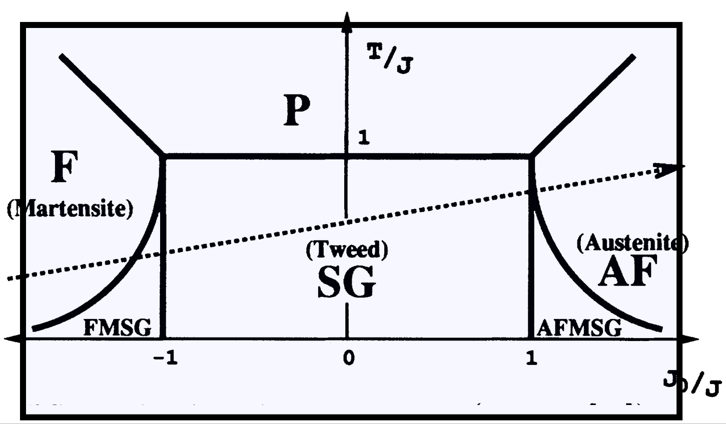
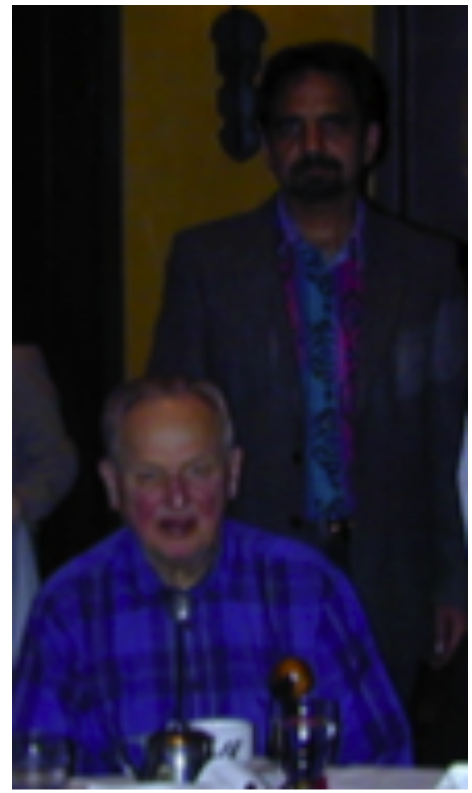
Heterogeneity and Phase Transformations in Ferroelastics: From JAK to Now.

Turab Lookman , Theoretical Division

JAK with G. Barsch: Nonlinear Physics of Materials

Free energy (C-T), FePd, 1D solutions '84;
Decaying fringing fields in austenite '87;
Tweed as spin glass' 91;
Shuffle order parameter '94;
Strain compatibility, tweed ' 95 ;



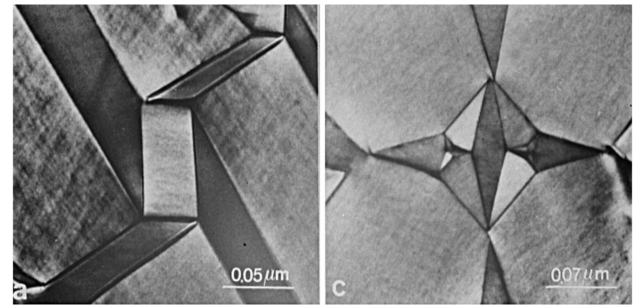


S. Kartha et al, PRL 67, 3630 (1991).

OUTLINE

- **Linear vs. Nonlinear Elasticity in Strain**

  (.. and rotation)



$Pb_3(VO_3)_2$

- **Glass Martensite: Continuum to Discrete Models**

  (MC, Mean Field Theory, RG)

  : Transition from Martensite to Glass

Acknowledgements

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: M. Porta (LANL), X. Ding (Xian), S.R. Shenoy (Trivadarum), A. Saxena (LANL)

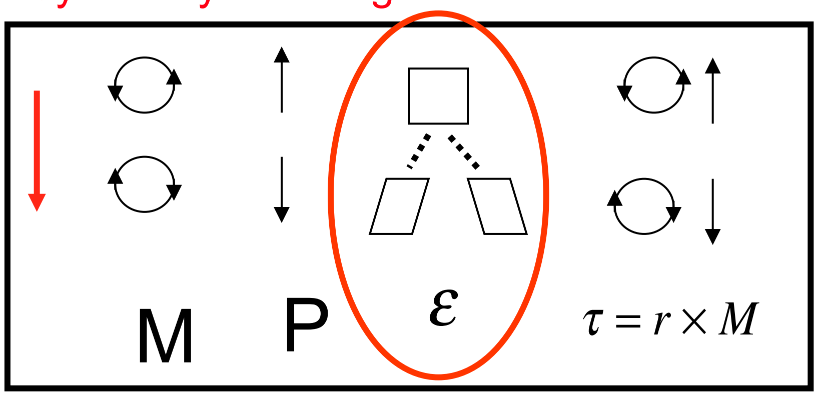
: D. Xue, Z. Zhang, X. Ren (NIMS)

Ferroic: "crystal with two or more distinct orientation states and switching between them by external fields" **(Aizu, 69)**

Multiferroic: two or more coexisting ferroic properties in the same phase

Ferromagnetic
Ferroelectric
Ferroelastic
Ferrotoroidal, ...

Symmetry breaking: Para → Ferroic



Ferroelastics

- Elastic heterogeneities: habit planes, twin boundaries :polydomains (aggregates)

Issues: orientations, specific microstructure,....
(Wechsler-Liebermann-Read, Bowles McKenzie, Eshelby, Sapriel, Roytburd, Khachaturyan,......)

- Example: Cubic to Tetragonal

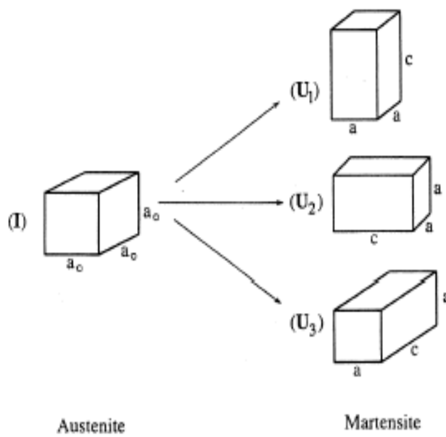
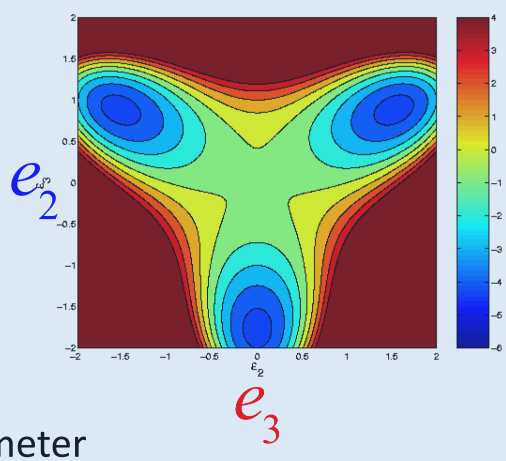


Figure 4.4: The three variants of martensite in a cubic to tetragonal transformation.

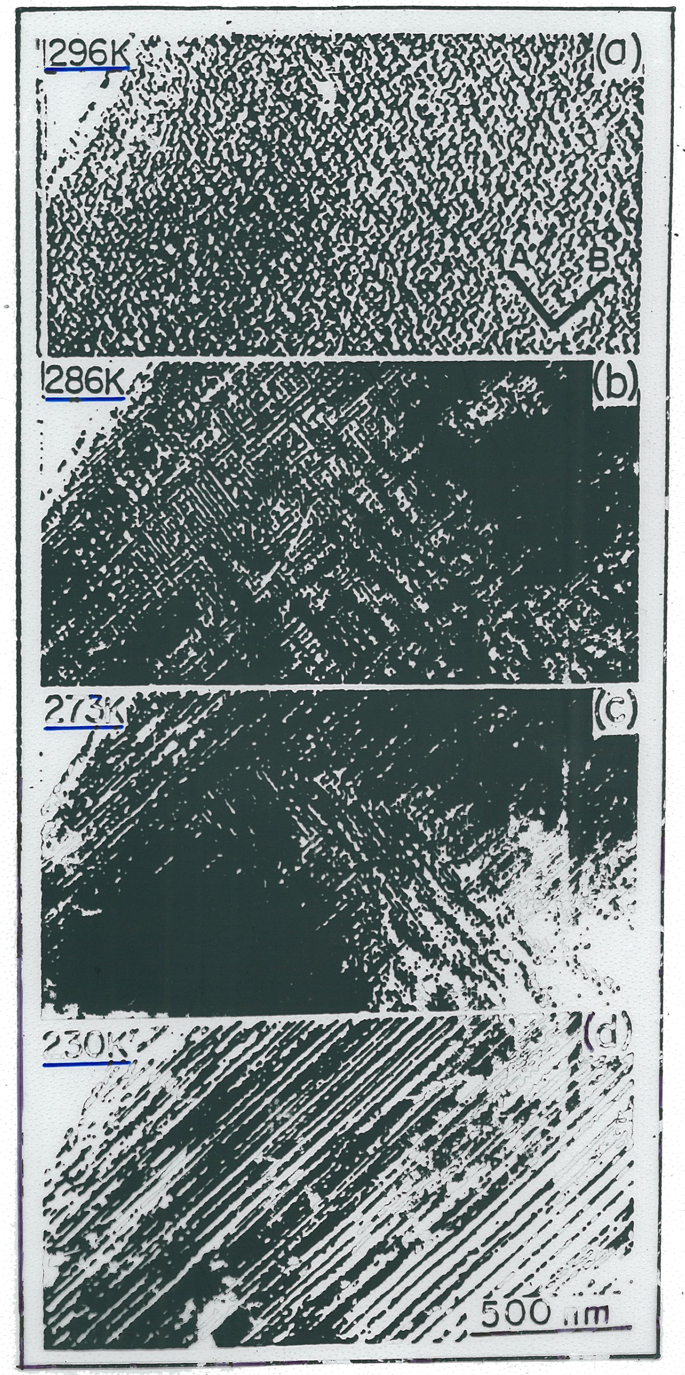
$F(e_2, e_3)$



Order parameter

$$\left(e_2 = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sqrt{2}}, e_3 = \frac{\varepsilon_{xx} + \varepsilon_{yy} - 2\varepsilon_{zz}}{\sqrt{6}} \right)$$

Barsch &Krumhansl, 84, Falk, 91



$Fe_{0.7}Pd_{0.3}$

AUSTENITE

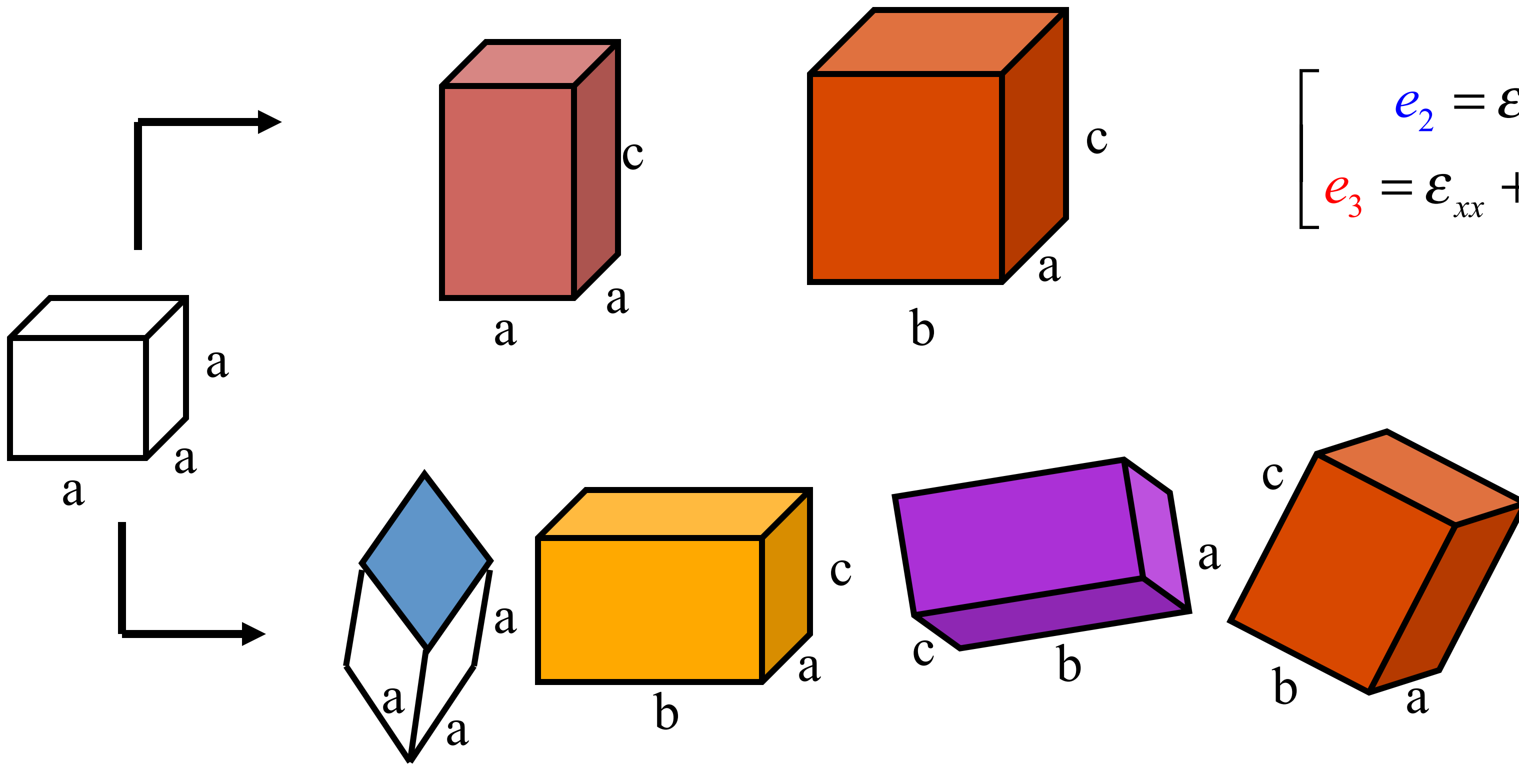
TWEED

$T_0 = 2.68$ K

(TWINNED) MARTENSITE

Sugiyama (1985)

