Zeros of FQH wavefunctions

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Kusmierz, SGJ, unpublished

Preliminary comments

Single particle states in a B field: $z^m e^{-\frac{|z|^2}{4}} \qquad m = 0, 1 \ldots$

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We will assume that the particles are bosonic: simplifies certain discussions.

Many particle states: Can be written in a basis of monomial symmetric polynomials.

$$|n_1, n_2\rangle \equiv m_\lambda = [z_1^{n_1} z_2^{n_2} + z_2^{n_1} z_1^{n_2}]$$

$$\lambda = (n_1, n_2)$$

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$$m_\lambda = \mathrm{Sym}[z_1^{\lambda_1} z_2^{\lambda_2} z_3^{\lambda_3} z_4^{\lambda_4} \ldots]$$

$$\lambda \equiv (\lambda_1, \lambda_2 \ldots \lambda_N) \text{ such that } \lambda_{i-1} \geq \lambda_i \geq 0$$

Preliminary comments

Repulsive interactions are minimized by diminishing the density of other particles at the location of each particle: ie through terms of the form

$$\prod_{i=2}^{N}(z_1 - z_i)^k$$

Holomorphic nature of the single particle functions + minimization of interaction energy ⇒ geometric phases of each particle around every other. Equivalently attachment of zeros of the wavefunction on every particle.

In general attachment of a set of zeros *m* to clusters of *k* particles.

Motivation

Coulomb interaction at filling fraction v=1/n

Gapped ground state

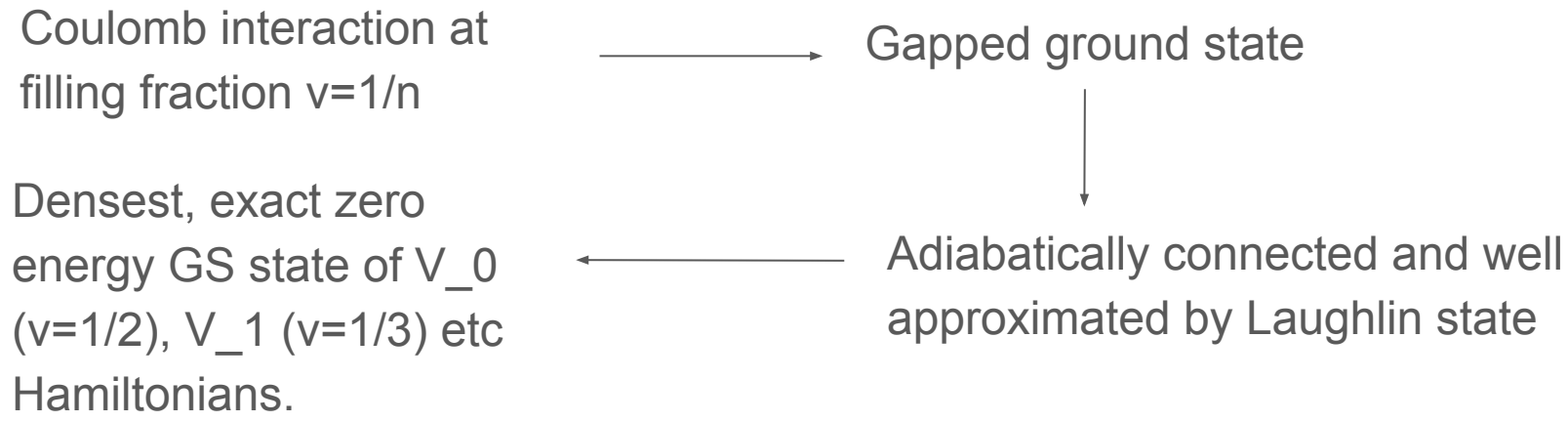
Adiabatically connected and well approximated by Laughlin state

Densest, exact zero energy GS state of V_0 (v=1/2), V_1 (v=1/3) etc Hamiltonians.

Similar exact Hamiltonians exist for Moore-Read state, Read-Rezayi states etc.

