Sedimenting lattices of discs

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Outline

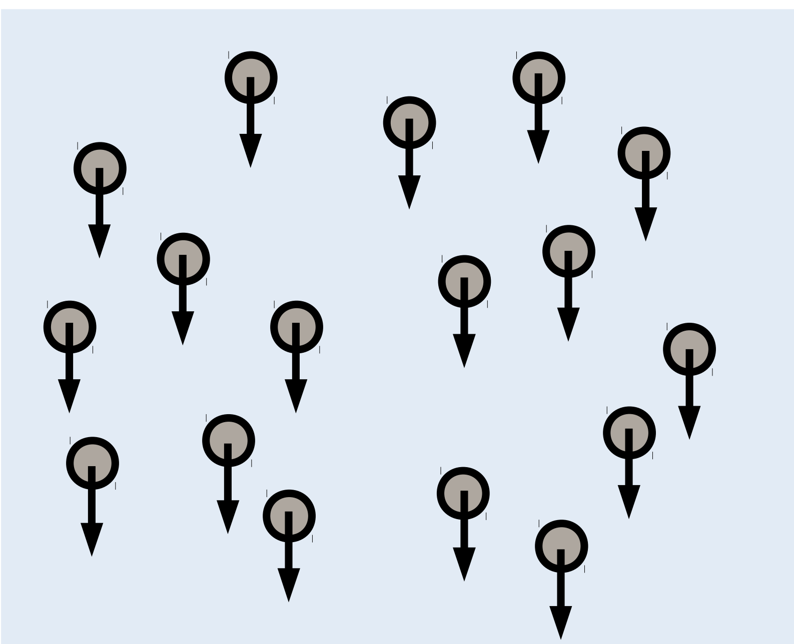
- Background
  - slow sedimentation stat-mech SR Adv Phys 2001
  - arrays unstable
- Disc arrays are different
  - tilt ~ momentum
  - linearly stable settling, waves
- Non-normal dynamics, transient growth
- Summary

Stokesian Settling

Purcell 1977

Navier - Stokes:

$$-\nabla p + \eta \nabla^2 \vec{v} = \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v}$$

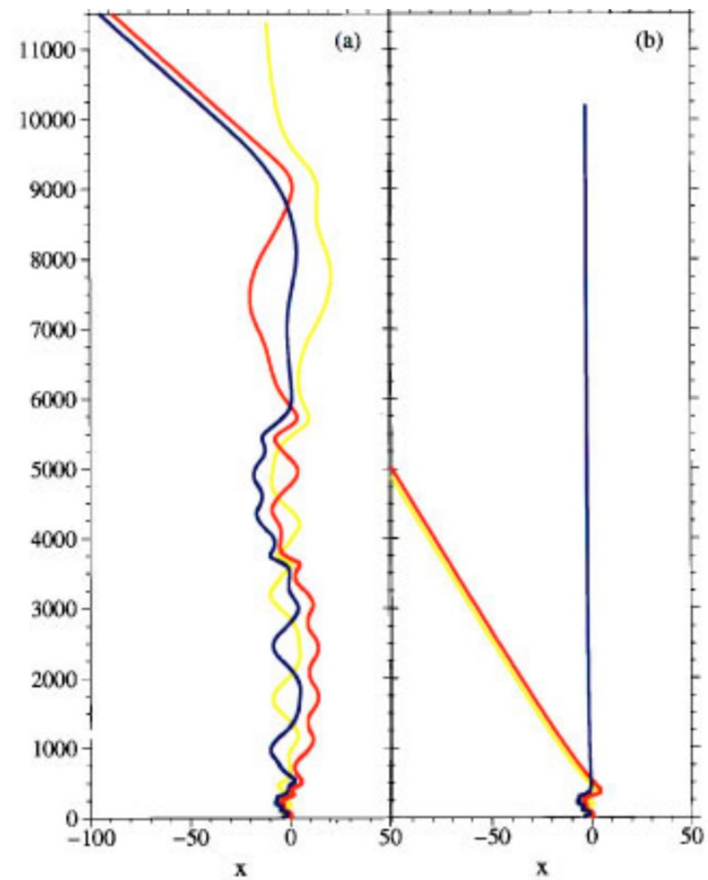


point force at the origin

$$\mathcal{G}_{ij}(r) \sim \frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3}$$

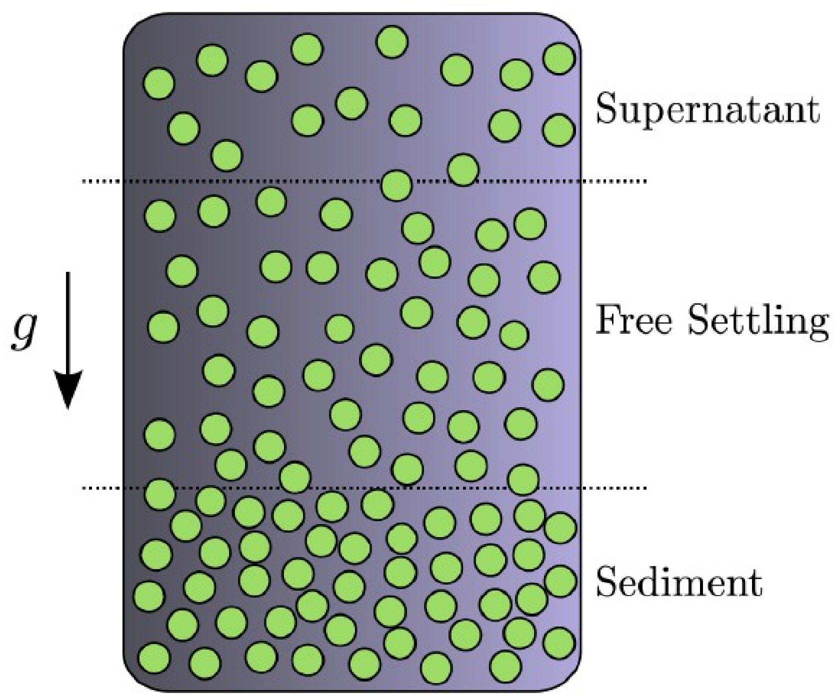
Chaos in 3-particle sedimentation
Janosi et al.
Phys Rev E 1997

