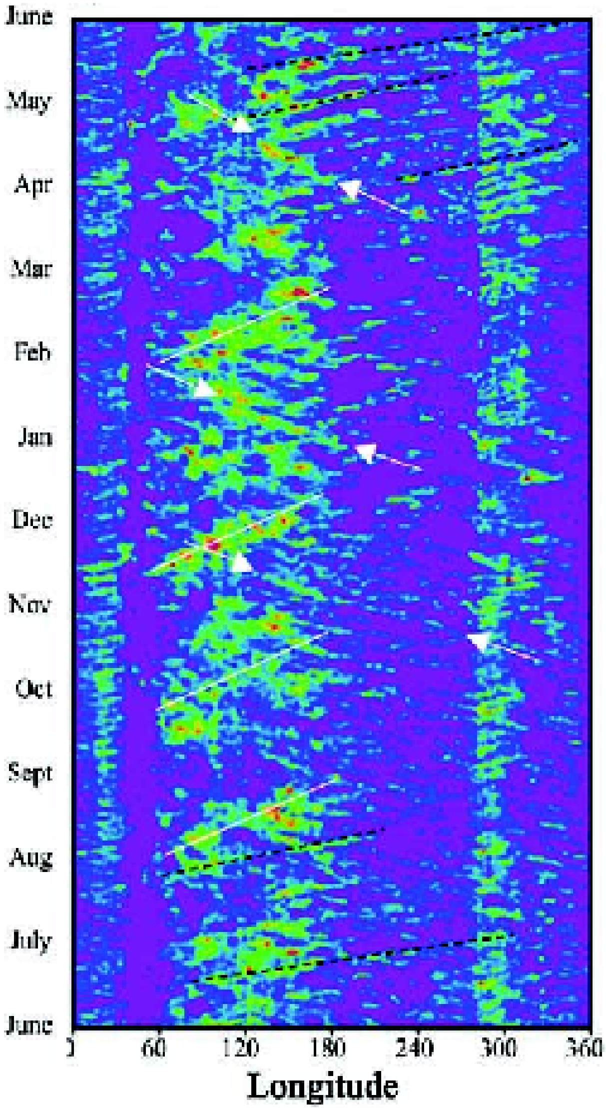
Chicken-and-egg question!

Need a two-way interaction model to sort this out:

CCW $\leftrightarrow$ mean wind

The Madden–Julian Oscillation (MJO) is an envelope of smaller-scale convection/waves

Precipitation



2000–2001 (from Zhang 2005)

How does the MJO envelope interact with the CCW within it?

- MJO $\longrightarrow$ CCW?
- MJO $\longleftarrow$ CCW?
- MJO $\longleftrightarrow$ CCW?

This Talk: momentum/wind shear interactions

Also (more) important: moist thermo. interactions

A model for convectively coupled wave–mean flow interaction

Majda & Stechmann 2009 JAS

"A simple dynamical model with features of convective momentum transport"

Dynamic multi-scale model for convectively coupled wave–mean flow interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables: $\bar{\Theta}$, θ', $\bar{Q}$, q', etc.)

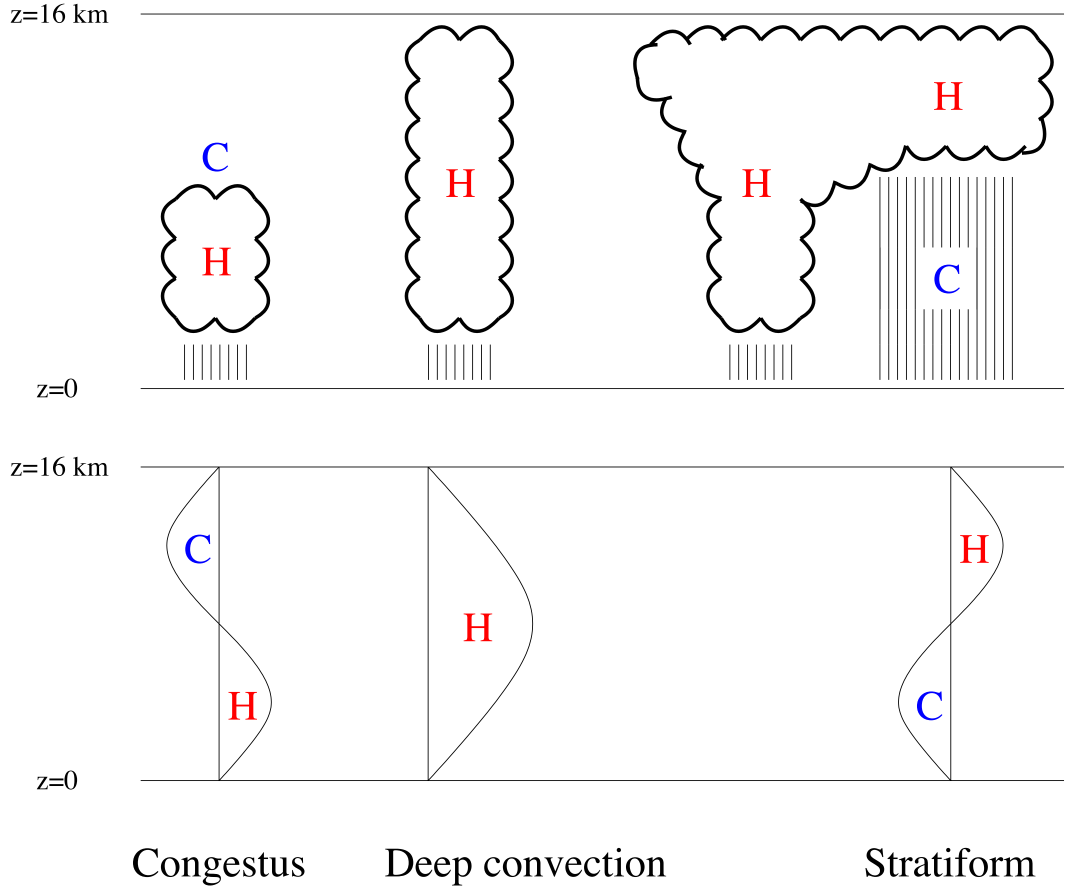
Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z\langle\overline{w'u'}\rangle$

The Multicloud Model (Khouider and Majda 2006)
(a model for CCW)



Two vertical baroclinic modes $\Rightarrow$ waves with vertical tilts

Multi-scale effects: add nonlinear advection and a 3rd baroclinic mode

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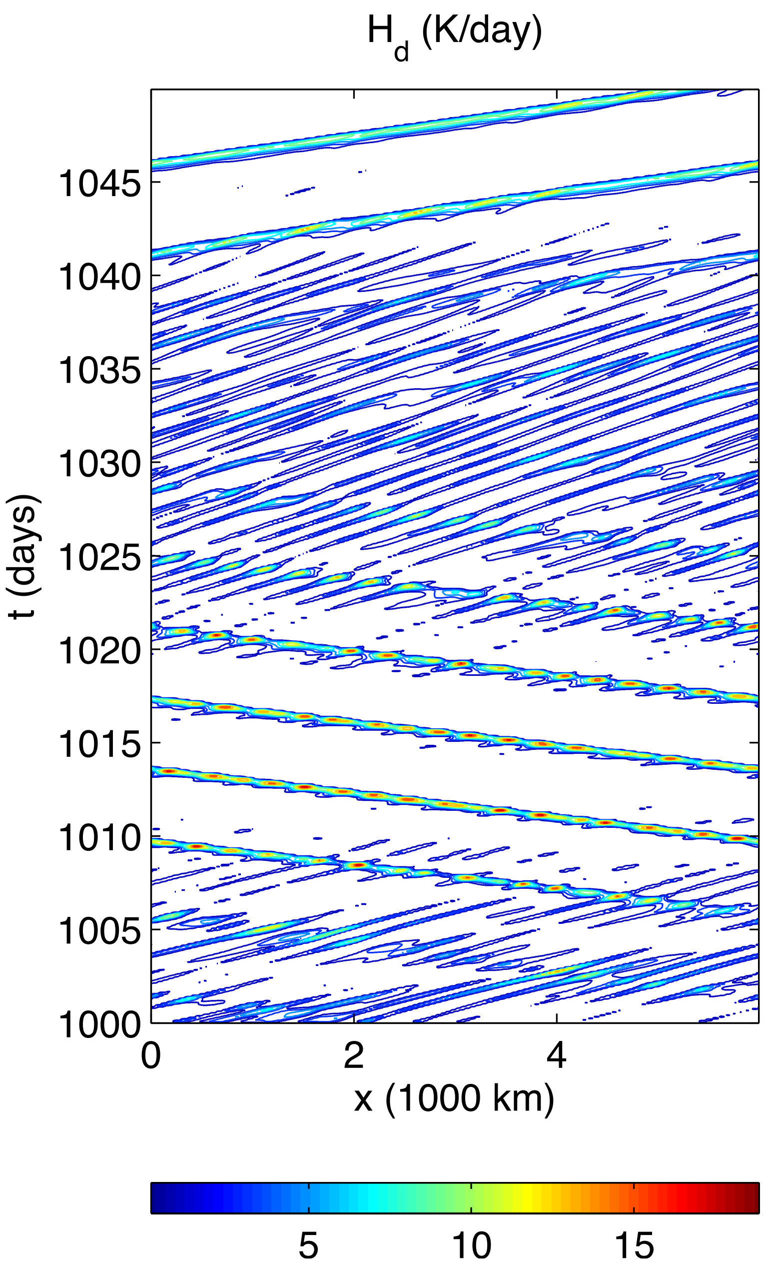
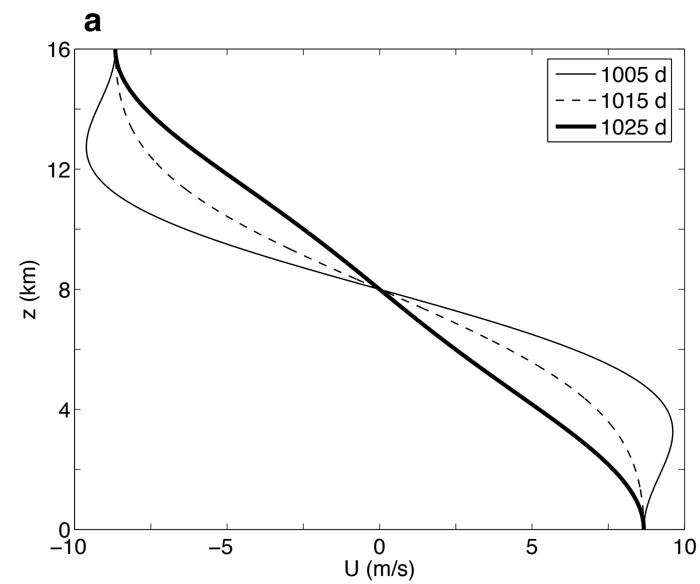
CCW–mean flow interactions on intraseasonal time scale



Mean wind $\bar{U}(z)$:



- Momentum transports from CCW drive changes in mean shear
- Changes in mean shear $\Rightarrow$ changes in wave propagation direction

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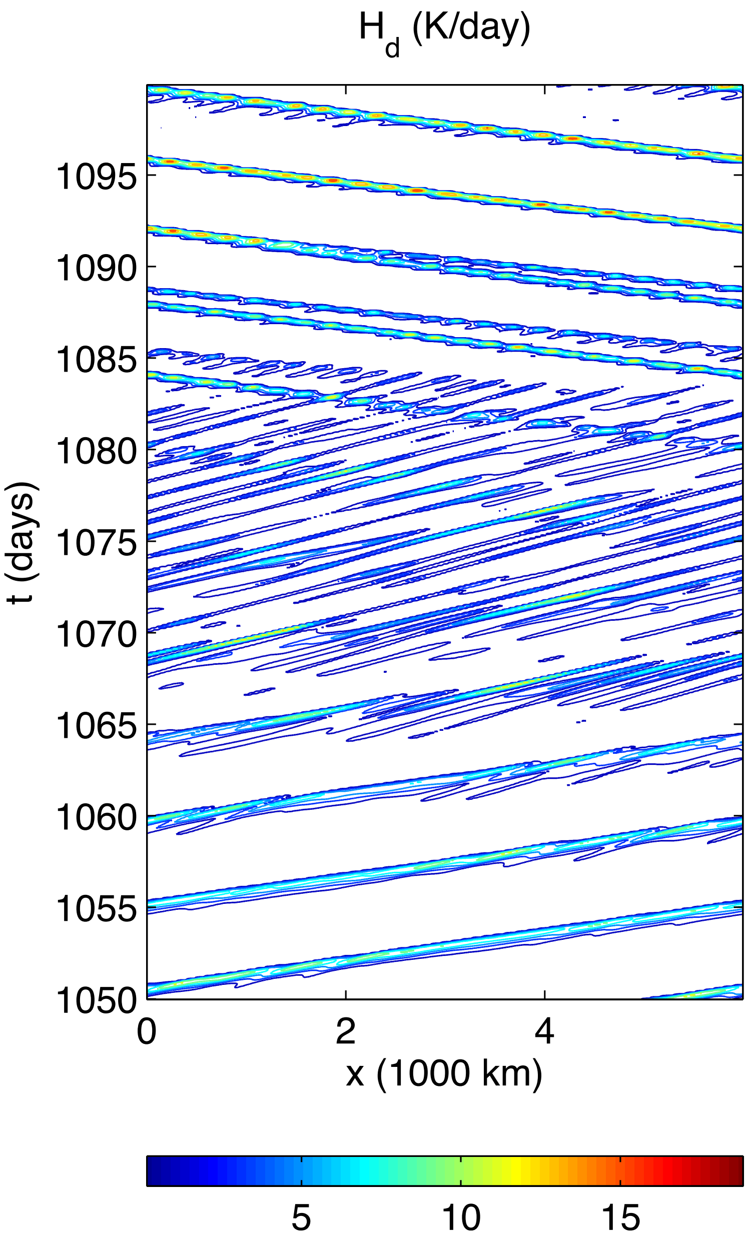
- Squall line-like fluctuations within wave envelope
- Energy transport *downscale* here

