Rheology and Segregation of Granular Mixtures in Dense Flows

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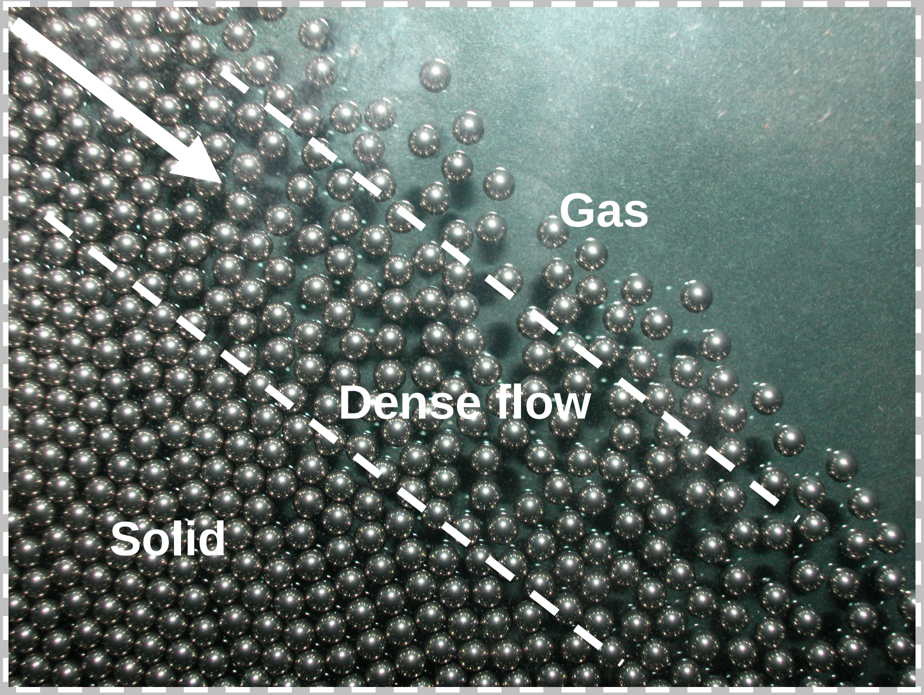
Flowing Granular Material

System

Particles (catalyst pellets, powders, gravel, rice grains, cement, coal, ores, sand, glass beads, ball bearings, …)

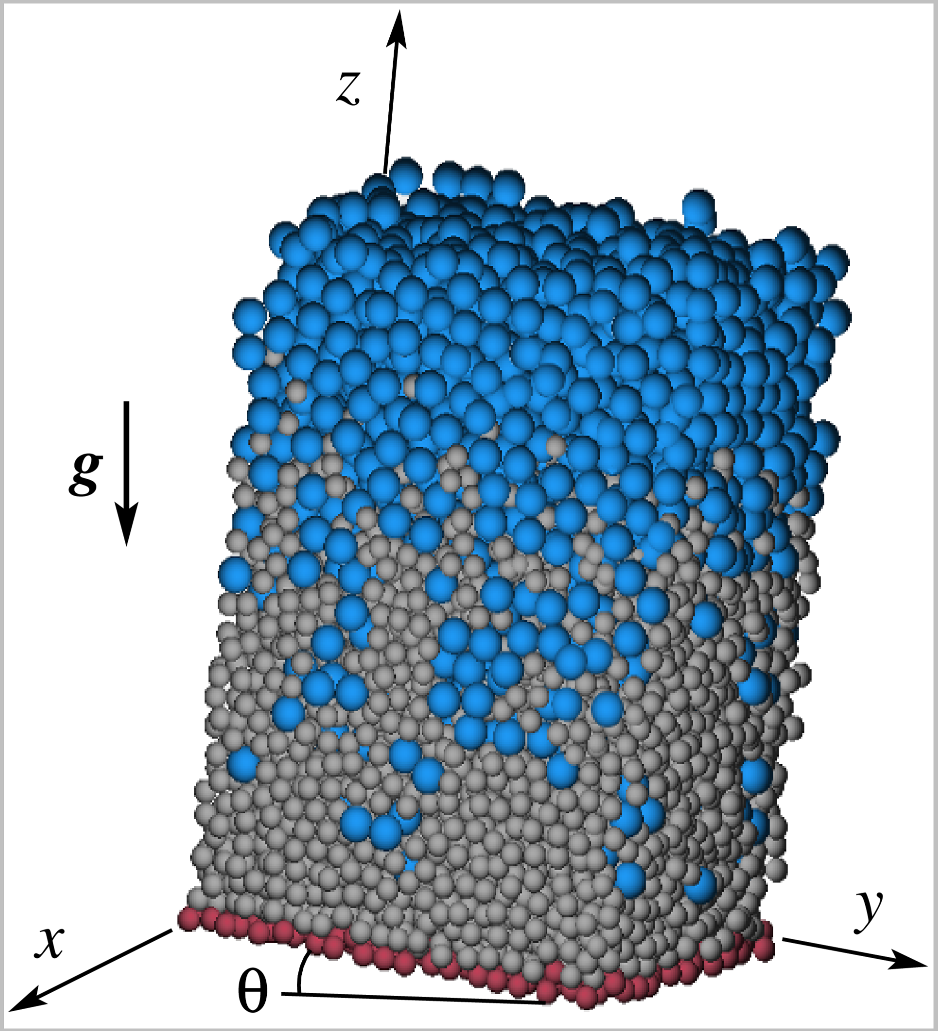
Medium (usually air)