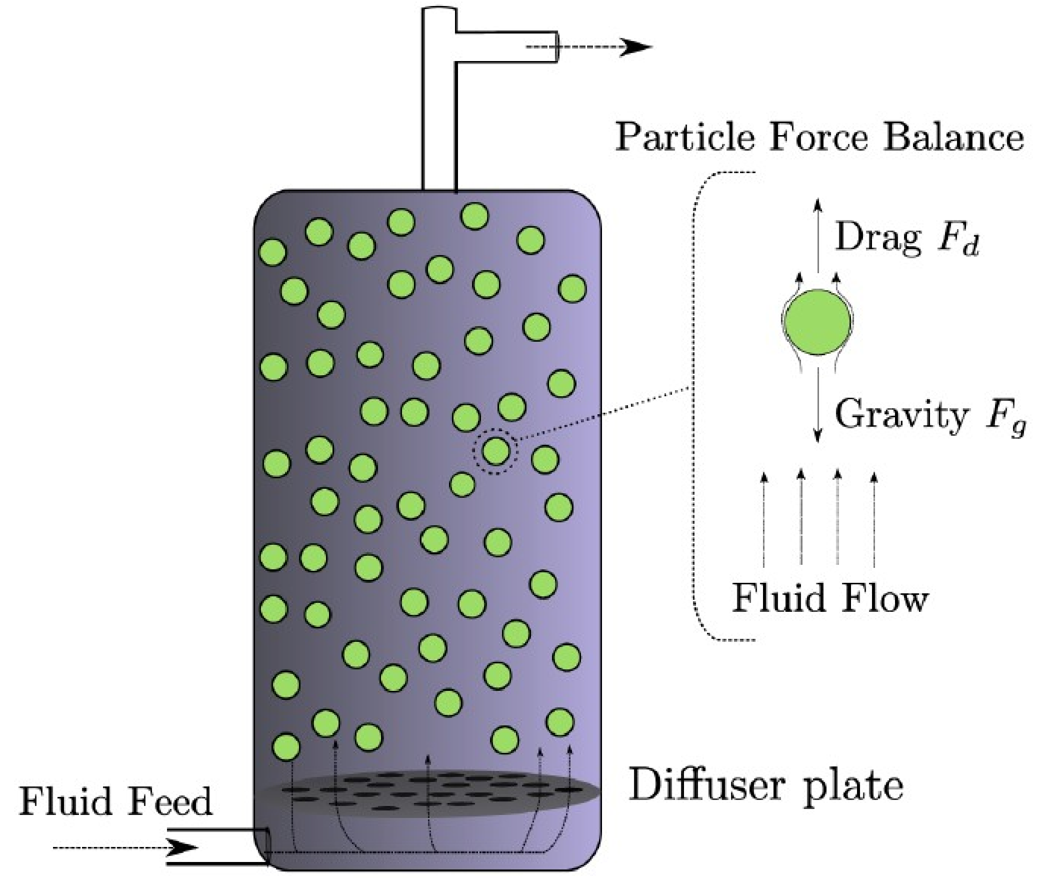
three discs



Sedimentation

many-body long-range statistical mechanics





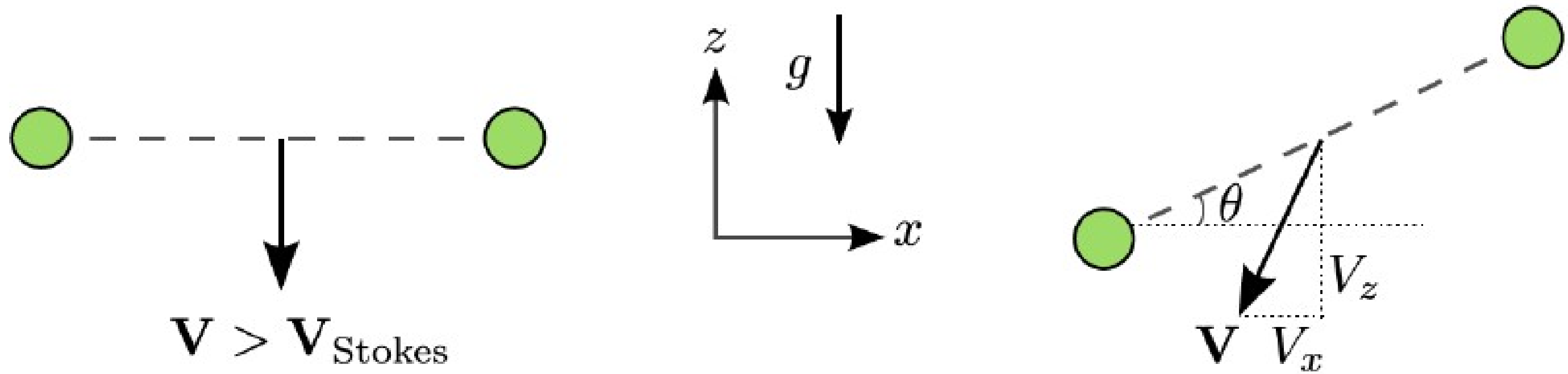
Batch settling

The velocity fluctuations problem
Caflisch-Luke 1985
Koch & Shaqfeh
Levine et al 1998
Ladd, Guazzelli, Hinch...

Fluidised bed

K Vijay Kumar, IISc PhD thesis 2010, modified from
https://upload.wikimedia.org/wikipedia/commons/9/96/FluidisedBed.svg