Westerly Wind Burst Intensification



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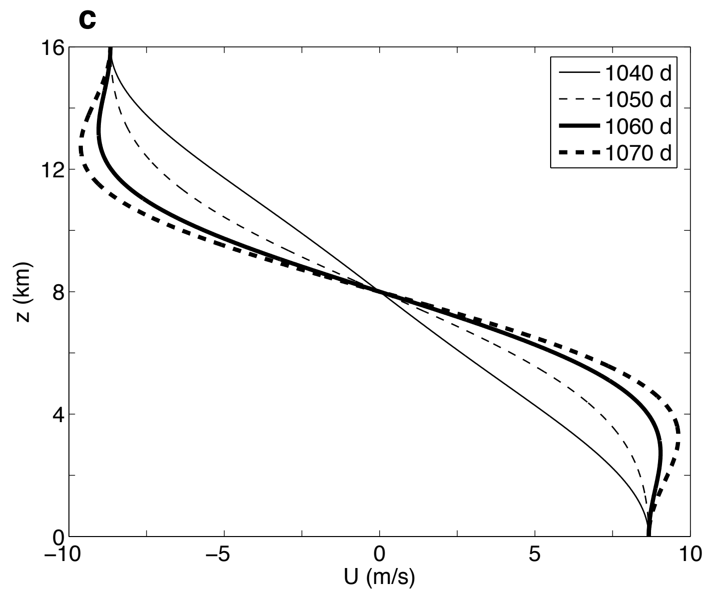


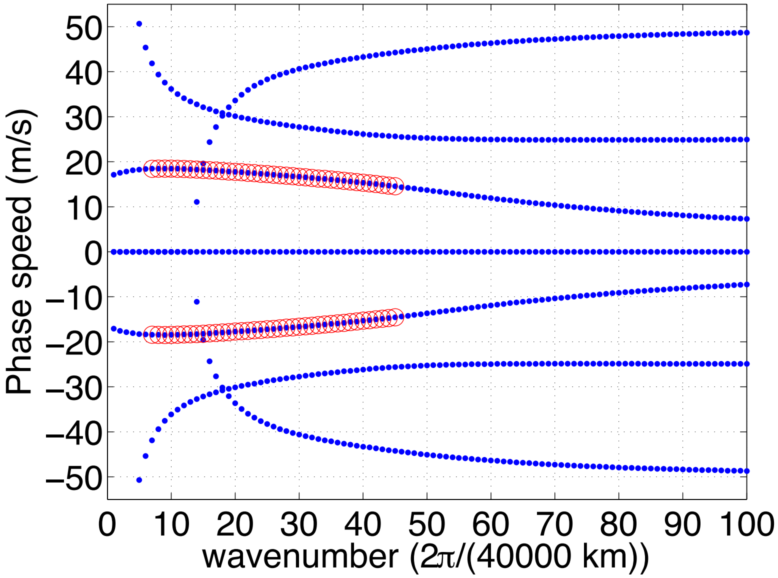
- CMT from eastward-moving CCW *accelerates* WWB aloft
- Energy transport *upscale* here

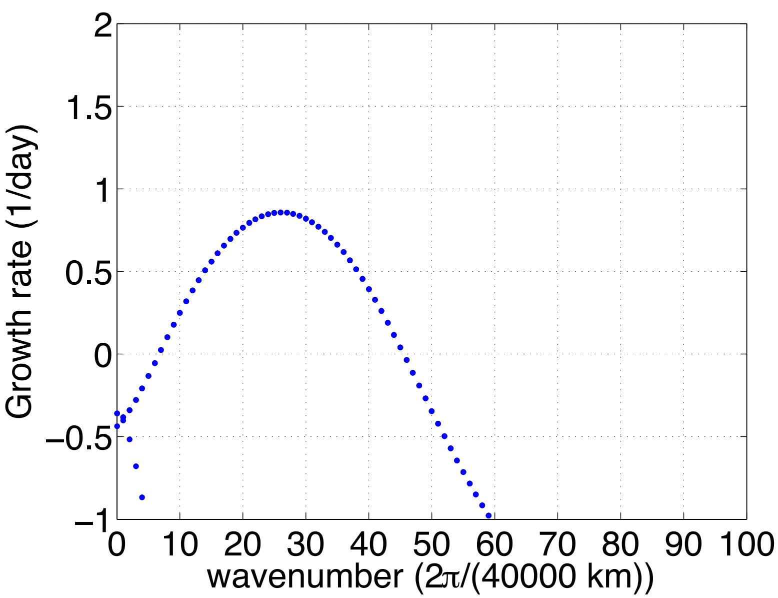
- CCW–mean flow interactions:
  sometimes energy transport *upscale*,
  sometimes energy transport *downscale*

Linear Stability Theory

Zero background wind:

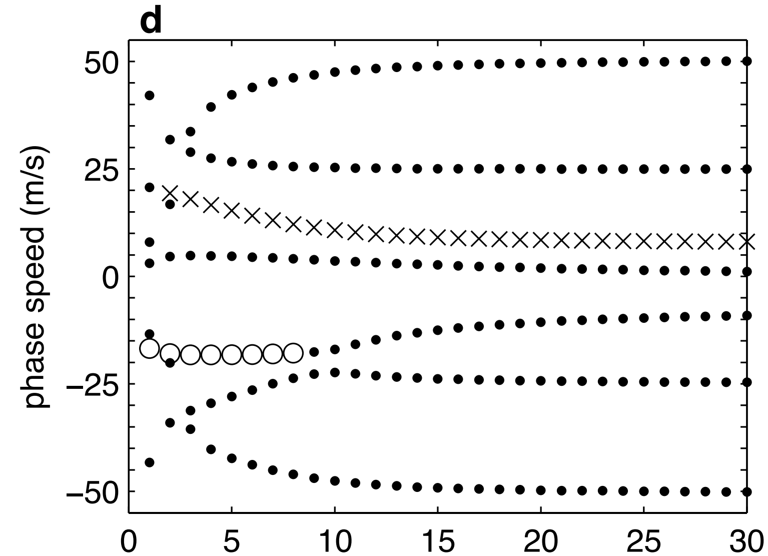
East–west *symmetry*

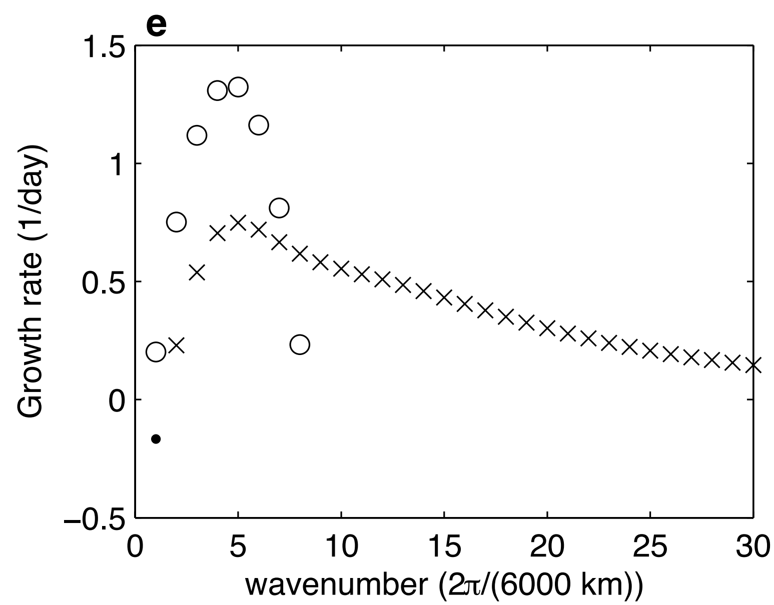




With background wind:

East–west *asymmetry*



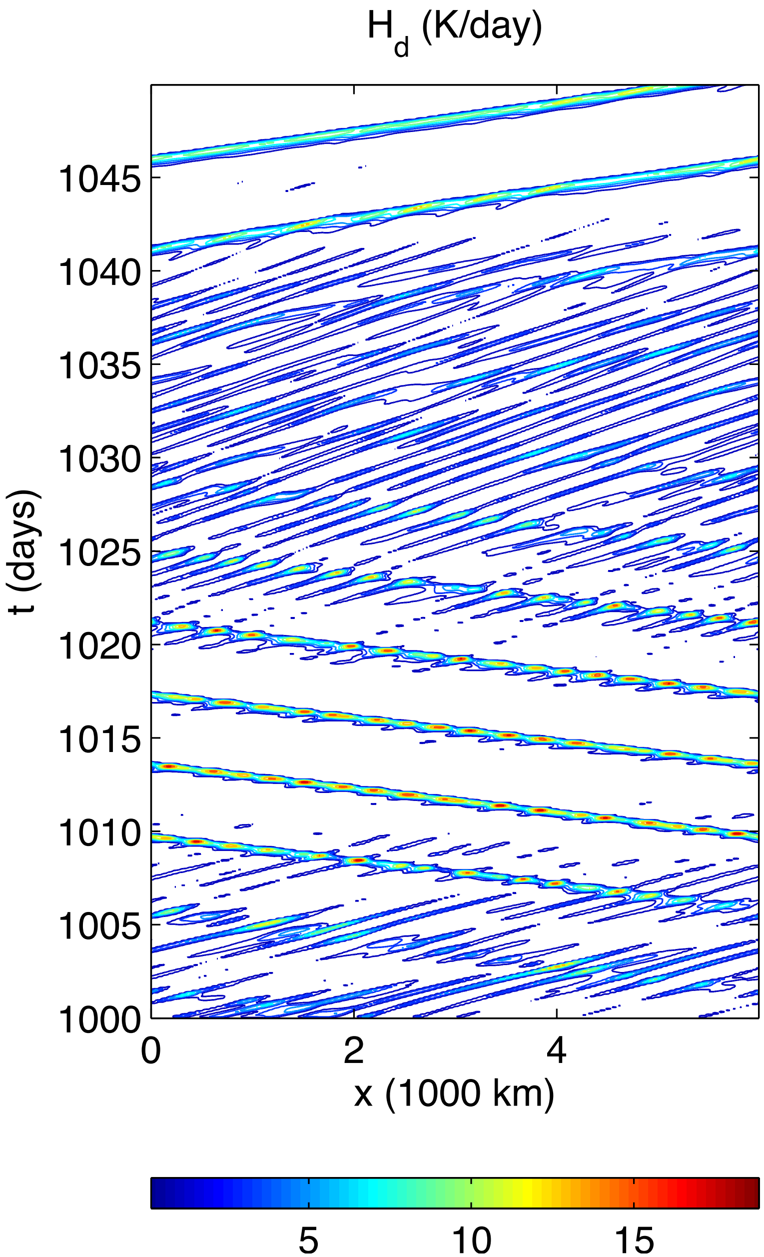


Kinetic energy transfers:

- organized convection $\longrightarrow$ large-scale wind (sometimes)
- organized convection $\longleftarrow$ large-scale wind (sometimes)

But what is effect of *multiscale organization of convection*?