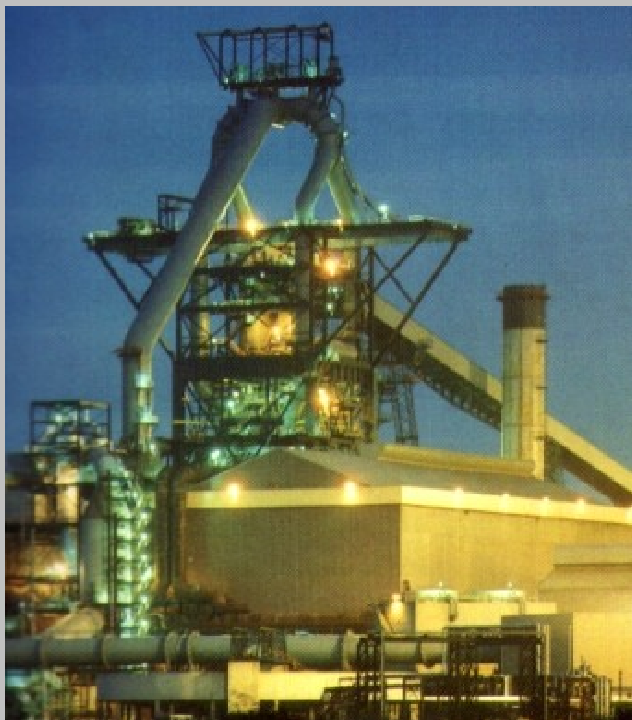
Interparticle interactions dominate – no effect of medium

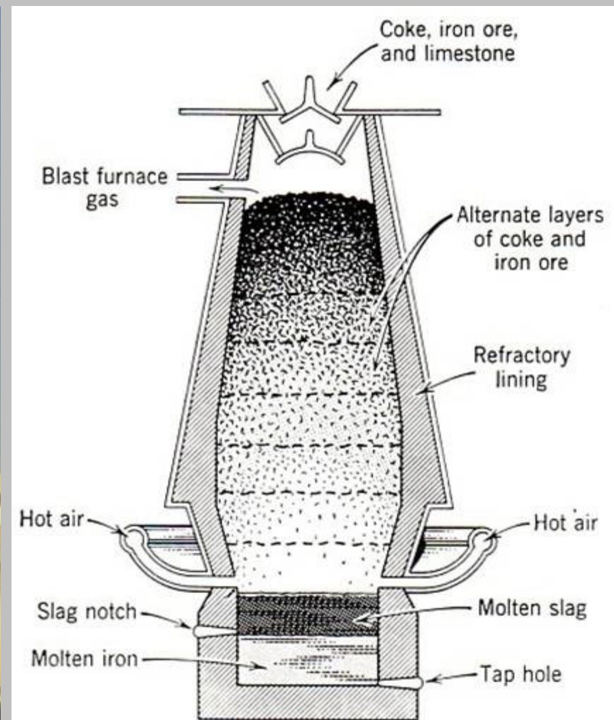
Flowing Granular Mixtures

- Spontaneous segregation due differences in particle properties (e.g., size, density)
- Rheology depends on local concentration of different species.
- *Segregation and rheology are coupled*



