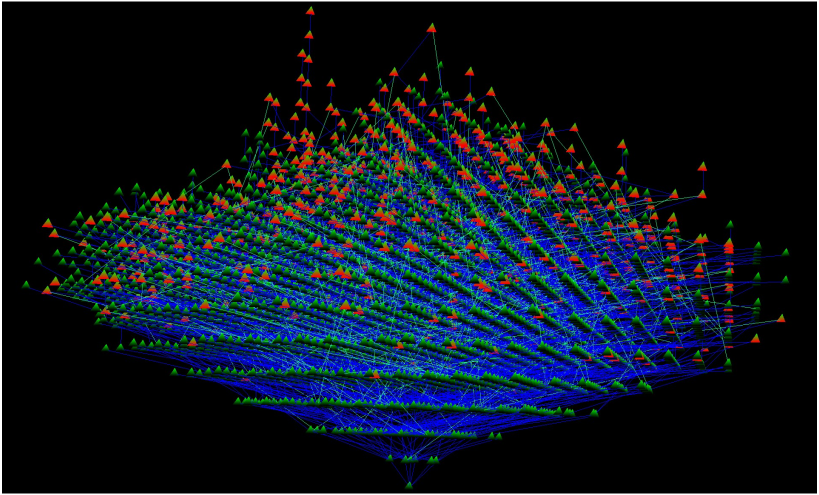

Gravity from Order and Number: Causal Sets

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work with

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references

PI mini-course: http://pirsa.org/10100038 – 10100042

Joe Henson's "invitation to invitation": http://pirsa.org/10090092

Fay Dowker's public lecture to PI (soon to be available at PIRSA)
http://www.perimeterinstitute.ca/Outreach/Public_Lectures/Public_Lectures/

I. What is QG? ("whole lot")

Among the various ideas put forward in the search for a theory of quantum gravity, the causal set hypothesis is distinguished by its logical simplicity and by the fact that it incorporates the assumption of an underlying spacetime discreteness organically and from the very beginning.

To put the programme in context, let me list some of the questions commonly asked in connection with quantum gravity. Causal sets represent a particular type of answer or have something interesting to say on almost all of them.

What is the deep structure of spacetime? (kinematics)

continuous? discrete? nothing new? other?

(what kind of discreteness? crystalline, fluid)

how does this structure produce a continuum with metric? (cf. Λ puzzle)

why is there a notion of length at all?

(is the metric a field like any other?)

What are the corresponding "deep laws of motion"?

in what sense need they be quantal?

what form should they take?

Schrödinger eq? constraint equations? path integral form?

other form, eg 3rd order interference?

(causet answer: a *histories-based:* soh of Schwinger-Kel'dysh type)

These are dynamical questions. But will QG unify kinematics with dynamics?

Can we stick with the "Copenhagen interp" with its external observers, or do we need a better — more *intrinsic* — grasp of microscopic physics? Are Quantum Gravity and Quantum Foundations intertwined?

Related theoretical questions

How to describe non-gravitational matter (fermions, gauge-fields)?

Can it arise spontaneously or must it be put in by hand?

Is topology-change part of the theory? (eg to describe birth of cosmos, geon pairs)

What becomes of unitarity and Relativistic Causality in QG?

What are the "observables" of QG ("problem of time")

Is nature "static" or "changing"?

Phenomenological questions

Why is Λ so small without being zero? (same as continuum problem!)

Role of nonlocality? (radical nonlocality)

Why $d = 4$?

Why isn't spacetime full of holes?

What is BH entropy counting? (horizon molecules?)

And why the generalized second law?

Does discreteness affect Hawking radiation?

Are there observable signatures of discreteness?

- Lorentz breaking?
- New effects like swerves?

Initial conditions for cosmos?

- why cosmos would have been so big and homogeneous at Planck time?