SYMMETRY ADAPTED STRAINS FOR CUBIC SYSTEM



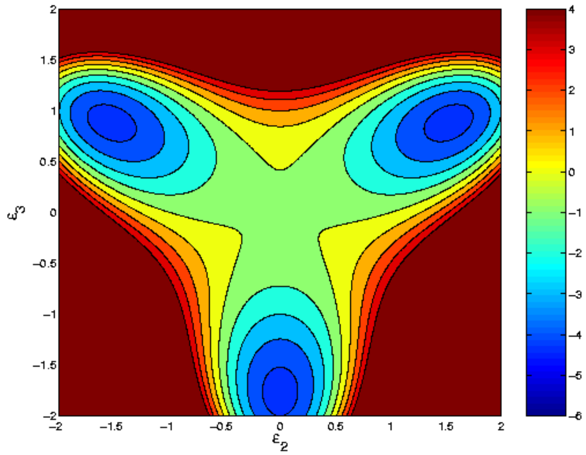
$$\begin{bmatrix} e_2 = \varepsilon_{xx} - \varepsilon_{yy} \\ e_3 = \varepsilon_{xx} + \varepsilon_{yy} - 2\varepsilon_{zz} \end{bmatrix}$$

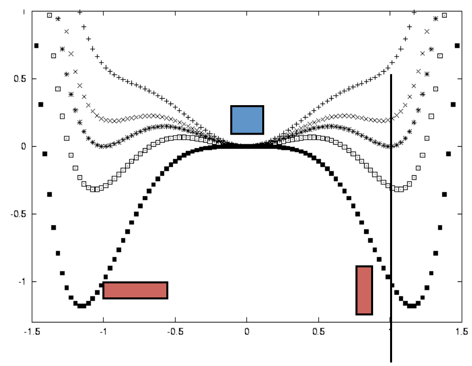
$$\begin{bmatrix} e_4 = \varepsilon_{xy} \\ e_5 = \varepsilon_{xz} \\ e_6 = \varepsilon_{yz} \end{bmatrix}$$

$e_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$

	e_1	e_2, e_3	e_4, e_5, e_6
3[111]	e_1	$-\frac{1}{2}e_2 + \frac{\sqrt{3}}{2}e_3, -\frac{\sqrt{3}}{2}e_2 - \frac{1}{2}e_3$	e_6, e_4, e_5
4[001]	e_1	$-e_2, e_3$	$-e_4, e_6, -e_5$
2[110]	e_1	$-e_2, e_3$	$e_4, -e_6, -e_5$

Free energy for Cubic → Tetragonal transition





Order Parameter

$$\left(e_2 = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sqrt{2}}, e_3 = \frac{\varepsilon_{xx} + \varepsilon_{yy} - 2\varepsilon_{zz}}{\sqrt{6}} \right)$$

$$e_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}{\sqrt{3}}$$

Barsch &Krumhansl'84

$$F(e_2, e_3) = \mathbf{A_o}(T - T_0)(e_2^{\;2} + e_3^{\;2}) + \mathbf{B}e_3(e_3^{\;2} - 3e_2^{\;2}) + \mathbf{C}(e_2^{\;2} + e_3^{\;2})^2$$

← Transformation

$$+\mathbf{E}e_1(e_2^{\;2} + e_3^{\;2})$$

← Volume change

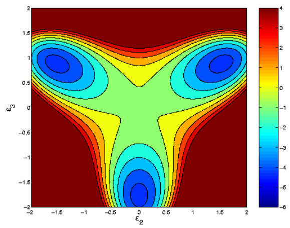
$$+\frac{\mathbf{g}}{2}\left(\left|\nabla e_2\right|^2 + \left|\nabla e_3\right|^2\right)$$

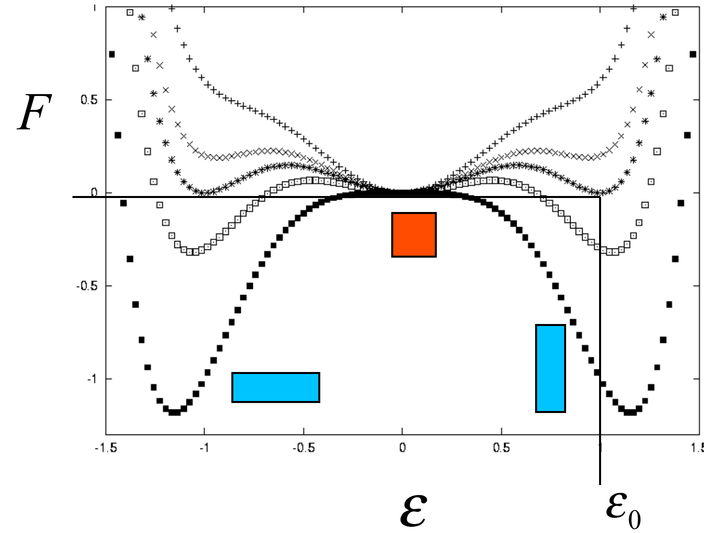
← Nonlocal strain inhomogeneity

$$+\frac{A_B}{2}e_1^{\;2} + \frac{A_S}{2}(\varepsilon_{xy}^{\;2} + \varepsilon_{xz}^{\;2} + \varepsilon_{yz}^{\;2})$$

← Compression-shear

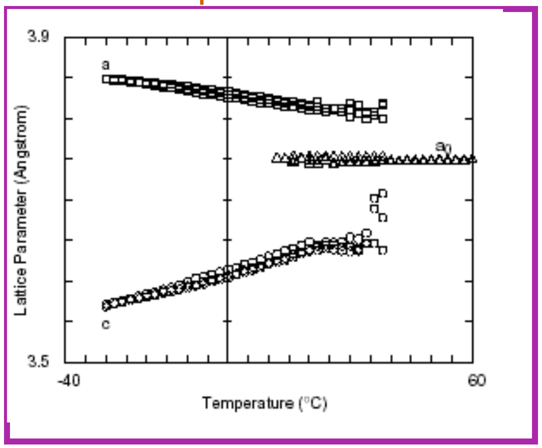
Parameters from experiment





Experiment: DSC, Ultrasonic
X-ray Scattering
Neutron Scattering

- Lattice parameters



Cui,Shield, James, 2004

$$\varepsilon_{xx} = \varepsilon_{yy} = \frac{a_t - a_o}{a_o} = .0176 \qquad \varepsilon_{zz} = \frac{a_c - a_o}{a_o} = -.0335$$

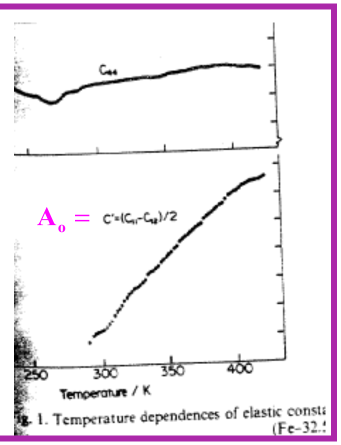
$$e_1 = -\frac{\mathbf{E}}{A_B}(e_2{}^2 + e_3{}^2)$$

$$e_2 = 0$$

$$e_3 = \varepsilon_0$$

$$e_1 = \frac{2\varepsilon_{xx} + \varepsilon_{zz}}{\sqrt{3}}$$

- Elastic constants



1. Temperature dependences of elastic consta (Fe-32.

Sato, 91

$$\mathbf{A_o} = \frac{\mathbf{B}^2}{4\left(\mathbf{C} - \frac{\mathbf{E}^2}{A_B}\right)} \qquad \varepsilon_0 = -\frac{\mathbf{B}}{\left(\mathbf{C} - \frac{\mathbf{E}^2}{A_B}\right)}$$

ENERGY MINIMIZATION: Solve for displacement fields

Monte Carlo: u_x, u_y Kartha et al., 95

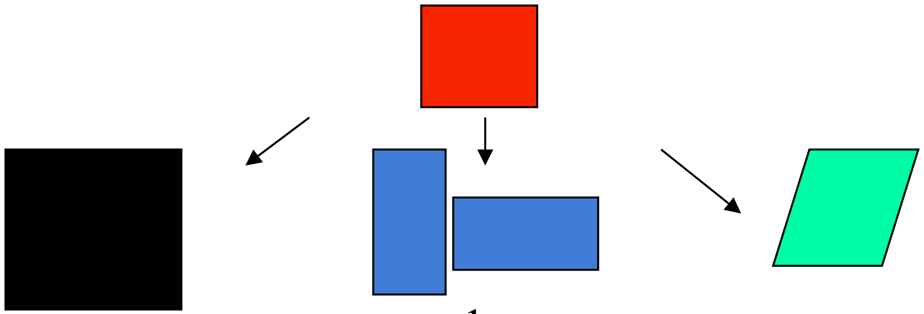
Relaxation, steepest descent: u_x, u_y Jacobs, 85, 92

Newton's equations: u_x, u_y, u_z Ahluwalia et al., 06

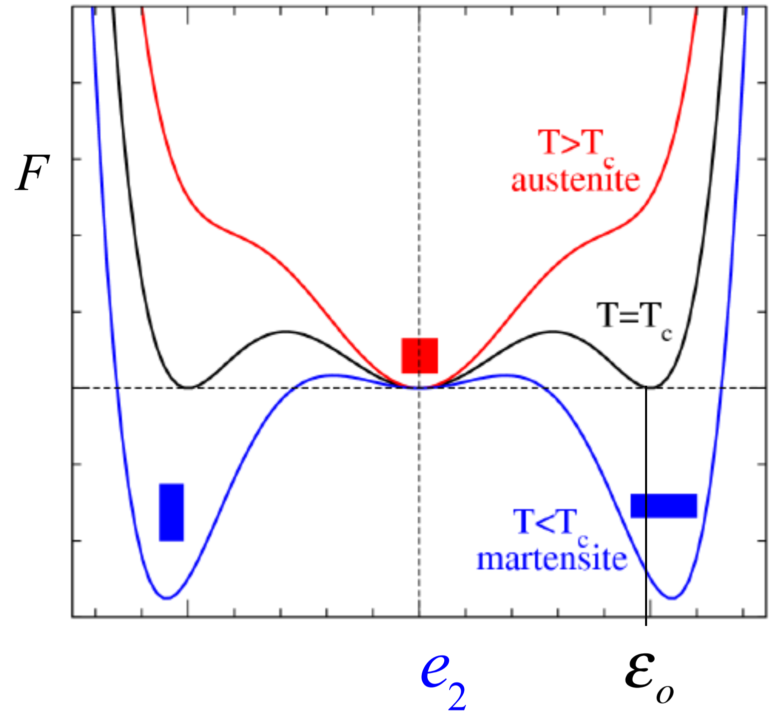
$$\rho \ddot{\vec{u}} = \vec{\nabla} \cdot \overleftrightarrow{\sigma}_{elastic} + \vec{\nabla} \cdot \overleftrightarrow{\sigma}_{dissipative}$$

$$\sigma_{ij}^{elastic} = \frac{\delta F}{\delta \varepsilon_{ij}} \qquad \sigma_{ij}^{dissipative} = \frac{\delta R}{\delta \dot{\varepsilon}_{ij}} \qquad R = \int d\vec{r} \mathrm{A}_1' \dot{\varepsilon}_{ij} \dot{\varepsilon}_{kl}$$

2D: Square → Rectangle



$e_1 = \frac{1}{\sqrt{2}}(\varepsilon_{xx} + \varepsilon_{yy})$ $e_2 = \frac{1}{\sqrt{2}}(\varepsilon_{xx} - \varepsilon_{yy})$ $e_3 = \varepsilon_{xy}$



$$f_{struct}(e_2) = \frac{a(T - T_0)}{2} e_2^{\ 2} - \frac{B}{4} e_2^{\ 4} + \frac{C}{6} e_2^{\ 6}$$

$$f_{grad} = \frac{g}{2} |\nabla e_2|^2$$

$$f_{compress-shear} = \frac{A_1}{2} e_1^{\ 2} + \frac{A_3}{2} e_3^{\ 2}$$

$$f_{load} = -\sigma\varepsilon_{xx} = -\frac{\sigma}{\sqrt{2}}(e_1 + e_2)$$

$$F = \int d\vec{r}\,[f_{struct} + f_{grad} + f_{compress-shear} + f_{load}]$$

Energy minimization using strains

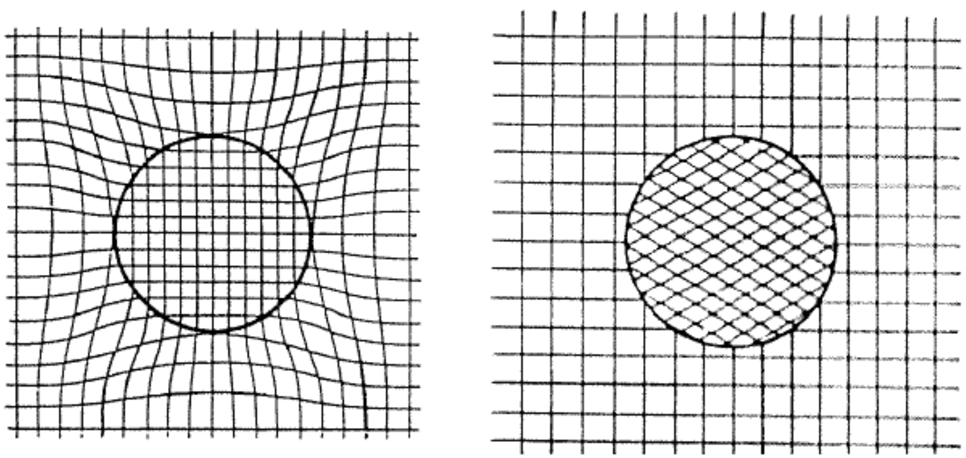
Conditions on strains for u to be single valued, continuous ?