Blast Furnace Feeding



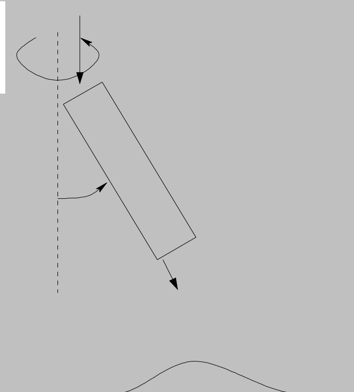


Control of blast furnace: pattern of pouring coke and ore by rotating chute.

Bed porosity determines air flow and temperature distribution.

Segregation of particles during flow affects bed porosity

Blast furnace: steel manufacture



Theoretical Approaches

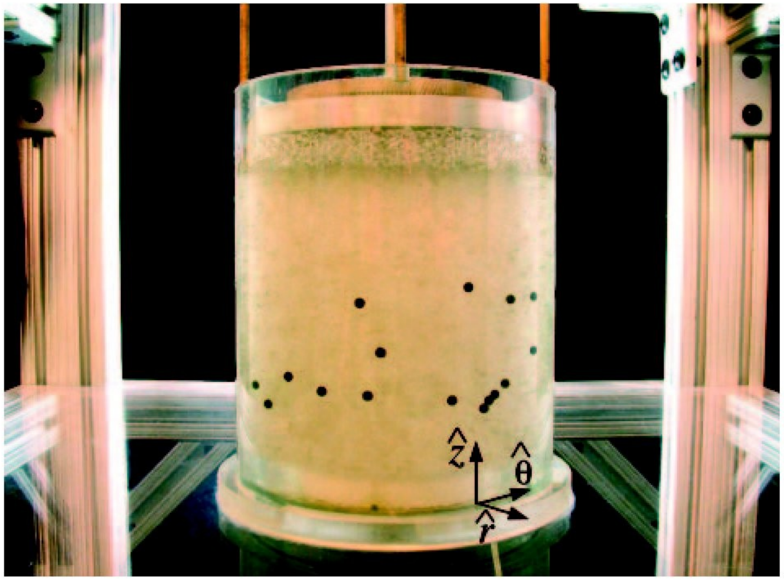
- Kinetic theory
  - JT Jenkins and F Mancini, *Phys. Fluids A* **1**, 2050 (1989); L Trujillo, M Alam,and HJ Herrmann, *Europhys. Lett.* **64**, 190 (2003).
- Partial stresses
  - JMNT Gray and AR Thornton, *Proc. R. Soc. A* **461** 1447 (2005); Y Fan and KM Hill, *New J. Phys.* **13**, 095009 (2011).
- Single particle motion
  - DV Khakhar, JJ McCarthy and JM Ottino, *Phys. Fluids* **9** 3600 (1997)

