Peeling of heterogeneous adhesives

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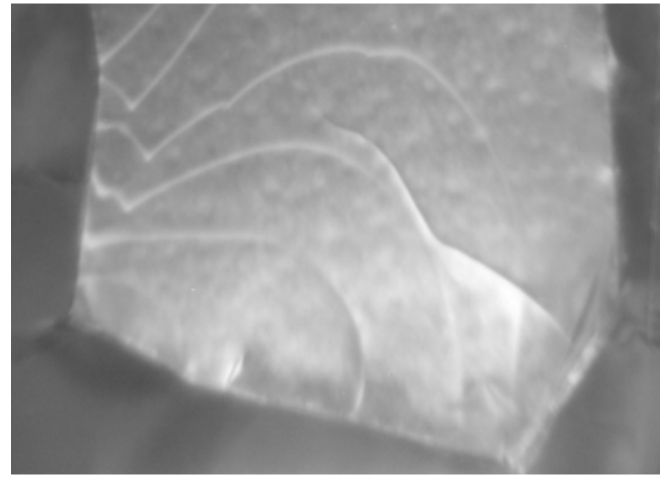
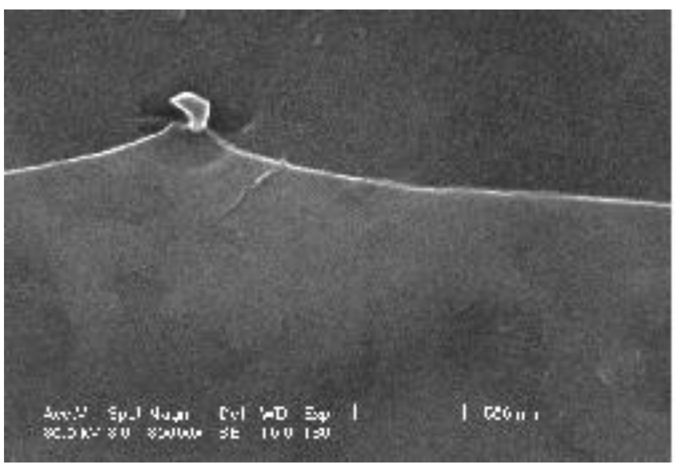
Phenomena

Fracture (Tate)



Dislocations (Arzt)



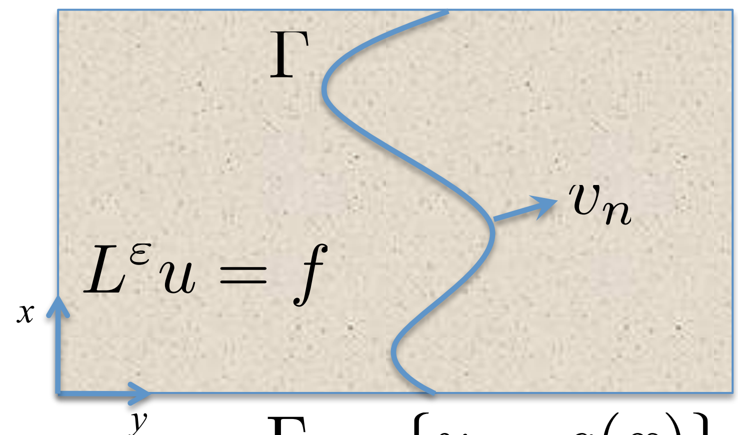
Phase boundary (Moelans)

Adhesive film

Theoretical Setting

- Free boundary coupled to a pde/ free discontinuity

$$v_n = F^{\varepsilon}(x) + c\kappa + N_u$$



$$\Gamma = \{y = g(x)\}$$

- `Gradient flow'

$$\int_\Gamma v_n \xi dA = -\delta_\Gamma \mathcal{E}, \qquad \mathcal{E} = \mathcal{E}^{\varepsilon}(u, \Gamma)$$

- Shallow interface

$$g_t = \eta(x, g) + c\Delta g + \mathbb{A} g$$

- Large literature in random setting
  - Roughness, Pinning-depinning
  - Macroscopic behavior

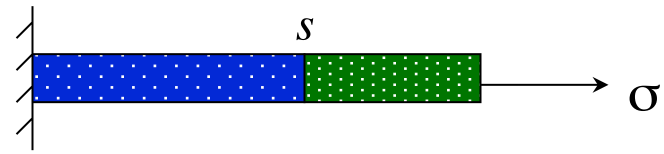
$$g_t = (F - F^{\star})^{\beta}$$

My motivation:
- Microstructure and *F**
- Not only random
- Possibly high contrast

This talk: Peeling

Example in one dimension

$$\frac{ds}{dt} = \sigma - \sigma^*(s)$$

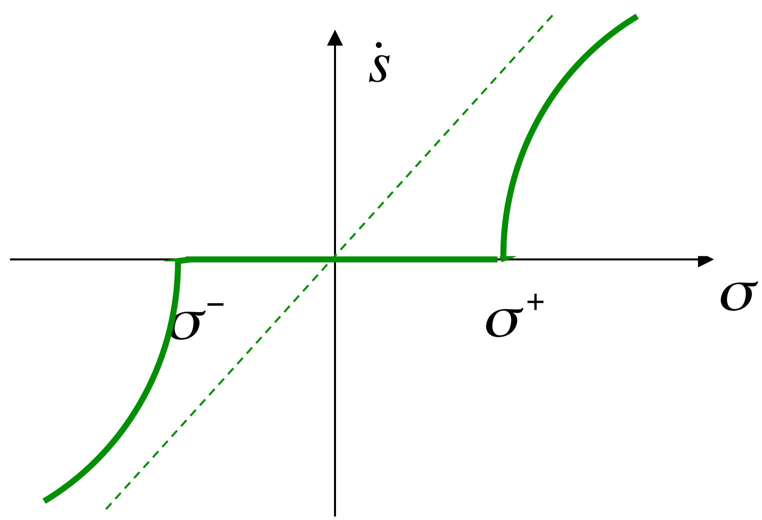


Formally,

$$\int_0^L \frac{ds}{\sigma - \sigma^*(s)} ds = T, \qquad \bar{v} = \frac{L}{T} = L \left(\int_0^L \frac{ds}{\sigma - \sigma^*(s)} ds \right)^{-1}$$

But, boundary may get stuck.