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$G = \vec{\nabla} \times (\vec{\nabla} \times \overleftrightarrow{\varepsilon})^T = 0$$

St. Venant, 1864

(a) coherent (b) incoherent



- **continuity at interfaces—no dislocations**
- **compatibility influences microstructure**

Problem:

$$\delta(F + \lambda G) = 0$$

$$F = \int d\vec{r}\,[f_{struct} + f_{grad} + f_{compress-shear} + f_{load}]$$

$$\frac{\partial^2 u_x}{\partial x \partial y} = \frac{\partial^2 u_x}{\partial y \partial x},$$

$$\frac{\partial^2 u_y}{\partial x \partial y} = \frac{\partial^2 u_y}{\partial y \partial x},$$

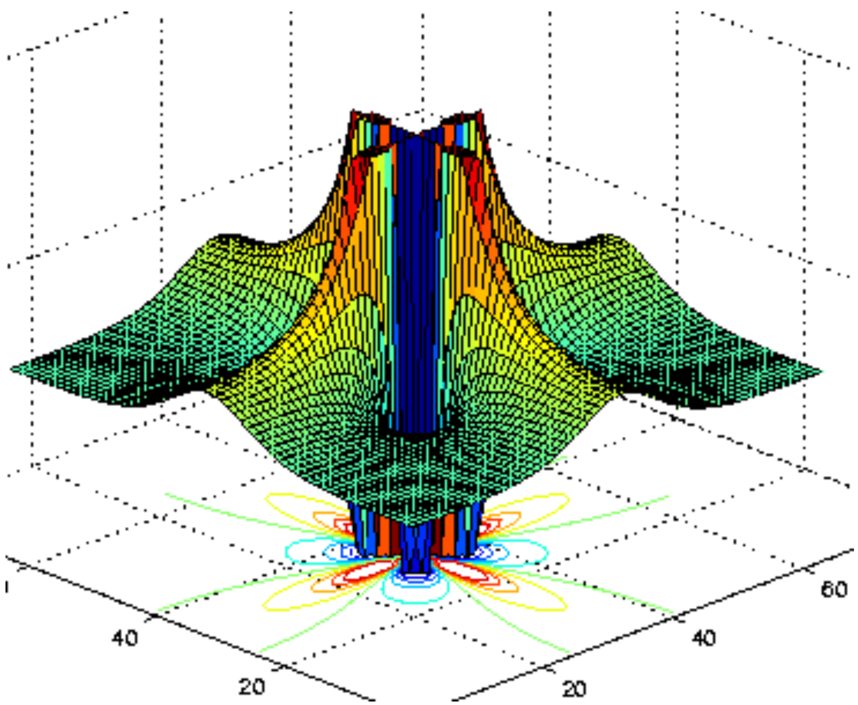
$$G = \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} - 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = 0$$

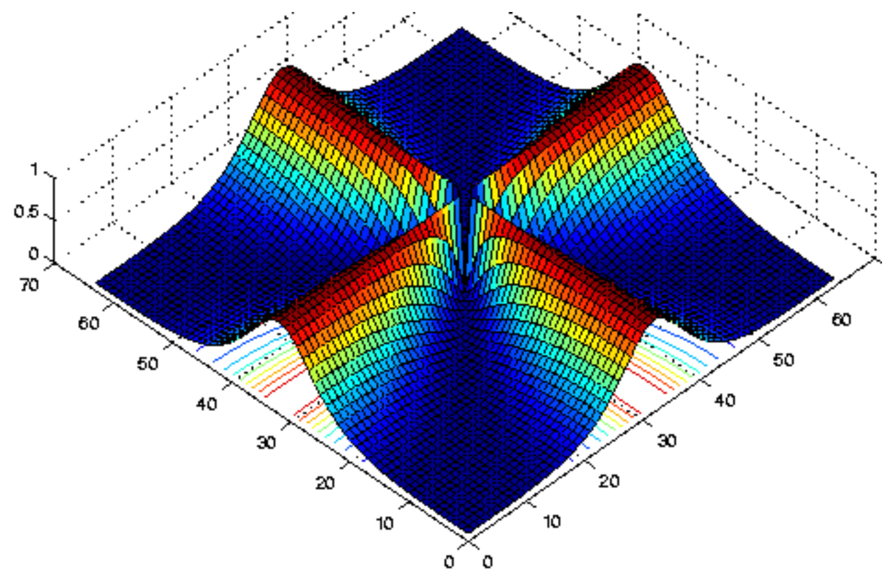
ANISOTROPIC LONG-RANGE INTERACTION

Elastic interaction

$$f_{\substack{compress-\\shear}} = \frac{A_1}{2} e_1^{\ 2} + \frac{A_3}{2} e_3^{\ 2} \quad \xrightarrow{\delta(f+\lambda G)=0} \quad \sum_{r,r'} \varepsilon_2(r) U(|r-r'|) \varepsilon_2(r')$$

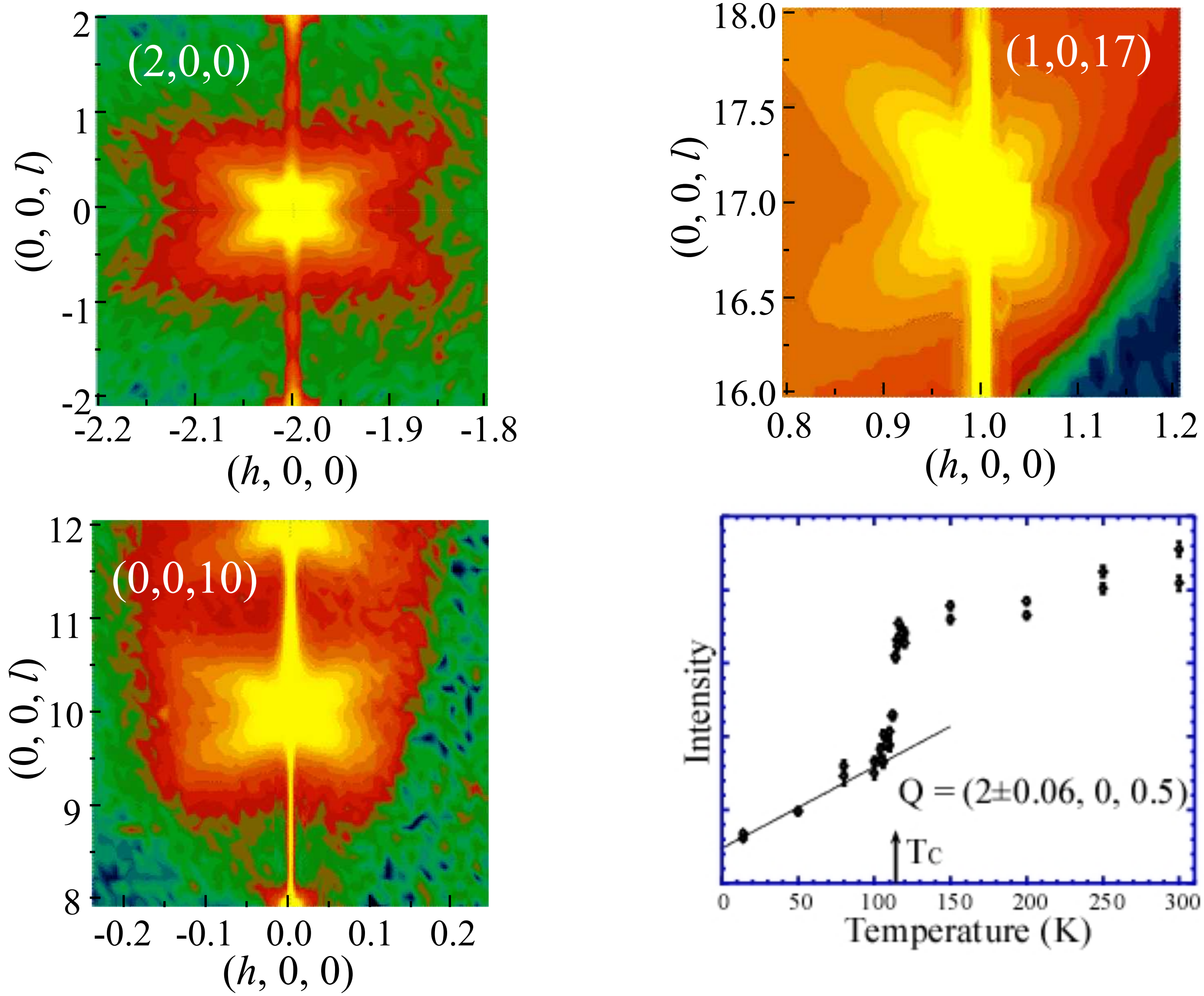
$$U(r) \sim \frac{\cos 4\theta}{r^2} \qquad U(k) = \frac{(k_x^{\ 2} - k_y^{\ 2})^2}{k^4 + \frac{A_1}{A_3} k_x^{\ 2} k_y^{\ 2}}$$





Kartha et al. PRB’ 95

Elastic Signatures in Polarons via single-crystal diffuse scattering



L. Vasiliu-Doloc *et al.*, PRL 83, 4393 (1999).

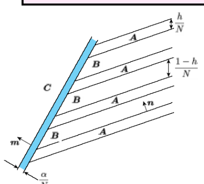
COMPARISON OF APPROACHES

$W[\phi];\phi(\nabla y(x_1,x_2))$

Ball and James' 87

$\{y^n\}$ s.t.

$\lim_{n\to\infty} W[\phi]=0$



$W\left(e_2,e_3,Q_1,Q_2\right);e_2,e_3,Q_1,Q_2$

$|\nabla e_2|^2,|\nabla Q_1|^2$

$$\rho\frac{\partial^2 e}{\partial t^2}=\frac{1}{2}\nabla^2\left[A_2'\frac{\partial e}{\partial t}+\frac{\delta W}{\delta e}\right]$$

$$\sigma(r)=C^o e(r)+\tau(r)$$
$$\nabla.\sigma=0$$

Hadamard jump

Compatibility

$\nabla\times\nabla\times\bar{\varepsilon}=0$

$\nabla\times\bar{F}=0$

STRAIN HETEROGENEITY

Eshelby inclusion

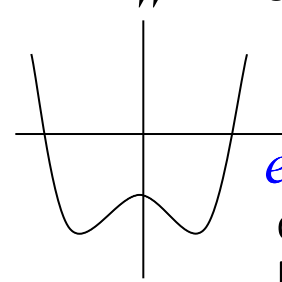
$W[\eta];\eta_1,\eta_2,\ldots.$

Khatchatcuryan '83

$|\nabla\eta|^2$

$$\frac{\partial\eta}{\partial t}=A'\frac{\delta W}{\delta\eta}$$

HOMOGENEOUS



COMPARISON OF APPROACHES

$W[\phi];\phi(\nabla y(x_1,x_2))$

Ball and James' 87

$\{y^n\}$ s.t.