II. Status of the causet programme

kinematics: good understanding of

- dimension
- length
- homology
- horizon area/entropy (both equilibrium *and* dynamical)
- "problem of time" is solved

dynamics *in* a background causet

- classical (retarded) wave equation
- QFT for ϕ

dynamics *of* causet

- a classical dynamics of "sequential growth"
- a "bilocal" action-functional S
  - in 2D we recover flat spacetime purely entropically
  - exploratory Monte Carlo simulations are being done for 2D, 4D

phenomenology

predicted $\Lambda \sim 10^{-120}$ (batting 1000 (500?), 4 neutrinos??)

toy model of Tolman-Boltzmann "cyclic" cosmology in CSG

model of swerves, model of propagation of "scalar light" along links

effect of discreteness on particle propagation ($m = 0$, $m > 0$)

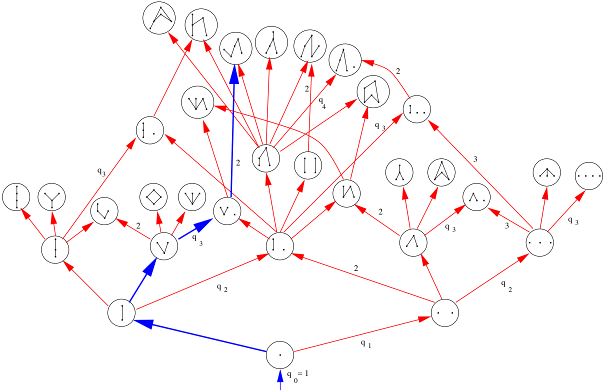
(bounds on phenomenological parameters)

two proposals for black hole entropy, one based on the new action-functional S

high frequency transparency

The recent progress on dynamics (in causet and of causet) stems from attempt to devise classical and quantum theories for a scalar field. First we need some definitions.

III. What is a causet?



The poset of finite causal sets

define link

How does spacetime emerge?

geometry = order + number ($N = V$)

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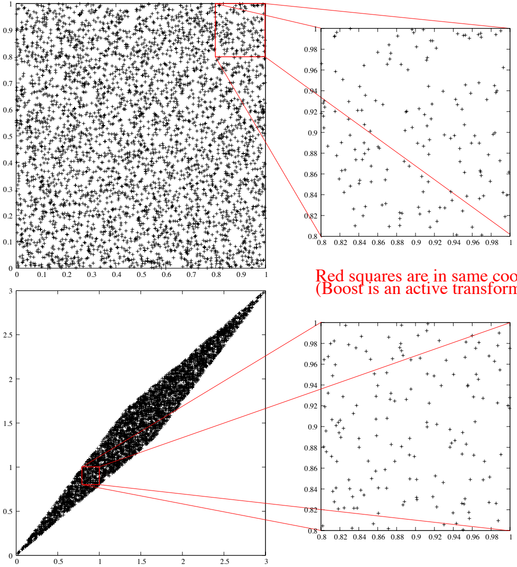
Meaning of $M \approx C$ is not as obvious as for fluid $(- + ++)$

Role of sprinkling to obtain $N = V$ (Poisson)

Sprinklings respect Lorentz symmetry (strictly)

Theorem If f is a measurable equivariant partially defined mapping from Poisson sprinklings of $\mathbb{M}^d$ to unit timelike vectors, then $\mathrm{dom}(f)$ has measure zero.

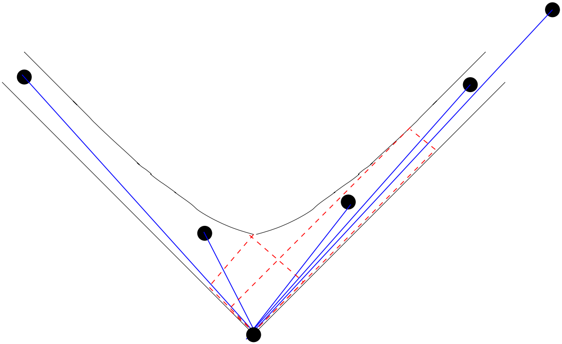
In other words the probability that a sprinkling will break Lorentz symmetry is zero



Red squares are in same coordinate location.
(Boost is an active transformation.)

Observe the **radical nonlocality**: numerous “nearest neighbors” (links)

This is why it’s so hard to write down something like a local wave equation.



IV. The wave-equation and the causet action

Two roads to causet dynamics: principled and opportunistic
The first has given us CSG but so far not QSG. So try the second.