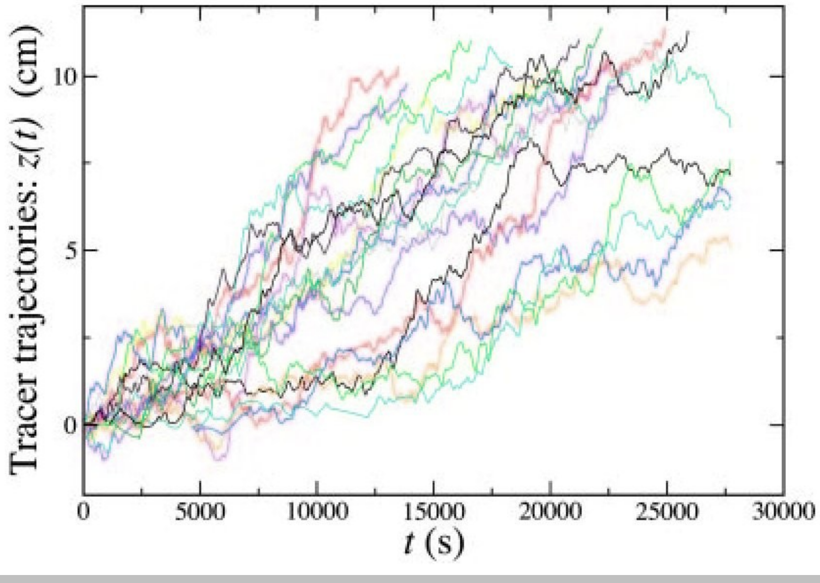
Outline

- Single particle motion (buoyancy, drag force)
- Density segregation of mixtures
- Rheology of mixtures (size and density)
- Combined model predictions
- Conclusions

Acknowledgments
- Anurag Tripathi
- Department of Science and Technology

Single Particle Motion





Lower density tracer particles in sheared annulus

Force $F = (\rho_p - \rho_{pt})Vg$

Song *et al.*, *PNAS*, **102**, 2299-2304 (2005)

Effective Temperature

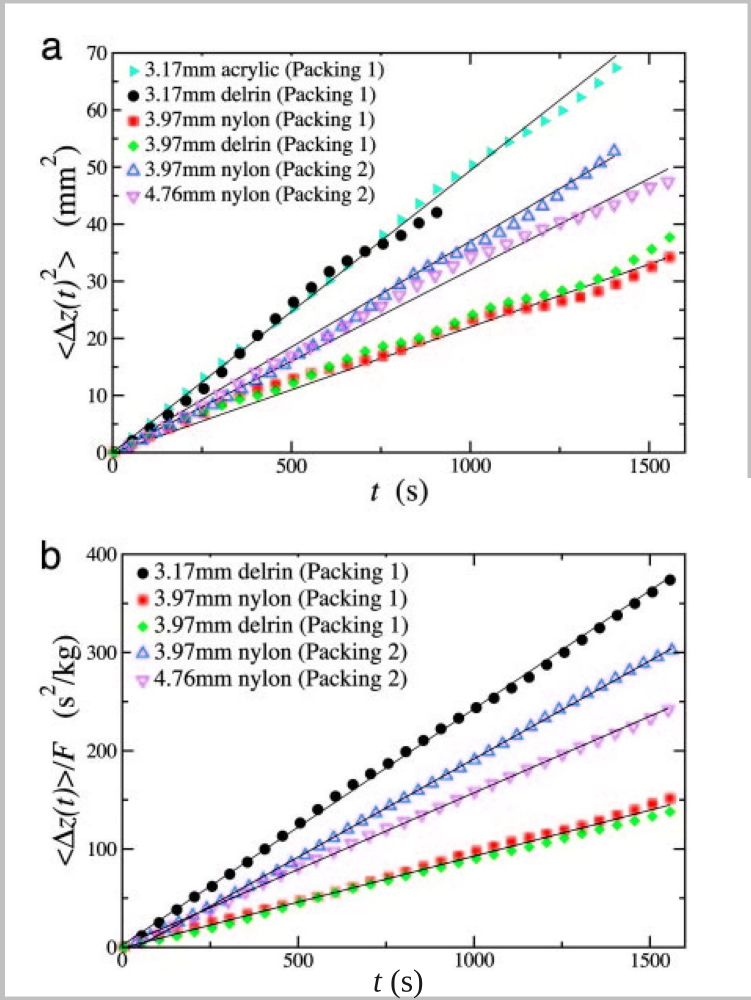
(Not granular temperature)

