Spontaneous Symmetry Breaking

2008 Nobel Pize for Physics

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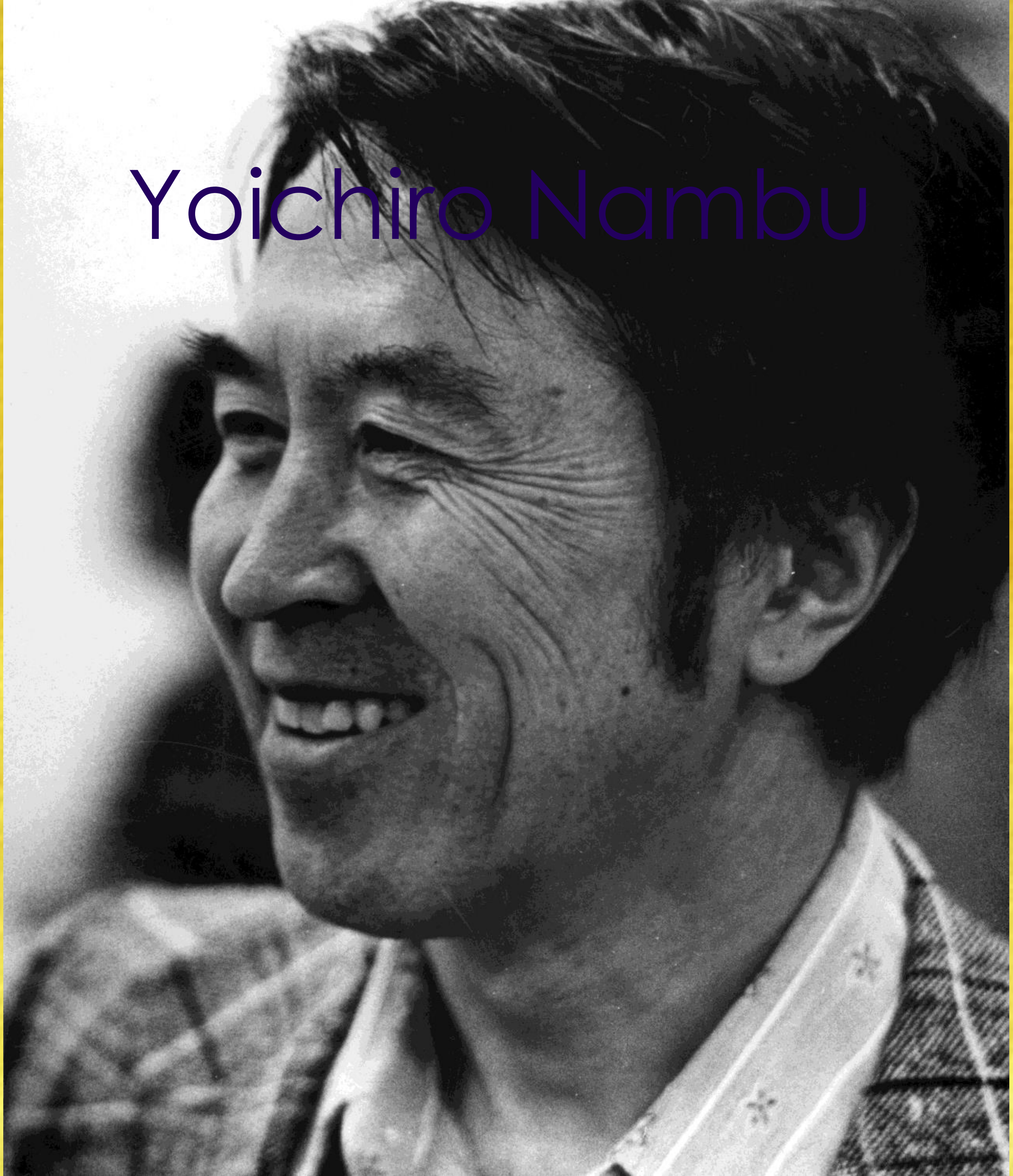


Yoichiro Nambu was awarded the Nobel Prize for "the discovery of Spontaneous Symmetry breaking in sub-atomic physics"