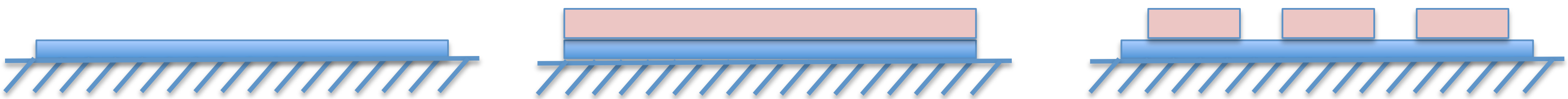
$$\bar{v} = \begin{cases} 0 & \sigma^- \leq \sigma \leq \sigma^+ \\ \bar{v}(\sigma) & \text{else} \end{cases}$$

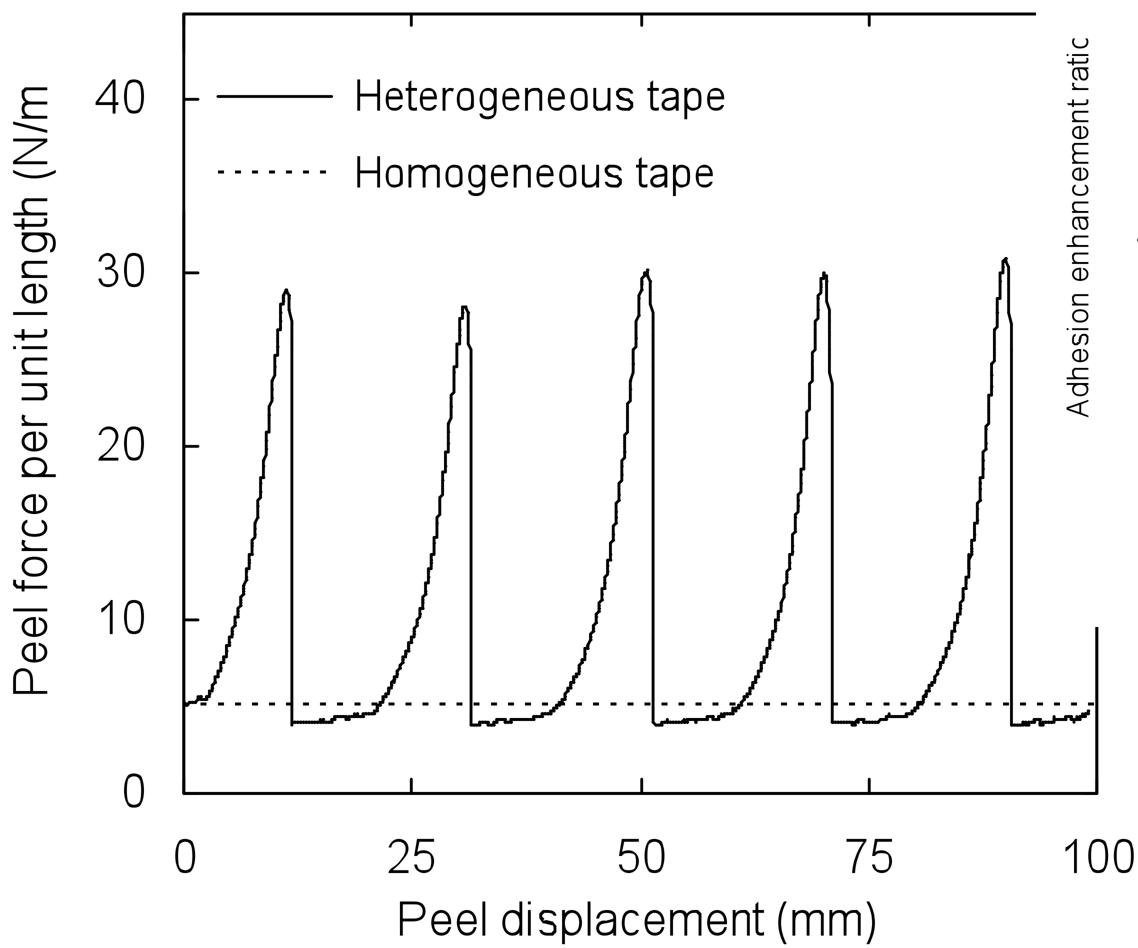


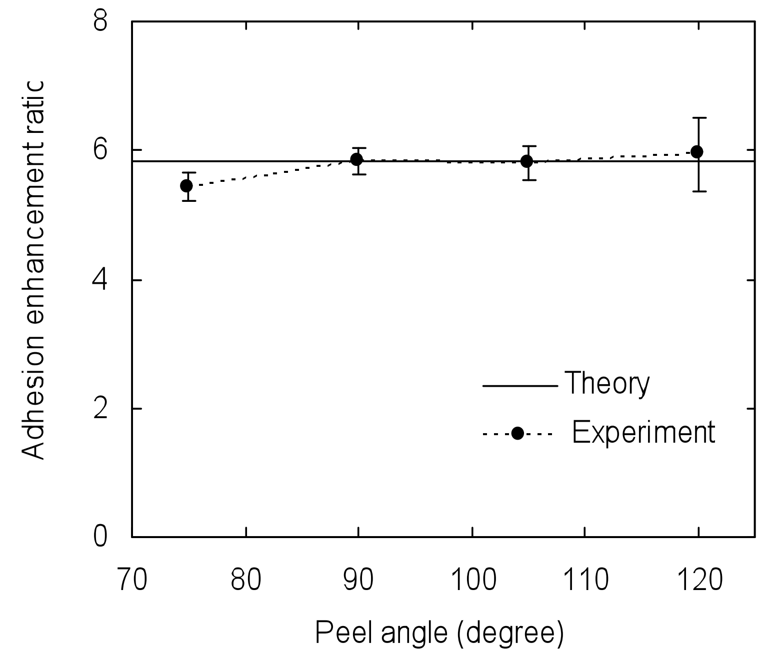
Asymptotically near critical forces,

$$\bar{v} = (\sigma - \sigma^+)^{1/2}$$

Stiffness heterogeneity



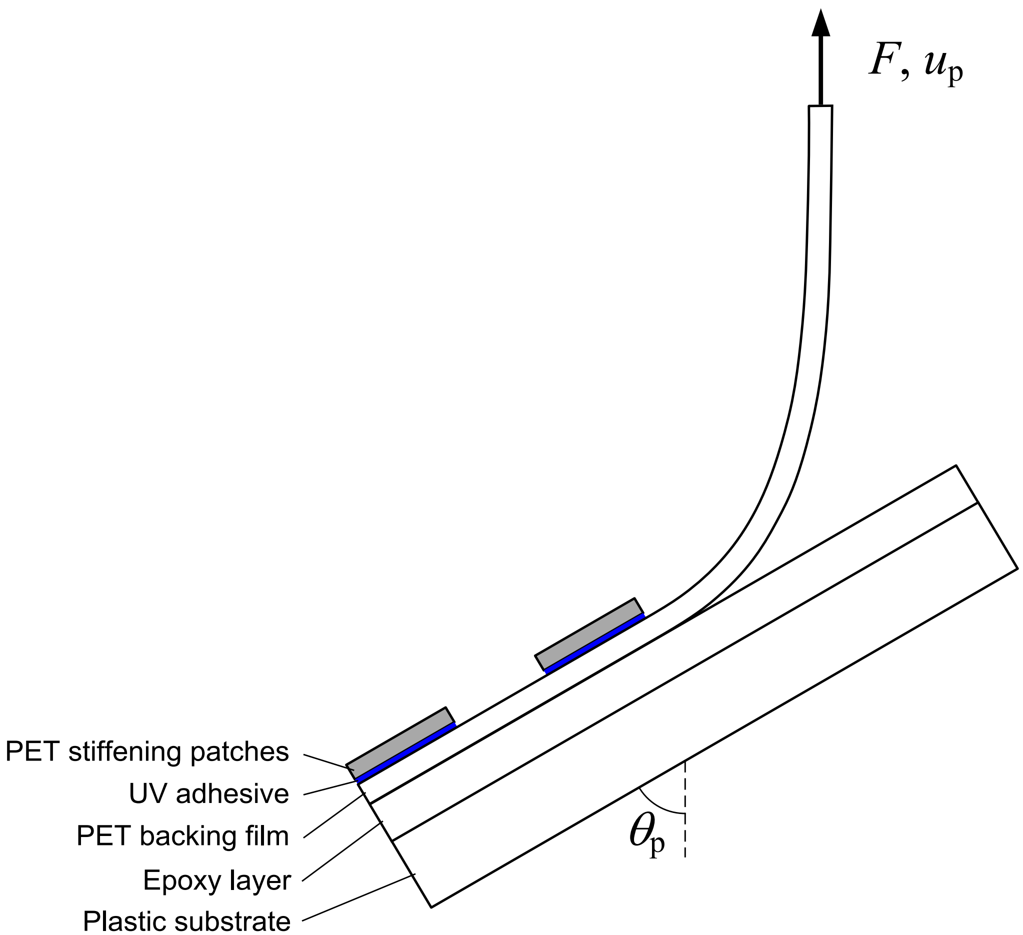


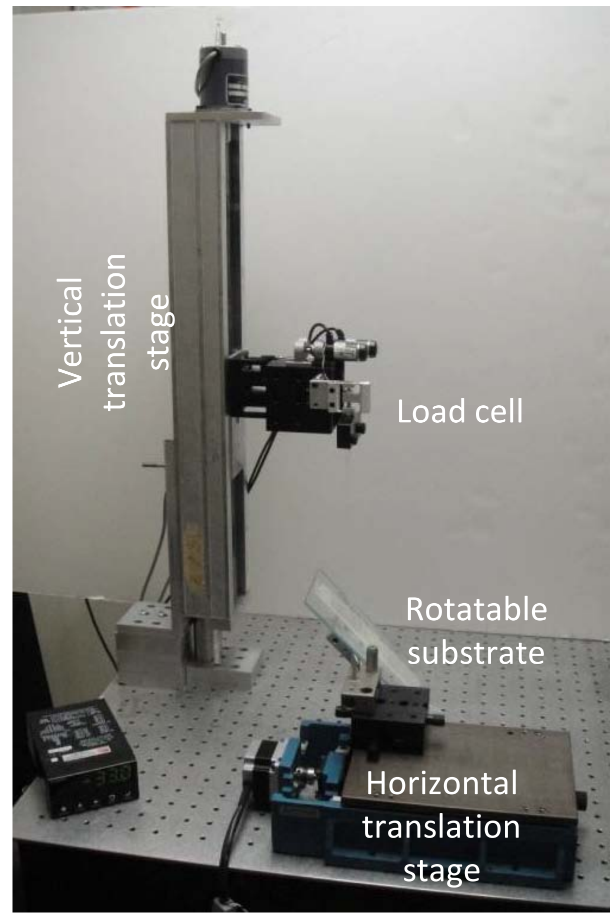


