High Temperature Superconductivity in the Cuprates

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Work done with S Banerjee and C Dasgupta, IISc

(two papers published in PR B 2011, some others in preparation)

- Materials: Ternary copper oxides, generally doped

Examples: $La_{2-x}Sr_xCuO_4$; $YBa_2Cu_3O_{6+x}$

(LSCO) (YBCO)

Discovered, starting 1986, to be superconducting at unprecedentedly high temperatures (T_c (max) ~ 160 K)

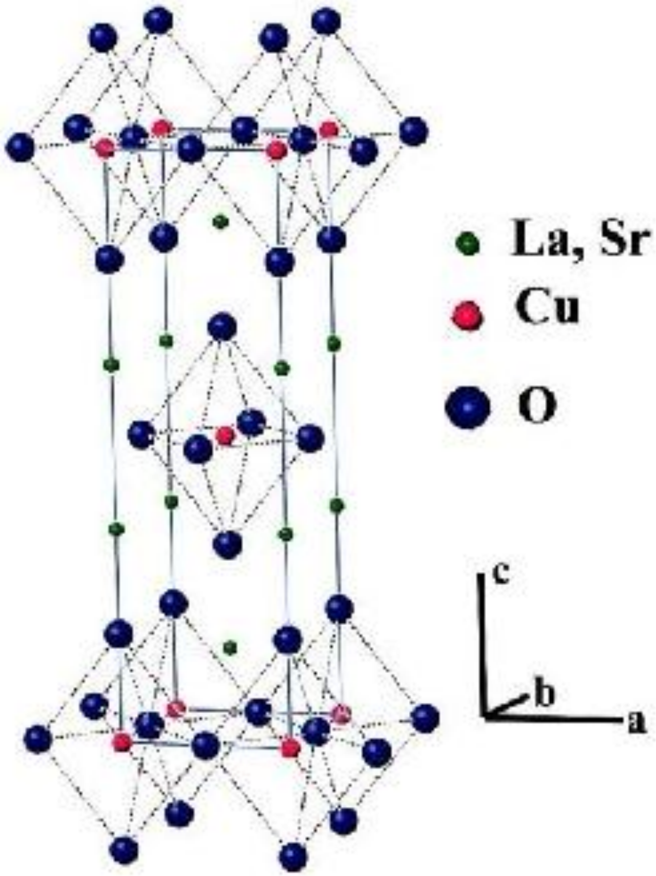
About thirty chemically different families

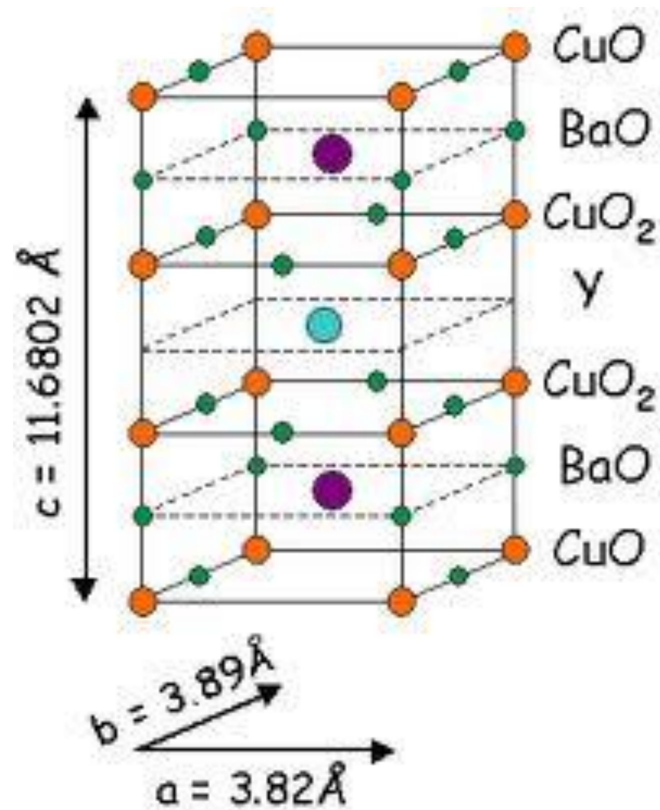
Electronic properties determined by electrons in the unfilled d shell of the Cu atom