Laughlin 1983, Haldane 1983; Trugman Kivelson 1985; Read Rezayi 1999; Simon, Rezayi, Cooper 2007

Motivation



V_n interaction: Energy cost whenever 2 particles are in a state of relative angular momentum n.

Exact zero energy ⇒ Electrons in that state “move around” strictly avoiding certain specific configurations.

Laughlin 1983, Haldane 1983; Trugman Kivelson 1985; Read Rezayi 1999; Simon, Rezayi, Cooper 2007

Motivation

Coulomb interaction at filling fraction ~~1/n~~ n/(pn+1)

→ Gapped ground state

↓

Adiabatically connected and well approximated by Jain CF states

← No known exact Hamiltonians for n>1

⇒ Constraints in the electronic states are not fully understood.

$$P_{LLL}\Phi_n\Phi_1^p$$

Jain 1989

Patterns of zeros

Example, consider the polynomial: $m_{4200} = \mathrm{Sym}[z_1^4 z_2^2 z_3^0 z_4^0]$

Send all but 1 particles (z) far away from origin:
Has no zeros near origin.

$$m_{4200} \sim z^0 \times const.$$

Send all but 2 particles (z,e1) far away from origin:
z sees no zeros near the origin

$$m_{4200} \sim z^0 e_1^0 \times const.$$

Send all but 3 particles (z,e1,e2) far away from origin:
z sees 2 zeros near the origin

$$m_{4200} \sim z^2 + a e_1^2 + b e_2^2 + ..$$

Pattern is simply 0,0,2,4

Bernevig Haldane PRL 2007, Bernevig Haldane PRL 2008, Wen, Wang 2008,

Patterns of zeros: Linear combinations of *m*

Example, keep in mind the polynomial: m_{4200} + m_{6000} + m_{3210}

Construct a “cumulative” partition

6000 → 6 0 0 0
4200 → 6 2 0 0
3210 → 6 3 1 0

$$(\lambda_1, \ldots \lambda_n) \rightarrow (K_1, K_2, \ldots K_n)$$

Where K_i is the sum $\lambda_i + \lambda_{i+1} + ..\lambda_n$

Define an ordering in the space of partitions. $(\lambda'_1, \lambda'_2 \ldots) > (\lambda_1, \lambda_2 \ldots)$

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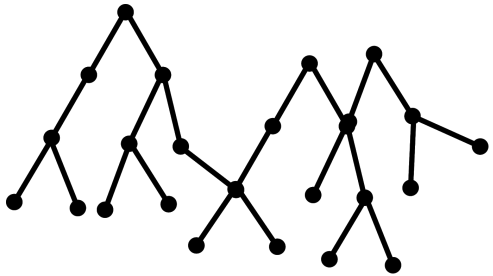
ZERO PATTERN OF

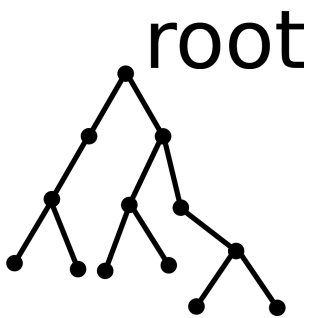
$$m_{\lambda^1} + m_{\lambda^2} + \ldots \qquad \rightarrow \max(\lambda^1, \lambda^2 \ldots)$$

6000 in the example

Bernevig Haldane PRL 2007, Bernevig Haldane PRL 2008, Wen, Wang 2008,

Root partition





Laughlin state at 1/2 : 0,2,4,6,8,10 ….

Laughlin state at 1/3 : 0,3,6,9,12…

⅔ Jain CF state : 0, 0, 2, 4, 5, 7,8, 10,11, 13,14, 16,17 …..

Bernevig Haldane PRL 2007, Bernevig Haldane PRL 2008, Rodriguez, Sterdyniak, Hermanns, Slingerland, Regnault (2012), SGJ, Fremling, Jeon, Jain (2018), Regnault, Bernevig, Haldane (2009)

Constraint on clusters of few particles

Example: Consider the Jain CF ⅔ state with root configuration 0, 0, 2, **4,** 5, 7,8, 10,11, 13,14, 16,17..

Can a cluster of 4 particles be in a state

m_{6000} ? No.
m_{4200} ? Yes.
m_{3300} ? Yes.