$$\frac{F_{het}}{F_{hom}} \approx 5.8$$

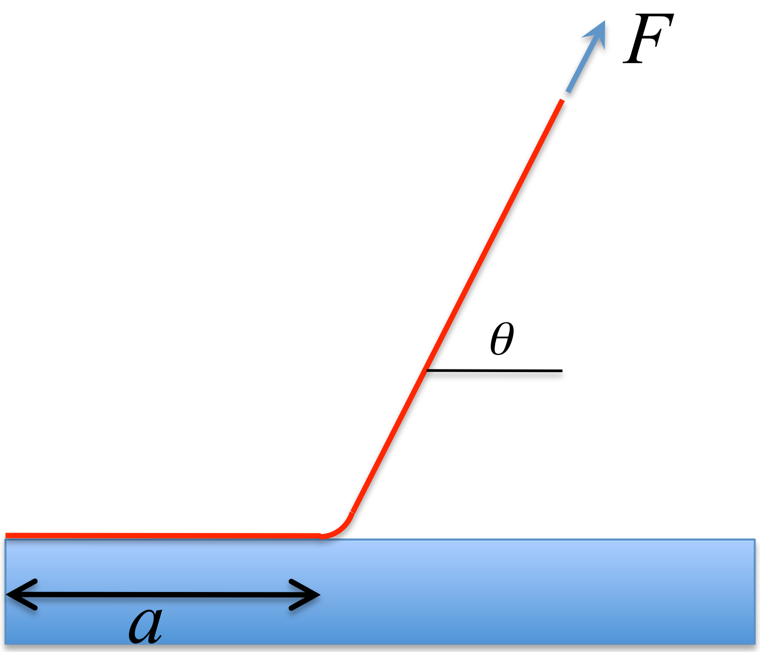
(independent of peel angle)

Experimental set up





Peeling



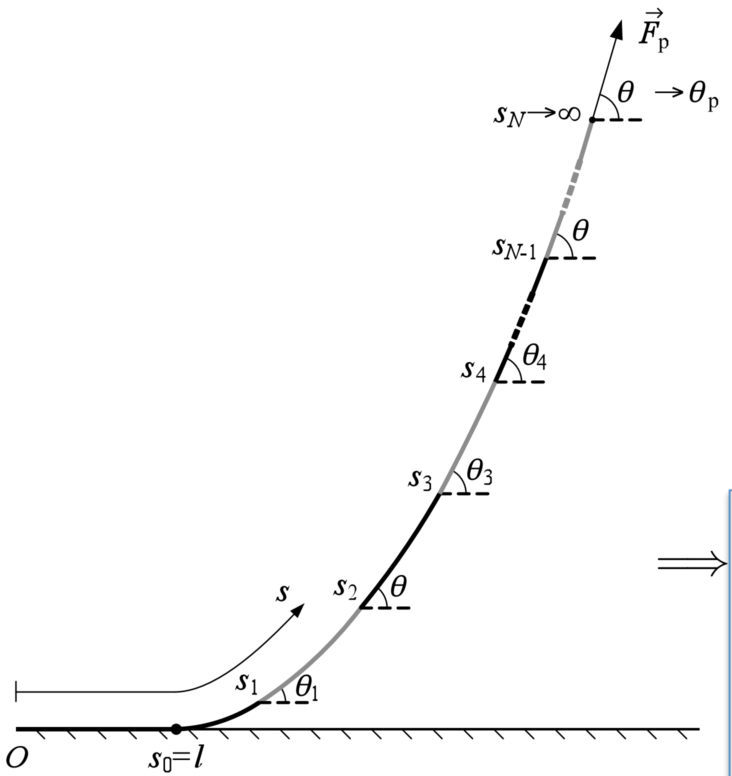
Rivlin (1944):

$$Fv = G\dot{a}$$

$$F = \frac{G}{1 - \cos\theta}$$

R. Rivlin, *J. Paint Tech 9*: 215, 1944

Stiffness heterogeneity



$$\mathcal{E} = \int_l^{s_N} \frac{1}{2} D(s)(\theta'(s))^2 ds - F \cdot u - \int_0^l G ds$$