$\lim_{n\to\infty} W[\phi]=0$

Hadamard jump

Compatibility

$\nabla\times\nabla\times\bar{\varepsilon}=\eta$

$\nabla\times\bar{F}=\alpha$

STRAIN HETEROGENEITY

Eshelby inclusion

$W\left(e_2,e_3,Q_1,Q_2\right);e_2,e_3,Q_1,Q_2$

$|\nabla e_2|^2,|\nabla Q_1|^2$

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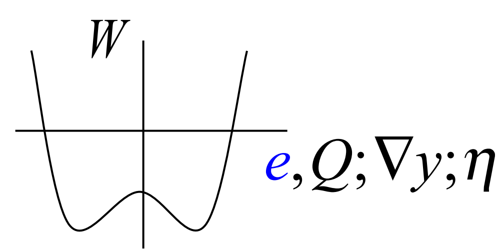
$W[\eta];\eta_1,\eta_2,....$

Khatchatcuryan' 83

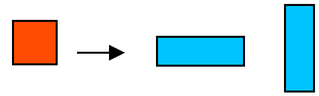
$|\nabla\eta|^2$

$$\frac{\partial\eta}{\partial t}=A'\frac{\delta W}{\delta\eta}$$

HOMOGENEOUS

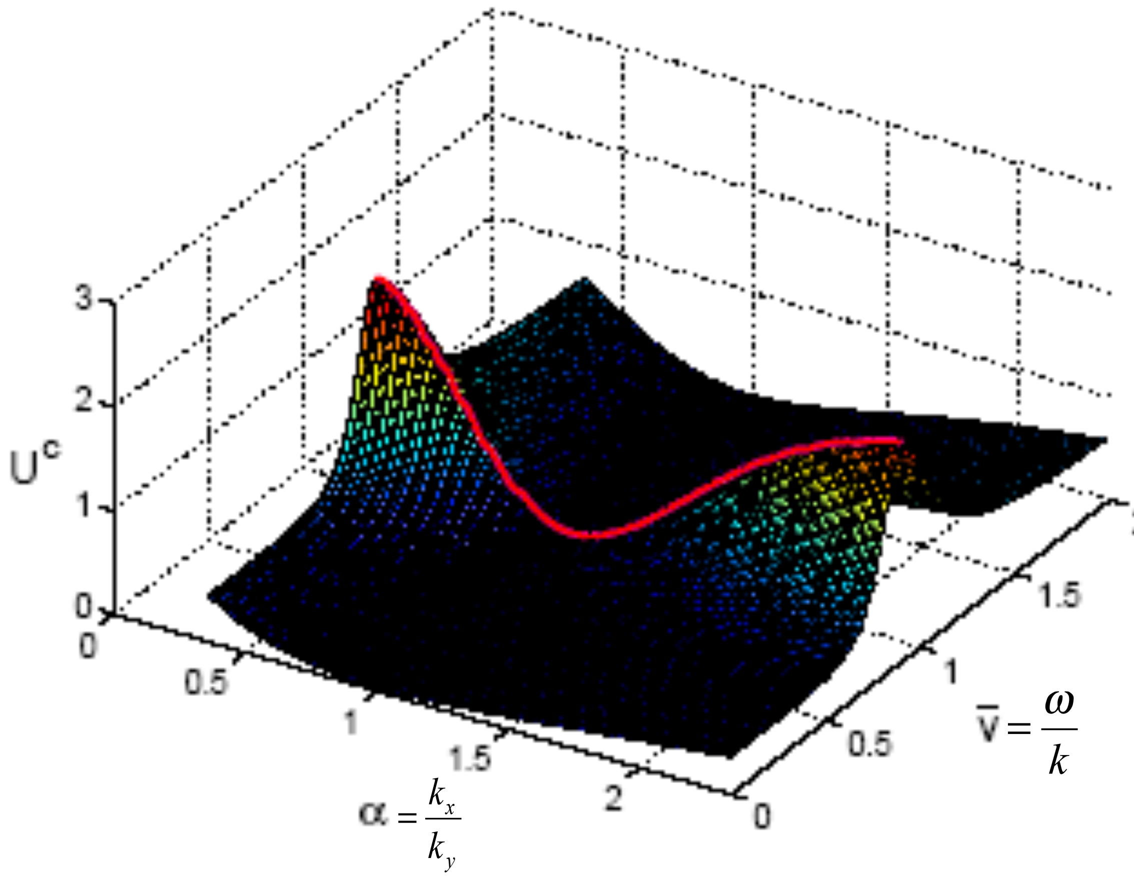


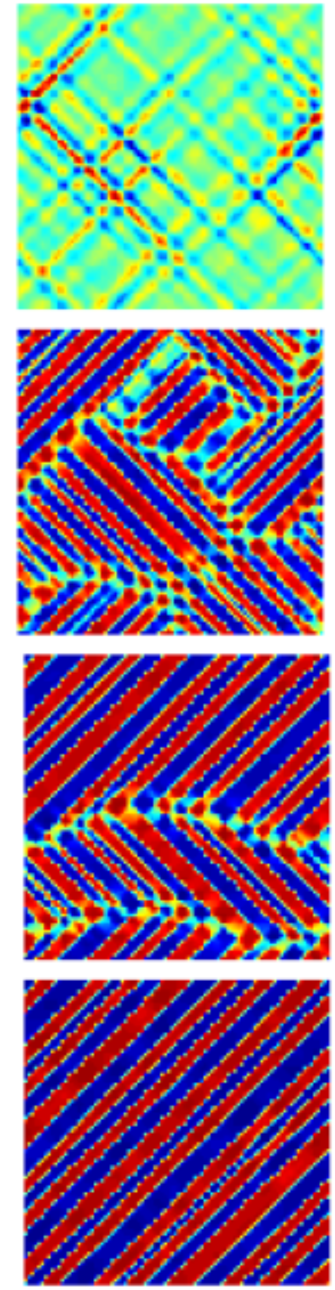
Repulsive potential drives interfaces



$$\rho\,\ddot{\varepsilon}_2 = \frac{1}{2}\nabla^2\left[\frac{a_{1\omega}(k_x^2-k_y^2)^2\varepsilon_2}{[k^4+b_{1\omega}8k_x^2k_y^2]} + A_2'\dot{\varepsilon}_2 + \frac{\delta F(\varepsilon_2)}{\delta\varepsilon_2}\right] \quad b_{1\omega} = \frac{a_{1\omega}}{[a_3 - i\omega a'_3 - \rho(2\omega/k)^2]}$$

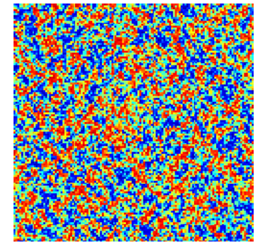
$$k = 2\sqrt{\rho/a_3}\,\omega$$





Solve for equilibrium microstructure using compatibility of strains

$$\sigma(x) = C(x) : \varepsilon(x)$$



Moulinec and Suquet, 94, 98
'Response of Elastically Heterogeneous Microstructure'

$$\delta C = C(x) - C^0(x)$$

$$\sigma(x) = C^0(x) : \varepsilon(x) + \delta C : \varepsilon(x)$$

$$\tau(x) = \delta C : \varepsilon(x)$$

$$\sigma(x) = C^0(x) : \varepsilon(x) + \tau(x)$$

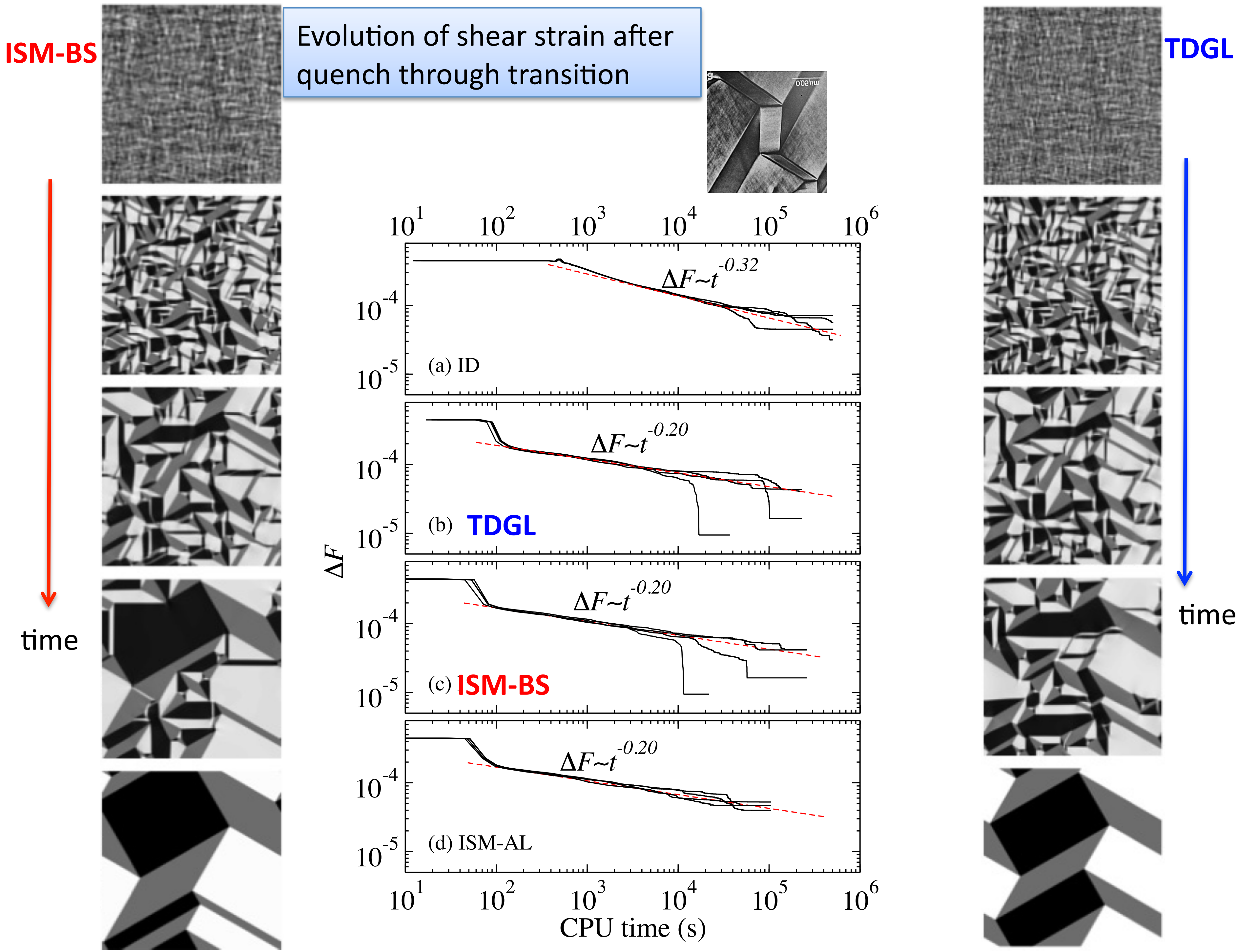
Linear problem

$$\nabla \cdot \sigma = 0$$

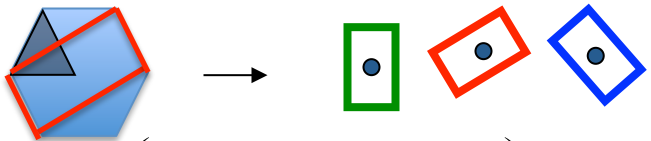
$$\varepsilon(u(x)) = \Gamma^0(x) * \tau(x)$$

$$\varepsilon(k) = \Gamma^0(k)\tau(k)$$

Iteratively solve for $\sigma(x), \varepsilon(x)$

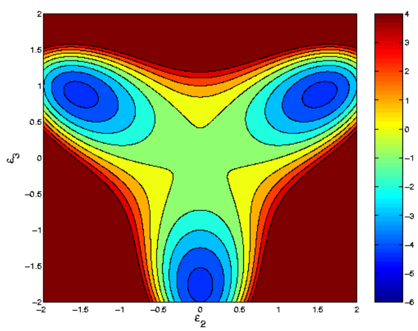

Evolution of shear strain after quench through transition
ISM-BS
time
TDGL
time
ΔF~t^-0.32
(a) ID
ΔF~t^-0.20
(b) TDGL
ΔF~t^-0.20
(c) ISM-BS
ΔF~t^-0.20
(d) ISM-AL
ΔF
10^-4
10^-5
10^1 10^2 10^3 10^4 10^5 10^6
CPU time (s)