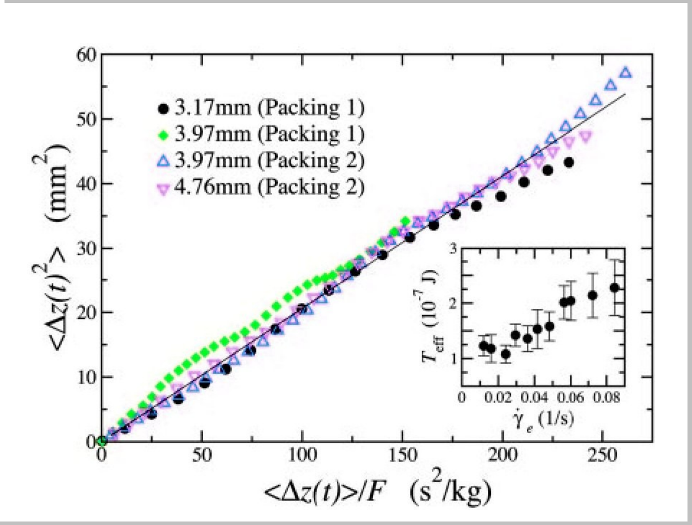
$$\langle \Delta z^2 \rangle = 2Dt$$

Stokes-Einstein

$$D = \frac{T_E}{\xi}$$

Parisi, *PRL*, 1997
Berthier and Barrat, *JCP*, 2002



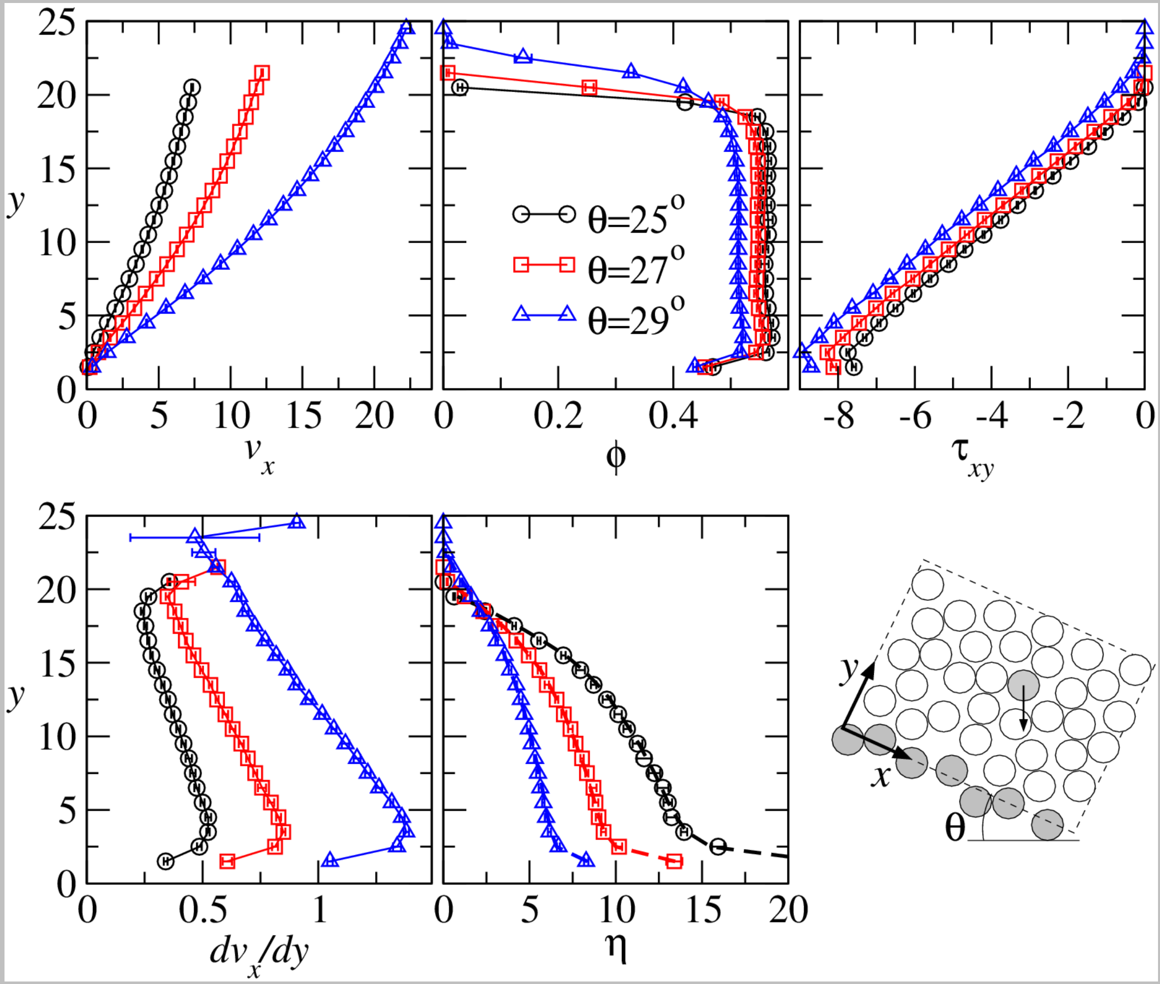


$$\langle \Delta z \rangle = Ft/\xi$$

$$\langle \Delta z^2 \rangle = 2T_E \frac{\langle \Delta z \rangle}{F}$$

System Definition

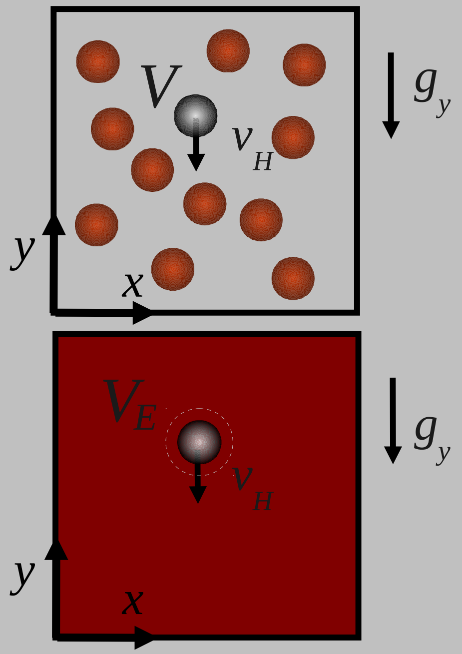
Sedimentation of a higher mass particle in a flowing layer of otherwise identical particles with diameter *d*



- DEM simulations in 3d – soft particles
- Viscoelastic force model (L3 model of Silbert *et al.* 2001)