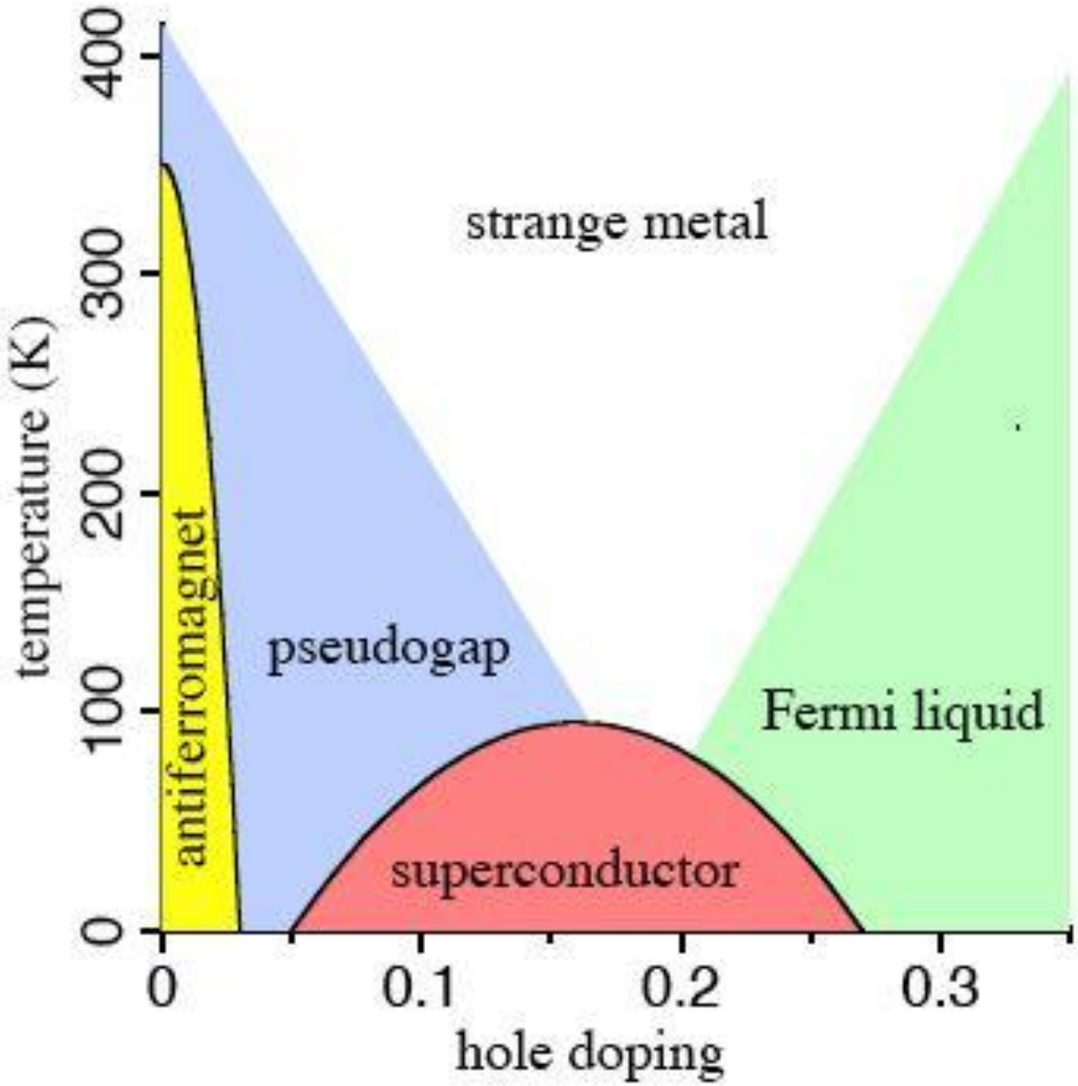
The thing consists of distorted corner sharing octahedra with Cu/RE ions at the centre and the O ions at the corners. It is most simply and (correctly) regarded as weakly coupled square lattice Cu-O planes, for electronic purposes

LSCO

YBCO







Idealized, universal, ‘phase’ diagram of hole doped cuprates in the hole doping (x) and temperature (T) plane

Background : The low energy degrees of freedom relevant for superconductivity in a continuum model are described by a complex field $\psi(\mathbf{r})$

(Ginzburg and Landau, 1950; identified by Gor'kov in 1956-7 as

$$\psi(\mathbf{r}) = \langle a^{+}_{\uparrow}(\mathbf{r})\, a^{+}_{\downarrow}(\mathbf{r})\rangle \;)$$

$\langle\psi\rangle \neq 0$ below T_c and $= 0$ above T_c in a uniform superconductor

The free energy is $F = \int d\mathbf{r}\, \{a|\psi(\mathbf{r})|^2 + b\,|\psi(\mathbf{r})|^4 + c\,|\partial\psi(\mathbf{r})|^2\}$

where a, b and c are inspired by experiment.