Propagation criteria:

$$(D\theta')' - F\sin(\theta - \theta_p) = 0$$

$$\frac{1}{2} D\theta'^2 \Big|_{l+} - G = 0$$

$$\Longrightarrow \quad F = \frac{D_1 G}{\sum_{i=1}^{N} D_i \left(\cos(\theta_i - \theta_p) - \cos(\theta_{i-1} - \theta_p)\right)}$$

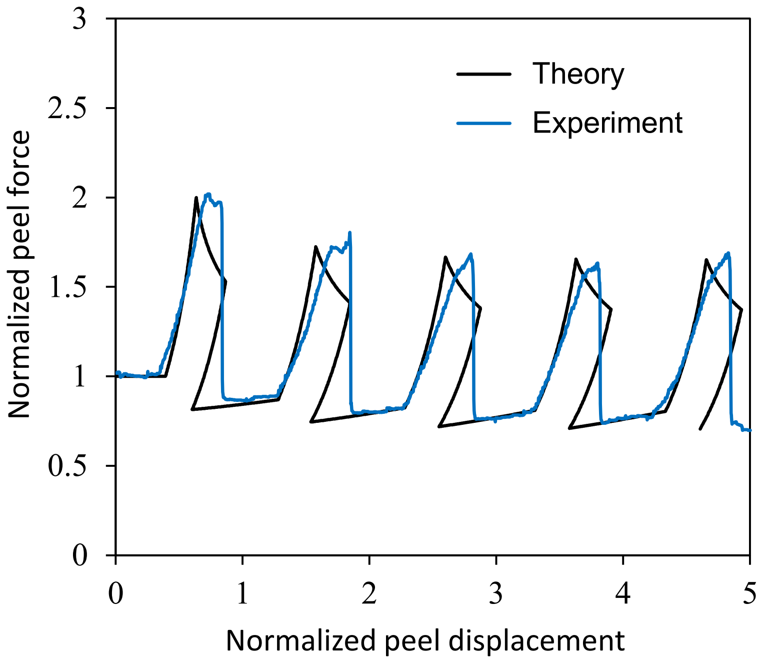
$$\int_{\theta_{k-1}}^{\theta_k} \left(D_1 G - F \left\{ \sum_{i=1}^{k-1} D_i f(\theta_i, \theta_{i-1}) + D_k f(\theta, \theta_{k-1}) \right\} \right)^{-1/2} d\theta = \frac{\sqrt{2}}{D_k}(s_k - s_{k-1})$$

Uniform: $F = \dfrac{G}{1 - \cos\theta}$

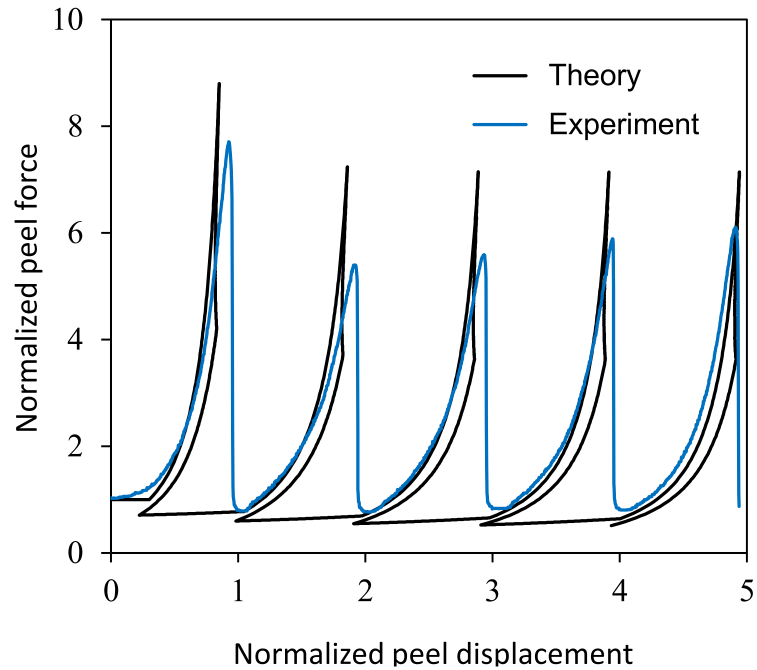
Interesting length-scale: $\lambda = \sqrt{\dfrac{D}{2G}}$

mce

Stiffness heterogeneity



$$D_s/D_c = 2$$



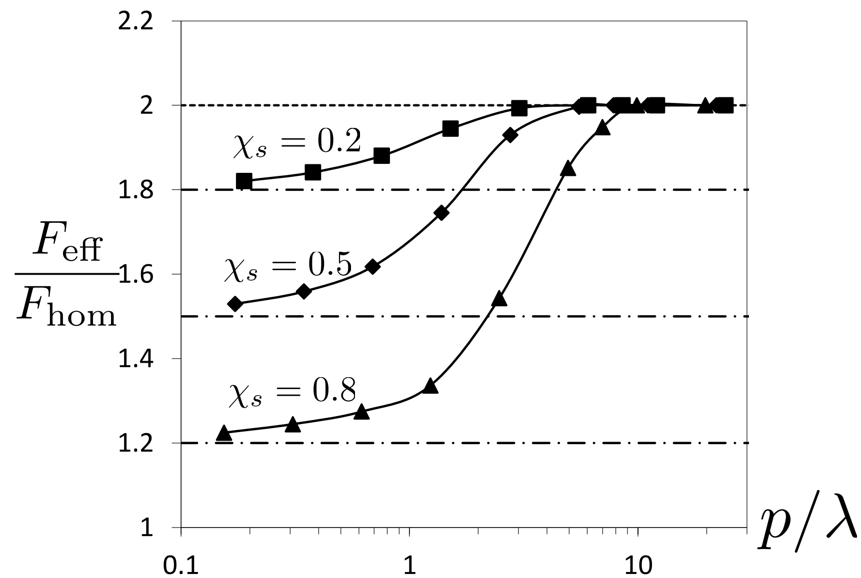
$$D_s/D_c = 8.8$$

Stiffness heterogeneity

Two-segment strip:

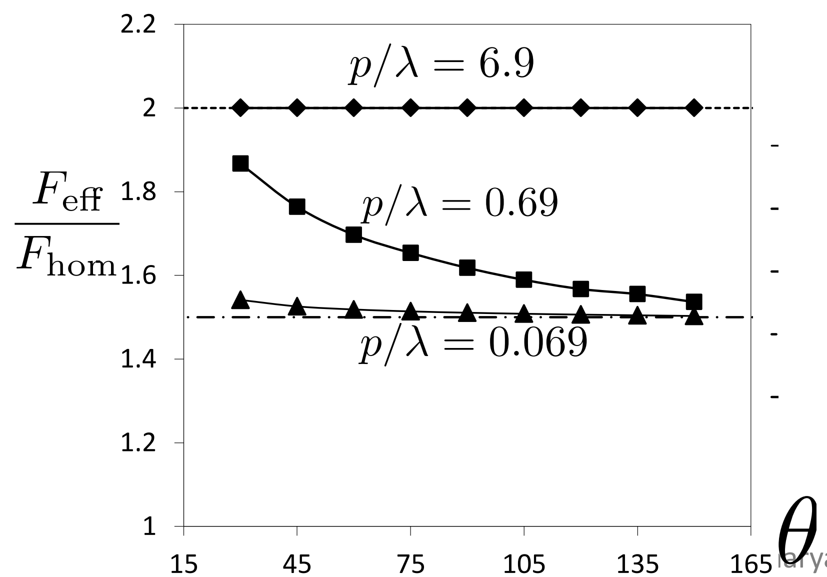
$$F = \frac{GD_1}{D_2 + (D_2 - D_1)\cos(\theta_1 - \theta_p) - \cos(\theta_p)}$$