1. Quasi-Particles and Gauge Invriance in the Theory of Superconductivity, Phys. Rev. Feb. 1960

2. Axial Vector Current Conservation in Weak Interactions, PRL April 1960

3. A Superconductor Model of Elementary Particles and its consequences
Conference April 1960

4. Dynamical Model of Elementary Particles based on an Analogy with Superconductivity I and II,
April 1961, with G. Jona-Lasinio

Nambu (1977):

- “the name Spontaneous Symmetry Breaking is due to Baker and Glashow.The concept actually consists of three elements :

1. Degeneracy of vacuum (ground state)
2. The associated massless (collective) mode
3. Subsuming of the zero mode and the coulomb field into the plasmon mode.

- It was my contention that the BCS theory had all these three elements. The next step was an easy one, to reach the analogy between energy gap in superconductors and the mass of the Dirac equation. It was before the quark model so the main question was how to choose a model in such a way as to show that symmetry breaking can actually happen”.

Nambu-Jona Lasinio Model

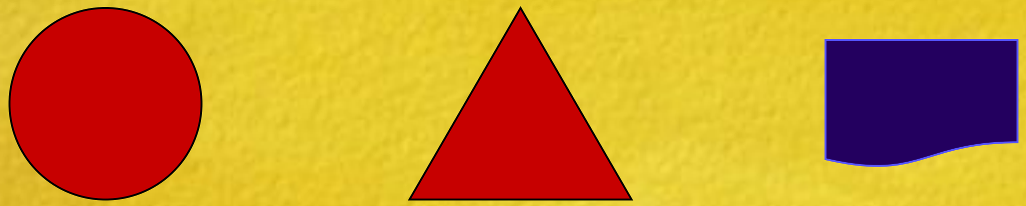
Nambu-Jona Lasinio Model is a non-linear model of nucleons in which chiral symmetry is spontaneously broken. It dynamically generates mass for them and describes pions as a Nambu-Goldstone collective mode (bound state). It thus explained the small mass of the pion relative to the nucleons, gave a novel derivation of the Goldberger-Treiman relations and derived universal results for low energy pions and nucleons in terms of the pion decay constant. It contained the germ of current algebra and sowed the seeds that eventually led to the Standard Model of Elementary particles.

Nambu: other works

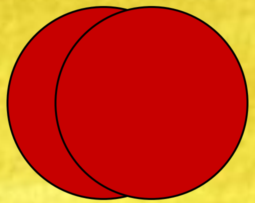
- Introduced the color quantum number for quarks (1965) and suggested that the force between quarks is mediated by an octet of color gluons
- Proposed the string model and the `area law’. (~1970)

Symmetries

- Symmetrical shapes of geometrical objects



- Approximate symmetry of geometric shapes



Symmetry in art and the natural world

Symmetry in Laws of Physics

- Symmetry principles are cornerstones of the laws of physics
- There are various types of symmetries: space-time, internal, continous, discrete, global and local

Continous Space-time Symmetries

- Translations in time
- Translations in space
- Rotations in space

- Galilean relativity
- Einstein special relativity

- Energy conservation
- Momentum conservation
- Angular momentum

- Classical Mech.
- Electromagnetism

Space rotations..an example of continous symmetry

- Physical laws do not depend upon which rotations in space
- Implies that the 3 components of angular momentum are conserved in any process (J_x, J_y, J_z)
- In atomic physics e.g this enables a book keeping of atomic spectra
- Rotational symmetry is broken by turning on a magnetic field; change in spectrum can be calculated (Zeemann effect)

Discrete Space-time Symmetries of Elementary particles

- Parity: space reflections
- Charge conjugation: particle anti-particle interchange
- Time reversal
- Only CPT is conserved in all interactions

Continous Internal Symmetry

- In nuclear physics if you ignore neutron-proton mass difference: Isospin symmetry: rotations in (n,p) space. 3 isopin quantum numbers analogous to 3 angular momentum quantum numbers

- Nuclear spectra organized by energy, isospin and angular momentum

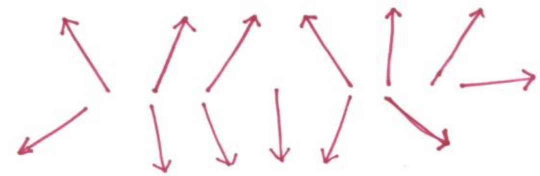
- e.g. I=J isobar series (meson-nucleon bound states)