SGJ, Fremling, Jeon, Jain (2018)

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m_{6000} ? No.
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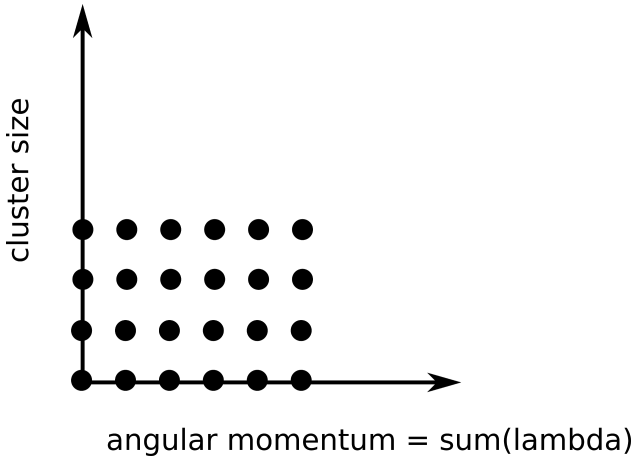
m_{3300} ? Yes.

If the configuration m_{6000} occurs in a 4-particle cluster, collection of 4 particles with be associated with 6 zeros, this is in contradiction to the root partition which says only 4 zeros are attached.

m_{3300} can occur in a coherent combination with m_{4200} as its zero pattern will be dominated by the pattern 4200.

If an n-particle configuration lambda cannot occur, the state is annihilated by the n-particle projector on to the state lambda. SGJ, Fremling, Jeon, Jain (2018)

Enumerate for clusters of n particles



We can enumerate constraints in n-particles sectors and within them, in different angular momentum.

Some constraints are mutually dependent. These can also be analyzed.

SGJ, Fremling, Jeon, Jain (2018)

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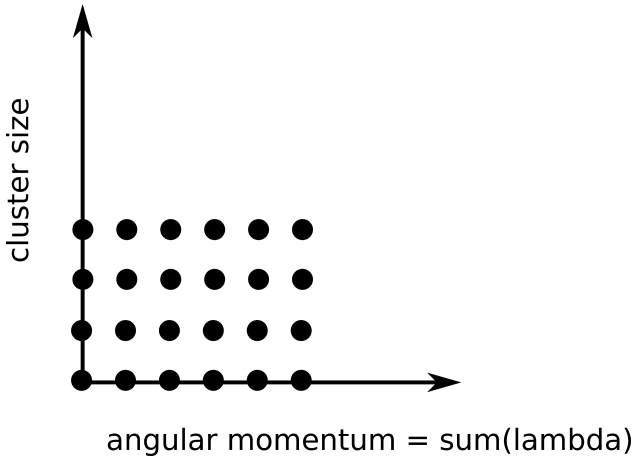

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Expectation values of the projection on to the missing states →

Same constraints are present the Coulomb GS not just in the variational states.

$4Q-L\rightarrow$		0	2	3	4	5	6		
	$N_b\downarrow$						T_6	T_{42}	T_{33}
CF	6	0	0	0	0	0	0	0.27	2.7
	8	0	0	0	0	0	0	0.51	4.7
	10	0	0	0	0	0	0	1.2	10.2
Coulomb	6	0	0	0	0	0	10^{-4}	0.42	2.5
	8	10^{-12}	10^{-7}	10^{-5}	10^{-3}	10^{-4}	10^{-4}	0.69	4.6
	10	10^{-6}	10^{-6}	10^{-6}	10^{-3}	10^{-7}	10^{-4}	1.7	10.1

SGJ, Fremling, Jeon, Jain (2018)

Projection Hamiltonian for the ⅔ Jain CF state

$$H_{N=3,L=0} + H_{N=4,L=4}$$

How constraining is this Hamiltonian ? ie How good are the constraints implied ?

Can rewrite the whole calculation on to the sphere and diagonalize this Hamiltonian.

For the largest systems sizes that we studied (12 particles), the above Hamiltonian identified a space of 4 states that contained the desired state.