$$\implies F_{\text{eff}} = \frac{D_s}{D_c} F_{\text{hom}}$$

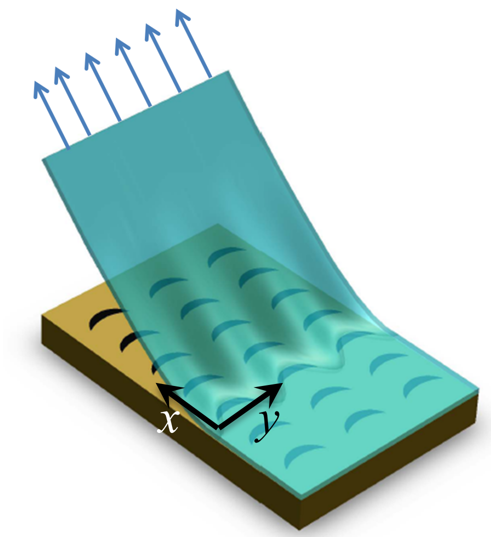


Periodic strip:

$$F_{\text{eff}} = \begin{cases} \dfrac{D_s}{D_e} F_{\text{hom}} & p << \lambda \\ \dfrac{D_s}{D_c} F_{\text{hom}} & p >> \lambda \end{cases}$$



Adhesive heterogeneity



$$\mathcal{E} = \frac{D}{2}\int_{\Omega_p} (\kappa_x^2 + \kappa_y^2 + 2\nu\kappa_x\kappa_y + (1-\nu)\kappa_{xy}^2)dA$$
$$- \int_{\Omega_b} G(x,y)dxdy - \lambda F_{\mathrm{p}} \cdot u_{\mathrm{p}}$$

Assume interface is almost straight;
Linearize about singly-bent film

$$D\nabla^4 w - F_{\mathrm{p}}\cos(\theta_x^0 - \theta_{\mathrm{p}})\frac{\partial^2 w}{\partial x^2} + F_{\mathrm{p}}\kappa_x^0 \sin(\theta_x^0 - \theta_{\mathrm{p}})\frac{\partial w}{\partial x} = 0$$

$$w|_{x=0} = 0, \quad \left.\frac{\partial w}{\partial x}\right|_{x=0} = -\kappa_x^0 f, \quad \left.\frac{\partial w}{\partial x}\right|_{x=\infty} = 0, \quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=\infty} = 0,$$

$$D\kappa_x^0 \frac{\partial^2 w}{\partial x^2}(0,y) + \frac{F_p}{1-\cos\theta} - G(f(y),y) = f_t(y)$$

$$\mathbb{A}f \qquad G_\infty$$

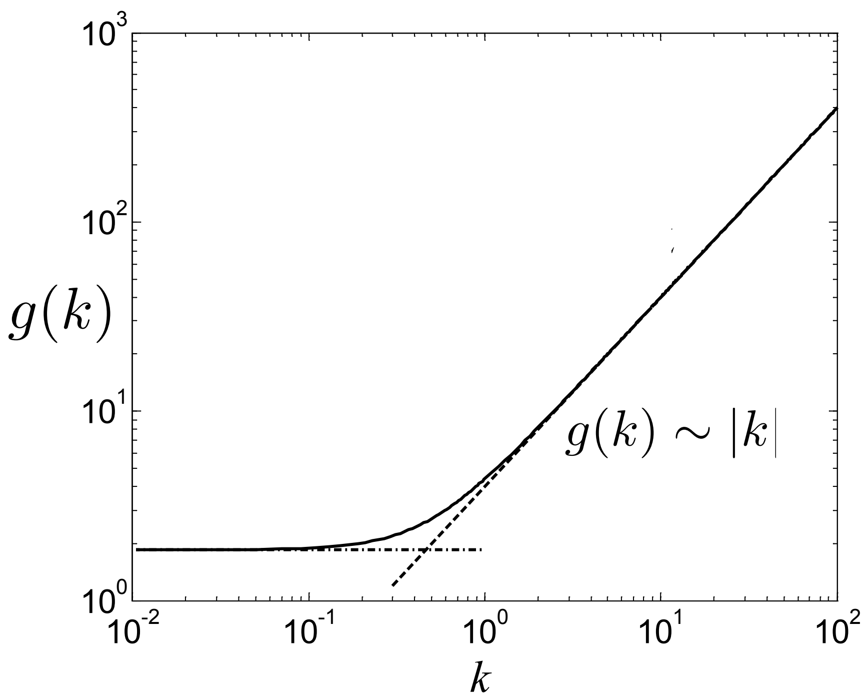
Peel-front evolution

- Evolution $f_t = \mathbb{A} f + G_\infty - G(f, y)$

$$\hat{f}_t(k) = -g(k)\hat{f}(k) + G_\infty - \widehat{G(f,y)}(k)$$

- Bending $(\hat{W},_{XXXX} - 2k^2\hat{W},_{XX} + k^4\hat{W}) - \cos(\theta^0_X - \theta_\mathrm{p})\hat{W},_{XX} + K^0_X \sin(\theta^0_X - \theta_\mathrm{p})\hat{W},_X = 0$

$$k >> 1, \xi = kX, \quad W_{,\xi\xi\xi\xi} - 2W_{,\xi\xi} + W = 0 \implies W = A\xi e^{-\xi} \quad \implies g(k) = |k|$$



Overall evolution

- Local evolution $\hat{f}_t(k) = -|k|\hat{f}(k) + G_\infty - \widehat{G(f,y)}(k)$

- Periodic setting

  Theorem

  There exist *G** such that there exist stationary solution for $G_\infty \leq G^*$.

  There exist pulsating solutions for $G_\infty > G^*$. In fact, for each $G_\infty > G^*$ there exists an unique *T* such that *f(y,t) = f(y,t+T) - 1.*