Spontaneous symmetry breaking

- Spins (rotors) interact locally
- No cost in energy if all spins are rotated by the same amount: Global internal continous symmetry
- Magnetic field breaks symmetry
- Bias persists for an ‘infinitely’ large system, even when magnetic field is switched off: Ground state breaks symmetry

Global Symmetry:

Rotate all spins by same amount



$$H \sim \sum_x \vec{S}_x \cdot \vec{S}_{x+\Delta} + B \sum_x S_x^z S_{x+\Delta}^z$$

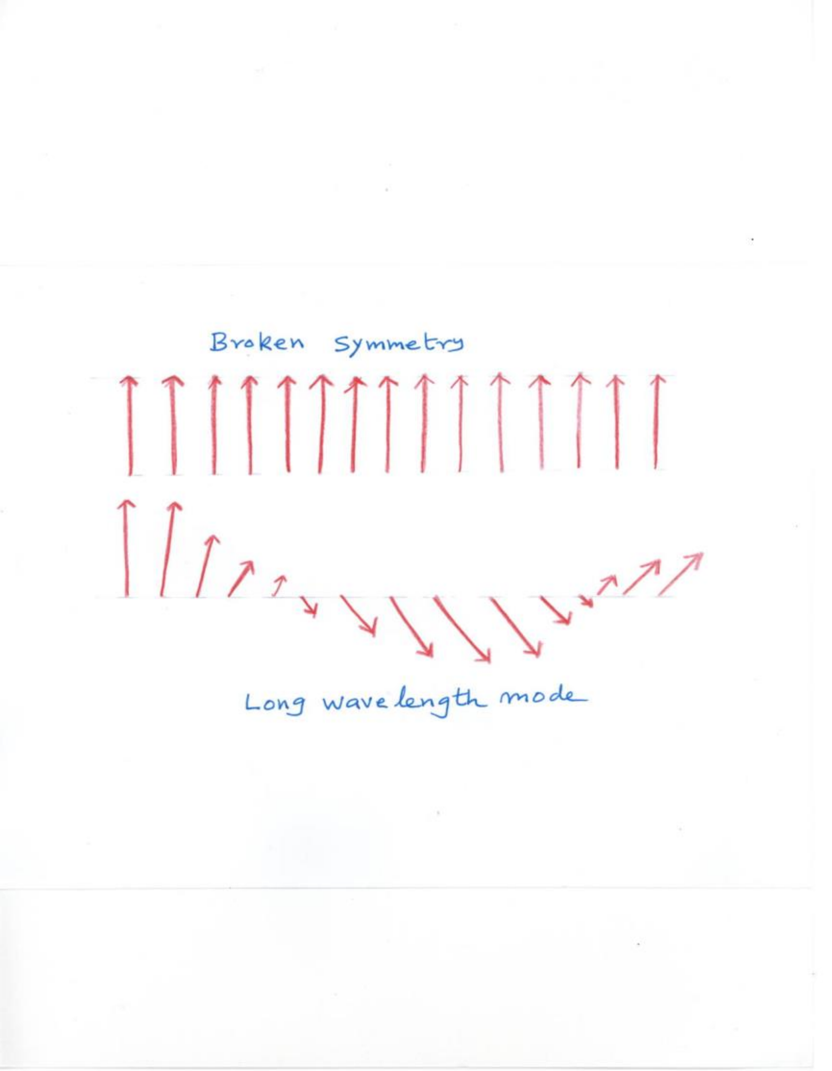
$B \neq 0$ breaks global symmetry

$T < T_c$, $B \to 0$ (infinite vol.)