Two settling spheres: the line–of–centres force



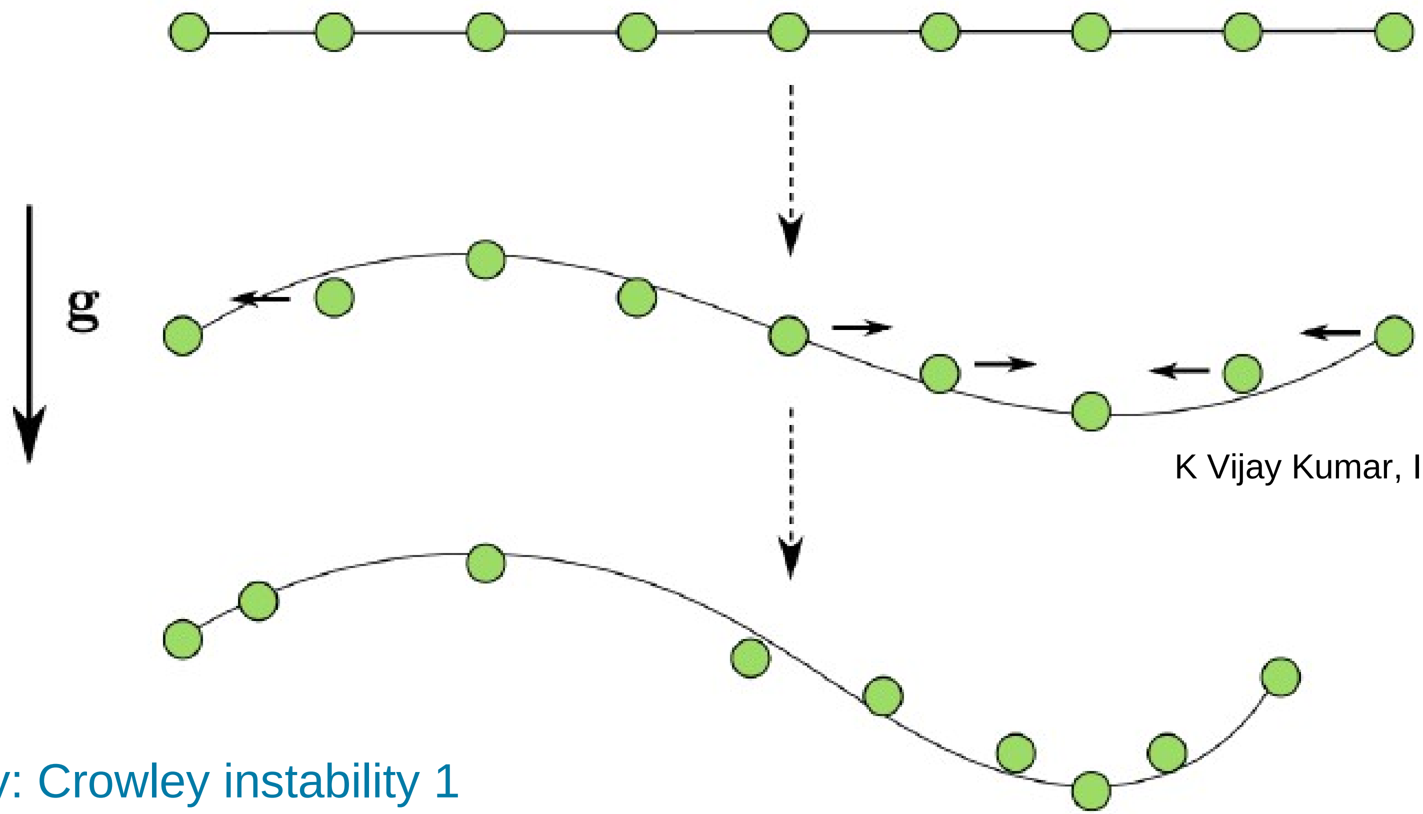
$$F_D = 6\pi\mu a U\{1 - \tfrac{3}{4}(a/d)\}$$

$$F_{LC} = 6\pi\mu a U\{\tfrac{3}{4}(a/d)\}\sin\theta$$

Russel, Savile & Showalter 1989
K Vijay Kumar, IISc PhD thesis 2010

CROWLEY'S INSTABILITY

Crowley JFM 1971, Phys Fluids 1976



K Vijay Kumar, IISc PhD thesis 2010

Sphere array: Crowley instability 1

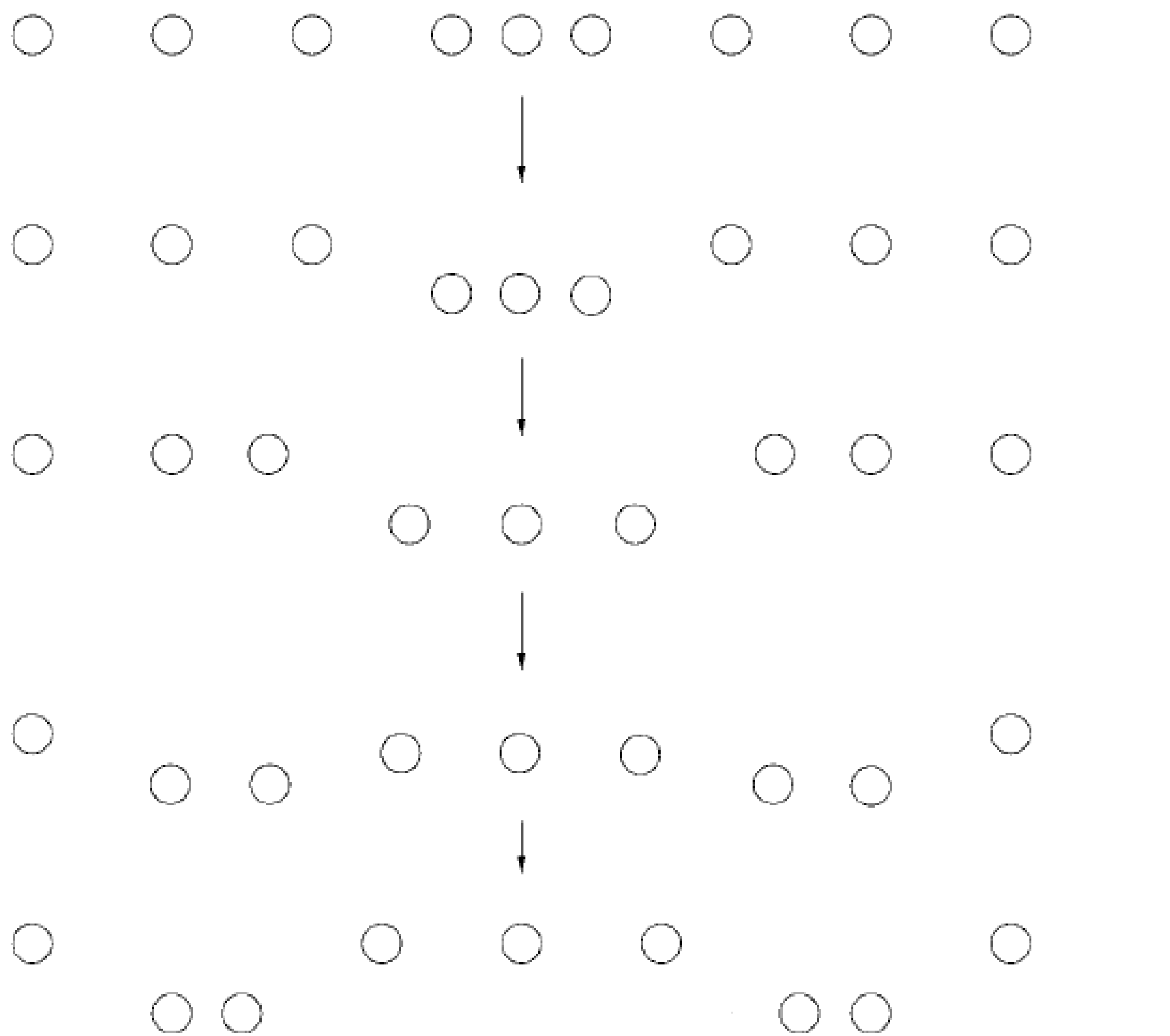
Rahul Chajwa, unpublished

Sphere array: Crowley instability 2

Pair dynamics is the building block

Strong phase separation: Lahiri, Barma, SR 1997-2000

Anti–Crowley challenge: a stably settling array?