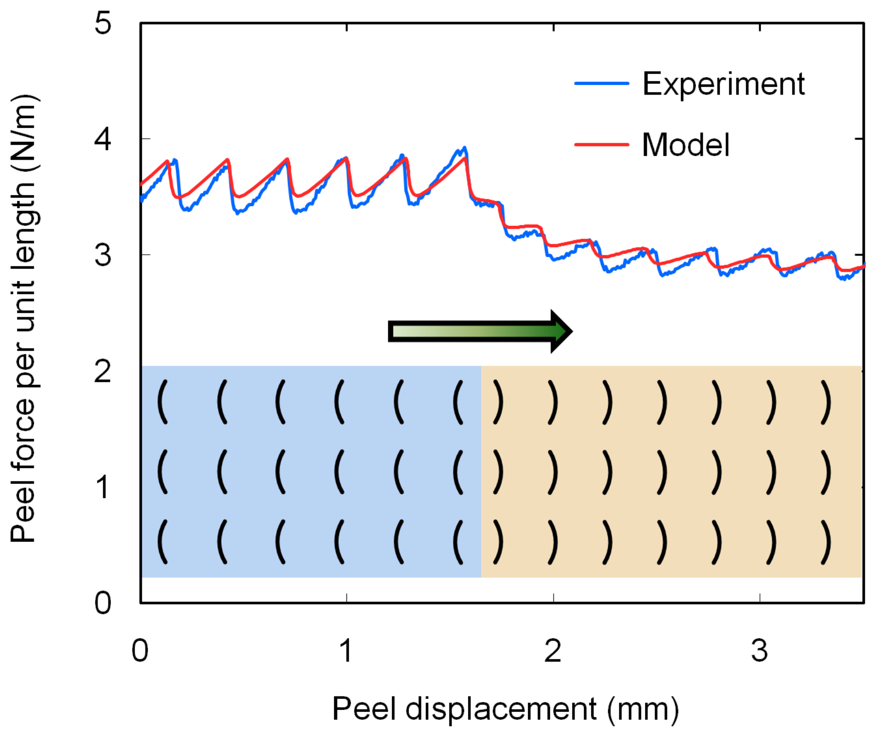
  For G_∞ near G^*, $v \sim (G_\infty - G^*)^{1/2}$

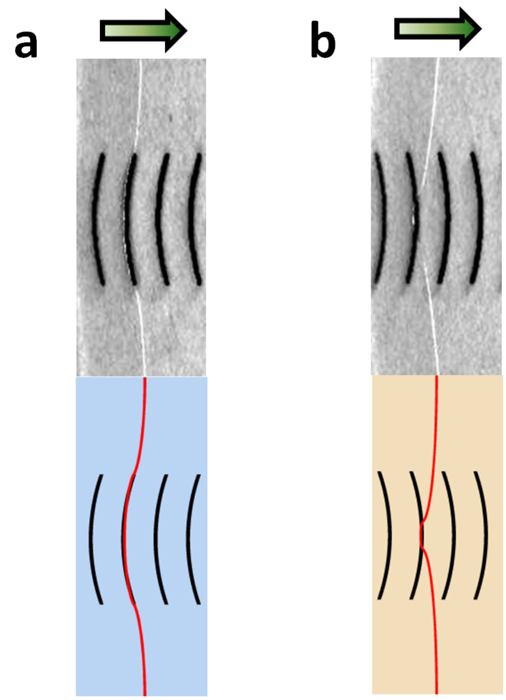
- Overall adhesive strength

  G(n)* – Potentially anisotropic!

N. Dirr, A. Yip, *Int. Free Boundaries* 8: 79-109, 2009
P. Dondl, K. Bhattacharya, Submitted, 2011

Asymmetric adhesive strength





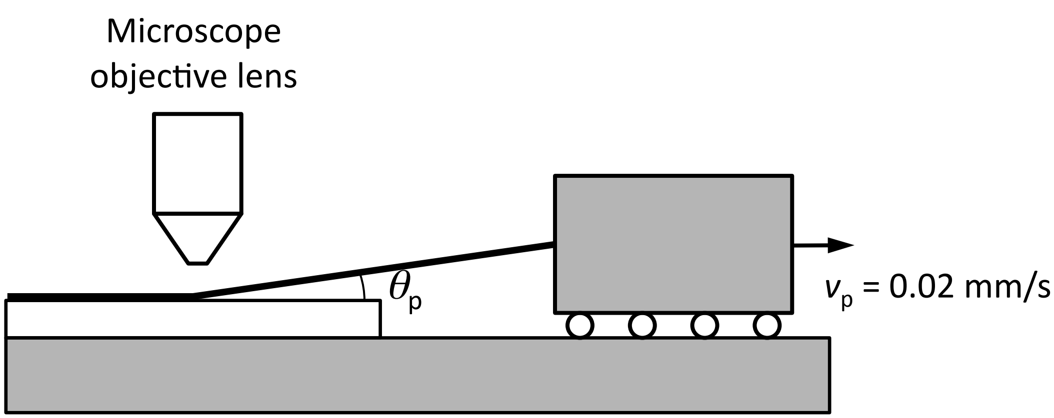
Small peel angle

- Observe Rivlin peel force increases as small angles

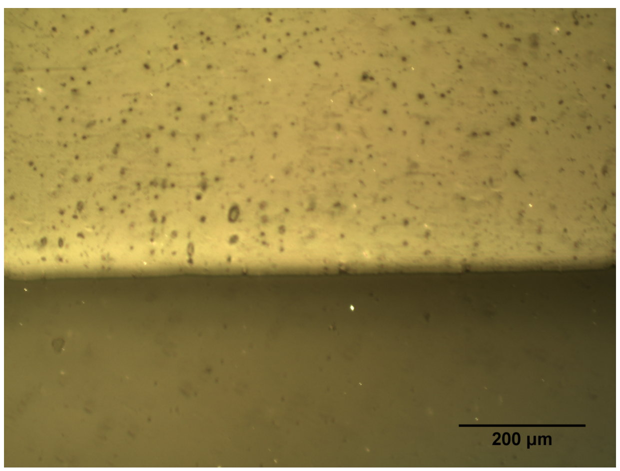
$$F = \frac{G}{1-\cos\theta} \longrightarrow \infty \quad \text{as} \quad \theta \to 0$$

  – Experiments do not show this singular behavior
  – Elastic stretching of the tape becomes important; However, corrections for elasticity due to Kendall still predicts this singular behavior

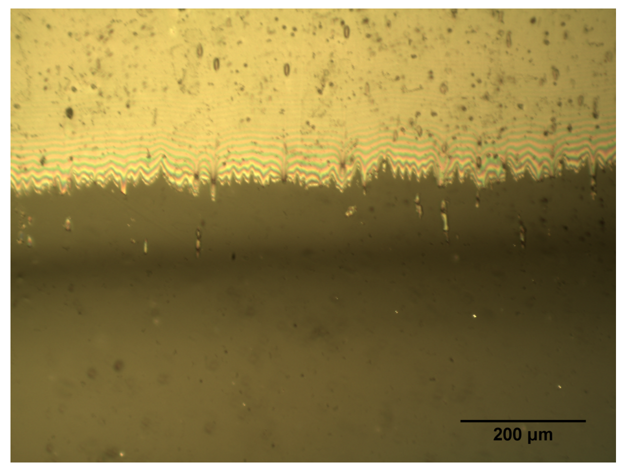
Shallow angle peeling



Rough peel front at small angles



$\theta_p = 11.5^o$



$\theta_p = 4.5^o$

Small peel angle

- Observe Rivlin peel force increases as small angles

$$F = \frac{G}{1 - \cos\theta} \quad \to \infty \quad \text{as} \quad \theta \to 0$$

  – Experiments do not show this singular behavior
  – Elastic stretching of the tape becomes important; However, corrections for elasticity due to Kendall still predicts this singular behavior
- Peel front becomes rough
- Model the tape as Föppl-von Kármán plate and study linear stability of the peel front … stable!
- Observed roughness seems to be related to a shear instability in the adhesive layer