Spontaneous Symmetry Breaking

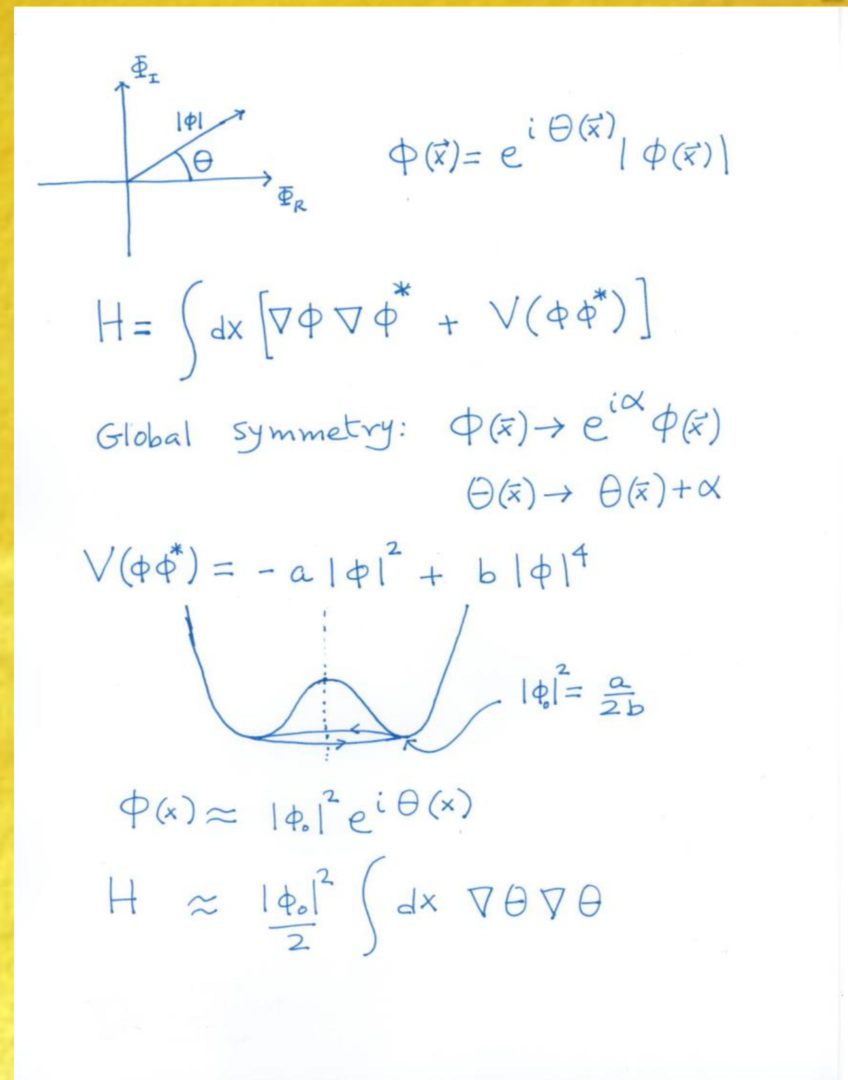
Nambu-Goldstone mode

- Because of the global continous symmetry small variations in the directions of spins will cost vanishingly small energy (as opposed to the cost when a magnetic field is turned on).
- One can imagine it as a `slowly varying', symmetry transformation
- This excitation is called a spin-wave or as part of a more general phenomenon: A Nambu-Goldstone mode of sponteneous symmetry breaking



Simple models of SSB

- Landau-Ginzburg order parameter
- Complex phase
- Global symmetry= global phase rotation
- Nambu-Goldstone mode is the slowly varying phase



BCS-Bogoliubov Theory of Superconductivity

- Electron-phonon interaction causes Cooper pairing of a large number of electrons,of opposite spin and opposite momentum, in the ground state. These being bosons condense in the ground state and lead to a band gap.

- Electrons are charged hence wave function has a phase
- Paired electrons have twice the phase; the ground state breaks '2e' charge symmetry spontaneously

Electron $\psi_{\uparrow\vec{k}}, \psi_{\downarrow\vec{k}}$

electric charge 'e'

$$\psi_{\uparrow\vec{k}} \to e^{-i\alpha e}\,\psi_{\uparrow\vec{k}}$$

U(1) phase rotation

Cooper pairing in ground state

$$\phi_0 = \frac{1}{V}\sum_{k} \langle \psi_{k\uparrow}\,\psi_{-k\downarrow} \rangle$$

$$\phi_0 \to e^{-2ie\alpha}\,\phi_0$$

SSB

Energy gap, Quasi-particles and Meissner effect

- There is a small energy gap that gives mass to the quasi particles, which are not particles/holes with definitive charge.
- Collective mode due to SSB subsumed by Coulomb interaction leading to the massive plasmon and Meissner effect (in particle physics this is called the Higgs mechanism)