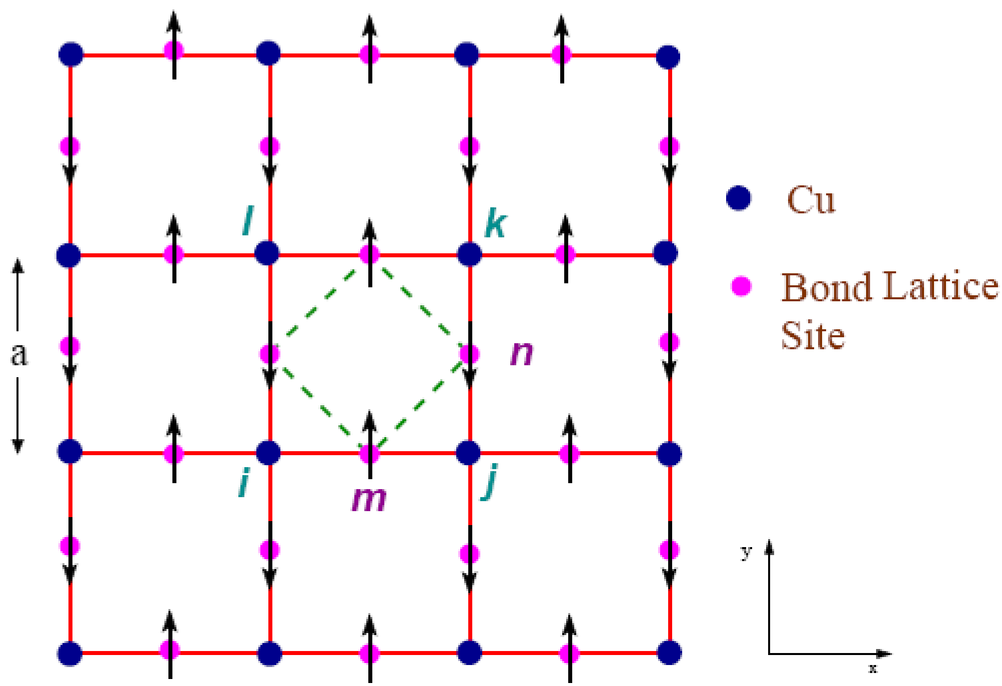
(After BCS in 1956, Gor'kov identified them with microscopic parameters of the metal)

Mean field theory: a(T) changes sign at T_c (locates T_c)

(Very successful not only for 'conventional' superconductors, but also for all continuous phase transitions with an identified order parameter)

Our approach is similar in spirit and superficially

similar in the form of the functional



Lattice version . The basic degree of freedom is

$$\psi_{ij} = \Delta_{ij} \exp(i\phi_{ij}) \quad (ij = m)$$

(Electrons at i and at j form a spin singlet Cooper pair)

The free energy is written as a sum of two terms;

$a\,\Delta_m^{\,2} + b\,\Delta_m^{\,4}$

depends only on the magnitude,

$c\,\Delta_m \Delta_n \cos(\phi_m - \phi_n)$

also depends on the phase.

superconductivity (macroscopic phase coherence) means the system is stiff with respect to fluctuations of the phase at two points very far apart

In mean field theory (eg ignore the c term, because the bare coherence length ξ_o is large and $c \sim \xi_o^{-2}$), $a(x,T)=0$ is where superconductivity begins

In the cuprates, ξ_o is small (~ 15-20 A rather than 10,000 A)

c term is not negligible

Nonzero superfluid (phase) stiffness is due to it; the magnitude term cannot lead to superfluidity

If c term is positive, one has d-wave symmetry phase stiffness below a transition temperature T_c

Superconductivity is not due to pair formation, but due to interaction between pairs.

(Spin analogy: Spin formation and long range spin order; 2d-XY spin)

Origins:

i) Nearest neighbour pairing: For spin (1/2) systems, the nearest neighbour AF superexchange interaction (known to be large in cuprates, ~ 1500K) is **identical** with nn singlet pairing attraction

ii) In a strong coupling picture, the c term can arise to linear order in hole density from the hopping of a hole to the diagonally opposite side (t'). Indeed, Pavarini, Dasgupta, Dasgupta and Andersen observed a correlation between t' and T_c (for optimum hole density)

Our picture is that there are nearest neighbour Cooper pairs with nonzero thermal probability at all temperatures, but the probability distribution changes character at a=0 (local Cooper formation temperature, pseudogap temperature

d-wave symmetry Cooper pairing **emerges** as a result of short range interactions

The two temperature scales are quite distinct for low doping, but indistinguishably merge beyond optimum doping (eg the BCS limit)

Unlike looking for a glue or pairing interaction which produces d-wave superconductivity

- **Input** :

$a = \{T - T_o(1 - x/0.3)\} \exp(T/T_o)$

$b = T_o/8$ and

$c = xT_o/3$

(energy units $T_o \sim T^*$ (x=0))

Output :