Theory: Single Particle Motion

Effective medium approach



m: mass, *V*: volume of particles
H: Heavy; *L*: Light

Net force on heavy particle

$$F_H = m_H g_y - \rho V_E g_y$$

weight buoyancy

Density: $\rho = (m_L / V)\phi$

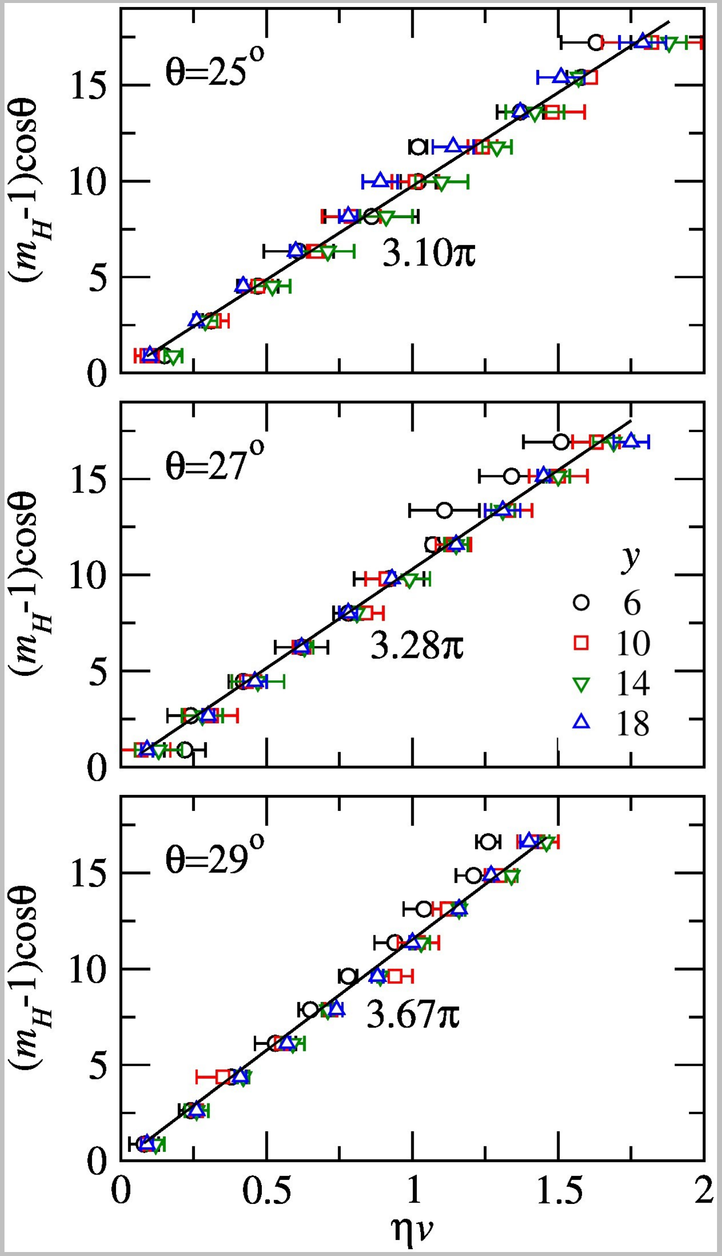
Effective volume: $V_E = V / \phi$

$$F_H = (m_H - m_L) g_y$$

Drag force: Modified Stokes Law

$$F_d = c \pi \eta v_H d$$

Drag Force



Terminal velocity $\left(F_H = F_d\right)$

$$\left(m_H - m_L\right) g_y = c\,\pi\,\eta\,v_H\,d$$

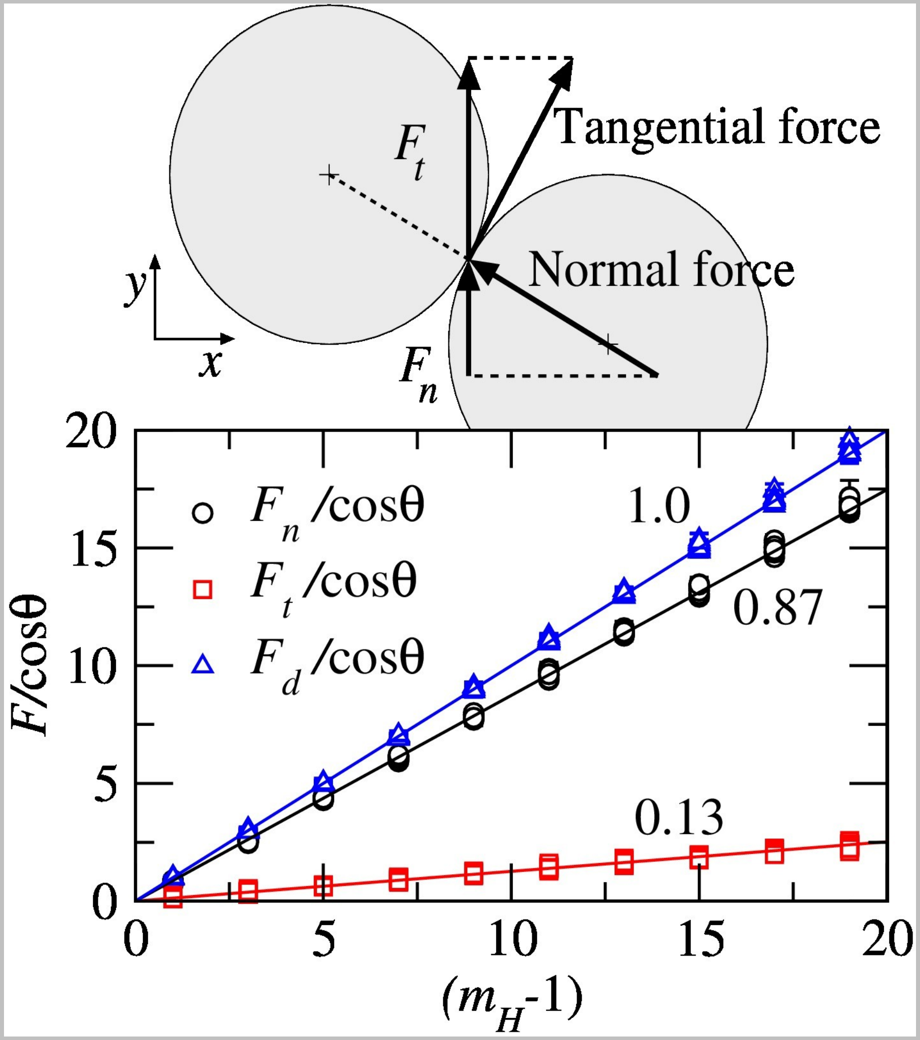
Dimensionless form

$$\left(\bar{m}_H - 1\right)\cos\theta = c\,\pi\,\bar{\eta}\,\bar{v}_H$$

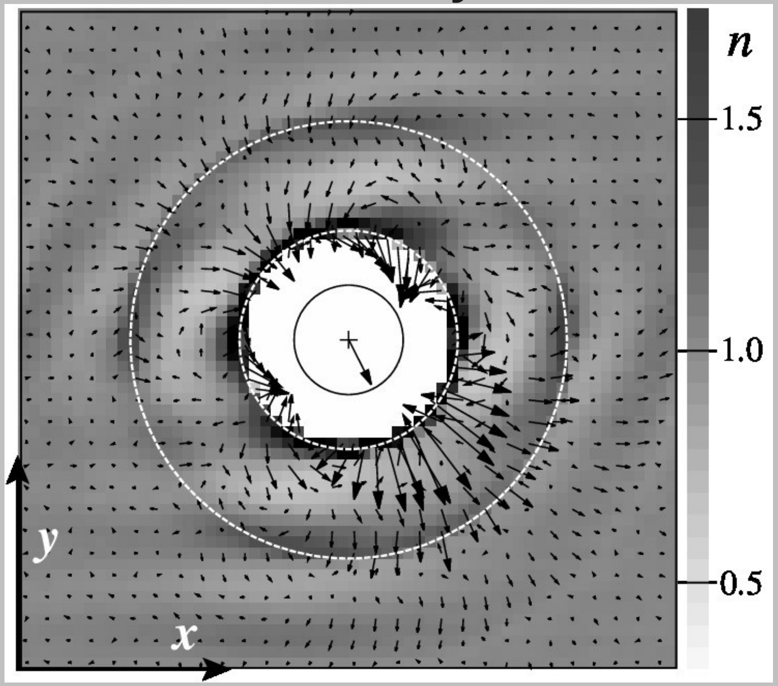
Low Re (0.01-0.18)

Tripathi and Khakhar, *PRL*, 2011