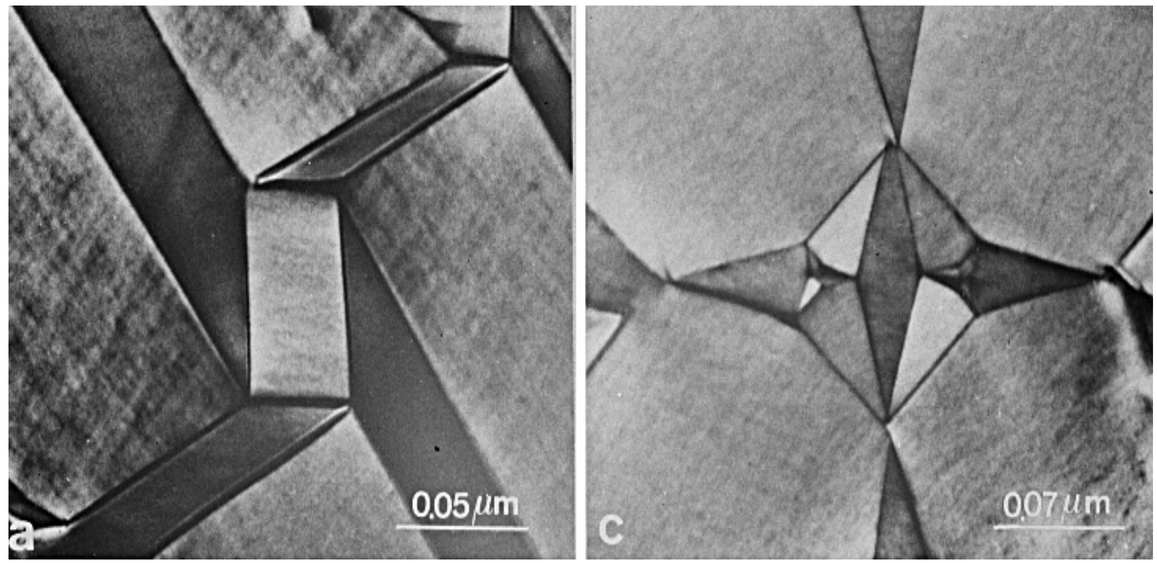
Hexagonal → **Orthorhombic** Lead Orthovanadate $Pb_3(VO_3)_2$



$$\left(e_2 = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sqrt{2}}, e_3 = \varepsilon_{xy} \right)$$

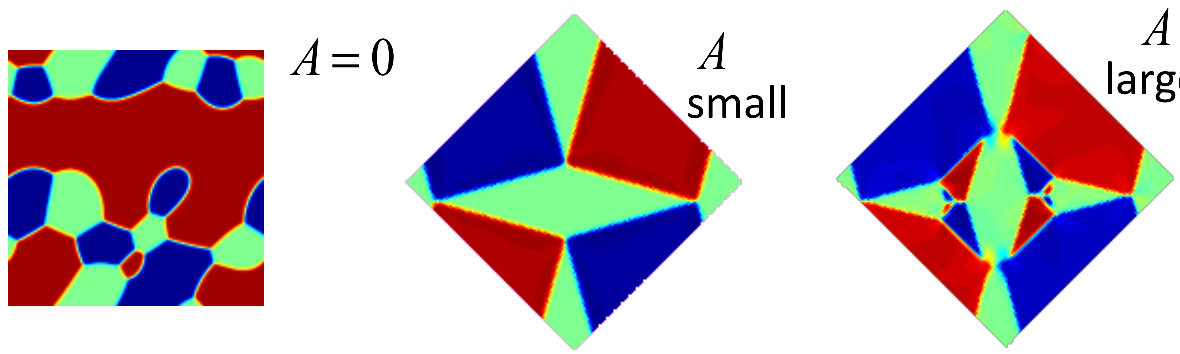
$$e_1 = \frac{1}{\sqrt{2}}(\varepsilon_{xx} + \varepsilon_{yy})$$





$$F = Ae_1^2 + T(e_2^2 + e_3^2) - 1/3(e_2^3 - 3e_2e_3^2) + 1/4(e_2^2 + e_3^2)^2$$

$$\hat{e}_1 = \frac{k_x^2 - k_y^2}{k_x^2 + k_y^2}\,\hat{e}_2 + \frac{2k_xk_y}{k_x^2 + k_y^2}\,\hat{e}_3$$

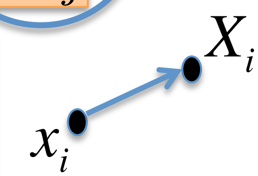


GEOMETRICAL NONLINEARITY: linear strains ⟶ displacement gradients

$$F_{ij} = \frac{\partial x_i}{\partial X_j} = \frac{\partial\left(x_i - X_i + X_i\right)}{\partial X_j} = \frac{\partial\left(u_i + X_i\right)}{\partial X_j} = \delta_{ij} + \frac{\partial u_i}{\partial X_j}$$

G-L strains

$$\epsilon_{ij} = \frac{1}{2}\left(F^T F - I\right) = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \sum_k \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right)$$



Displacement gradients ⟺ GL strains

$$D_{ij} \equiv \frac{\partial u_i}{\partial X_j}$$

$$d_1 = \frac{1}{2}(D_{xx} + D_{yy})$$

$$d_2 = \frac{1}{2}(D_{xx} - D_{yy})$$

$$d_3 = \frac{1}{2}(D_{xy} + D_{yx})$$

$$d_4 = \frac{1}{2}(D_{xy} - D_{yx})$$

$$\mathrm{e}_1 = d_1 + \frac{1}{2}\left[(d_1)^2 + (d_2)^2 + (d_3)^2 + (d_4)^2\right]$$

$$\mathrm{e}_2 = d_2 + d_1 d_2 - d_3 d_4,$$

$$\mathrm{e}_3 = d_3 + d_1 d_3 + d_2 d_4$$

Compatibility:

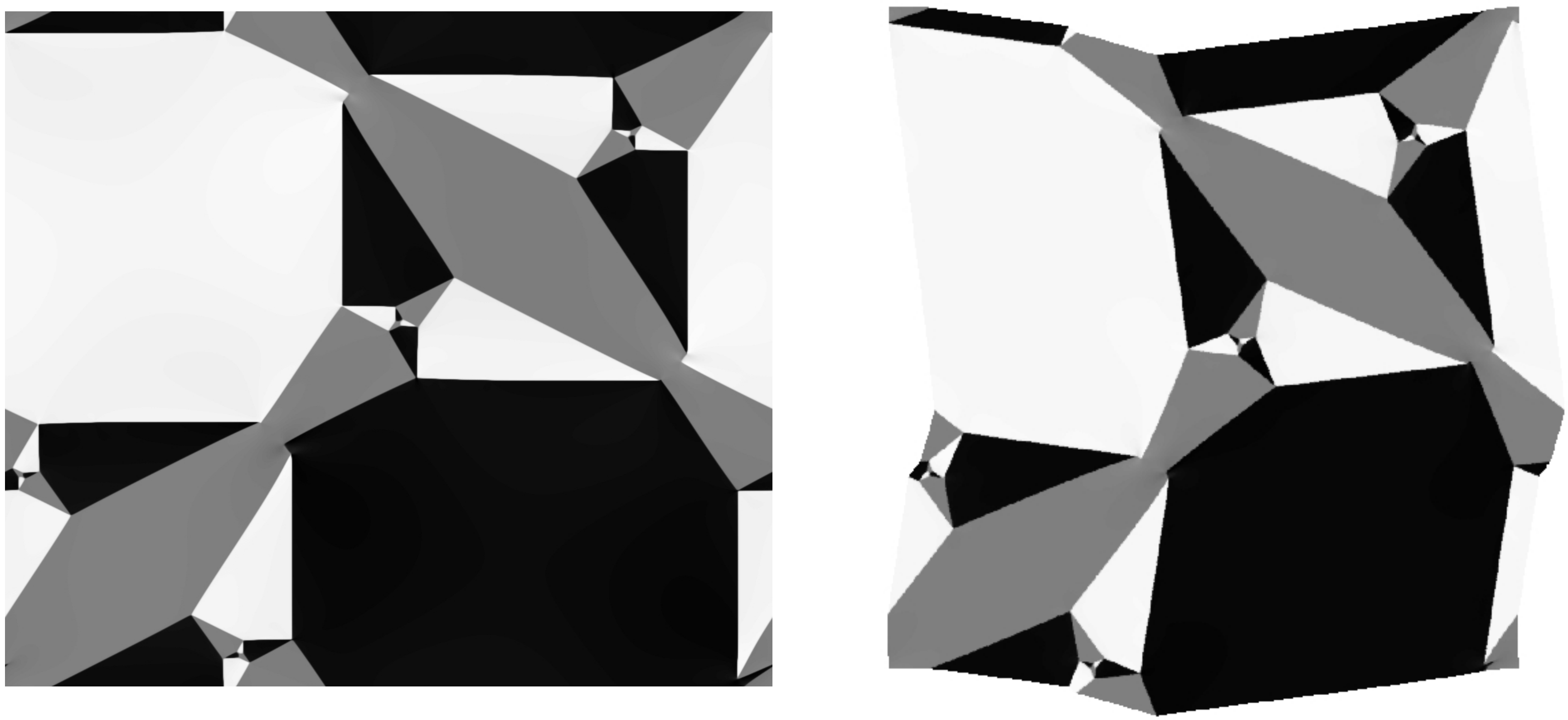
$$\frac{\partial D_{xy}}{\partial X} = \frac{\partial D_{xx}}{\partial Y}, \qquad \frac{\partial D_{yy}}{\partial X} = \frac{\partial D_{yx}}{\partial Y},$$

⟹

$$\hat{d}_1 = \frac{k_x^2 - k_y^2}{k_x^2 + k_y^2}\hat{d}_2 + \frac{2k_x k_y}{k_x^2 + k_y^2}\hat{d}_3,$$

$$\hat{d}_4 = \frac{2k_x k_y}{k_x^2 + k_y^2}\hat{d}_2 - \frac{k_x^2 - k_y^2}{k_x^2 + k_y^2}\hat{d}_3.$$

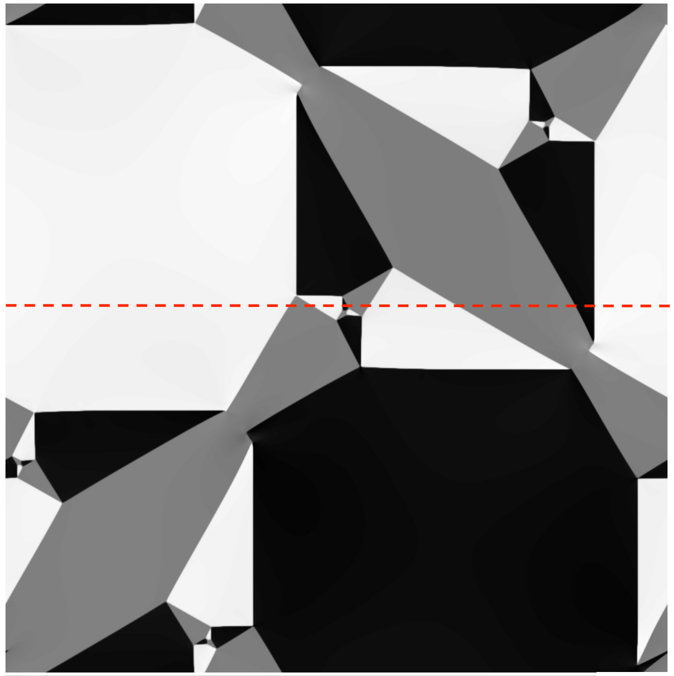
Steady State shear strain after quench through transition