A few comments on fracture

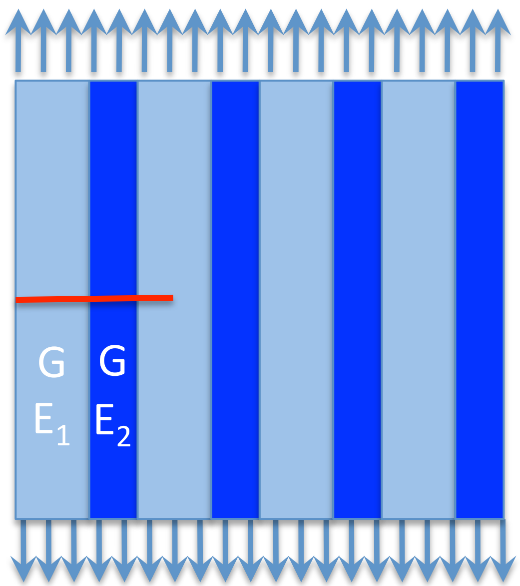
with

Chris Larsen (Worcester Polytechnic Institute)

B. Bourdin (Louisiana State University)

Layered media

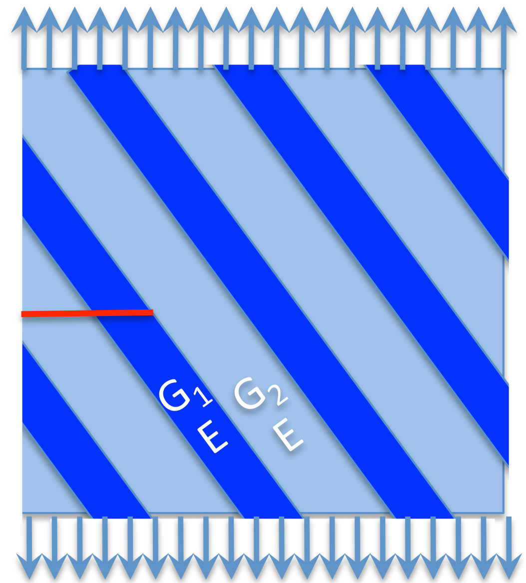
Example 1



Uniform toughness, different modulus

$$G_{\text{eff}} \begin{cases} = G & p << \sqrt{G/E} \\ > G & p >> \sqrt{G/E} \end{cases}$$

Example 2



Different toughness, uniform modulus:
Where will the crack go?

$$K_{II} = 0$$
$$\max J$$
$$\max(J - G)$$

K. Bhattacharya

Variational Approach to Fracture

- Recast Griffith as criticality of a total energy

$$-\frac{\partial E_b}{\partial l} = G_c \quad \leftrightarrow \quad \text{criticality of } E_b(l) + G_c l$$

- Reformulate Griffith in terms of minimization of a total energy with respect to any compatible displacement field and any crack set (Francfort and Marigo, 1993)

$$\min_{u,\Gamma} E(u,\Gamma) := E_b(u,\Gamma) + G_c \mathcal{H}^{N-1}(\Gamma)$$

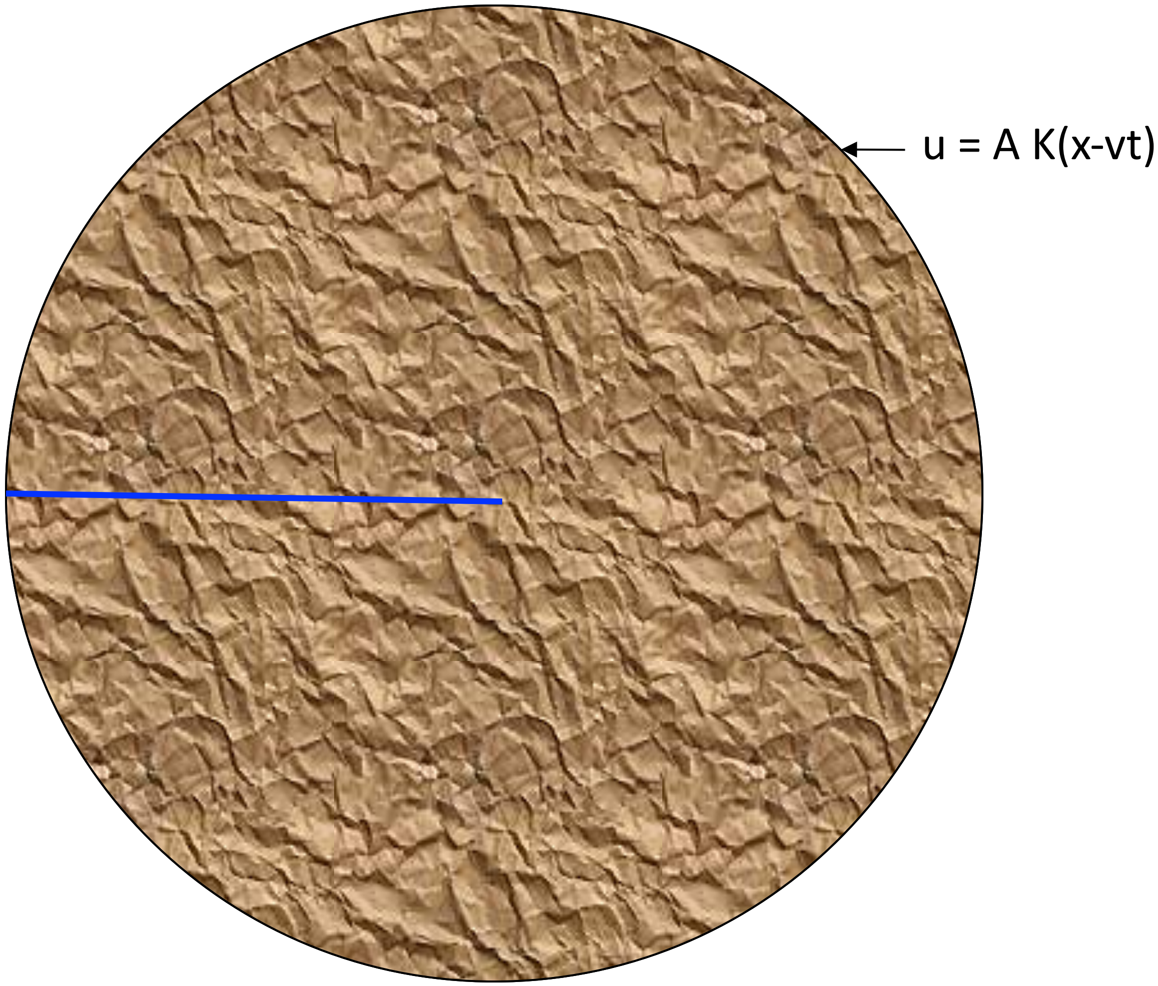
- Phase field approach

$$\mathcal{F}_\varepsilon(u,v) = \int_\Omega v^2 W(\mathrm{e}(u),\theta)\, dx + G_c \int_\Omega \frac{(1-v)^2}{4\varepsilon} + \varepsilon |\nabla v|^2 \, dx.$$