$\sqrt{<\Delta_m^2>}$

Onset of nonzero phase stiffness ('Neel' long range order) or T_c

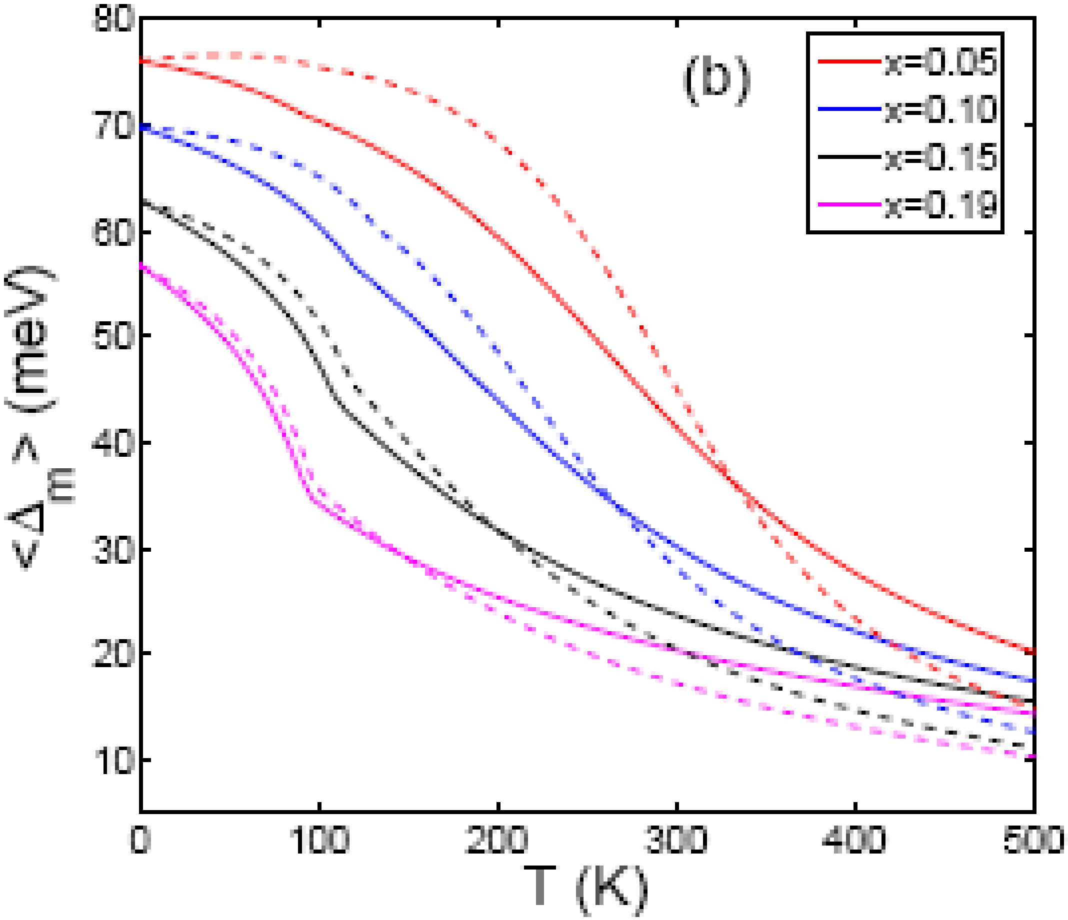
ρ_s

C_v

Vortex structure and energetics

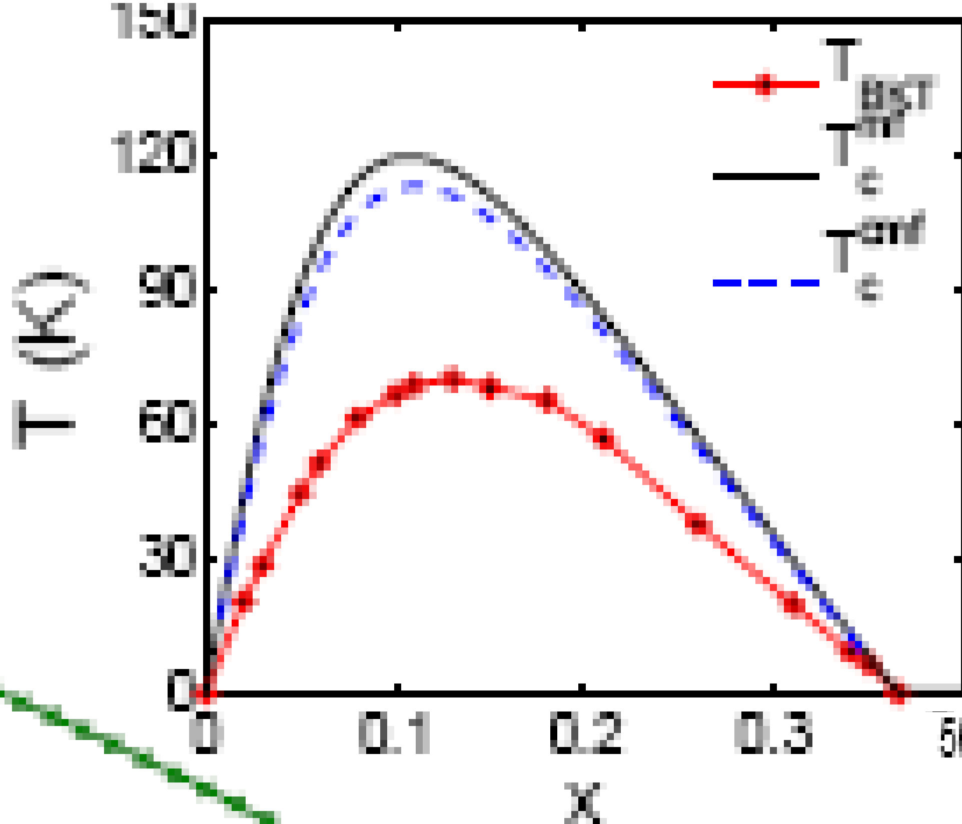
Electron Green's function using $< \psi_m^* \psi_n >$

coupled to electrons (especially useful for large $|m-n|$)