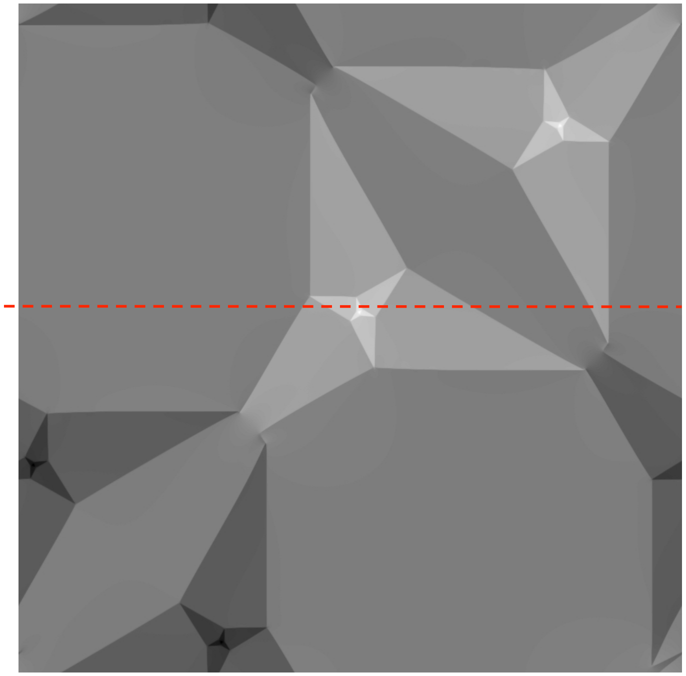


Reference **Current**

$$e_3 = 1/2(\varepsilon_{xy} + \varepsilon_{yx})$$

Shear Strain, e_3

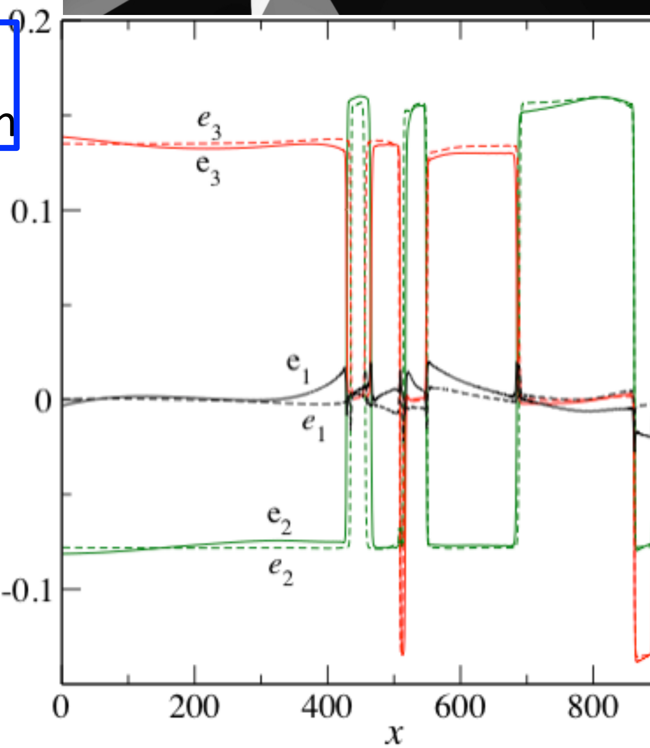


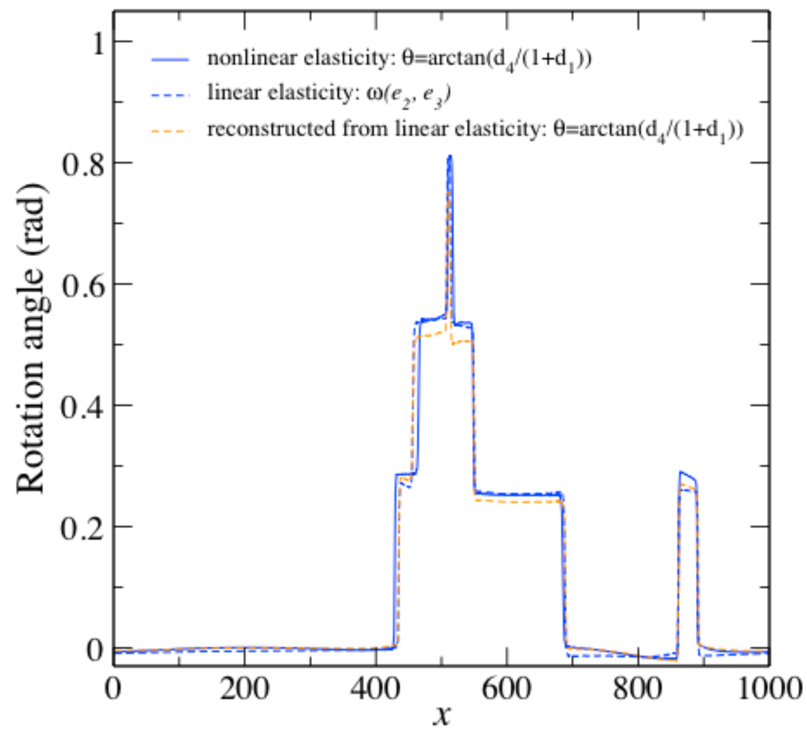


Orientation

e : G-L strain
e : Linear strain

Strains





$$e_1 \equiv \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}),$$
$$e_2 \equiv \frac{1}{2}(\varepsilon_{xx} - \varepsilon_{yy}),$$
$$e_3 \equiv \frac{1}{2}(\varepsilon_{xy} + \varepsilon_{yx}),$$

Rotations Follow Strains

Finite deformation

$$\nabla \times \bar{F} = 0$$

$$g(\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3) = 0$$

$$\nabla^2 \psi = h(\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3)$$

$$\psi = \tan^{-1} \frac{d_4}{1 + d_1}$$

Small deformation

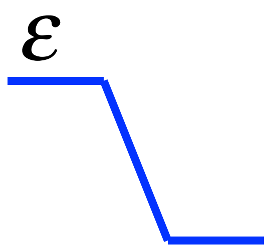
$$\nabla \times \nabla \times \bar{\varepsilon} = 0$$

$$\nabla^2 \psi = 2 \frac{\partial^2}{\partial x \partial y} \mathrm{e}_2 - \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \mathrm{e}_3$$

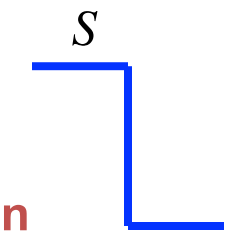
Coarsegraining:

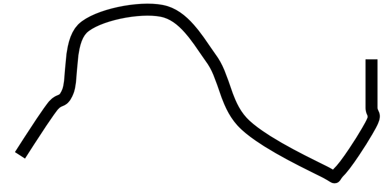
Landau Continuum model

$F(\varepsilon)$



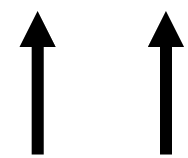
skeletal approximation





Discrete (pseudo spin model)

$$H = \sum S_i S_j$$

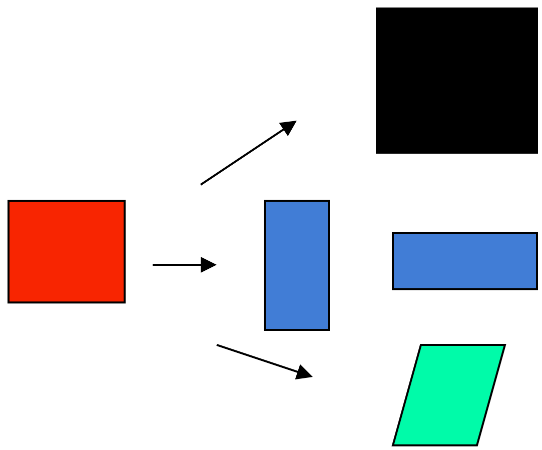


$$S_i = 0, \pm 1$$

(Mean Field Theory, Renormalization Group or Monte Carlo)

Microstructure, Glassy behavior,….

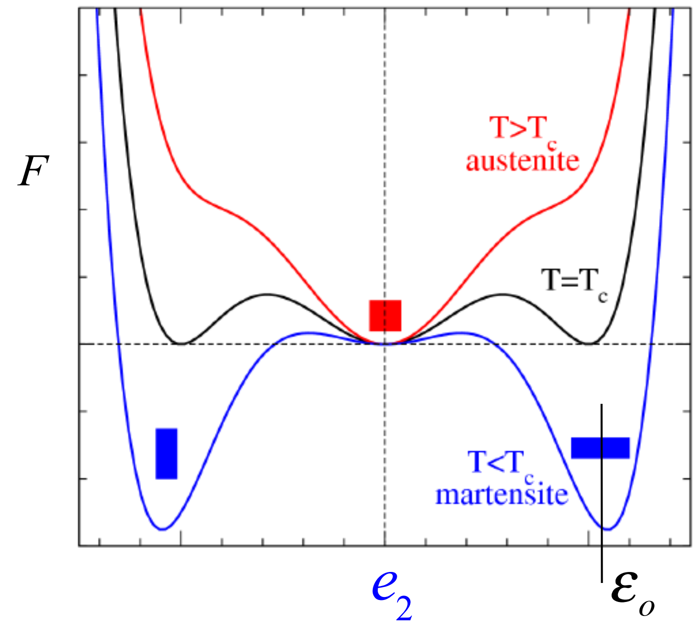
2D: Square → Rectangle



$$e_1 = \frac{1}{\sqrt{2}}(\varepsilon_{xx} + \varepsilon_{yy})$$

$$e_2 = \frac{1}{\sqrt{2}}(\varepsilon_{xx} - \varepsilon_{yy})$$

$$e_3 = \varepsilon_{xy}$$



$$f_{struct}\left(e_2\right) = \frac{a(T-T_0)}{2} e_2{}^2 - \frac{B}{4} e_2{}^4 + \frac{C}{6} e_2{}^6$$

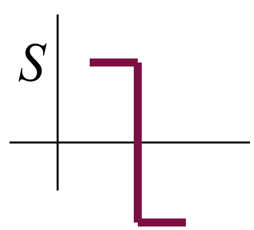
$$f_{grad} = \frac{g}{2}\left|\nabla e_2\right|^2$$

$$f_{compress-shear} = \frac{A_1}{2} e_1^2 + \frac{A_3}{2} e_3^2$$

$$\rightarrow e_2(r) \frac{\cos 4\theta}{r^2} e_2(r')$$

$$f_{load} = -\sigma e_2$$

$$e_2 = \varepsilon_o S$$



$$S_i = \pm 1, 0$$

$$S_i^6 = S_i^4 = S_i^2$$

$$\sum \left[(\varepsilon_o^2 - 1)^2 + (\tau - 1) \right] S_i^2$$

$$\varepsilon_o^2 \left[-\sum_{<ij>} S_i S_j + 2 \sum_i S_i^2 \right]$$

$$\sum_{<ij>} S_i \frac{\cos 4\theta}{r^2} S_j$$

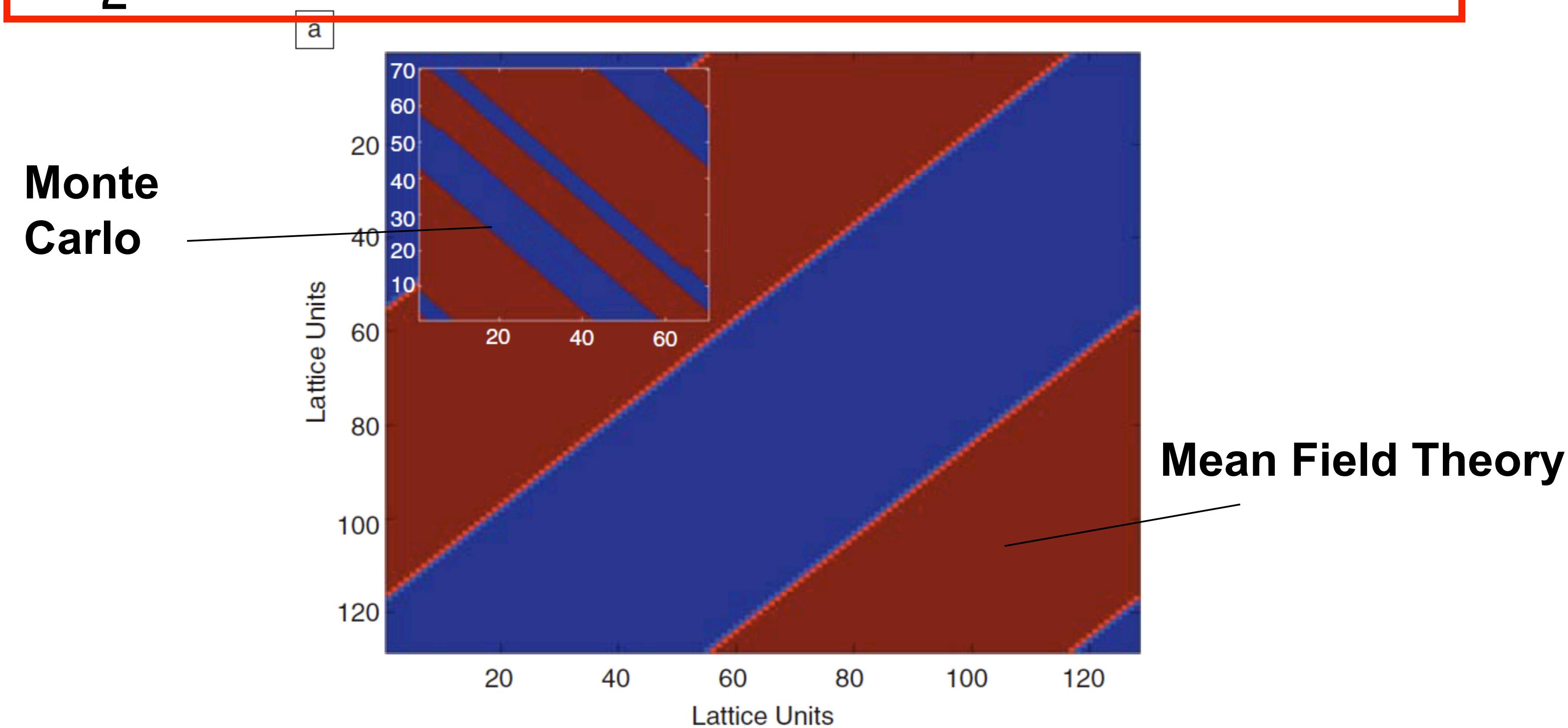
$$-\sum_i h_i S_i$$

Spin 1 (Blume-Capel) with long-range interactions

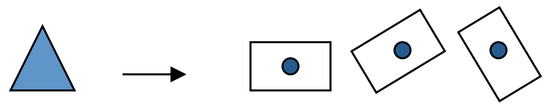
$$H = -J(\tau)\sum_{\langle ij \rangle} S_i S_j + J'(\tau)\sum_{i} S_i^2 + \sum_{i,j} S_i G(|i-j|) S_j$$

$$J' = \varepsilon_o^2[g + (\varepsilon_o^2 - 1)^2 + (\tau - 1)] \qquad S_i = 0, \pm 1$$

$$J = \frac{g}{2}\varepsilon_o^2$$

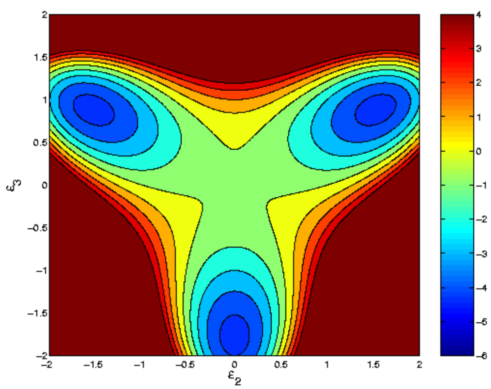


Hexagonal → Orthorhombic

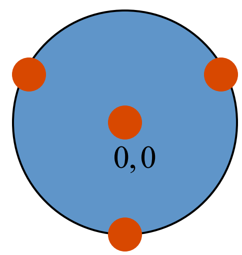


2D: Triangle → Centered rectangle

$$\left(e_2 = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sqrt{2}}, e_3 = \varepsilon_{xy} \right)$$