Multiscale Convection/Waves and CMT

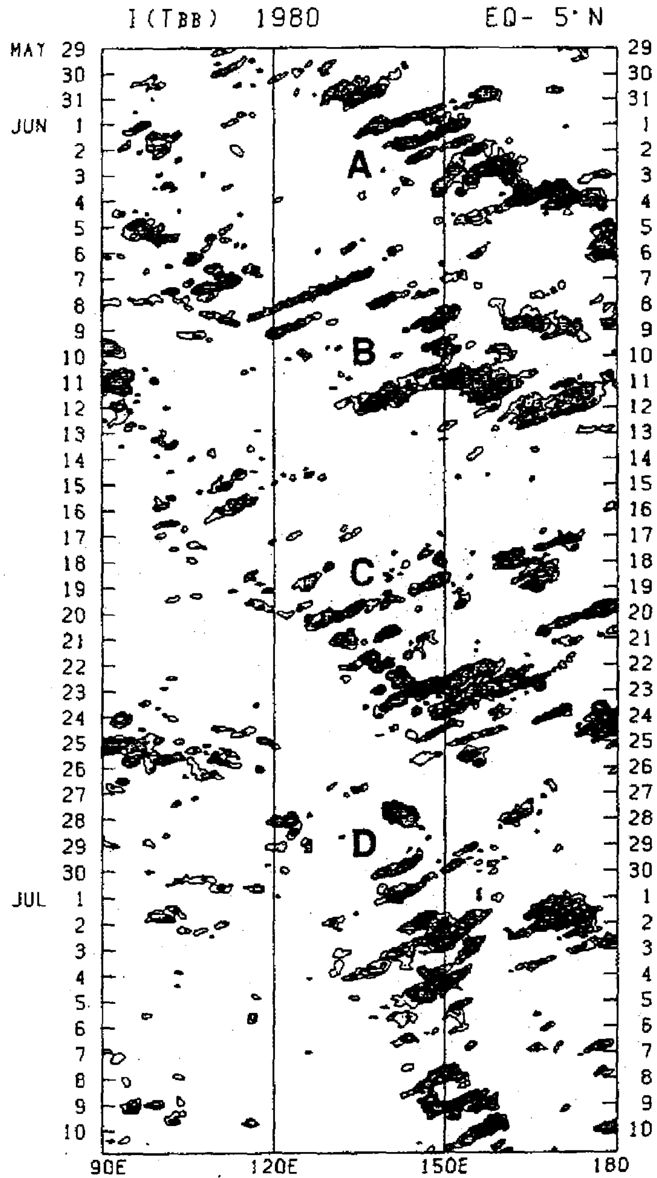


Question:
Competing effects of CMT due to synoptic-scale CCW vs. mesoscale squall lines?

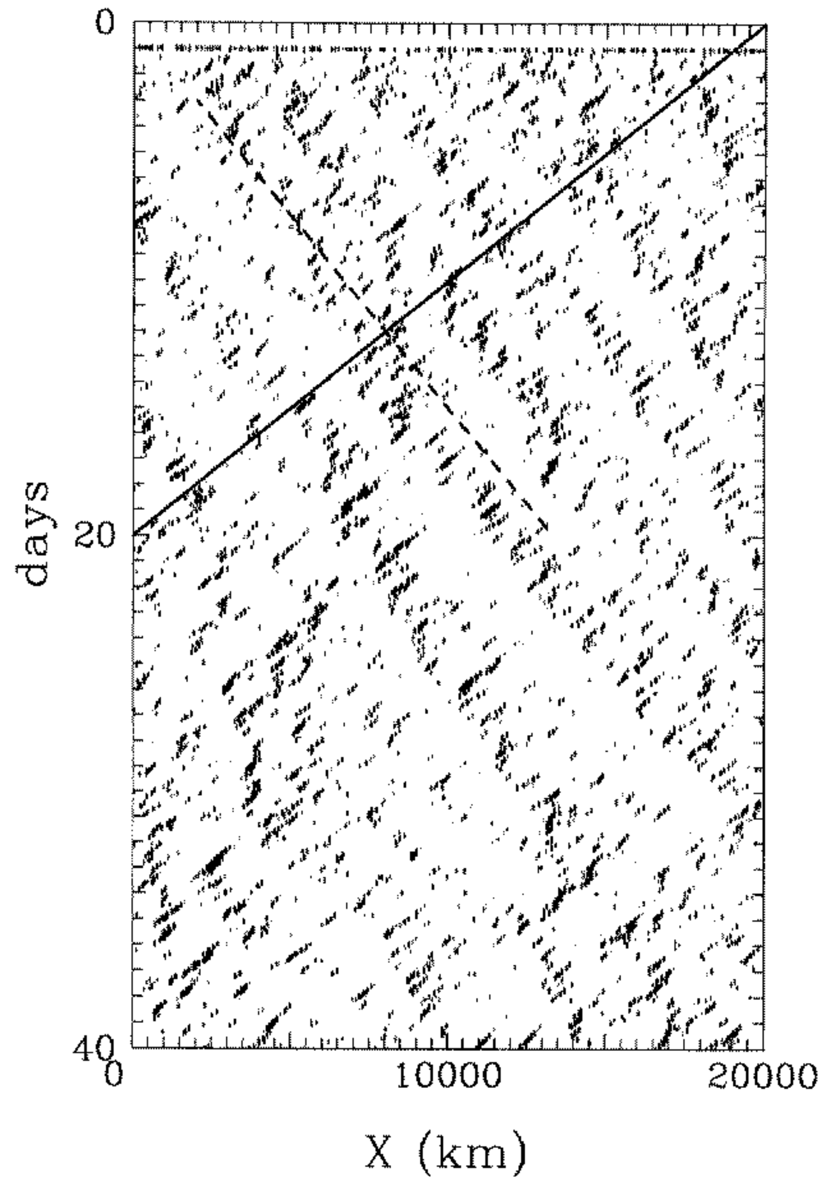
CCW with embedded squall lines for times 1000–1010 days

⟵

Multiscale Convection/Waves and CMT

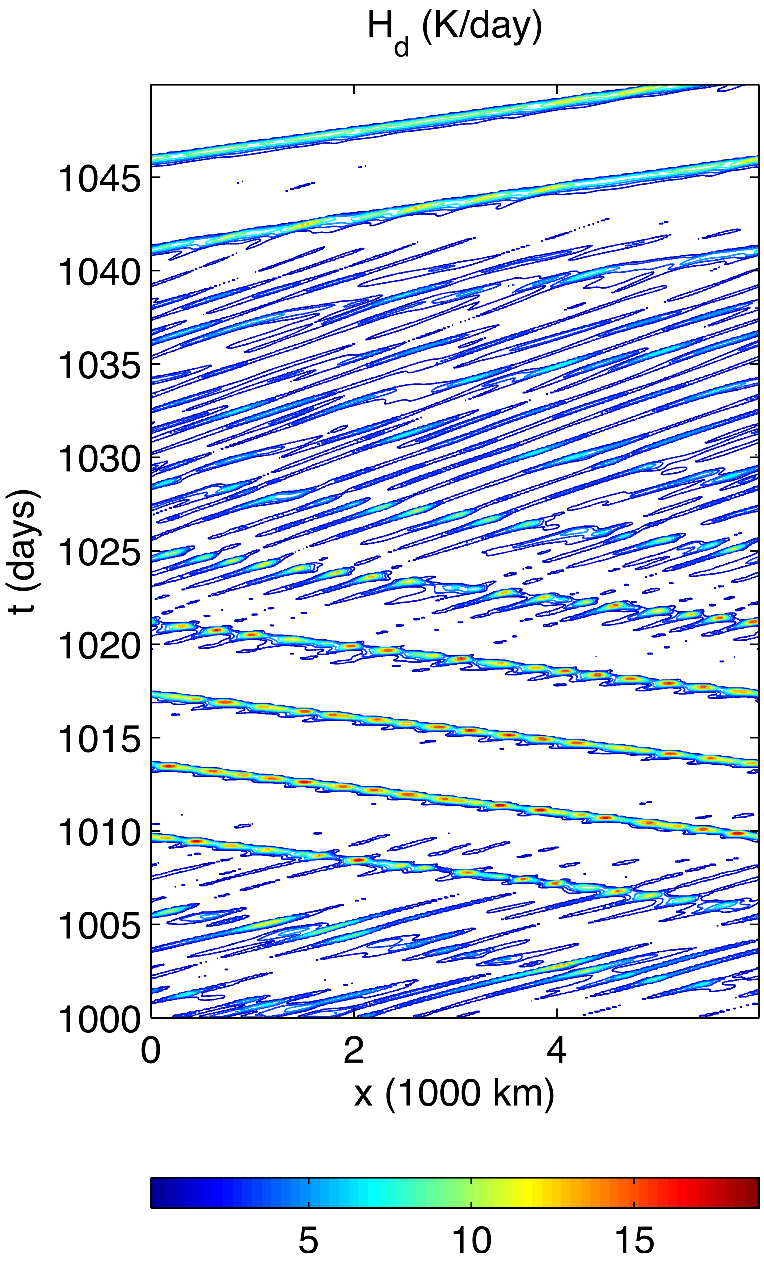


Nakazawa 1988



Grabowski & Moncrieff 2001

Multiscale Convection/Waves and CMT



Question:
Competing effects of CMT due to synoptic-scale CCW vs. mesoscale squall lines?

CCW with embedded squall lines for times 1000–1010 days

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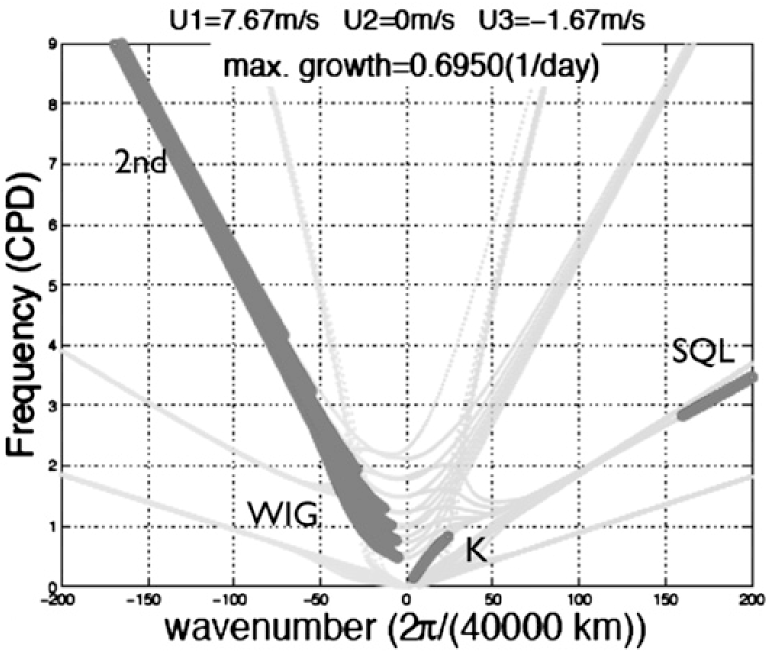
"Multiscale Waves in an MJO Background and Convective Momentum Transport Feedback"

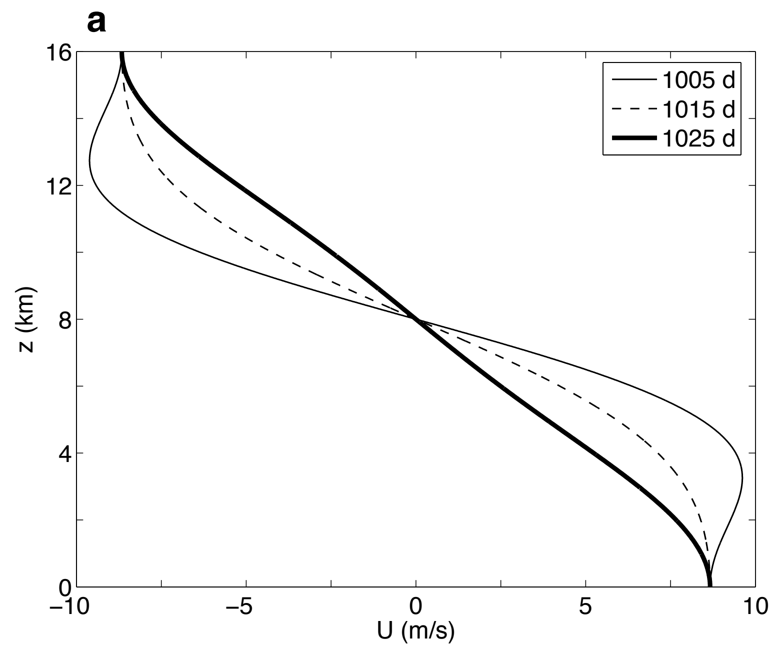
Khouider, Han, Majda, Stechmann 2012 JAS

Multiscale linear instabilities and CMT implications

Beta-plane multicloud model



Using jet shear from $t = 1005$ d



CMT Impact $\propto$ Growth Rate (as simple supposition, with caveats)