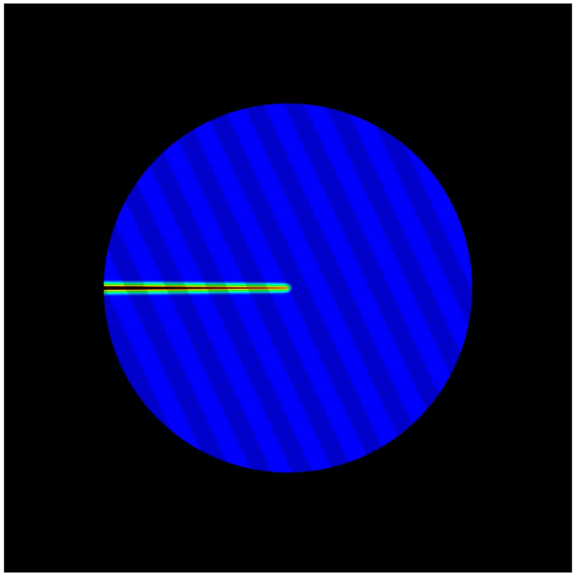
$\mathcal{F}_\varepsilon$ (and its FE approximation) Γ–converges to $\mathcal{E}$

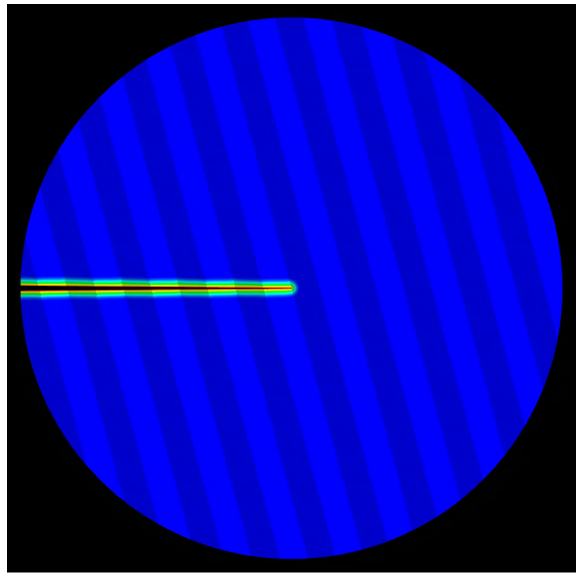
As $\varepsilon \to 0$, the minimizers of $\mathcal{F}_\varepsilon$ converge to that of $\mathcal{E}$

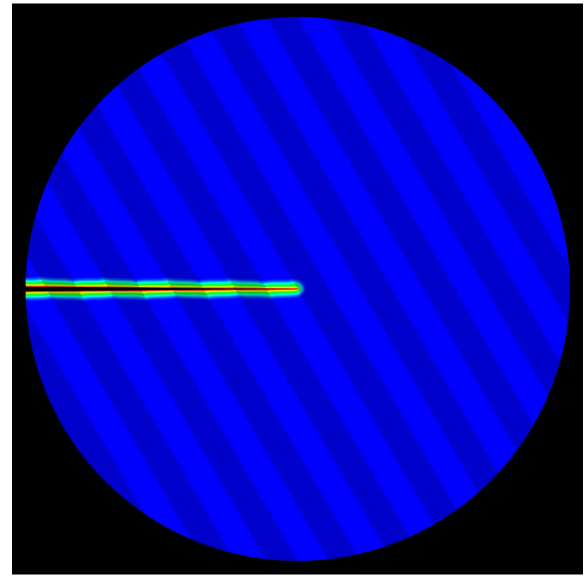
Surfing boundary conditions

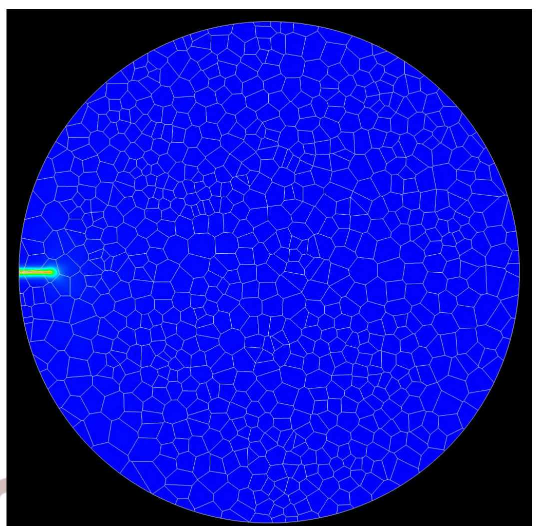


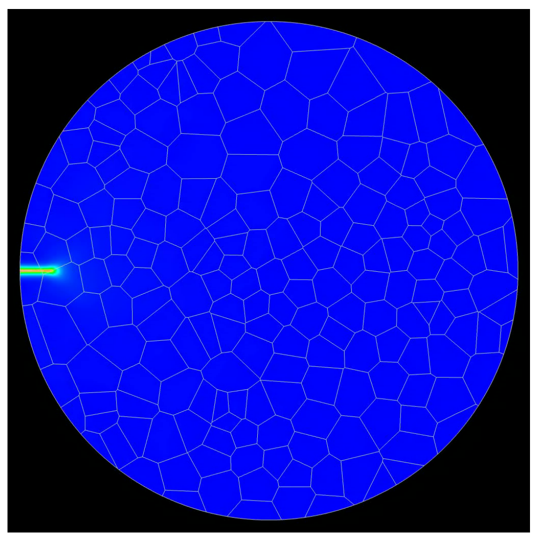
A few movies

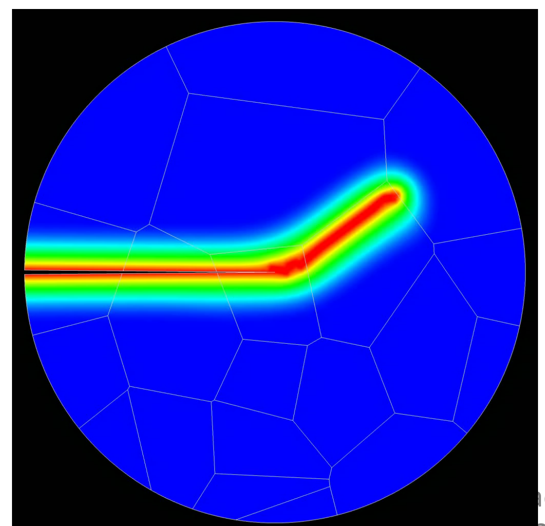










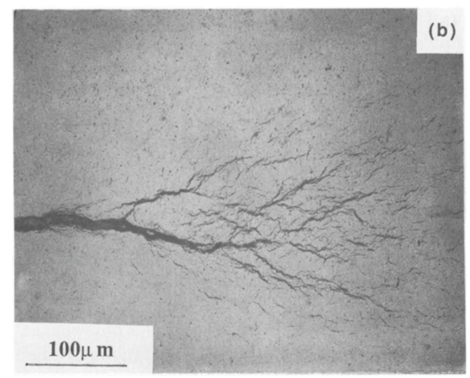


Concluding remarks

- Heterogeneities can give rise to significant enhancement, asymmetry and anisotropy in adhesive strength

- Current work:
  - Optimization
  - Free discontinuity problems

- Acknowledgement. NSF



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