$-\frac{1}{2}, \frac{\sqrt{3}}{2}$



$-\frac{1}{2}, -\frac{\sqrt{3}}{2}$

$1, 0$

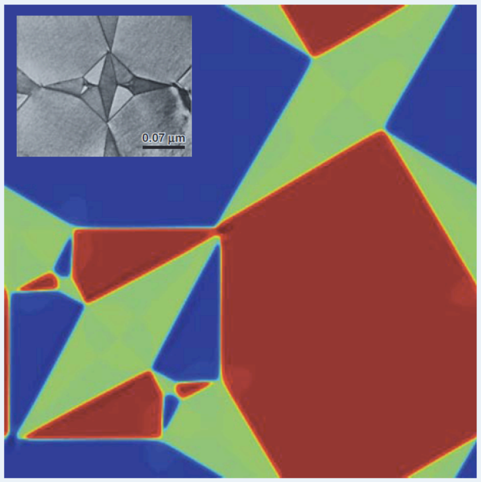
$$(\cos\frac{2\pi n}{3}, \sin\frac{2\pi n}{3})$$

(3+1) clock 0 model

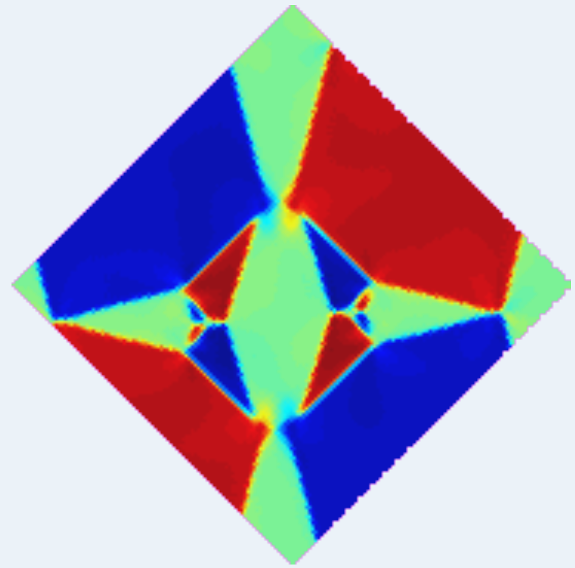
$n = 1, 2, 3$ and 0

Other structural transitions:

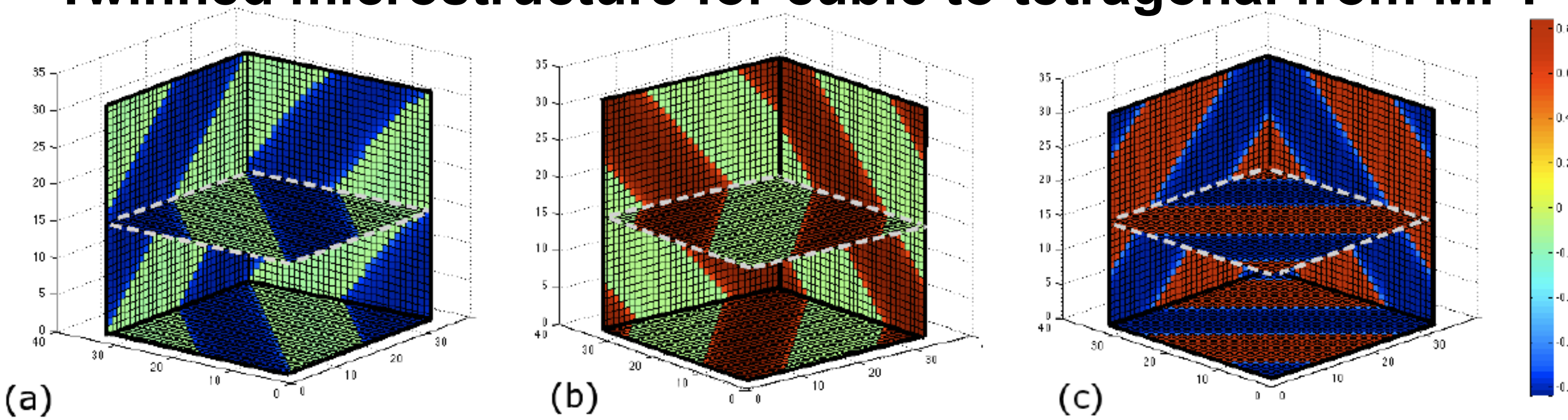
Spin model using MFT



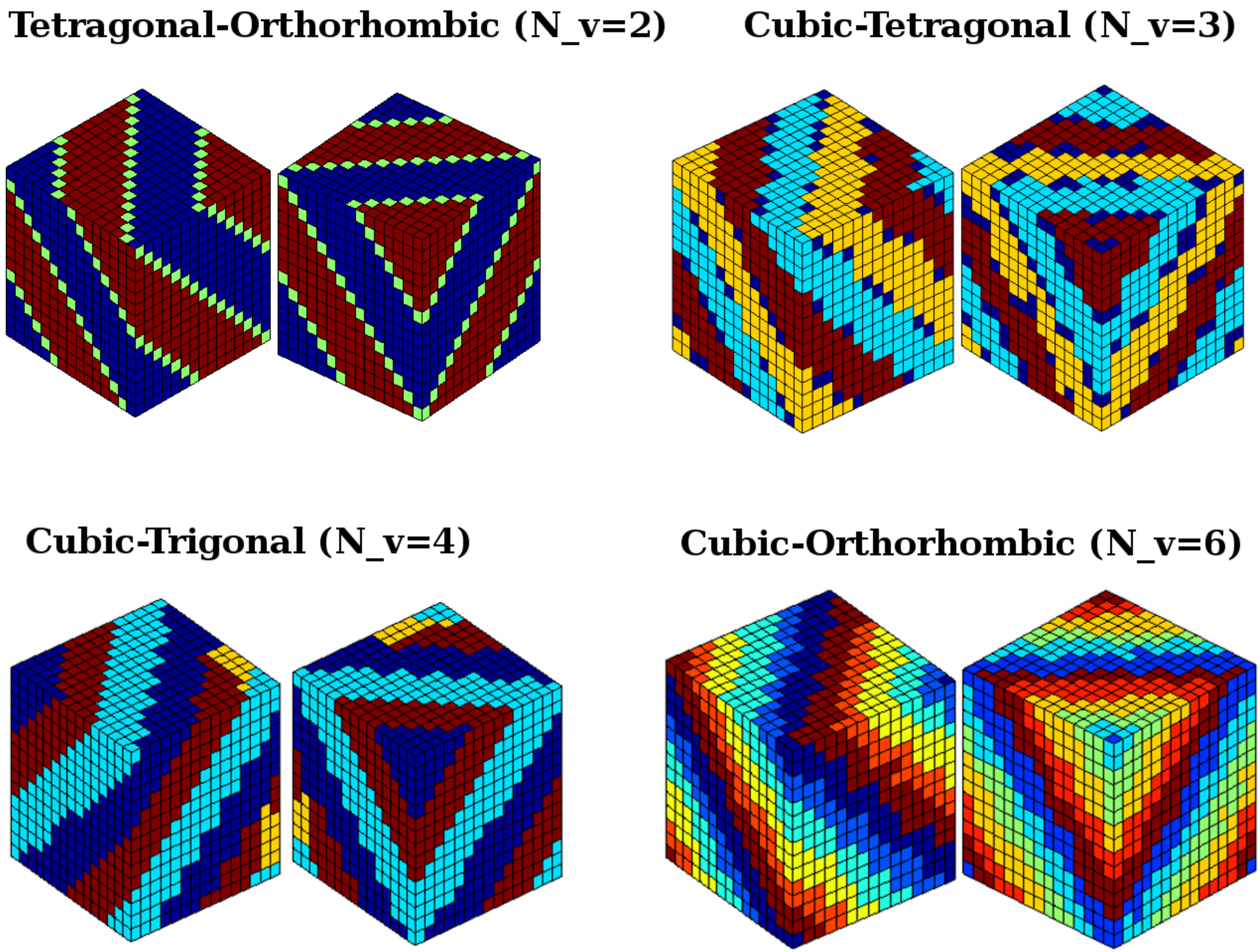
Continuum Landau theory



Twinned microstructure for cubic to tetragonal from MFT



Vasseur et al. (2010)


Tetragonal-Orthorhombic (N_v=2)
Cubic-Tetragonal (N_v=3)
Cubic-Trigonal (N_v=4)
Cubic-Orthorhombic (N_v=6)

Effects of disorder:

$$H = -\sum_{\langle i,j \rangle} J_{ij}(\tau) S_i S_j + \sum_i \Delta_i(\tau) S_i^2$$

$$\mathcal{P}(J_{ij}) = \frac{1}{\sqrt{2\pi}\sigma_J} \exp\left\{ -\frac{[J_{ij} - J_0(\tau)]^2}{2\sigma_J^2} \right\}$$

$$\mathcal{P}(\Delta_i) = \frac{1}{\sqrt{2\pi}\sigma_\Delta} \exp\left\{ -\frac{[\Delta_i - \Delta_0(\tau)]^2}{2\sigma_\Delta^2} \right\}$$

\+ long-range

Δ J distributions $J_o \neq 0$

Order Parameters:

$m = \overline{< S >}$ $p = \overline{< S^2 >}$

$q = \overline{< S >^2}$ Glass phase, Edwards-Anderson OP

a) Mean field: Replica

b) Monte Carlo

c) Renormalization Group

Thermodynamic Phases and RG fixed points

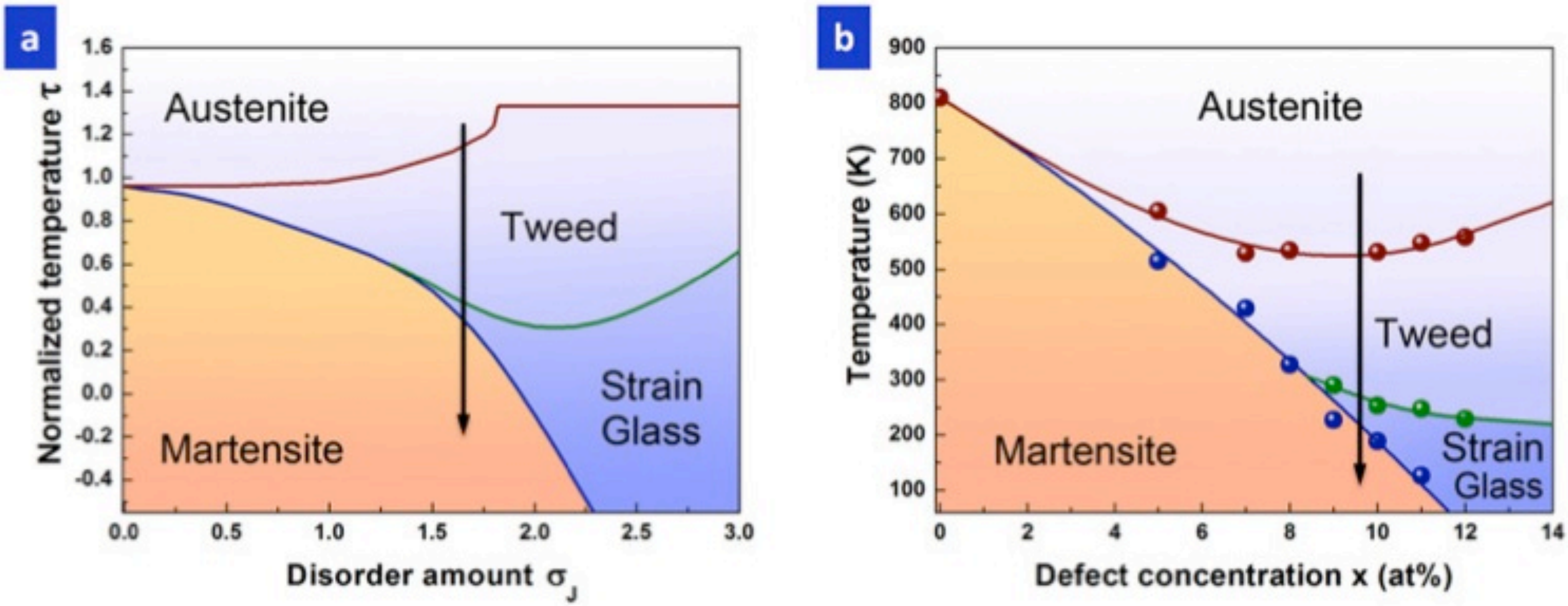
Phase	OP characterization	RG fixed point
Austenite	$m = q = 0$, p small	$\Delta^* = +\infty$, $J^* = 0$, $\sigma_J^* = 0$
Martensite	$m \neq 0$, $p \neq 0$, $q \neq 0$	$\Delta^* = -\infty$, $J^* = \infty$, $\frac{\sigma_J^*}{J^*} = 0$
Tweed	$m = q = 0$, p large	$\Delta^* = -\infty$, $J^* = 0$, $\sigma_J^* = 0$
Strain glass	$m = 0$, $p \neq 0$, $q \neq 0$	$\Delta^* = -\infty$, $J^* = 0$, $\sigma_J^* = \infty$