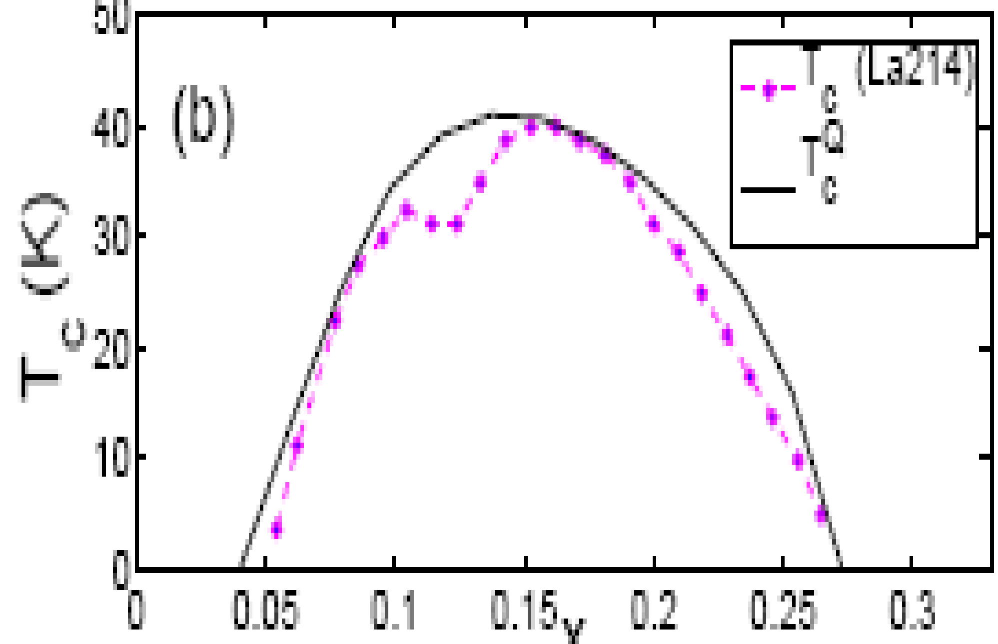


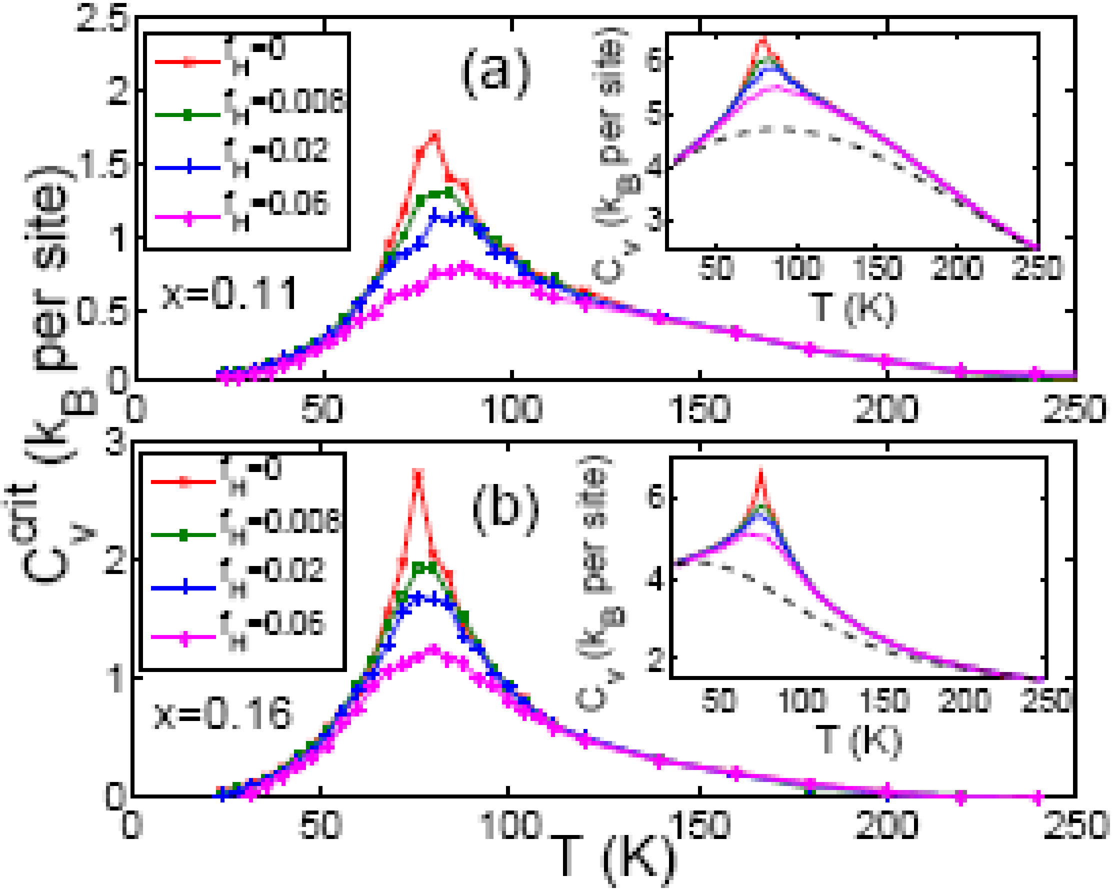
Local gap as a function of temperature for different doping values; the full and dotted curves are for slightly different values of T_o . From a maximum slope criterion, can infer T^*.