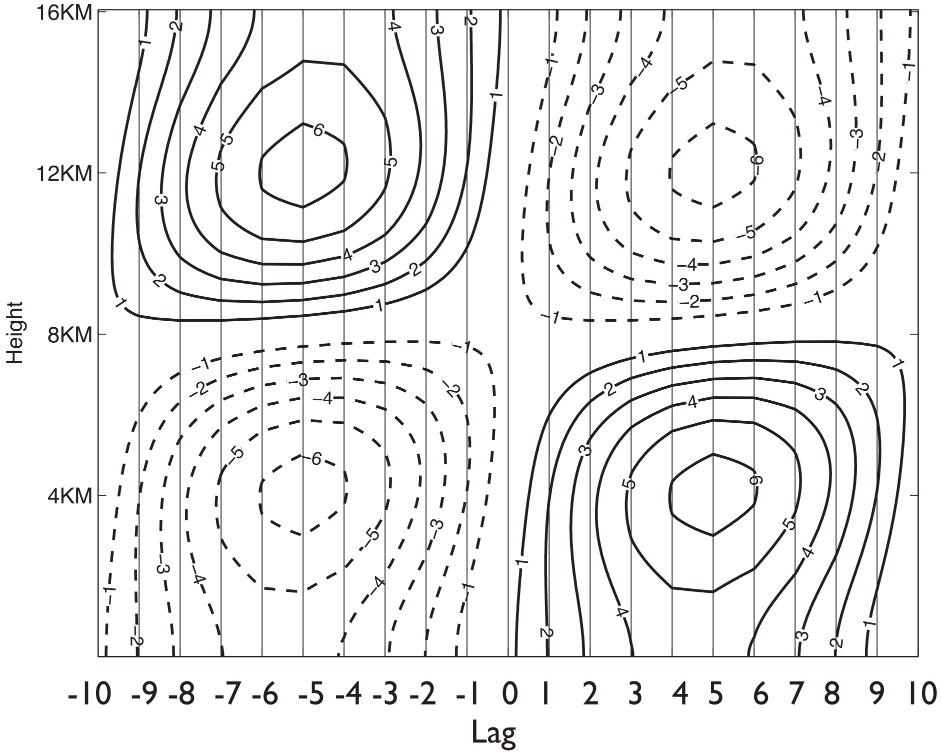
- WIG growth rate $\approx 0.7\ \mathrm{d}^{-1}$ $\leftarrow$ *WIG growth much larger here*
- SQL growth rate $\approx 0.1\ \mathrm{d}^{-1}$

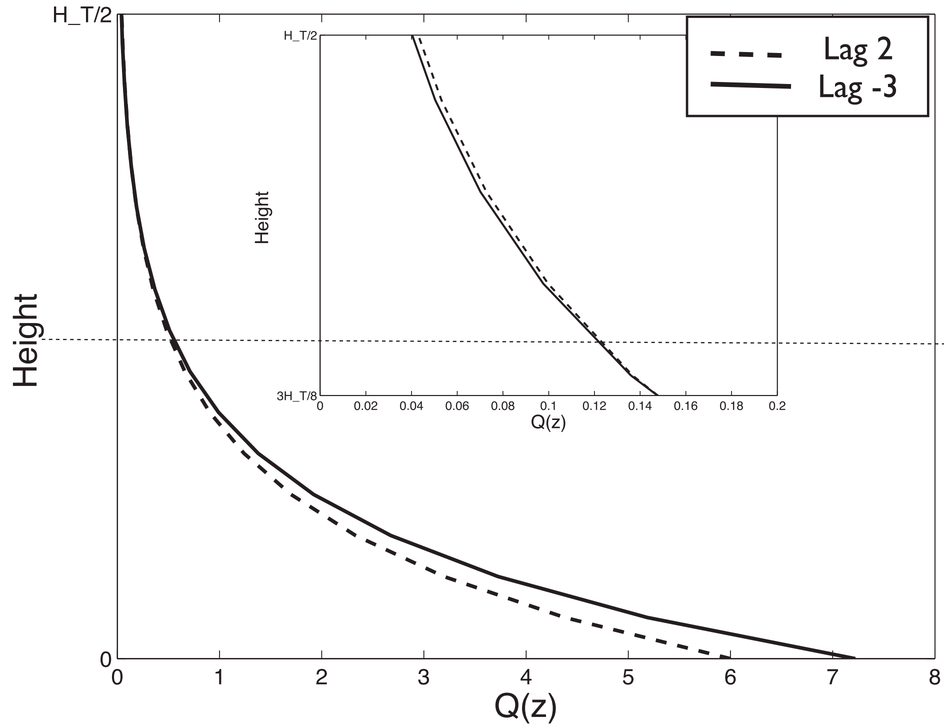
Use sequence of MJO background states instead ...

Sequence of MJO background states

Zonal wind

Moisture



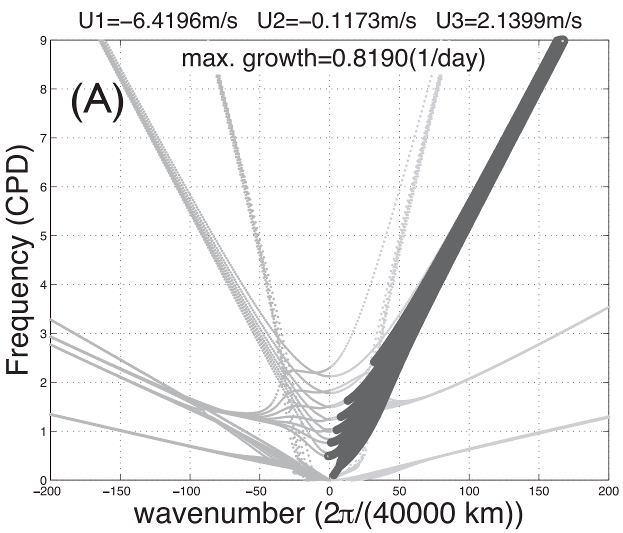


Moisture transitions from ...

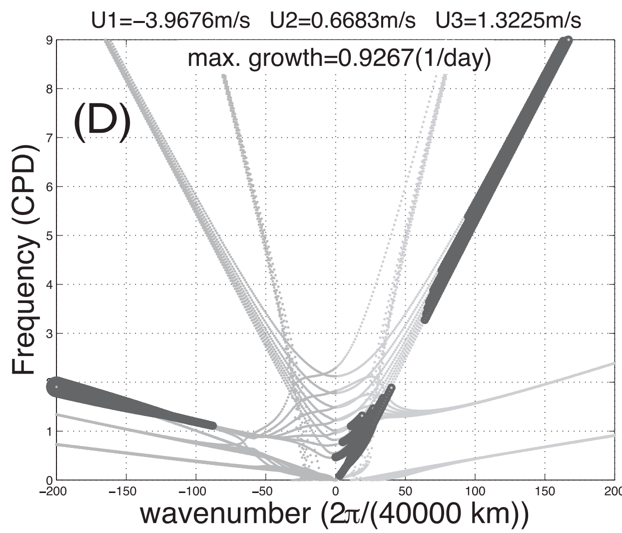
- Moist at low levels $\longrightarrow$ Moist at middle/upper levels

Multiscale linear instabilities and CMT implications

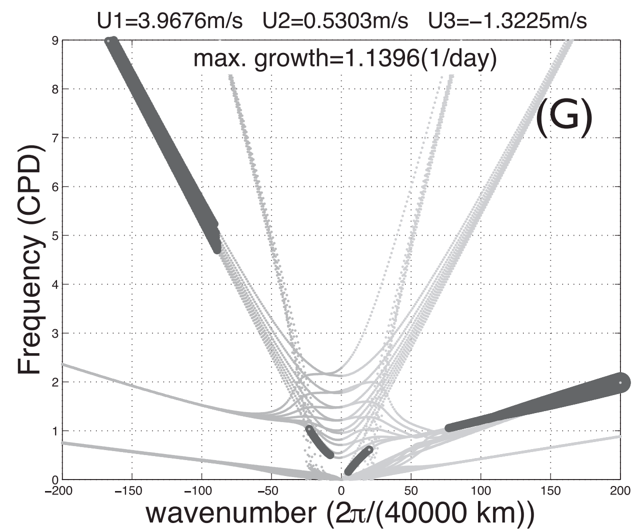
Convective onset



Mature convection



WWB/Convective decline



CMT Impact $\propto$ Growth Rate (as simple supposition, with caveats)

Kelvin strongest growth

(would decelerate)

Squall lines & Kelvin

(competing CMT)

Squall lines (& a little WIG)

(would accelerate)

For sequence of frozen background states:

Multicloud model provides linear waves and plausible CMT feedbacks

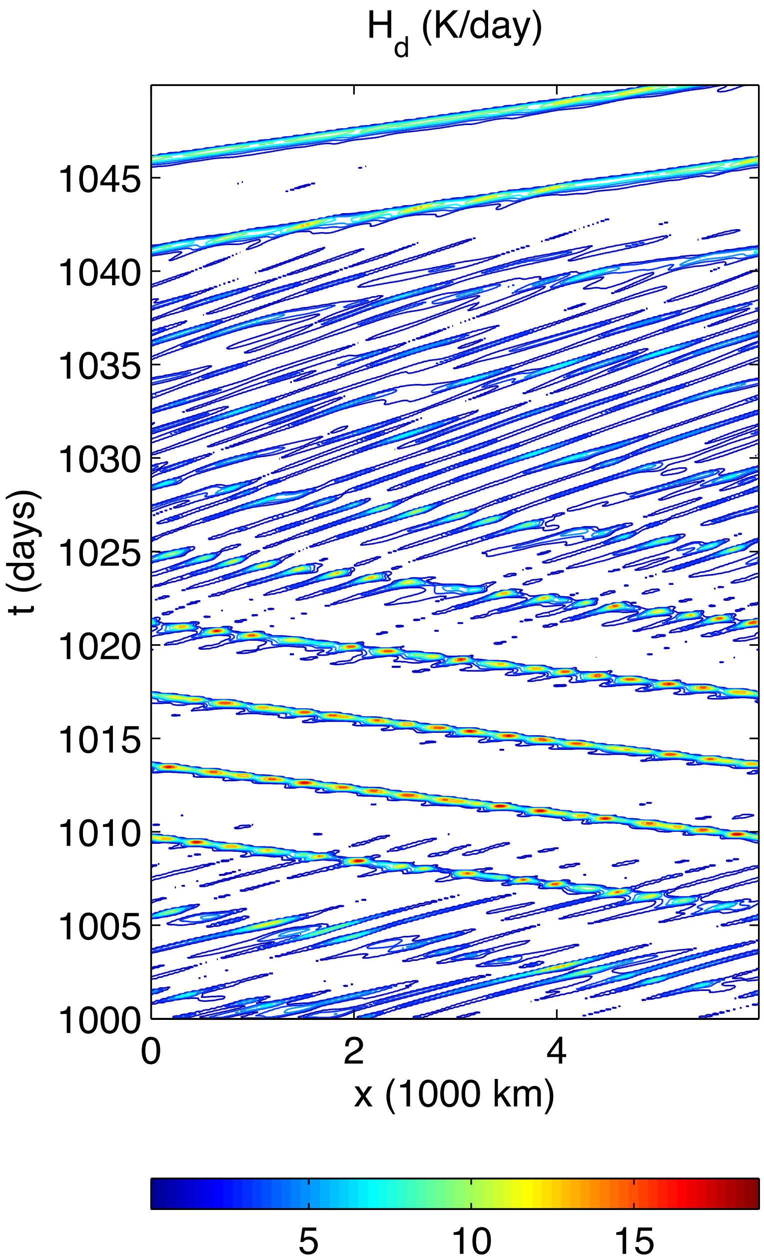
Caveat for interpretation of CMT Impact $\propto$ Growth Rate:

Strong growth rate $\Rightarrow$ Strong *growth* — not *necessarily* a strong *amplitude*

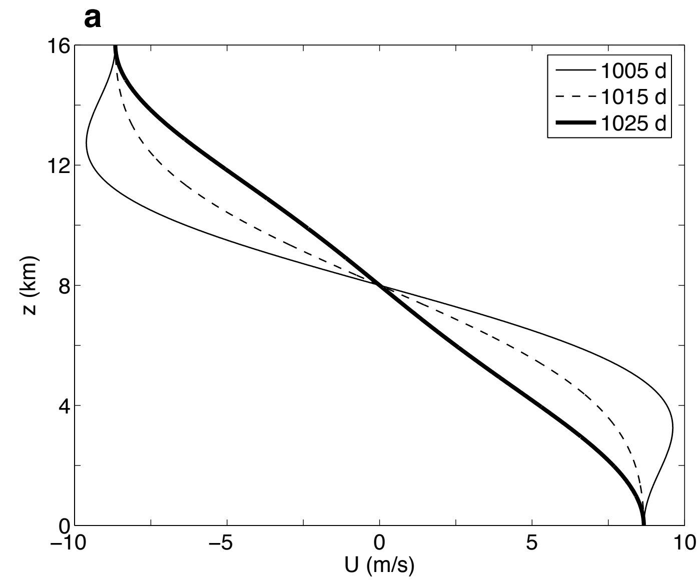
Time delay in convective response to background state