Electrons / holes near fermi-surface:

$\psi_{\vec{k}\uparrow} \equiv$ electron (charge e)

$\psi^{\dagger}_{-\vec{k}\downarrow} \equiv$ hole (charge $-e$)

$$E_k \psi_{\vec{k}\uparrow} = \epsilon_k \psi^{\dagger}_{-\vec{k}\downarrow} + \Delta \psi^{\dagger}_{-\vec{k}\downarrow}$$

$$E_k \psi_{-\vec{k}\downarrow} = \Delta \psi_{\vec{k}\uparrow} - \epsilon_k \psi^{\dagger}_{-\vec{k}\downarrow}$$

(Bogoliubov)

$$E_k = \pm\sqrt{\epsilon_k^2 + \Delta^2}$$ (quasi-particles)

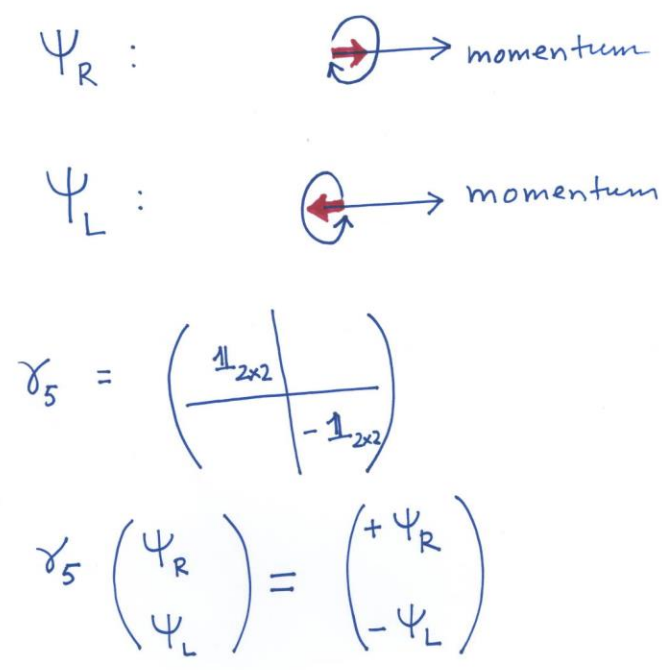
↓

not charge eigen states

$\Delta \sim |\phi_0|$

Particle physics: Chirality

- For a massless Dirac fermion, in 3+1 dims., the spin can be parallel or anti-parallel to the direction of motion
- Parallel=chirality+1(right handed)
- Anti-parallel = chirality -1(left handed)



Chiral Symmetry

- Dirac equation splits into 2 independent equations with 2 independent U(1) symmetries: U(1)xU(1)
- Diagonal sum corresponds to electric charge
- Mass term spoils additional 'chiral' U(1)

Massless Dirac Equation

$$E_p \Psi_R = \vec{\sigma}\cdot\vec{p}\,\Psi_R$$

$$E_p \Psi_L = -\vec{\sigma}\cdot\vec{p}\,\Psi_L$$

Invariance: $\Psi_R \to e^{i\alpha_R}\Psi_R$

U(1) x U(1) $\Psi_L \to e^{i\alpha_L}\Psi_L$

Massive Dirac Equation

$$E_p \Psi_R = \vec{\sigma}\cdot\vec{p}\,\Psi_R + m\Psi_L$$

$$E_p \Psi_L = -\vec{\sigma}\cdot\vec{p}\,\Psi_L + m\Psi_R$$

Symmetry if $\alpha_R = \alpha_L$

$$E_p = \pm\sqrt{p^2 + m^2}$$

"Quasi particles" do not have fixed Chirality.