$T_c(x)$ curve



Same, on including quantum phase fluctuations





Specific heat in a magnetic field (measured in units of flux quantum per unit cell) for two dopings x=0.11 and x=0.16, at differetnt values of the magnetic field

Electron moving in a medium of bond pairs

'AF' or d wave short range order ($T > T_c$)

Long range order and residual thermal or quantum fluctuations ($T < T_c$)

Electron (self energy) Σ in this medium :

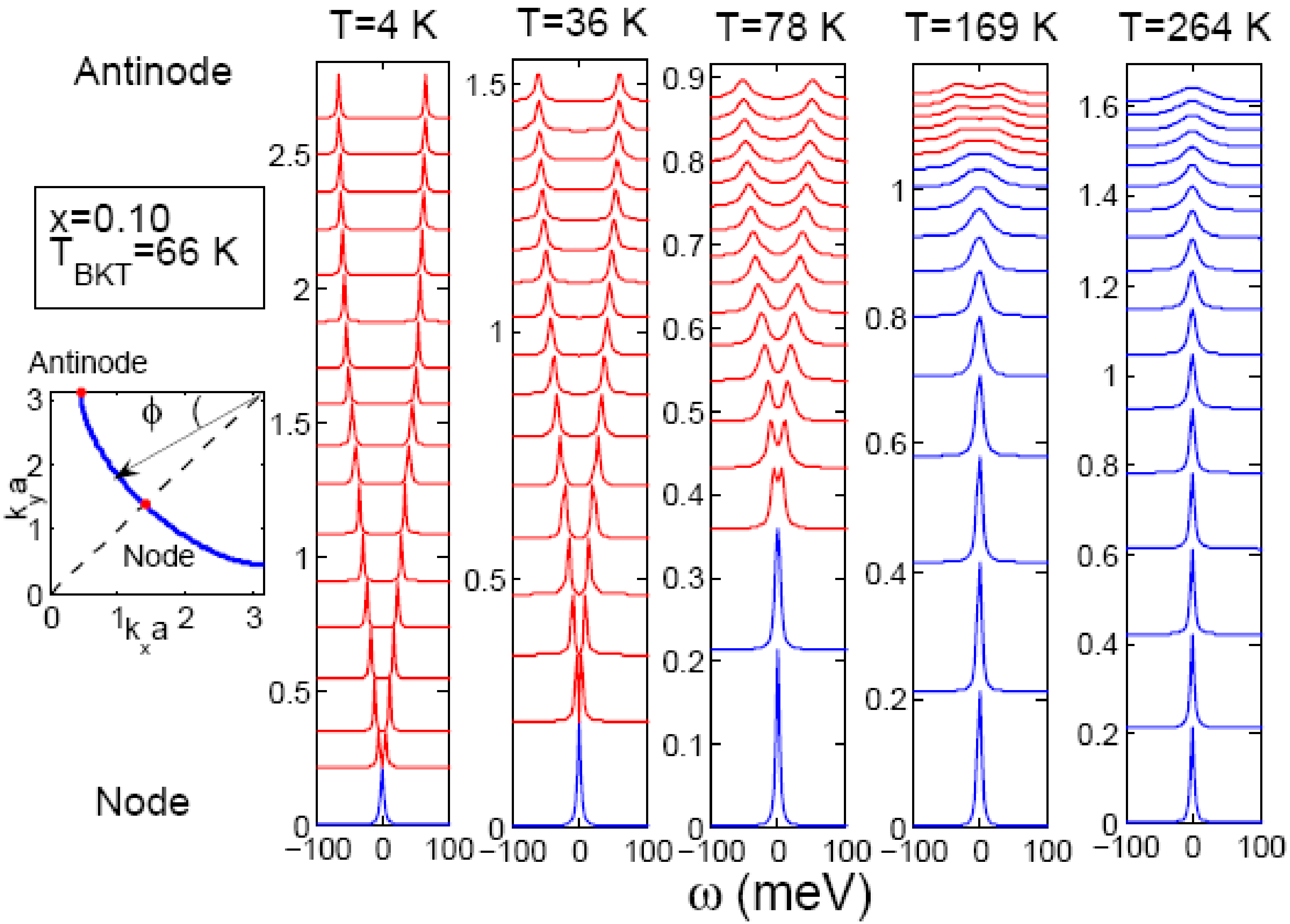
$$\Sigma \sim P\, D\, G$$

P : form factor (reduces to $\{\cos(k_x a) - \cos(k_y a)\}$ for long range superconductivity; Gor'kov propagators with
$\Delta_{\mathbf{k}} = (\Delta/2)\,(\cos(k_x a) - \cos(k_y a))$ ie d-wave)