Need 2-way interaction models to clarify

Time delay in response of organized convection



Mean wind $\bar{U}(z)$:



$\longleftarrow$

WIG takes > 10 days to decay *after* mean wind has become unfavorable ($t = 1025 - 1040$ days)

And same for *growth* of Kelvin wave

Illustration of Time Delay: Amplitude Equations

Stechmann, Majda, & Skjorshammer 2013 TCFD

"Convectively coupled wave–environment interactions"

Illustration of Time Delay: Amplitude Equations

U : mean shear

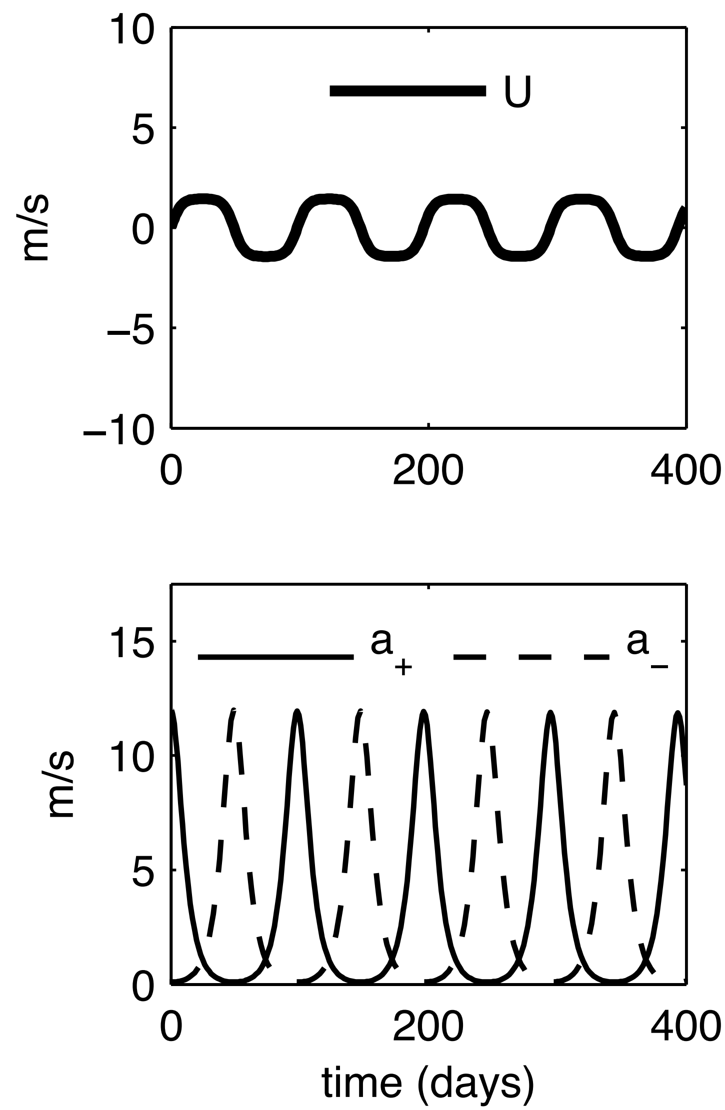
a_+ : eastward-moving CCW amplitude

a_- : westward-moving CCW amplitude

$$\frac{\mathrm{d}U}{\mathrm{d}T} = C(a_+^2 - a_-^2)$$
$$\frac{\mathrm{d}a_+}{\mathrm{d}T} = -\gamma U a_+$$
$$\frac{\mathrm{d}a_-}{\mathrm{d}T} = +\gamma U a_-$$

Mean wind U affects
growth/decay of CCWs

$\Rightarrow$ *time delay in CCW response*



(compare with multicloud model results)

Systematic Derivation of Amplitude Equations

(carried out in other contexts [see Craik 1985, Bourlioux & Majda 1995, ...], but not here)

Asymptotic wave ansatz:

$$\alpha_+(\epsilon^2 t)\mathbf{e}_+(k)\mathrm{e}^{\mathrm{i}[kx-\omega_+(k)t]} + \alpha_-(\epsilon^2 t)\mathbf{e}_-(k)\mathrm{e}^{\mathrm{i}[kx-\omega_-(k)t]} + \text{c.c.}$$

where $T = \epsilon^2 t$ is a slow time scale

Systematic asymptotics yields:

$$\frac{\mathrm{d}\alpha_+}{\mathrm{d}T} = \chi\alpha_+ + \beta|\alpha_+|^2\alpha_+ + \eta|\alpha_-|^2\alpha_+$$
$$\frac{\mathrm{d}\alpha_-}{\mathrm{d}T} = \chi\alpha_- + \beta|\alpha_-|^2\alpha_- + \eta|\alpha_+|^2\alpha_-$$

Includes nonlinear damping and nonlinear wave–wave interactions

Amplitude Equations with More Complexity

U : mean shear

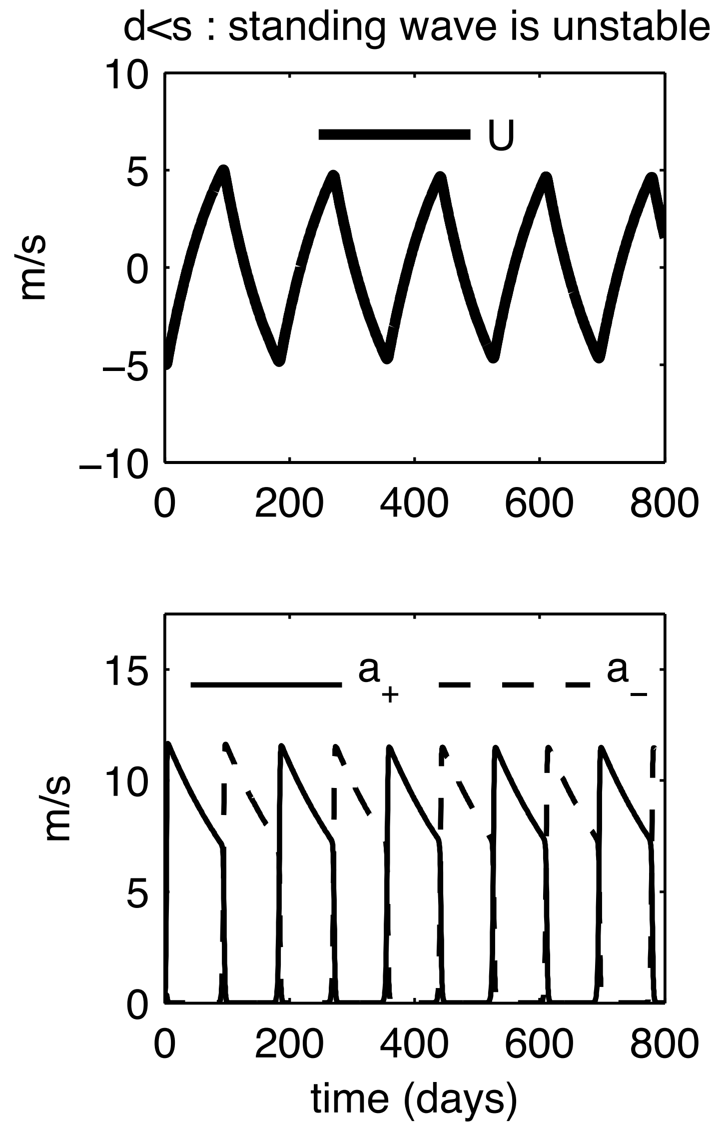
a_+ : eastward-moving CCW amplitude

a_- : westward-moving CCW amplitude

$$\frac{\mathrm{d}U}{\mathrm{d}T} = C(a_+^2 - a_-^2),$$
$$\frac{\mathrm{d}a_+}{\mathrm{d}T} = (\Gamma - \gamma U)a_+ - da_+^3 - sa_-^2 a_+$$
$$\frac{\mathrm{d}a_-}{\mathrm{d}T} = (\Gamma + \gamma U)a_- - da_-^3 - sa_+^2 a_-$$

Either a_+ or a_- present, but not both at same time $\longrightarrow$

(as in multicloud model simulations)



(compare with multicloud model results)

Other dynamics possible,

depending on climatological mean wind

(Hope bifurcation, etc.)

Ongoing effort to better understand two-way interactions:

$$\text{convection/CMT} \longleftrightarrow \text{mean wind}$$

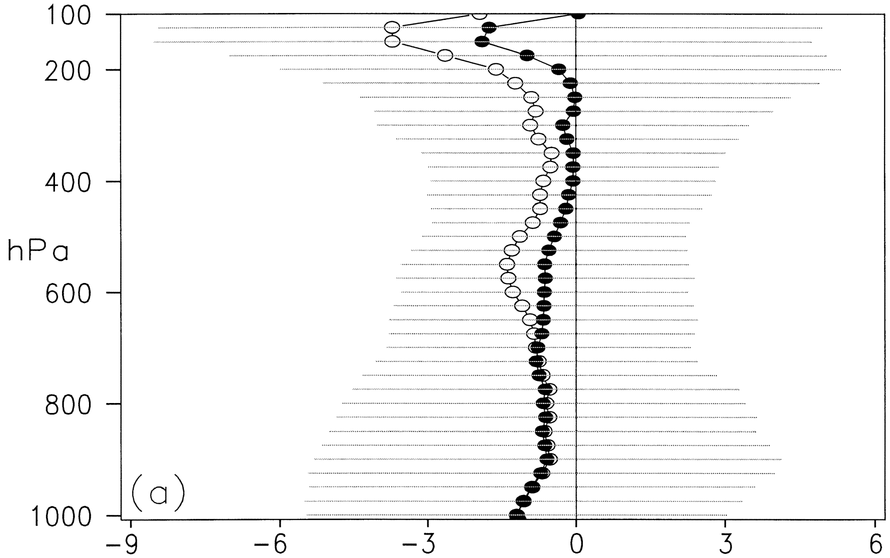
With sufficient understanding, could hope to faithfully *parameterize* in GCMs

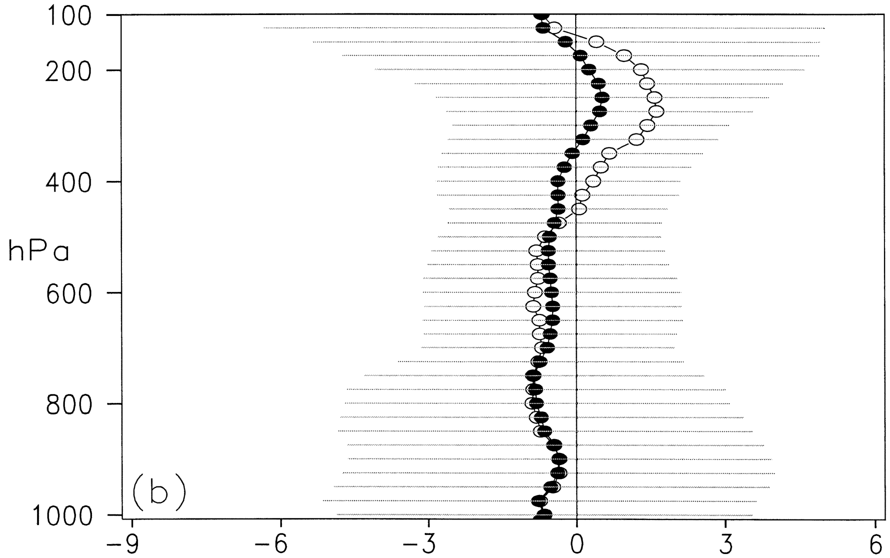
(Schneider & Lindzen 1976, Cheng & Yanai 1987, Moncrieff & Klinker 1997, ...)

Final topic: stochastic parameterization of CMT

motivated by ...