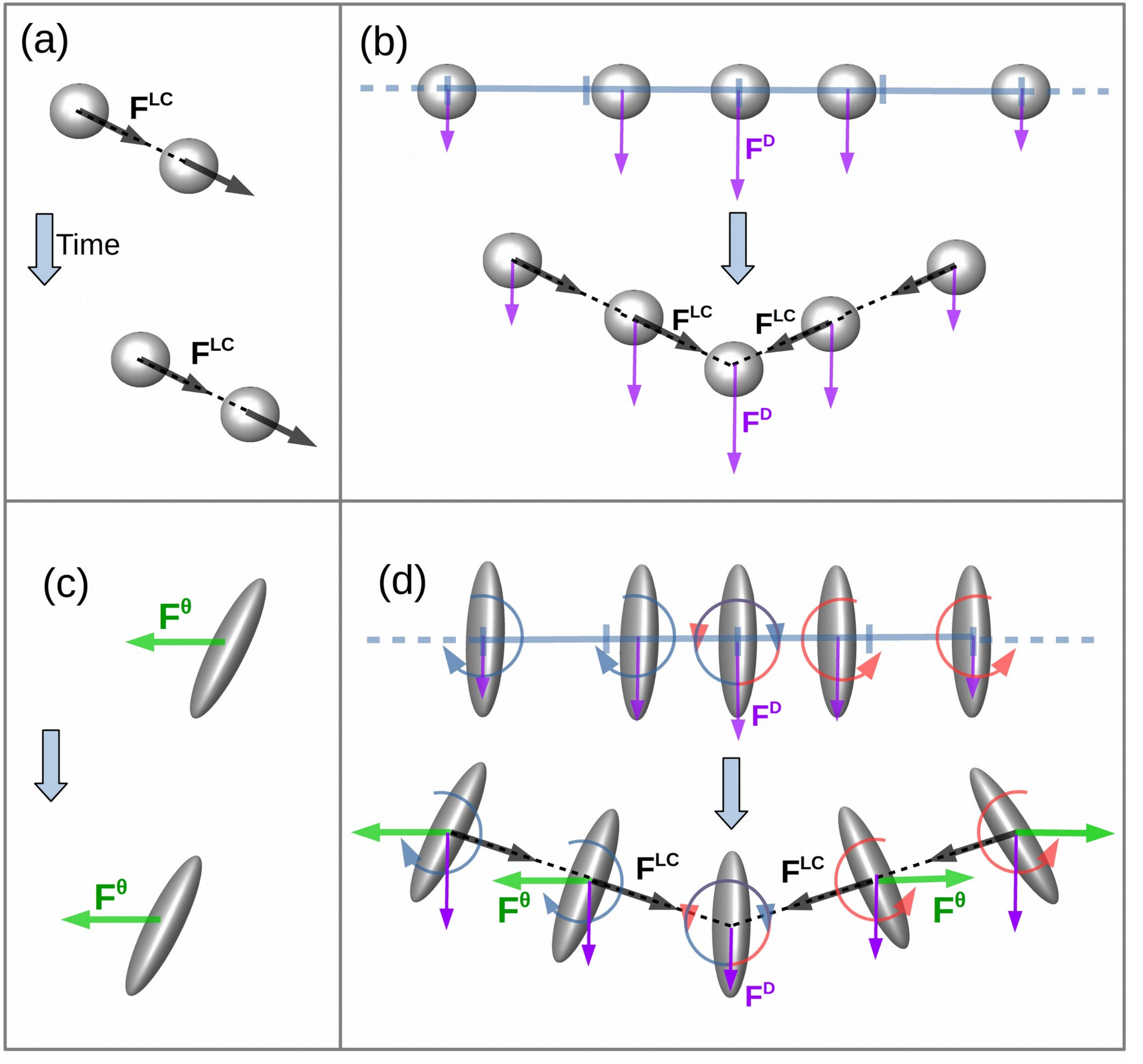
Waves without inertia in drifting flux lattices
Simha/SR PRL 1999

Competition: tilt vs line-of-centres

line-of-centres drift

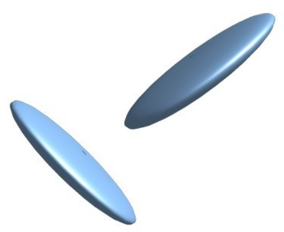
tilt ~ horizontal momentum

Chajwa et al.
Kepler Orbits in Pairs of Disks
Settling in a Viscous Fluid
Phys. Rev. Lett. 2019



SYMMETRY: GRAVITY INDIFFERENT TO DISC ORIENTATION

Ingredients for pair dynamics



- Orientational drift: $U_x^0 = \frac{F\alpha(e)}{12\pi\mu a}\sin 2\theta$
- Reduced drag:

$$U_z^1 = -\frac{F}{6\pi\mu a}\left(\frac{3a}{4r}\right)(1 + \frac{(y_1 - y_2)^2}{r^2})$$

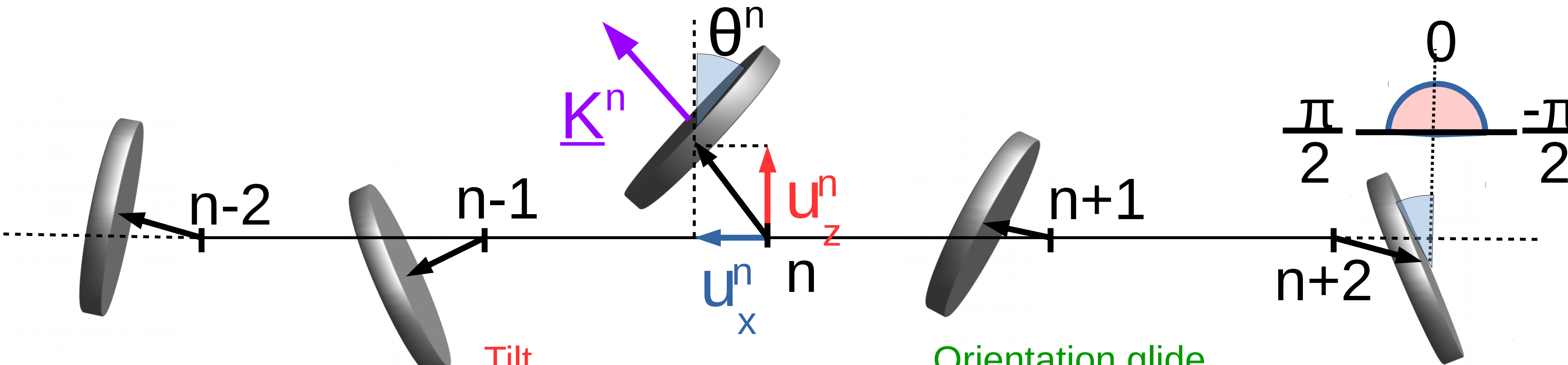
- Line-of-centres drift:

$$U_x^1 = -\frac{F}{6\pi\mu a}\left(\frac{3a}{4r^3}\right)(x_1 - x_2)(y_1 - y_2)$$