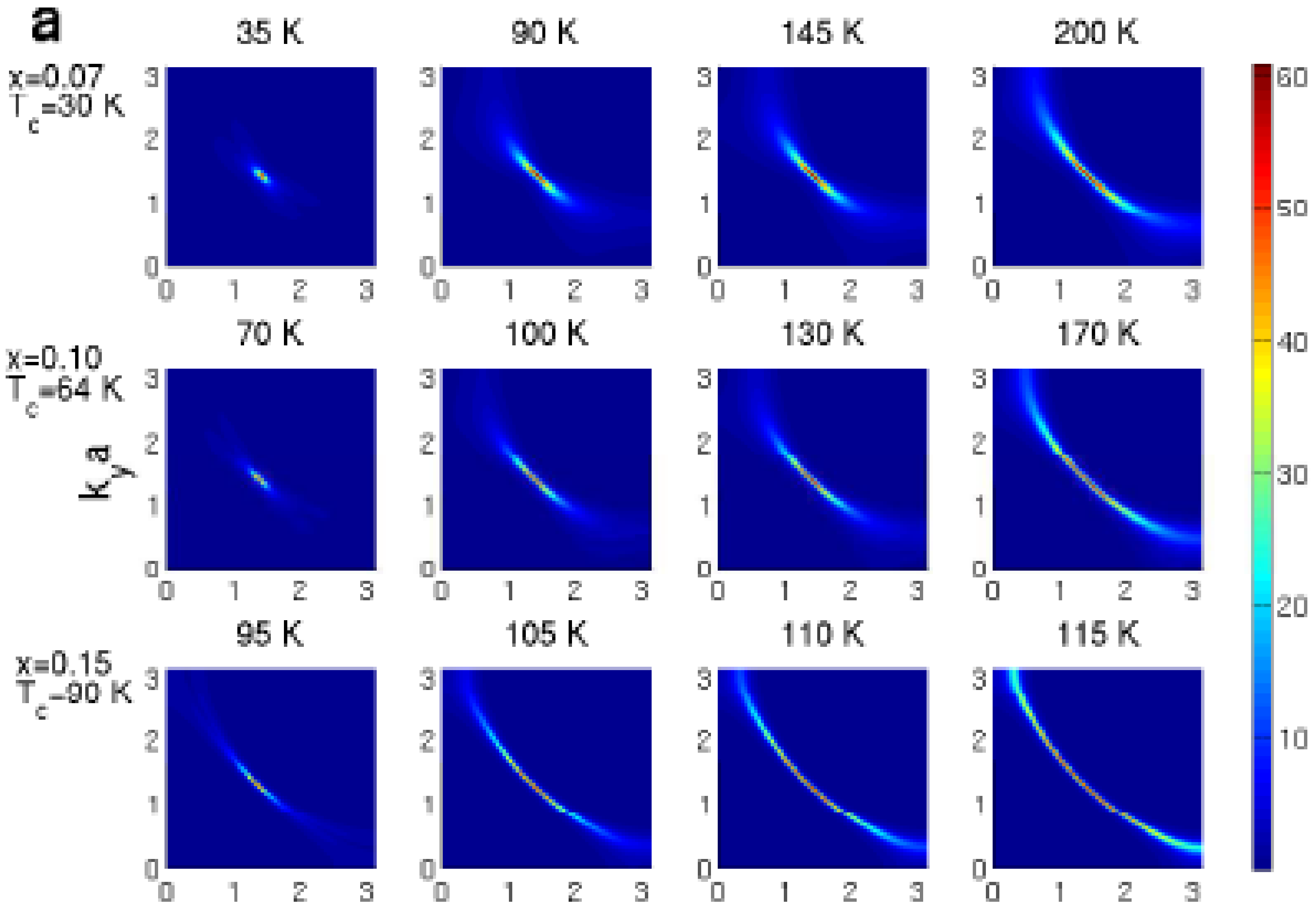
D : pair-pair correlation function (for small momentum transfer **q** with respect to the 'AF' ordering wavevector)