Statistics of convective momentum transport (CMT)





Tung and Yanai (2002a)

Top: $-(\overline{w'u'})_z \frac{\bar{U}}{|\bar{U}|}$

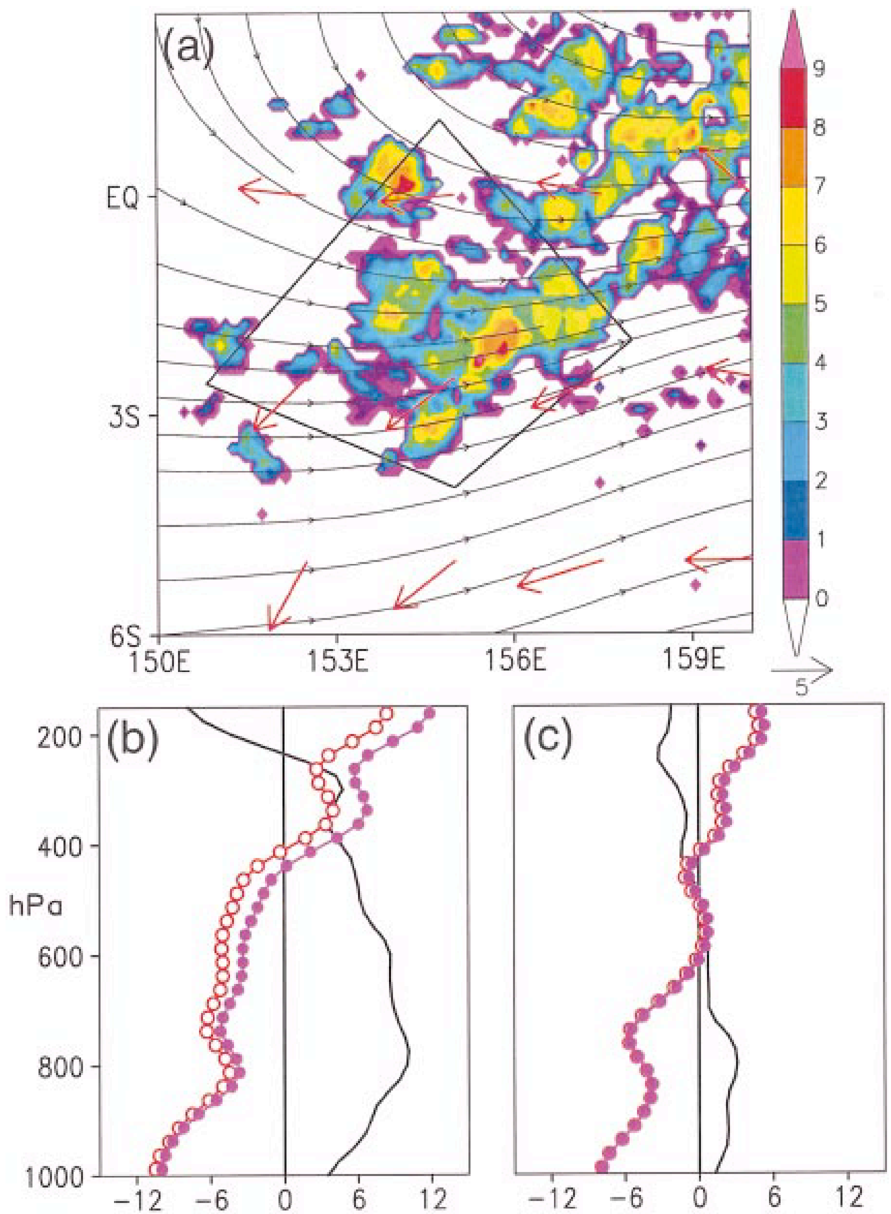
Bottom: $-(\overline{w'v'})_z \frac{\bar{V}}{|\bar{V}|}$

Circles: IOP mean

Horizontal lines: standard deviation

- Mean CMT: weak damping (cumulus friction)
- But standard dev. of CMT is huge!
- Examples demonstrate that both acceleration and deceleration can be intense

Examples of convective momentum transport (CMT)



Tung and Yanai (2002b)

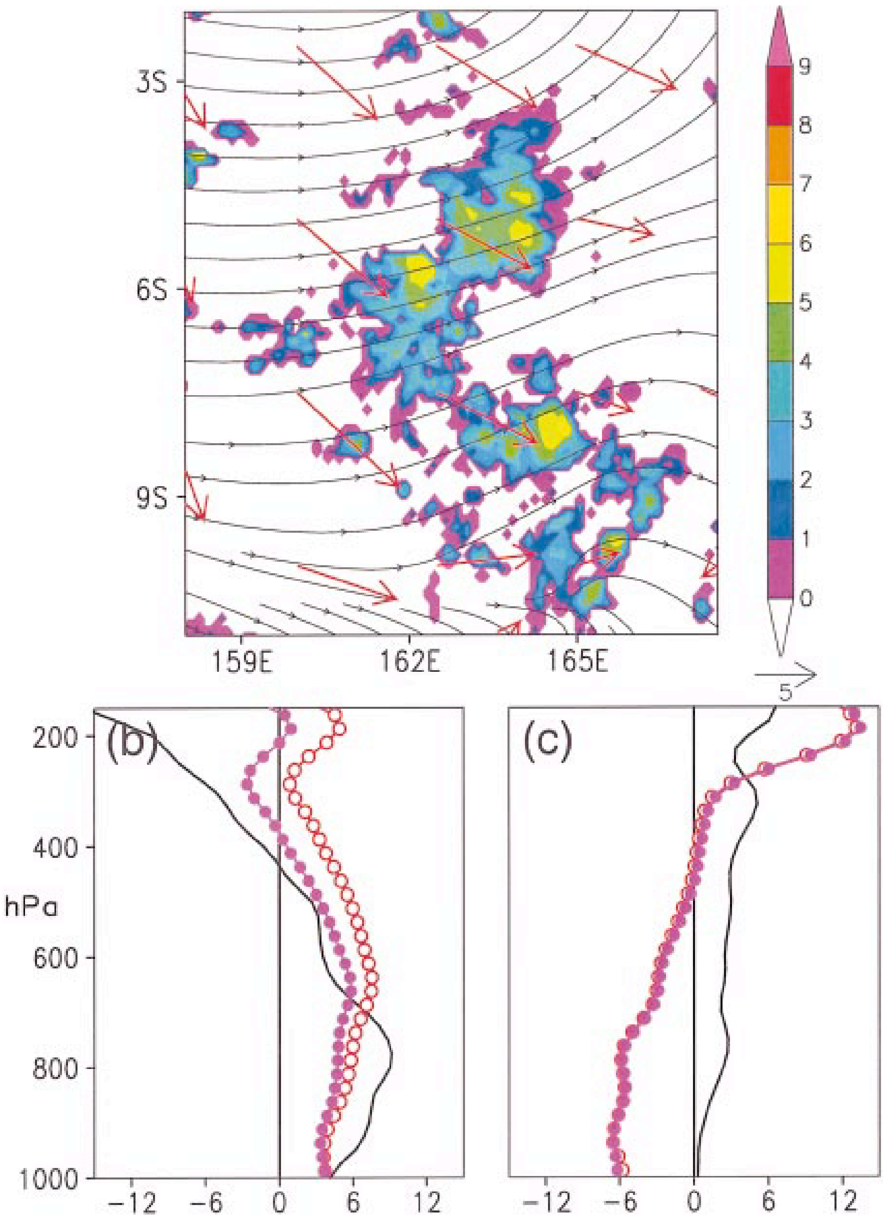
<u>Non-squall convective system:</u>

- CMT decelerates mean wind

(b) $\bar{U}$ (black), $-(\overline{w'u'})_z$ (color)

(c) $\bar{V}$ (black), $-(\overline{w'v'})_z$ (color)

Examples of convective momentum transport (CMT)



Tung and Yanai (2002b)

Squall line:

- CMT accelerates $\bar{U}$

(b) $\bar{U}$ (black), $-(\overline{w'u'})_z$ (color)

(c) $\bar{V}$ (black), $-(\overline{w'v'})_z$ (color)

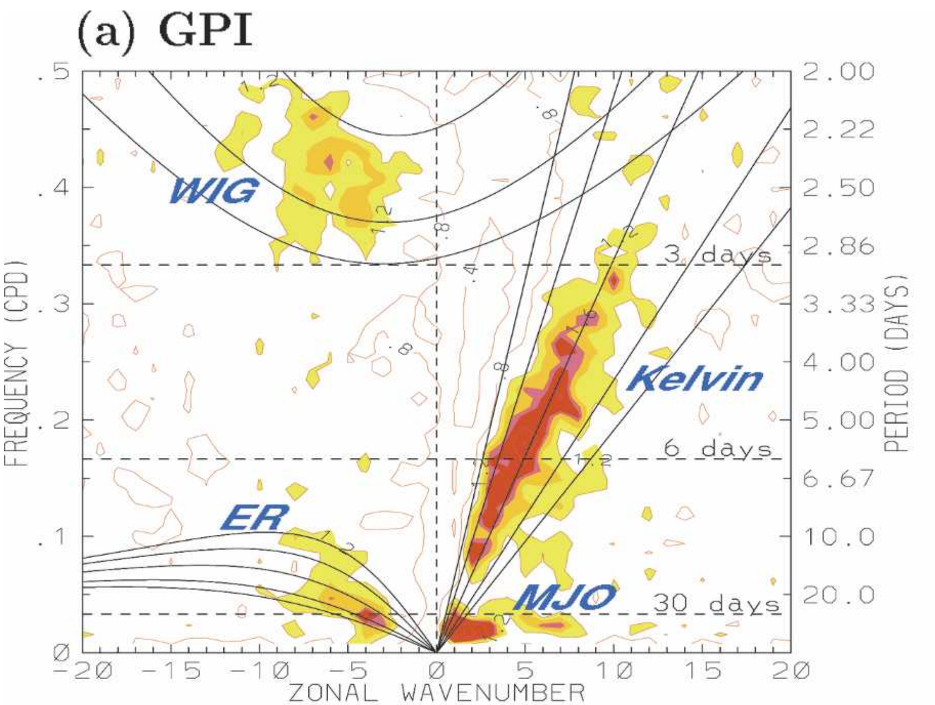
Motivation for stochastic models for CMT

To capture:

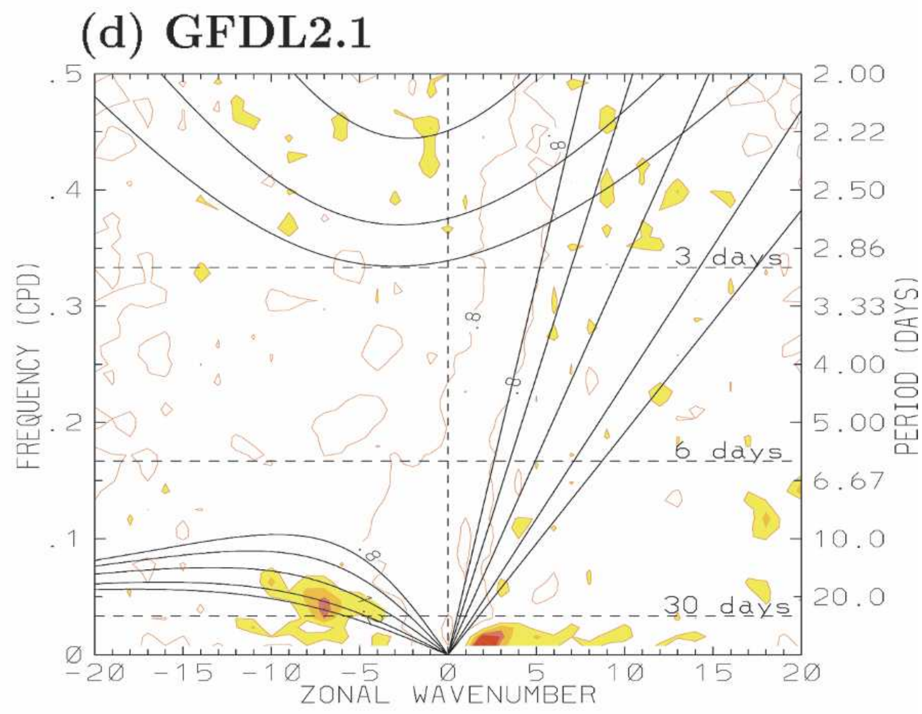
1. intermittent intense bursts of CMT as in observations
2. realistic variability of tropical convection

Spectral Power of Tropical Precipitation in Observations and GCMs

Observations



GCM



From Lin et al. (2006)

GCMs typically lack proper variability of tropical convection

Stochastic models for tropical convection

- Majda, Khouider (2002) Stochastic and mesoscopic models for tropical convection. *Proc. Natl. Acad. Sci.*
- Khouider, Majda, Katsoulakis (2003) Coarse-grained stochastic models for tropical convection and climate. *Proc. Natl. Acad. Sci.*
- Majda, Franzke, Khouider (2008) An applied mathematics perspective on stochastic modelling for climate. *Phil. Trans. Roy. Soc. A*
- Majda, Stechmann (2008) Stochastic models for convective momentum transport. *Proc. Natl. Acad. Sci.*
- Khouider, Majda, Biello (2010) A stochastic multicloud model for tropical convection. *Comm. Math. Sci.*
- Stechmann, Neelin (2011) A stochastic model for the transition to strong convection. *J. Atmos. Sci.*
- ...

3 Convective Regimes with Different CMT

1. Dry regime.
   - Weak or no cumulus friction.
   - Favored for dry environments, regardless of shear.

2. Upright convection regime.
   - Stronger cumulus friction.
   - Favored for moist, weakly sheared environments.

3. Squall line regime.
   - Intense CMT, either upscale or downscale depending on the shear.
   - Favored for moist, sheared environments.