Fix a causet $C = \{e_1, e_2, e_3, \ldots\}$ that embeds faithfully in some region of $\mathbb{M}^d$ via $e_j \to x_j$, and let $\phi : \mathbb{M}^d \to \mathbb{R}$.
We seek a matrix $\Box_{jk}$ such that $\Box_{jk}\phi_k \leftrightarrow (\Box\phi)(x_j)$.

The following works when $d = 2$ (overall scale-factor omitted):

$$\Box_{jj} = -1/2 \,,$$

$$\Box_{jk} = f(n_{kj}) \text{ when } e_k \prec e_j, \text{ where}$$

$$n_{kj} = \mathrm{card}\{e_\ell | e_k \prec e_\ell \prec e_j\} \text{ and}$$

$$f(n) = 1, \ -2, \ 1, \ 0, \ 0, \ 0 \ \cdots$$ [proof complete]

The same form works when $d = 4$ but with different coefficients:

$$\Box_{jj} = -1 \,,$$

$$f(n) = 1, \ -9, \ 16, \ -8, \ 0, \ 0 \ \cdots$$ [proof almost complete]

A 1-parameter generalization is also available if needed ($0 < \varepsilon \leq 1$):

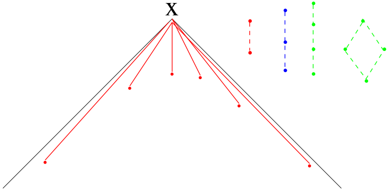
$$f(n) = \varepsilon(1-\varepsilon)^n \left[1 - 9\frac{n\varepsilon}{(1-\varepsilon)} + 8\frac{n(n-1)\varepsilon^2}{(1-\varepsilon)^2} - \frac{4n(n-1)(n-2)\varepsilon^3}{3(1-\varepsilon)^3} \right]$$

In curved spacetime our "D'Alembertian" becomes (even dim) $\Box - \frac{1}{2}R$ (Benincasa)
Applying it to $\phi \equiv -1$ (and $\varepsilon = 1$), we obtain

$$\frac{1}{2}\int R dV \leftrightarrow -\sum_{jk} f(n_{jk})$$

Just a weighted sum over a finite number of "building blocks",
eg the diamond order-interval has weight $f(2) = 16$ for $d = 4$.

(There even seems to be a kind of Gauss-Bonnet result for $d = 2$.)



What is the physical significance of S_{causet}?

Even in continuum no one really knows how S_{grav} is to be used — not even formally!

Nevertheless, we can attempt Monte Carlo

(Many questions arise: What to “measure”? To Wick rotate the causet itself makes no sense. We can nevertheless rotate S (each term has a definite sign). Might *quantum* montecarlo handle complex amplitudes better?)

Does S also signify an entropy somehow: Fay’s “mutual information”!

V. Quantum field theory à la causet

Recall that the continuum theory uses

$$\text{both} \quad \Box\phi = 0 \qquad \textbf{and} \qquad [\hat{\phi}(x), \hat{\phi}(y)] = i\Delta(x, y)$$

where $\Delta(x, y) = G(x, y) - G(y, x)$ and $G = G^{retarded}$.

These are compatible since $\Box\Delta = 0$

On the causet they are not compatible, so use the commutator only (Noldus)

Can the commutator alone yield the full algebra of operators (and the "vacuum state")?

For a Gaussian (Wickian) field, everything follows from $W(x, y) = \langle\hat{\phi}(x)\hat{\phi}(y)\rangle$

(we assume also $\langle\hat{\phi}\rangle = 0$)

The antisymmetric part of W is just $i\Delta/2$

Choose the symmetric part to make $W \geq 0$ (Johnston):

$$2W = i\Delta + \sqrt{-\Delta^2}$$

We did this without ever being asked to define positive frequency!

Byproduct is a unique “ground state” for any compact region of spacetime

All these steps can be repeated in the causet once we have $G^{retarded}_{jk}$.

In $d = 2$ we know G in full generality, for $m^2 = 0$.

In flat $d = 4$ and $d = 2$ we know G for all m^2.

In general we might try inverting the matrix $\Box_{jk}$

Lots more to say: decoherence functional, interactions, classical limit, “causality” . . . . . .

Byproduct (workshop talk): new formula for entropy direct from $W(x, y)$

Now we can also define entanglement entropy of black hole à la causet