- Mutual rotation: $\dot{\theta}_1 = \frac{F(x_1 - x_2)}{8\pi\mu r^3}$

- S. Wakiya, J. Phys. Soc. Jpn. **20**, 1502 (1965)
- S. Kim, Int. J. Multiphase Flow **11**, 699 (1985)
- S. Jung et al. , Phys. Rev. E **74**, 035302(R) (2006).
- R. Chajwa et al., PRL **122**, 224501 (2019)

Sedimenting lattice of spheroids



$$\frac{du_x{}^n}{dt} = -\frac{F}{8\pi\mu}\sum_{l=1}^{\infty}\frac{u_z{}^{n+1}-u_z{}^{n-1}}{l^2d^2} + \frac{F\alpha(e)}{12\pi\mu a}\sin 2\theta^n$$

Reduced drag

$$\frac{du_z{}^n}{dt} = \frac{F}{8\pi\mu}\sum_{l=1}^{\infty}\frac{u_x{}^{n+1}-u_x{}^{n-1}}{l^2d^2} + \frac{F\alpha(e)}{6\pi\mu a}\sin^2\theta^n$$

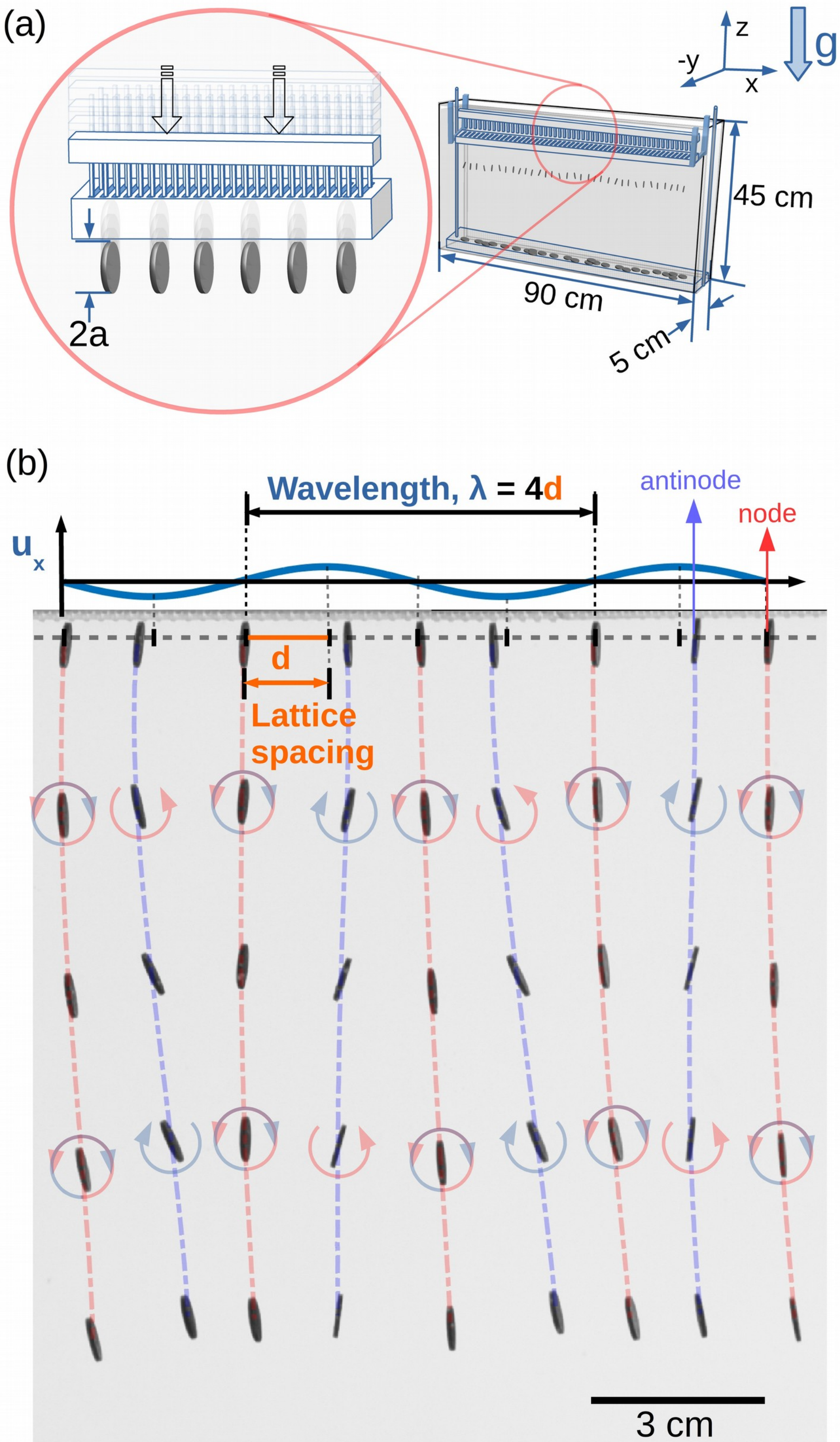
Rotational coupling

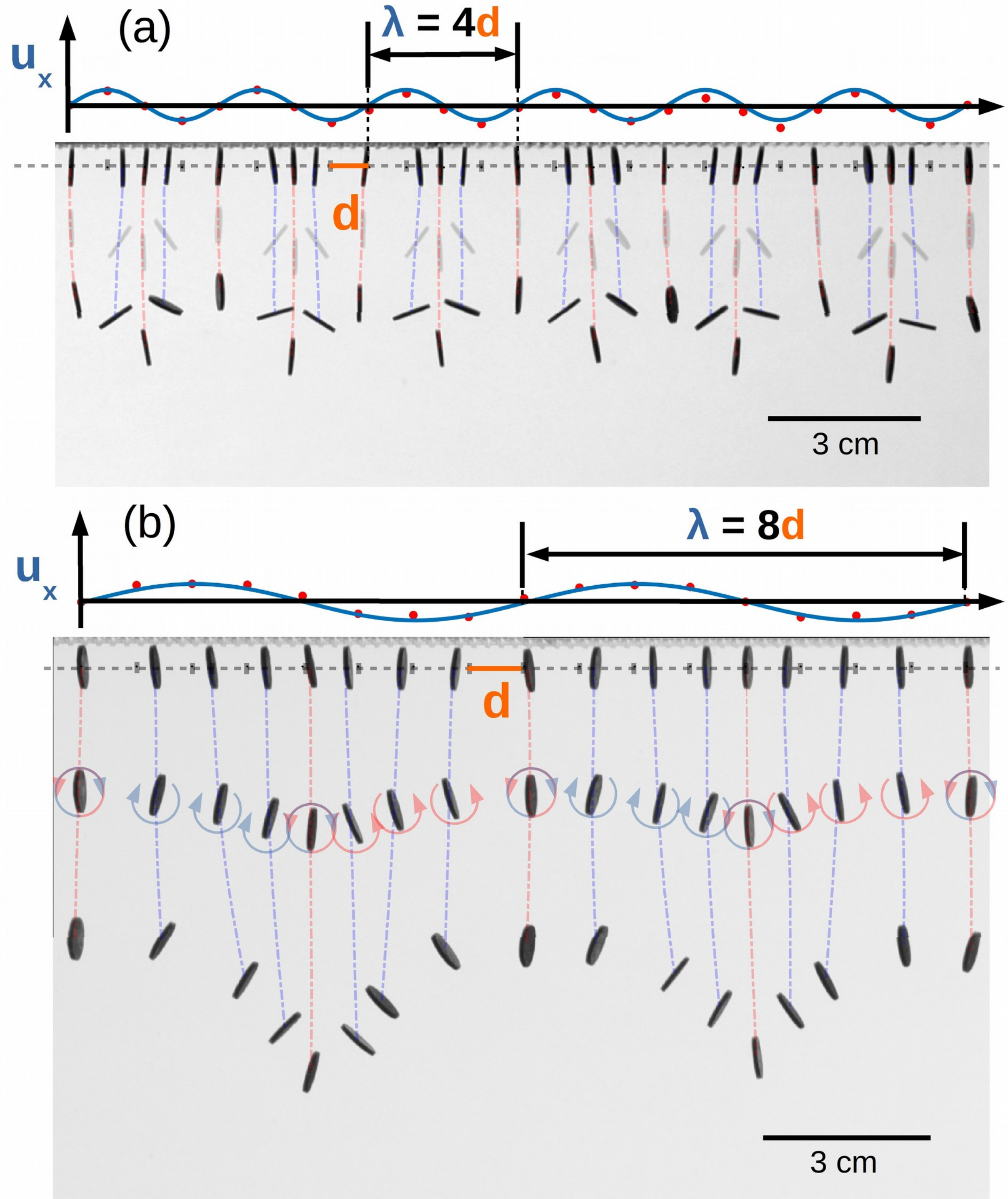
$$\frac{d\theta^n}{dt} = -\frac{F}{4\pi\mu}\sum_{l=1}^{\infty}\frac{u_x{}^{n+1}+u_x{}^{n-1}-2u_x{}^n}{l^3d^3}$$

truncate: nearest neighbour

NON-NORMAL DYNAMICAL MATRIX

Gravity indifferent to disc orientation


(a)
z
-y
x
g
45 cm
90 cm
5 cm
2a
(b)
Wavelength, λ = 4d
antinode
node
u x
d
Lattice spacing
3 cm