Markov jump process for transitions between regimes

3-state continuous-time Markov jump process

- at each large-scale spatio-temporal location (x, t)
- with transition rates depending on local values of large-scale variables at (x, t)

Denote the discrete, stochastic regime variable by

$$r_t = 1 \text{ (dry)}$$
$$r_t = 2 \text{ (conv.)}$$
$$r_t = 3 \text{ (squall)}$$

T_{ij}: transition rate from regime i to regime j

based on observations such as LeMone, Zipser, & Trier (1998)

Stochastic time delay in response to large-scale variables

Transition rates

$$
\begin{array}{lr}
T_{12} = \frac{1}{\tau_r}\mathcal{H}(Q_d)e^{\beta_\Lambda(1-\Lambda)}e^{\beta_Q Q_d} & \text{dry} \;\longrightarrow\; \text{conv.} \\
T_{13} = 0 & \text{dry} \;\longrightarrow\; \text{squall} \\
T_{21} = \frac{1}{\tau_r}e^{\beta_\Lambda \Lambda}e^{\beta_Q(Q_{d,ref}-Q_d)} & \text{conv.} \;\longrightarrow\; \text{dry} \\
T_{23} = \frac{1}{\tau_r}\mathcal{H}(|\Delta U_{low}| - |\Delta U|_{min})e^{\beta_U|\Delta U_{low}|}e^{\beta_Q Q_c} & \text{conv.} \;\longrightarrow\; \text{squall} \\
T_{31} = T_{21} & \text{squall} \;\longrightarrow\; \text{dry} \\
T_{32} = \frac{1}{\tau_r}e^{\beta_U(|\Delta U|_{ref}-|\Delta U_{low}|)}e^{\beta_Q(Q_{c,ref}-Q_c)} & \text{squall} \;\longrightarrow\; \text{conv.}
\end{array}
$$

- Exponentials capture sensitive dependence on large-scale variables
- τ_r, β: model parameters
- Q: cloud heating
- Λ: measures dryness of lower-mid troposphere relative to boundary layer
- ΔU: vertical wind shear

Different convective regimes have different CMT

$$F_{CMT} = -\partial_z(\overline{w'u'}) = \begin{cases} -d_1(U - \hat{U}) & \text{for} \quad r_t = 1 \\ -d_2(U - \hat{U}) & \text{for} \quad r_t = 2 \\ F_3 & \text{for} \quad r_t = 3 \end{cases}$$

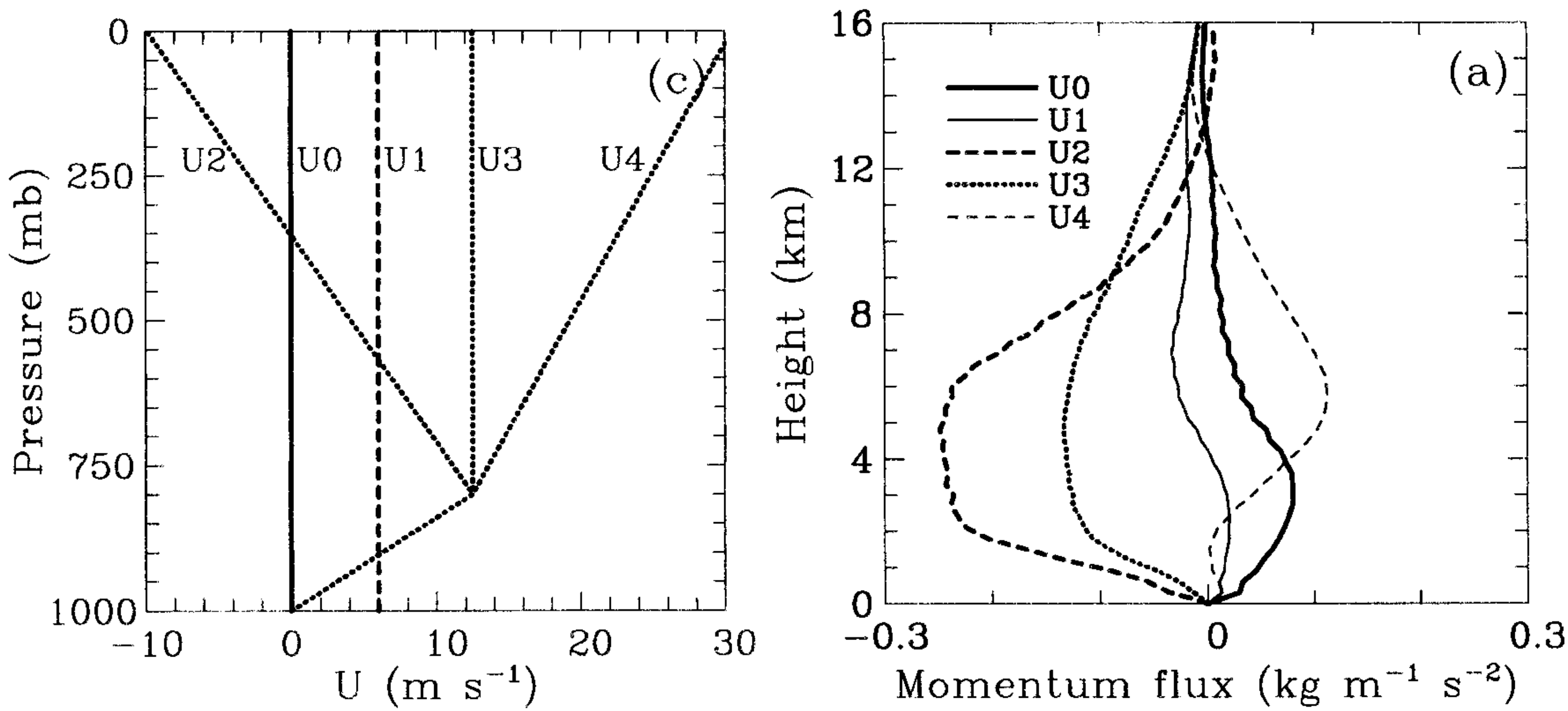
$$F_3 = -\partial_z(\overline{w'u'}) = \kappa[\cos(z) - \cos(3z)].$$

$$\kappa = \begin{cases} -\left(\frac{Q_d}{Q_{d,ref}}\right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if} \quad \Delta U_{mid} \Delta U_{low} < 0 \\ 0 & \text{if} \quad \Delta U_{mid} \Delta U_{low} > 0 \end{cases}$$

Formulas for F_3 and κ motivated by observations, CRM simulations, and a simple multi-scale model ...

$$\text{Formula for } \kappa = \begin{cases} -\left(\frac{Q_d}{Q_{d,ref}}\right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if} \quad \Delta U_{mid}\Delta U_{low} < 0 \\ 0 & \text{if} \quad \Delta U_{mid}\Delta U_{low} > 0 \end{cases}$$

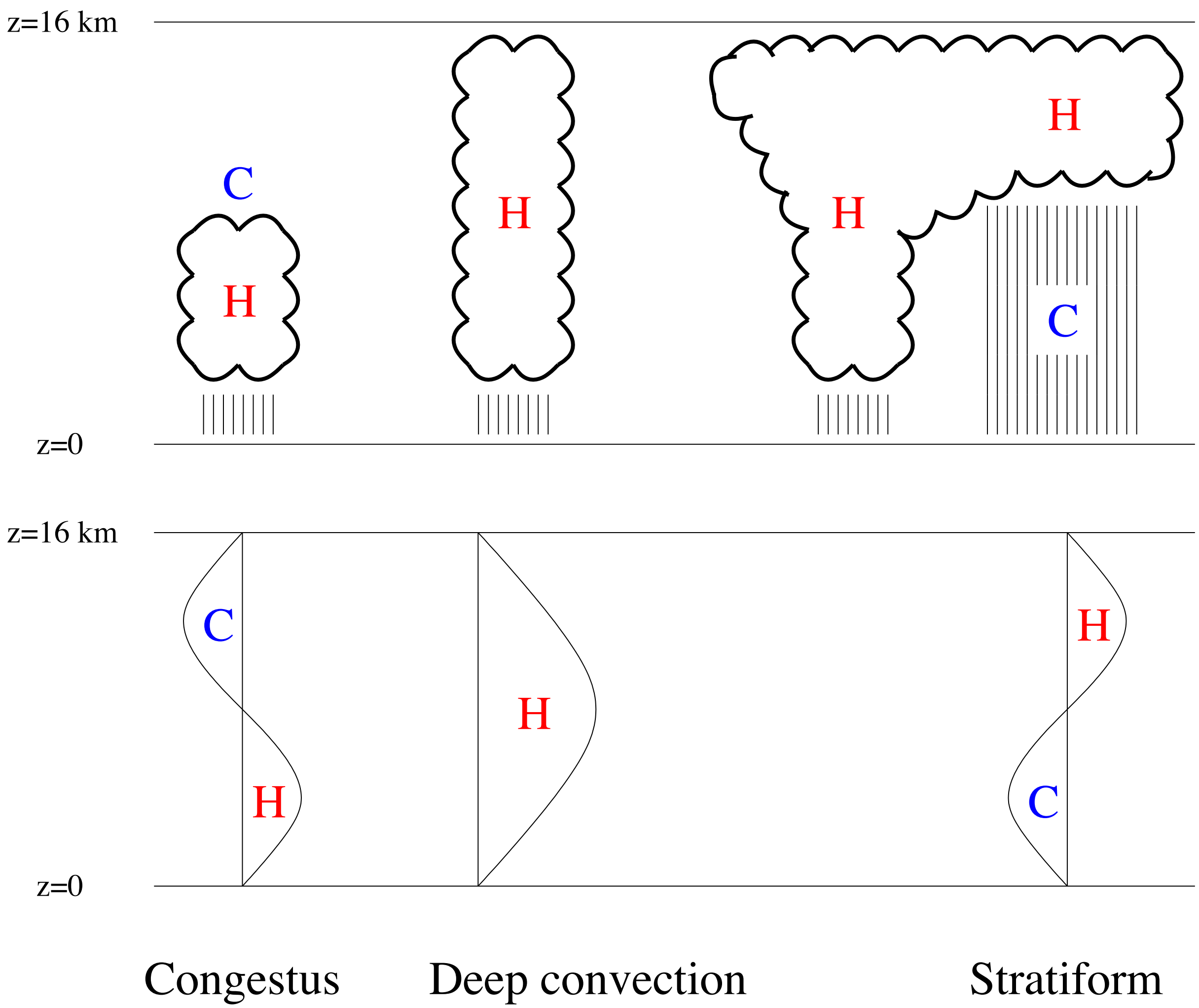
CRM results: Liu and Moncrieff (2001)



Vertical tilts of squall lines, and their CMT, depend on the mid-level shear

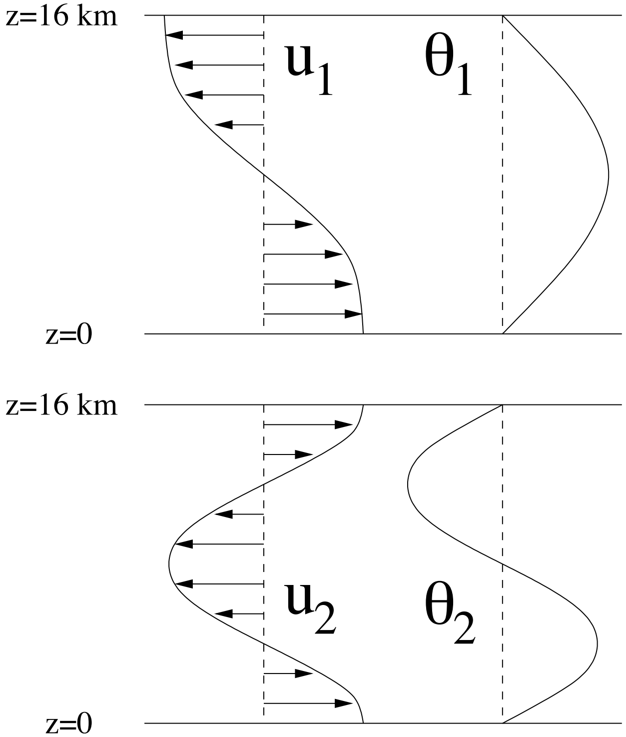
Model for convectively coupled waves

Multi-cloud Model of Khouider & Majda (2006,2008)



Model for convectively coupled waves

Multi-cloud Model of Khouider & Majda (2006,2008) + Stochastic CMT Model



$$\frac{\partial u_1}{\partial t} - \frac{\partial \theta_1}{\partial x} = F^1_{CMT}$$
$$\frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} = F^2_{CMT}$$
$$\frac{\partial u_3}{\partial t} = F^3_{CMT}$$
$$\frac{\partial \theta_1}{\partial t} - \frac{\partial u_1}{\partial x} = H_d + \xi_s H_s + \xi_c H_c - R_1$$
$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\frac{\partial u_2}{\partial x} = H_c - H_s - R_2$$