G : Electron propagator

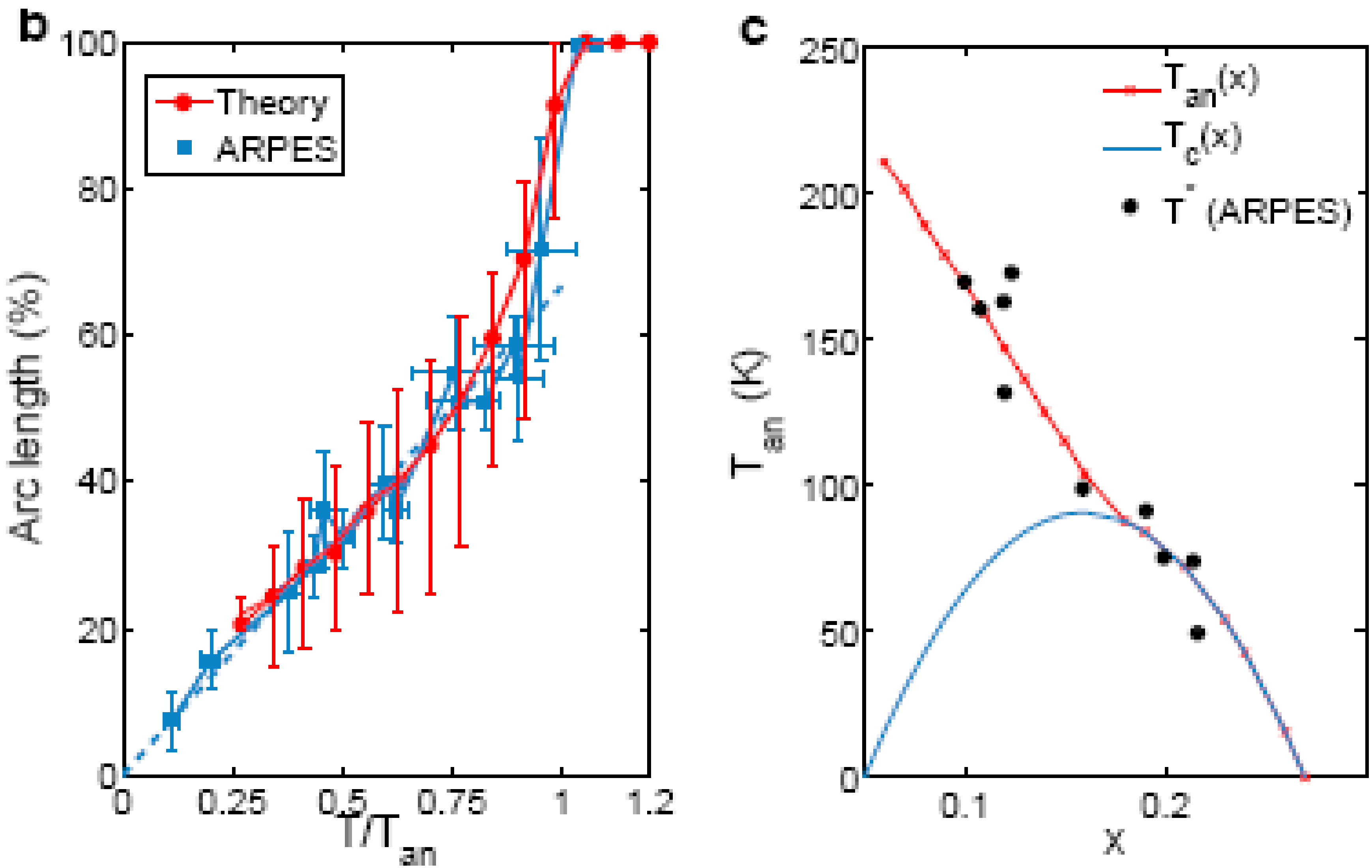
- An electron ‘becomes’ an electron pair and a hole and then recombines to become an electron
- Simplest vertex correction vanishes above T_c because the anomalous propagator vanishes
- Below T_c , this correction is of relative order $(T_c / \varepsilon_F) \ll 1$. This is the Migdal like theorem here
- Boson (Cooper pair fluctuation) propagator fully renormalized
- Use bare intermediate state electron propagator (closed form expressions possible with the dressed propagator)
- Lot of unbiased numerical evidence that there is a low energy quasiparticle part to the electron propagator; the residue z can be absorbed in the definition of the unobserved bare gap)



T=4 K
T=36 K
T=78 K
T=169 K
T=264 K
Antinode
x=0.10
T_BKT=66 K
Antinode
Node
k_y a
k_x a
φ
Node
ω (meV)



a
x=0.07
T_c=30 K
x=0.10
T_c=64 K
x=0.15
T_c=90 K
35 K
90 K
145 K
200 K
70 K
100 K
130 K
170 K
95 K
105 K
110 K
115 K
k_y a
0
1
2
3
10
20
30
40
50
60



b
Theory
ARPES
Arc length (%)
T/T_an
c
T_an(x)
T_c(x)
T* (ARPES)
T_an (K)
x

- Ignores:

Other interactions (even within the phenomenological scheme) eg $\psi_m^* \psi_{m'}$ where m and m' are both 'x' or 'y' bonds

Other 'bosonic' fields eg spins $\mathbf{S_i}$ (and of course its interaction with ψ_m)

Time dependence of ψ_m

Existence and relevance of other low energy degrees of freedom,eg lattice vibrations (bosonic) and unpaired fermions(except in the part on ARPES) ……..

- Cannot address: (in its present form)

competition (changing dramatically with x) between superconductivity and antiferromagnetism, stripes, 4X4 superstructure, liquid crystalline correlations, isotope effects ……

- Expect to do: (incomplete wish list)

Calculate $\xi(x,T)$ in the phenomenological theory

Nernst effect in the (x,T) plane, esp. for $x<x_{opt}$(Subroto Mukerjee, IISc)

Effect of coupling to neutrons, photons(Raman), more carefully the coupling to electrons (for STM, STS)

Quantum oscillations

- What about a **real**, microscopic theory?
- A phenomenological, Ginzburg Landau like, lattice theory for high T_c superconductivity in the cuprates, with nearest neighbour Cooper pairs as the basic low energy degrees of freedom has been proposed and its consequences have been compared with experiment.
- For nearest neighbour antiferromagnetic interactions, d-wave symmetry superconductivity (‘Neel’ long range order) emerges.
- For electrons moving on the same square lattice in which the nearest neighbour electrons constitute Cooper pairs, their inevitable coupling with the latter leads to low excitation energy (near Fermi energy)

  spectral function features that have been widely seen in high resolution ARPES experiments.
- The approach is also likely to be useful for phenomena connected with the coupling of Cooper pairs with electrons, photons, neutrons etc.(eg quantum oscillations, STS, Raman spectra, ‘41 meV’ peak) in cuprates.