(a)
λ = 4d
u x
d
3 cm
(b)
λ = 8d
u x
d
3 cm

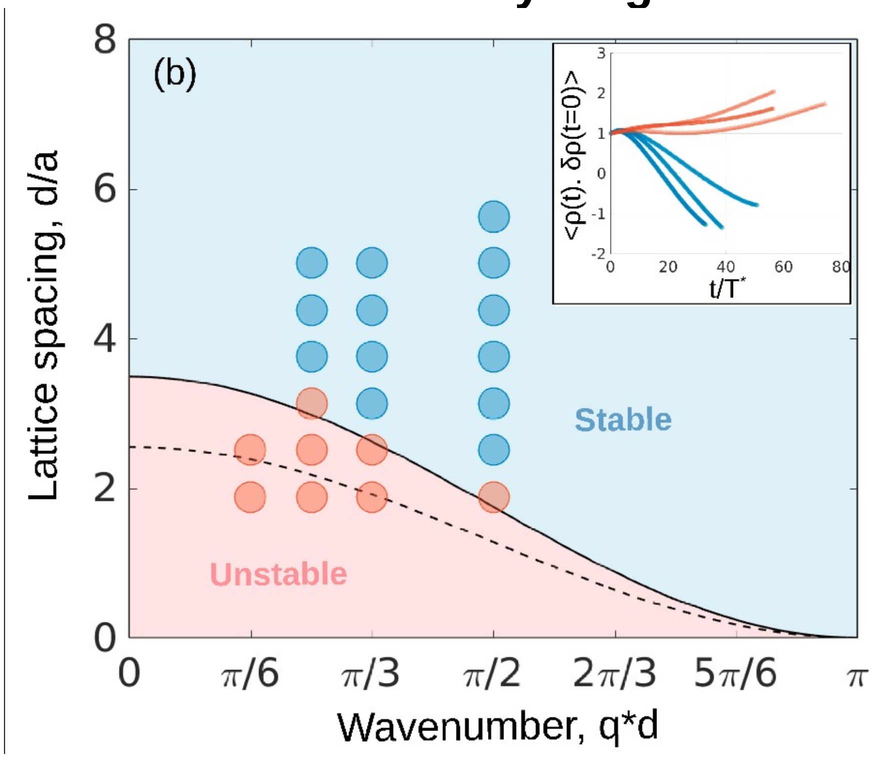
Varieties of dynamics

linearly stable wavelike mode

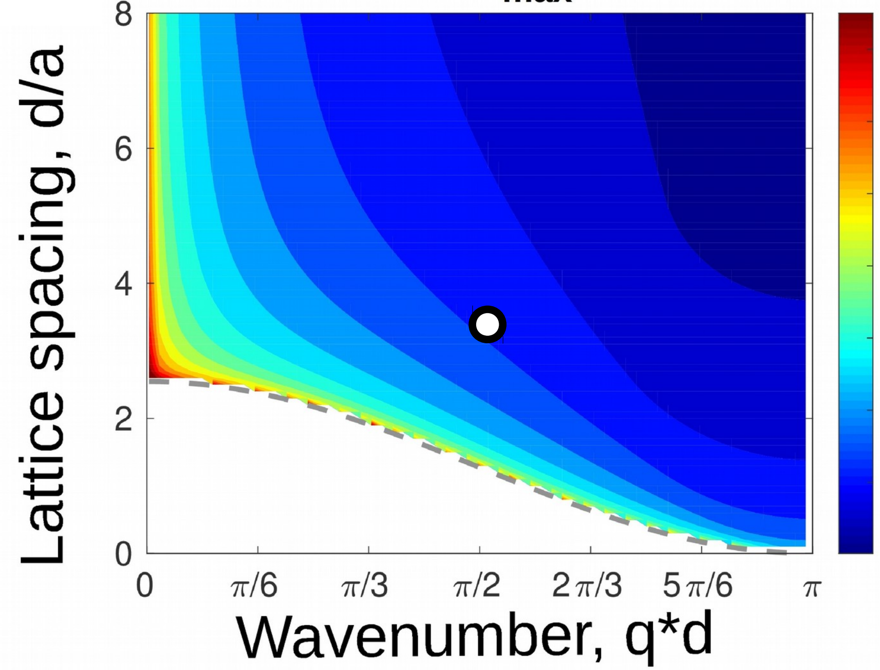
linear instability to clumping

linear instability to clumping at late times

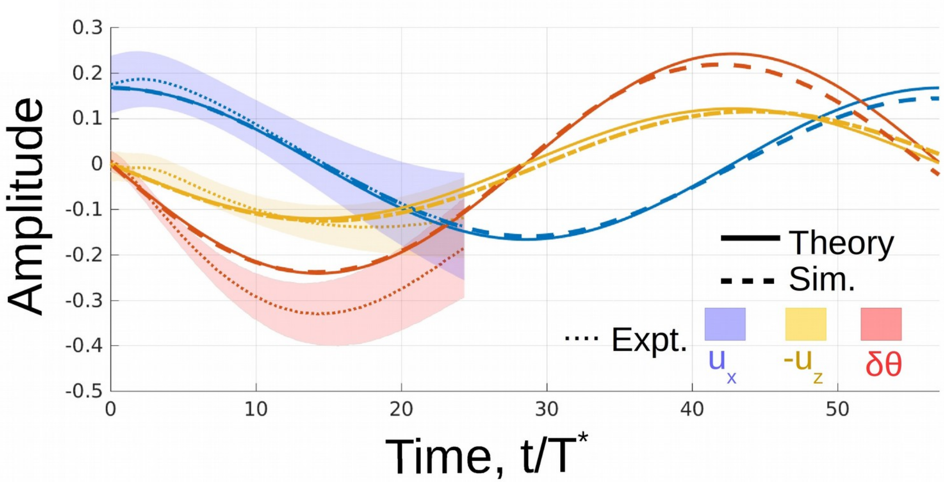
Linear stability diagram



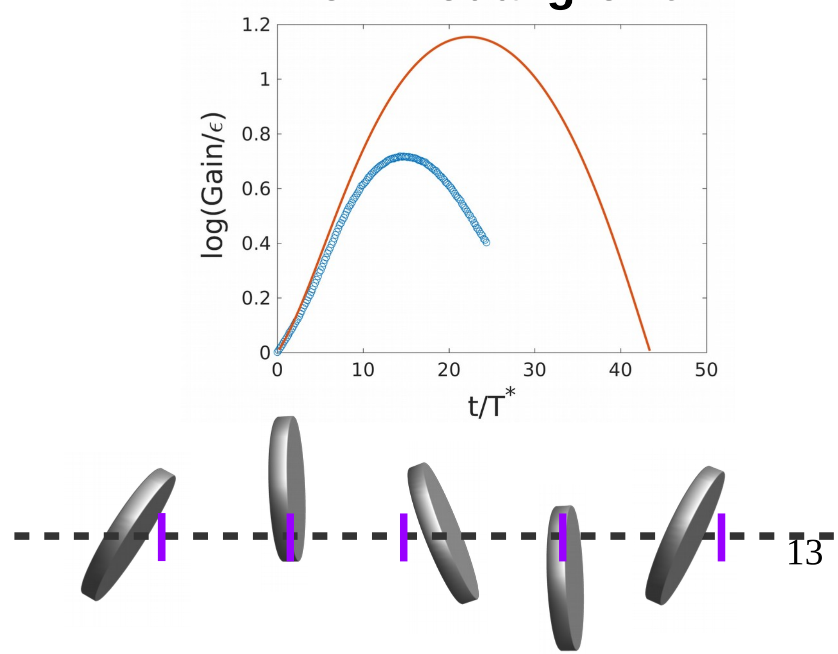
log(G_{max})