+ evolution equations for θ_{eb}, q, H_s

and formulas for nonlinear interactive source terms

such as convective heating, downdrafts, etc.

and r_t evolves as 3-state Markov jump process with transition rates T_{ij}

Convectively coupled wave simulation

6000-km periodic domain

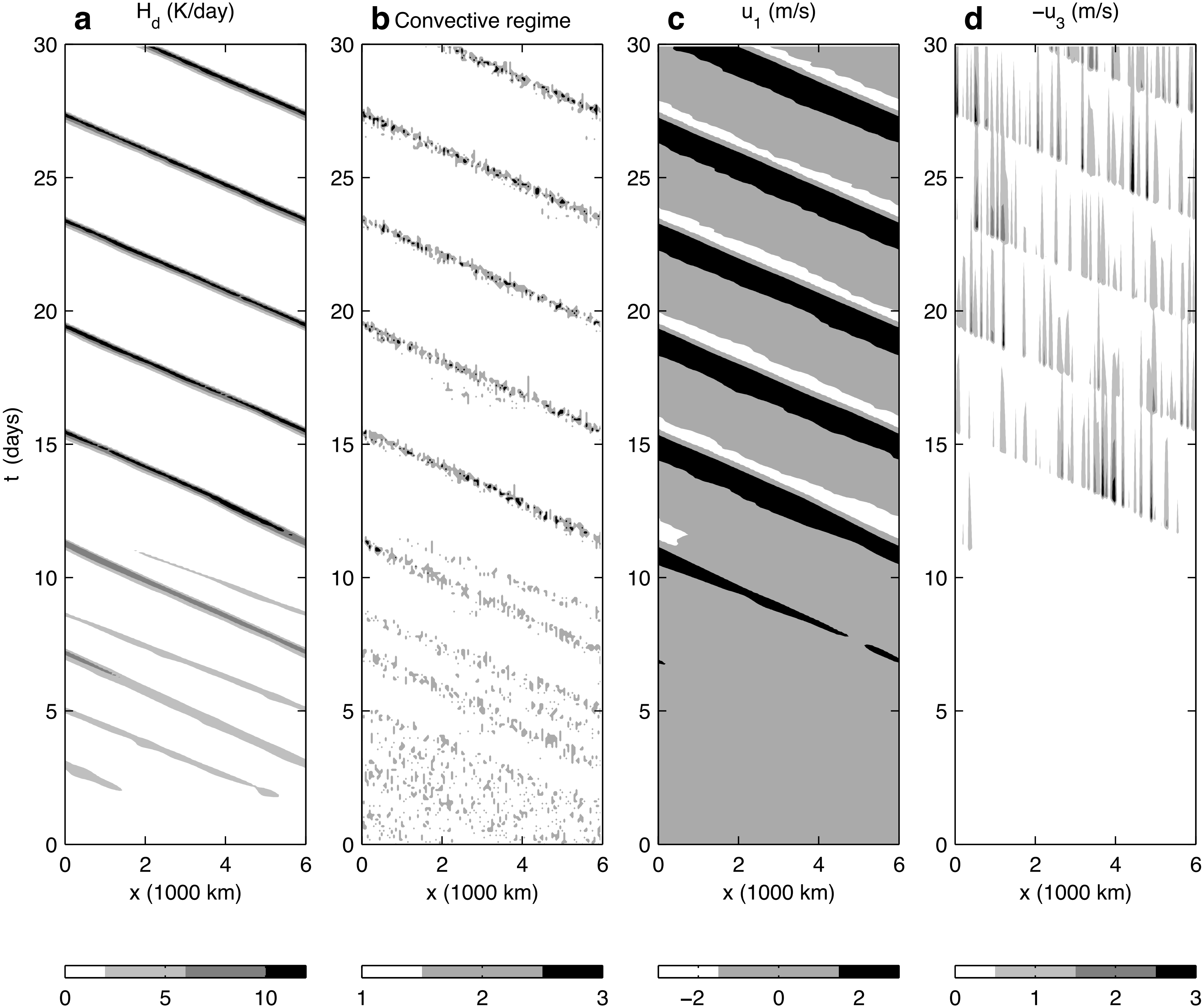
- to capture a single convectively coupled wave

$\Delta x = 50$ km

- representative of a GCM's grid spacing

Initial conditions:

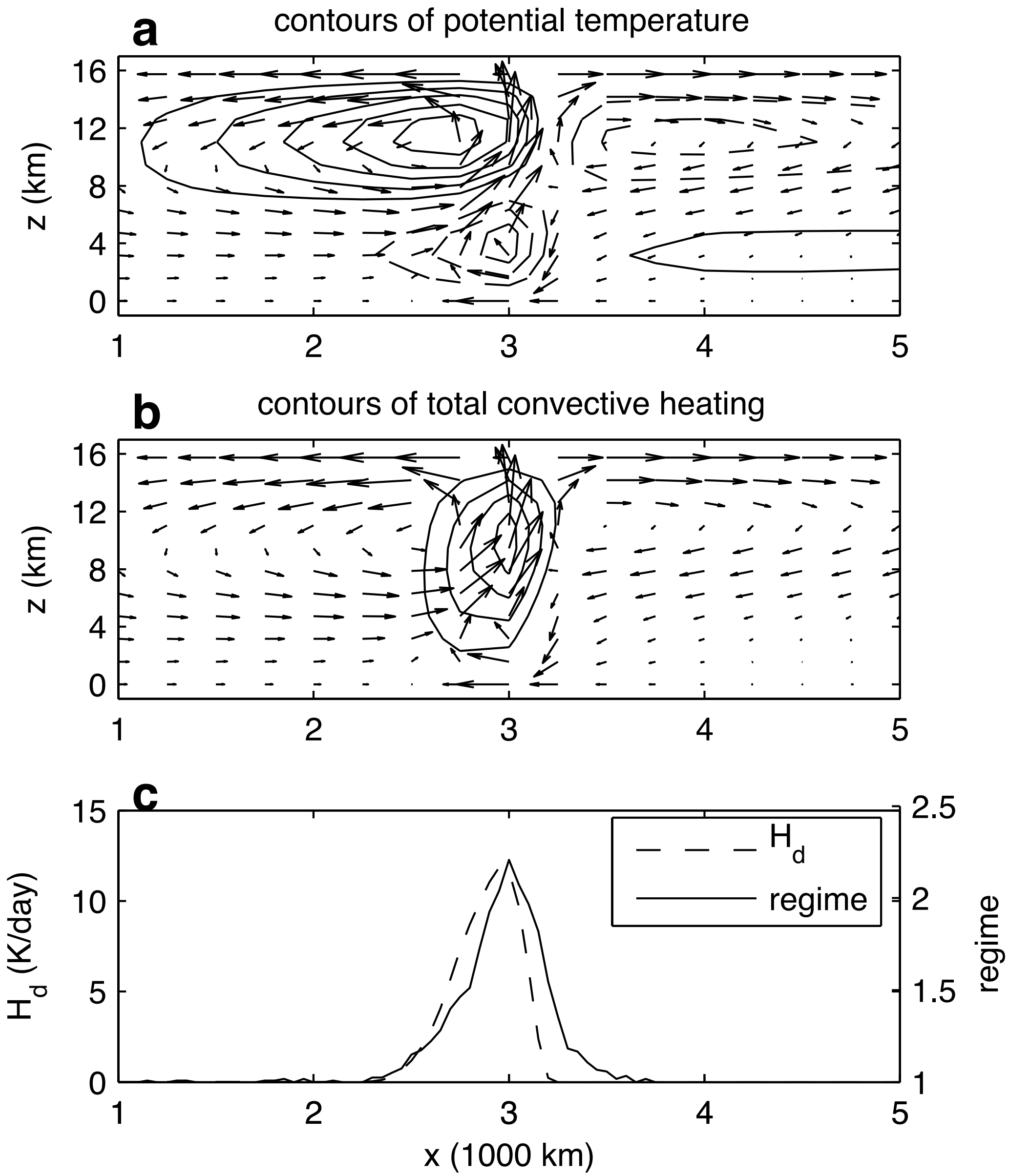
- small perturbation to uniform radiative–convective equilibrium solution



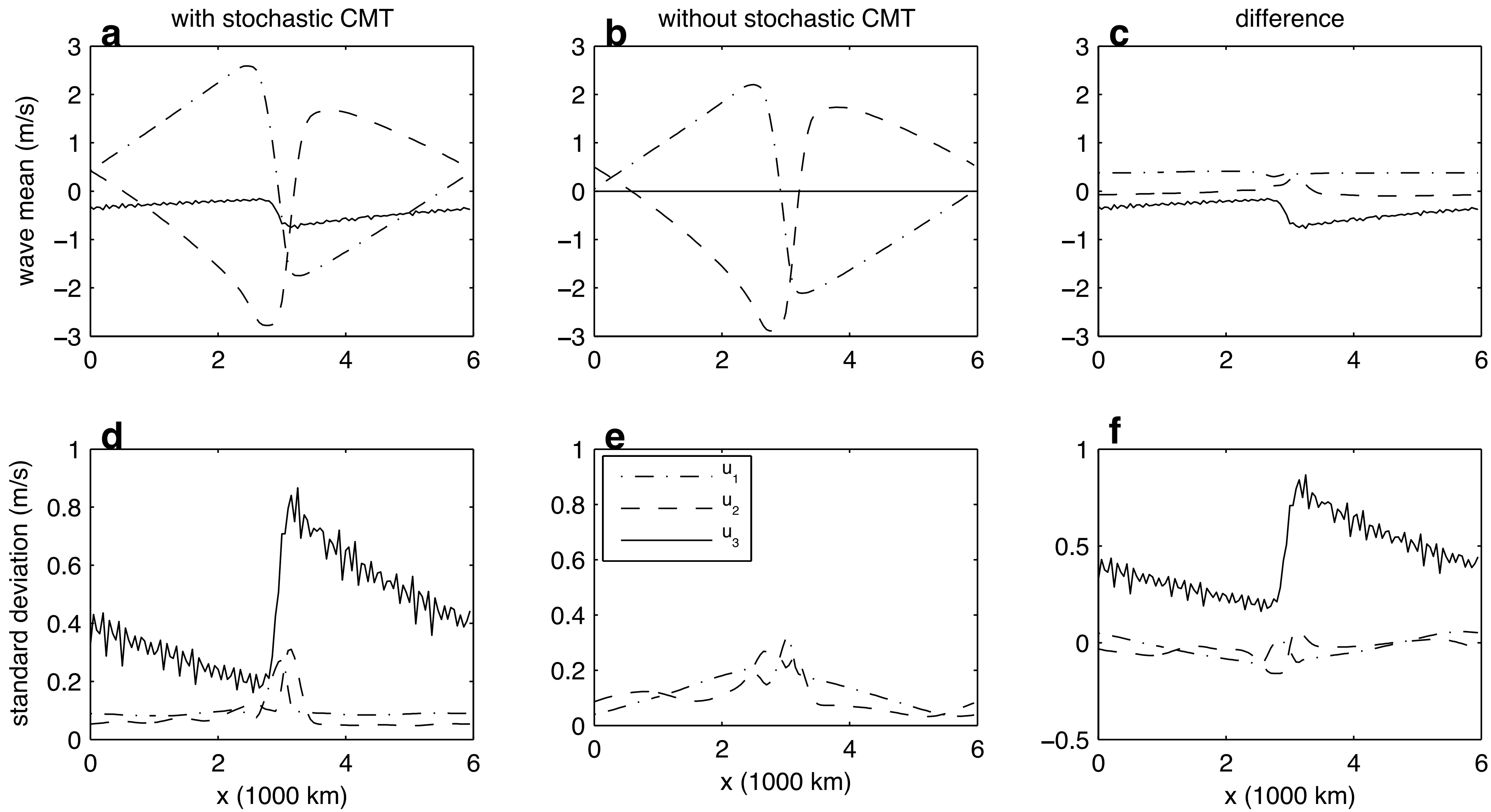
a
H_d (K/day)
b Convective regime
c
u_1 (m/s)
d
$-u_3$ (m/s)
t (days)
x (1000 km)
0
5
10
15
20
25
30
0
2
4
6
1
2
3
−2

Wave-mean structure

Average in a reference frame moving with the wave at -17.5 m/s



Comparison: with and without stochastic CMT



u_1: dash-dot

u_2: dash

u_3: solid

Stochastic CMT generates a nontrivial mean flow that can interact with the wave (see Majda & Stechmann 2009 JAS)

Summary

Multiscale modeling to understand and illustrate features of CMT:

1. CMT energy transfers:
   sometimes upscale, sometimes downscale

2. Multiscale convection/waves
   $\Rightarrow$ competition between CMT effects from mesoscale and synoptic-scale

3. Time delays:
   organized convection does not respond immediately to large-scale state

4. Stochastic CMT parameterization:
   captures *intermittent* aspects of CMT