Superconductivity Elementary Particles

Electrons/holes near fermi-surface:

$\psi_{\vec{k}\uparrow} \equiv$ electron (charge e)

$\psi^{\dagger}_{-\vec{k}\downarrow} \equiv$ hole (charge −e)

$$E_k \psi_{\vec{k}\uparrow} = \epsilon_k \psi^{\dagger}_{-\vec{k}\downarrow} + \Delta \psi^{\dagger}_{-\vec{k}\downarrow}$$

$$E_k \psi_{-\vec{k}\downarrow} = \Delta \psi_{\vec{k}\uparrow} - \epsilon_k \psi^{\dagger}_{-\vec{k}\downarrow}$$

(Bogoliubov)

$E_k = \pm\sqrt{\epsilon_k^2 + \Delta^2}$ (quasi-particles) → not charge eigen states

$\Delta \sim |\phi_0|$

Massless Dirac Equation

$$E_p \psi_R = \vec{\sigma}\cdot\vec{p}\, \psi_R$$

$$E_p \psi_L = -\vec{\sigma}\cdot\vec{p}\, \psi_L$$

Invariance: $\psi_R \to e^{i\alpha_R}\psi_R$

$U(1)\times U(1)$ $\quad \psi_L \to e^{i\alpha_L}\psi_L$

Massive Dirac Equation

$$E_p \psi_R = \vec{\sigma}\cdot\vec{p}\, \psi_R + m\psi_L$$

$$E_p \psi_L = -\vec{\sigma}\cdot\vec{p}\, \psi_L + m\psi_R$$

Symmetry if $\alpha_R = \alpha_L$

$E_p = \pm\sqrt{p^2 + m^2}$

"Quasi particles" do not have fixed chirality.

NJL model: A model of elementary particles in analogy with BCS-Bogoliubov theory of superconductivity

Superconductivity:

1. Free electrons
2. Phonon interaction
3. Energy gap
4. Collective excitation
5. Charge
6. Gauge symmetry (global)

Elementary particles:

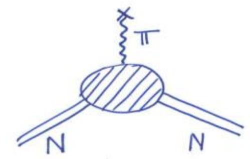
1. Bare fermion (nucleon)
2. Some unknown interaction
3. Observed mass (nucleon)
4. Meson (bound nucleon pair)
5. Chirality
6. Chiral symmetry

Chiral Symmetry in Elmentary Particle Physics (pre-Quark Model)

- Elementary particles (lightest ones): pions, neutrons and protons
- Assuming equal neutron, proton mass: SU(2) isotopic-spin symmetry…CVC
- Assuming neutron, proton mass = 0: additional chiral SU(2) symmetry…PCAC

Application of the superconductor model of elementary particles

- Goldberger-Treiman relations
- Processes involving soft pions



Chirality conservation is allowed in the sense: change in fermion chirality (~ helicity × velocity) is compensated by emission of a pion at (nearly) zero energy.

$$g_A = \frac{f_\pi}{m_N} g_{NN\pi}$$

Soft Pions:

$e + N \to N + e + \pi$

$\nu + N \to N + e + \pi$

$N + \pi \to N + \pi$ or $N + 2\pi$

NJL model and QCD

- A scenario to obtain a NJL type of model involving quarks rather than nucleons was proposed in the early 1980s, combining the large N limit and the mass gap in the pure gluon sector, which acts as a short distance cut-off.
- This model reproduces all known results of current algebra including (and most importantly) current algebra anomalies and current algebra solitons.
- The first works in this direction were done at TIFR (1984):

A. Dhar, S. Gupta, R. Shankar and SRW

Spontaneous Symmetry Breaking in String Theory

- This question is a tough one in string theory but there is recent progress using the AdS/CFT correspondence.

- T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD" Prog.Theor. Phys.113, 843 (2005), hep-th/0412141.
- O. Bergmann, S. Seki and J. Sonnenschein, "Quark mass and condensate in HQCD", JHEP 0712 (2007) 037, arXiv:0708.2839.
- A. Dhar and P. Nag, "Sakai-Sugimoto model, Tachyon Condensation and Chiral symmetry Breaking", JHEP 0801 (2008) 055, arXiv:0708.3233.
- K. Hashimoto, T. Hirayama, F. Lin and H. Yee, "Quark Mass Deformation of Holographic Massless QCD", arXiv:0803.4192.
- O. Aharony and D. Kutasov, "Holographic Duals of Long Open Strings", arXiv:0803.3547.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