SGJ, Fremling, Jeon, Jain (2018)

Concluding remarks

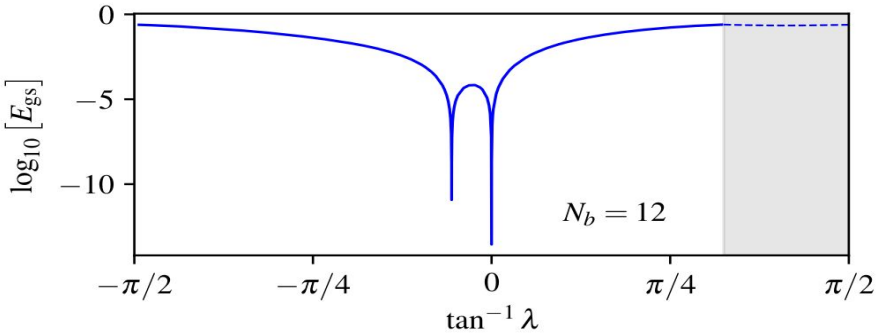
We can construct a sequence of constraints that partially pin down a Hamiltonian that annihilates the general state at n/(2pn+1).

When there are multiple *allowed/admissible* configurations possible in a given angular momentum. We find that Hamiltonian produces a 0-energy states only for special values of their coherent states.

Under what set of such constraints can a 0-energy state occur ?

Can the ⅔ or more generally n/(pn+1) state be realized as a zero energy state ?

$$H = H_0 + P(T_{42} + \lambda T_{33})$$

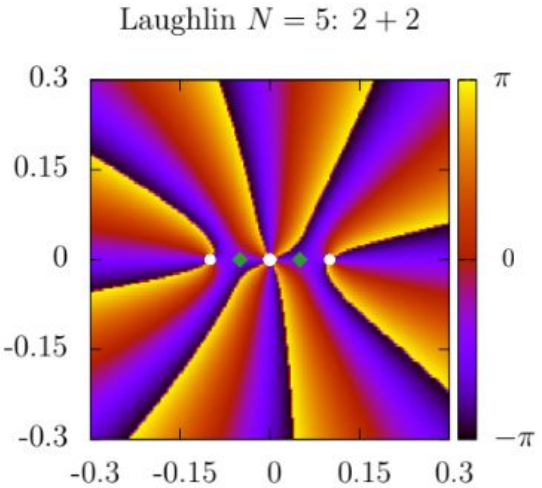


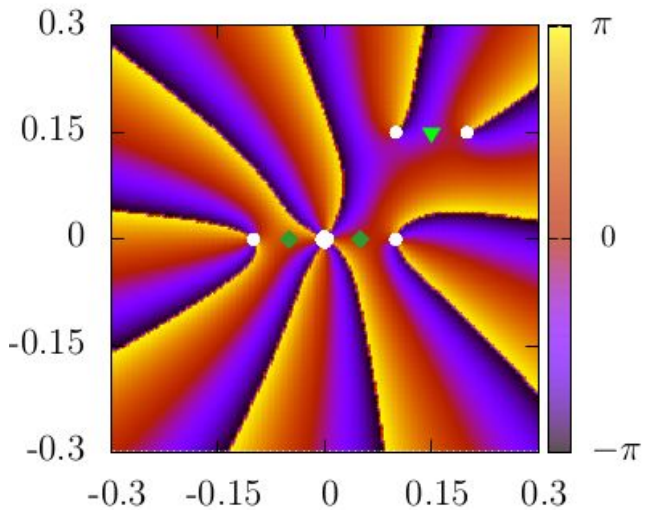
SGJ, Fremling, Jeon, Jain (2018), Regnault, Bernevig, Haldane (2009)

Concluding remarks

A more general class of questions can be asked about (1) zeros of a centroid of a cluster of particles attached to another cluster of particles, (2) Clustering in exact Coulomb GS, (3) States that live in multiple Landau levels etc.
Algorithms to numerically explore this can be constructed.

Also, one can go beyond GS and ask how the clustering properties of zeros are destroyed ie constraints are violated in the vicinity of localized excitations.





Kusmierz, SGJ unpublished