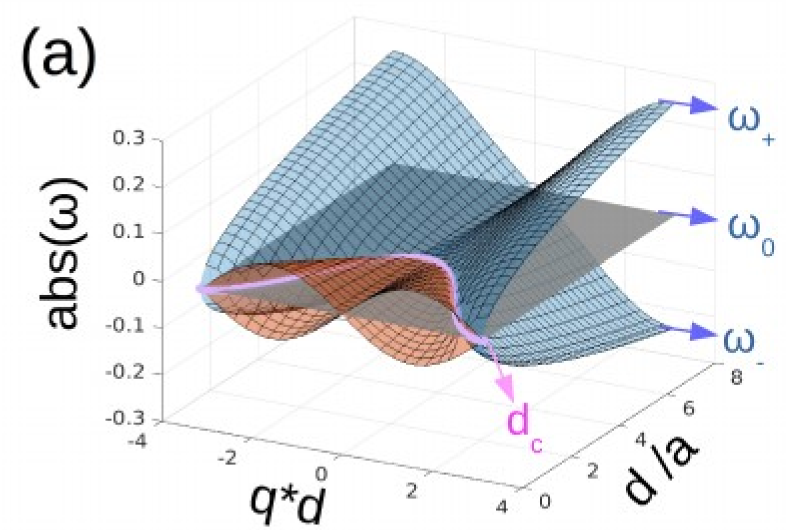
Non-modal wave



Non-modal growth

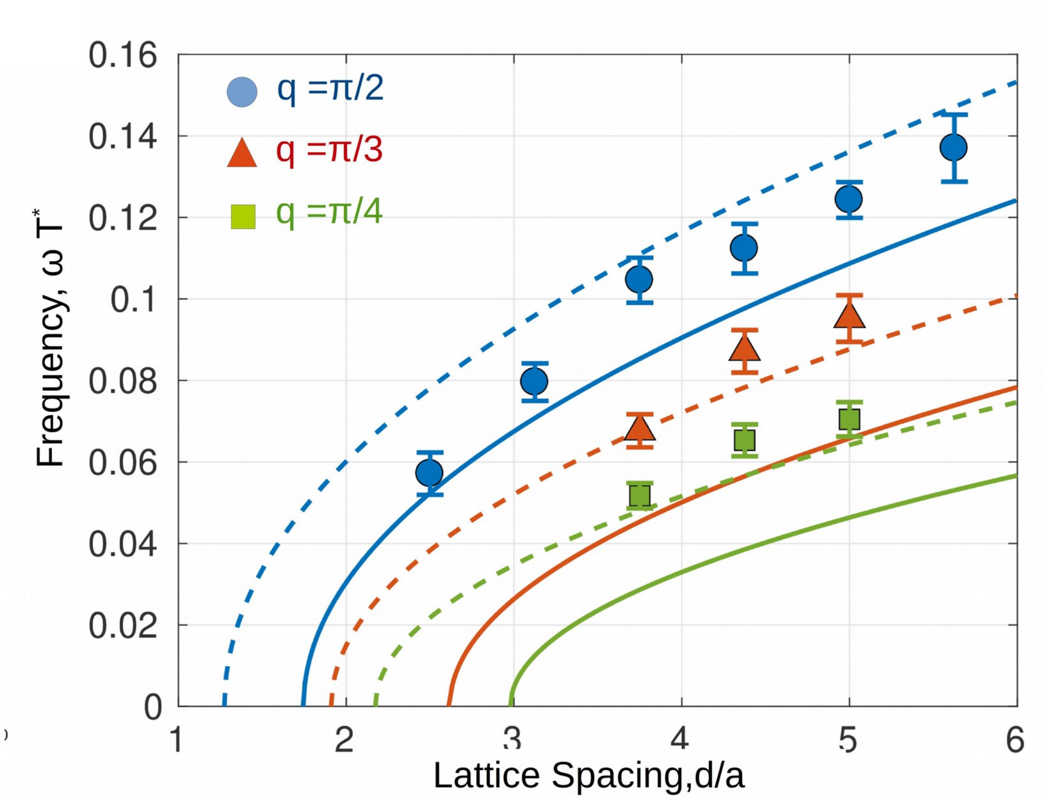


Gapless modes



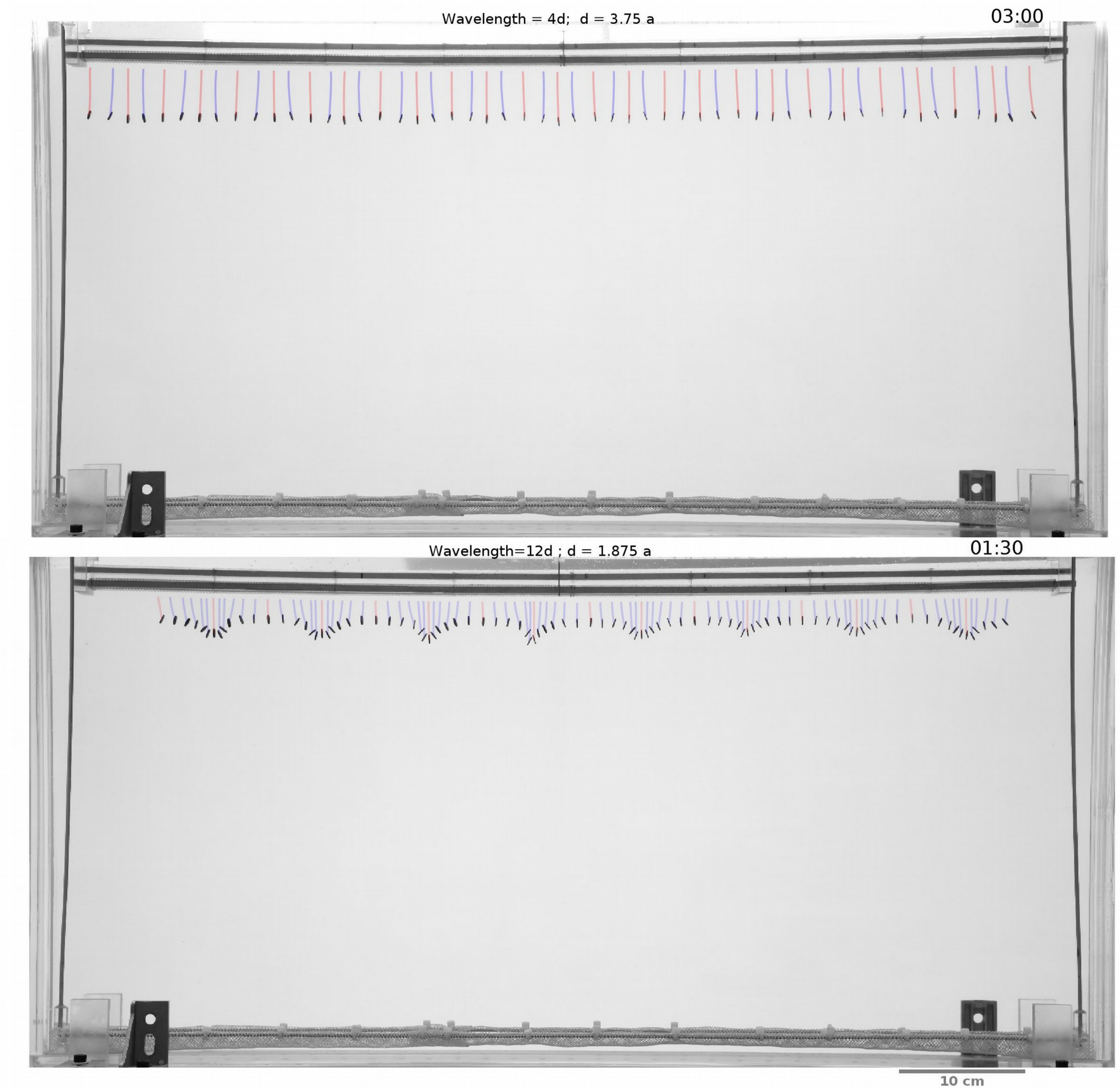
$$i\omega_{\pm}(q) = \pm\frac{1}{4\pi}\sqrt{\sin^2\left(\frac{q}{2}\right)\left(-\frac{d\pi}{2a} + 4\cos^2\frac{q}{2}\right)}$$

Orientations act like an conserved momentum not a Nambu-Goldstone field



Ref. Chajwa et. al. arXiv:2002.04168 [cond-mat.soft] (2020)

Non-linear instabilities



Summary

- Linearly stable sedimenting disc arrays
- Underdamped waves in experiments and theory
- Dynamical matrix non-normal: linear stability deceptive
- Complete stability with orientational elasticity?
- Polar particles?
- Statistical mechanics of disc suspensions?

Summary

- Linearly stable sedimenting disc arrays
  - tilt-induced drift beats line-of-centres drift, Crowley clumping averted
- Underdamped waves in experiments and theory
  - no inertia, but tilt ~ horizontal momentum
  - gravitational energy indep of disc orientation: small-q tilt undamped
- Dynamical matrix non-normal: linear stability deceptive
  - transient growth permitted, predicted, strongly confirmed
  - periodic BC simulations: control initial perturbation, postpone instability
- Complete stability with orientational elasticity?
- Polar particles?
- Statistical mechanics of disc suspensions?