$m = \overline{< S >}$ $\quad p = \overline{< S^2 >}$ $\quad q = \overline{< S >^2}$

$$\beta H = -\sum_{<i,j>} J_{ij}(\tau) S_i S_j + \Delta(\tau) \sum_i S_i^2 - K \sum_{<i,j>} S_i^2 S_j^2$$

$$\{\mathcal{P}'(J'_{ij}), \mathcal{P}'(\Delta'_i), \mathcal{P}'(K'_{ij})\} = \mathcal{R}\left[\{\mathcal{P}(J_{ij}), \mathcal{P}(\Delta_i), \mathcal{P}(K_{ij})\}\right]$$

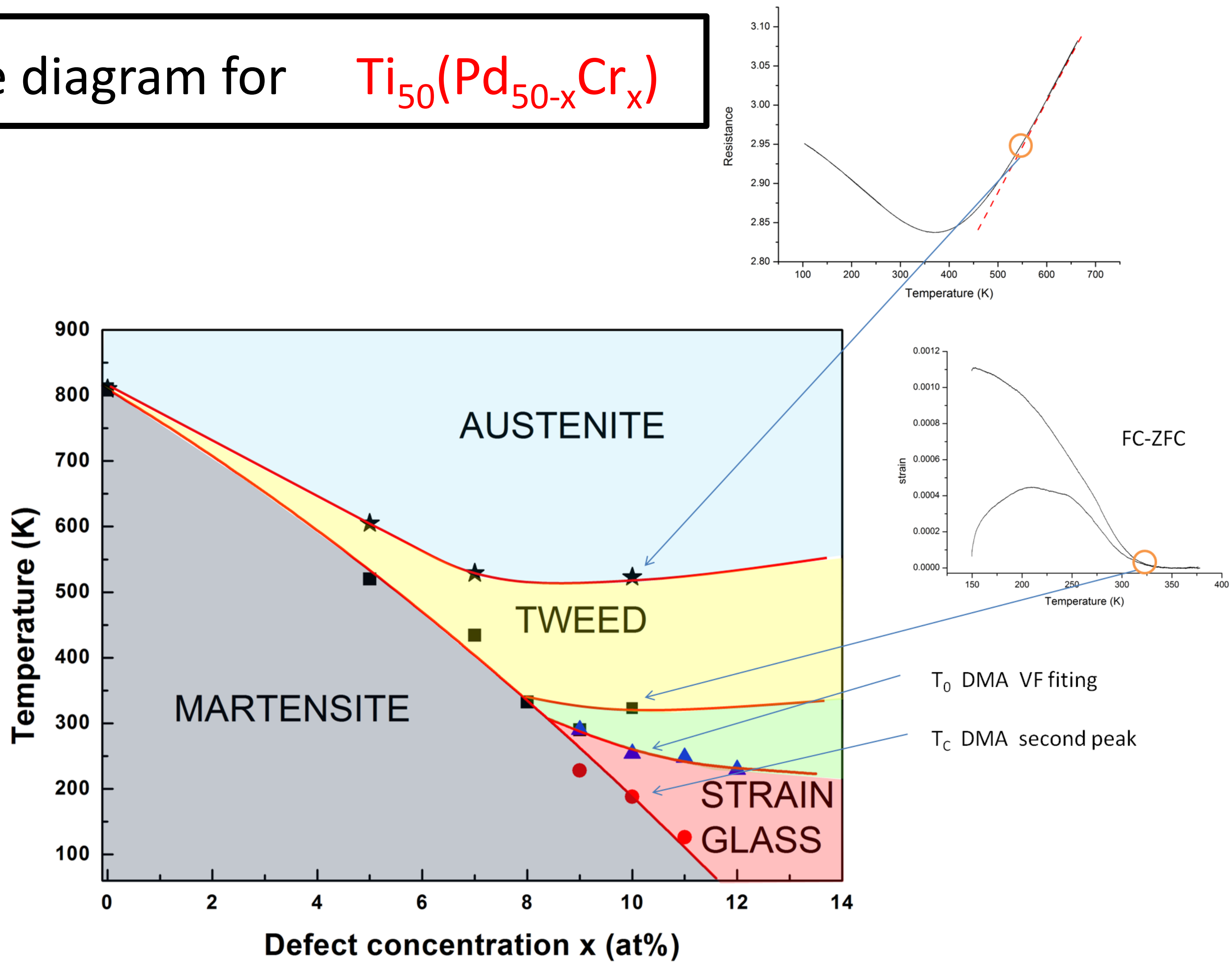
Phase diagram



RG phase diagram

$Ti_{50}(Pd_{50-x}Cr_x)$

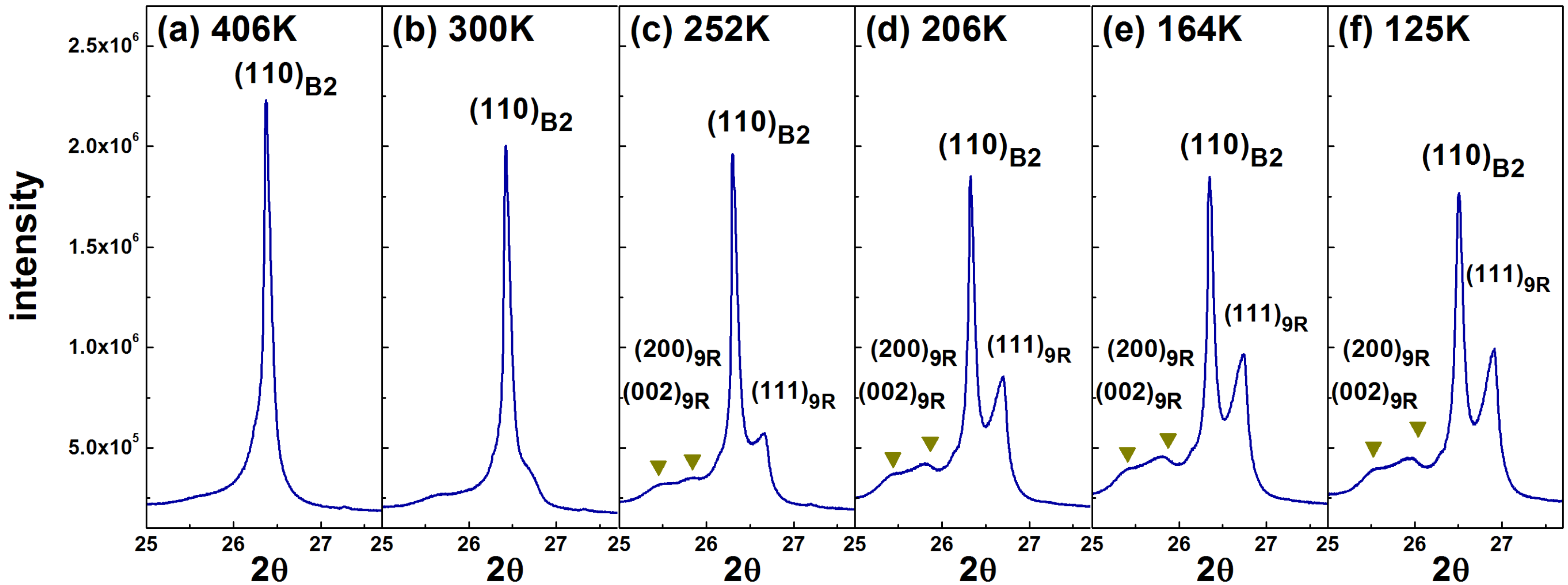
Phase diagram for $Ti_{50}(Pd_{50-x}Cr_x)$



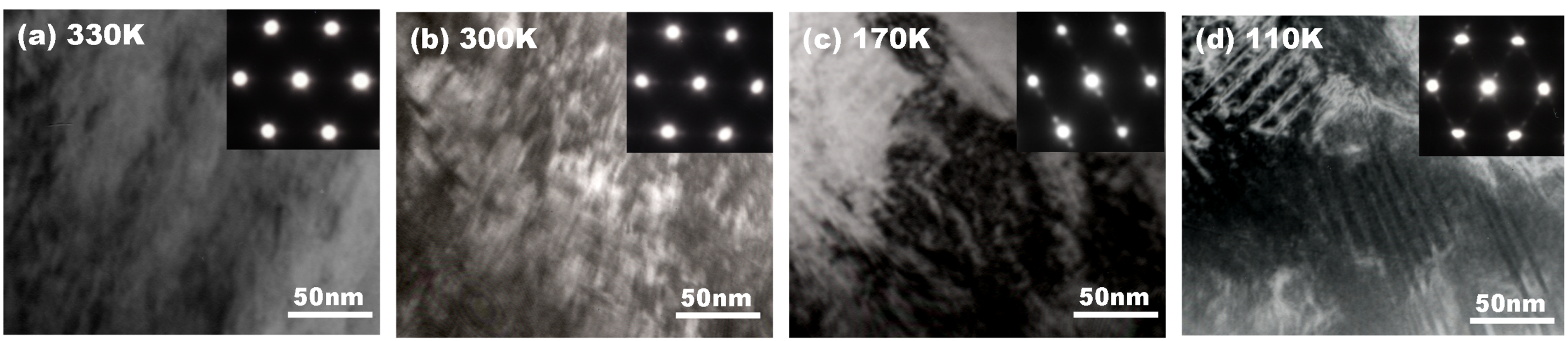
Xue, Zhou et al. (2011)

Transition from glass to long range ordered martensite phase in $Ti_{50}(Pd_{40}Cr_{10})$ alloy

In-situ synchrotron XRD from 400K to 100K



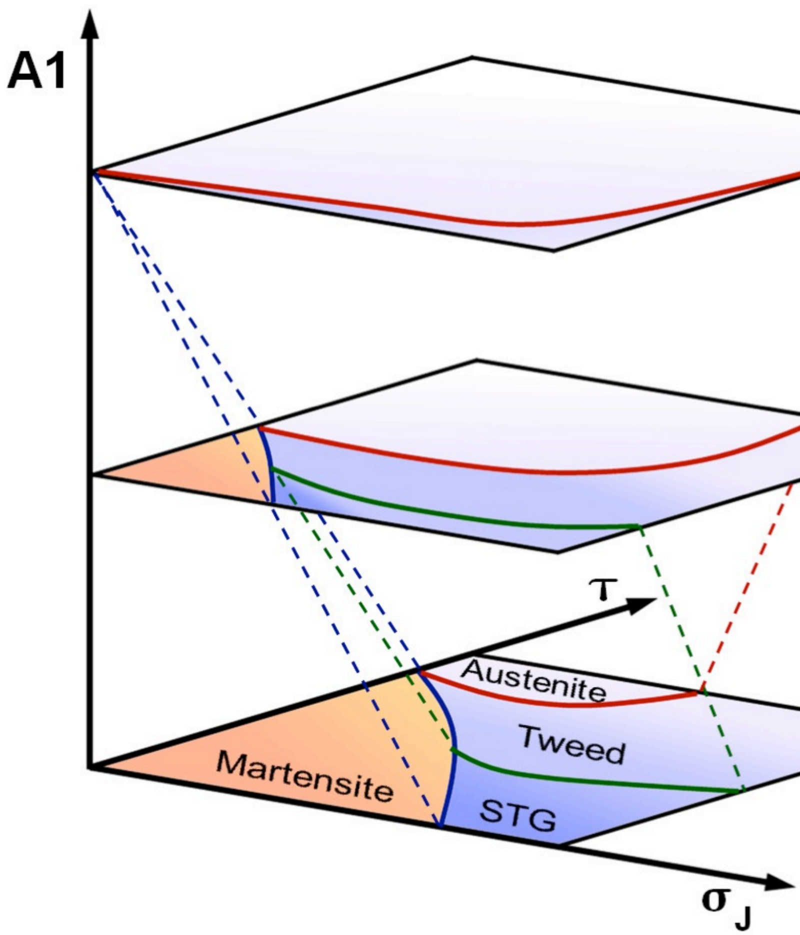
In-situ TEM from 400K to 100K



Xue, Zhou et al. (2011)

Interplay of disorder and long-range

$$\beta H = -\sum_{<i,j>} J_{ij}(\tau) S_i S_j + \Delta(\tau) \sum_i S_i^2 + \frac{\beta A_1}{2} \sum_{ij} S_i U_{ij} S_j .$$



Summary:

- **JAK with G. Barsch: Nonlinear Physics of Materials**

  Free energy (C-T), FePd, 1D solutions '84;
  Decaying fringing fields in austenite '87;
  Tweed as spin glass' 91;
  Shuffle order parameter '94;
  Strain compatibility, tweed ' 95 ;