Details



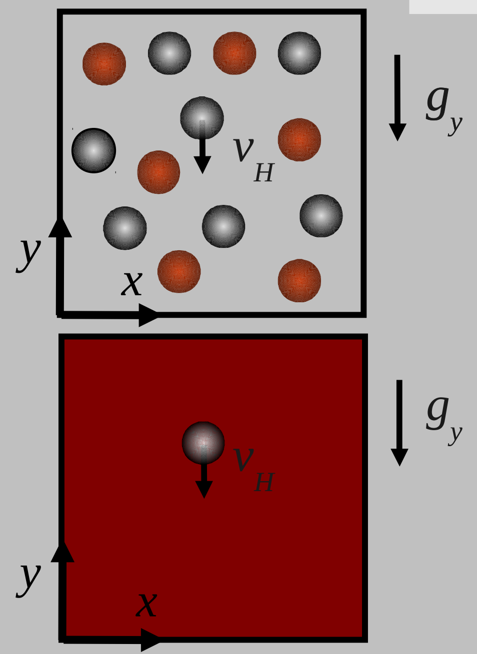
Disturbance velocity and Number density



Tripathi and Khakhar, *PRL*, 2011

Theory: Mixtures

Effective medium approach



m: mass, V: volume of particles
H: Heavy; L: Light
f: number fraction of H

Net force on heavy particle

$$F_H = m_H g_y - \langle\rho\rangle V_E g_y$$

weight buoyancy

Density: $\langle\rho\rangle = [m_H f + m_L(1-f)]\phi/V$

Effective volume: $V_E = V/\phi$

$$F_H = (1-f)(m_H - m_L) g_y$$

Segregation velocity

$$v_H = F_H / c\pi\eta d$$

Theory

Segregation flux:

$$J_H^s = nfv_H = -\frac{nD}{T_E}(m_H - m_L)g_y f(1-f)$$

$$T_E = D[c\pi\eta d]$$

Diffusion flux:

$$J_H = -nD\frac{df}{dy}$$

Equilibrium:

$$J_H^s + J_H = 0$$

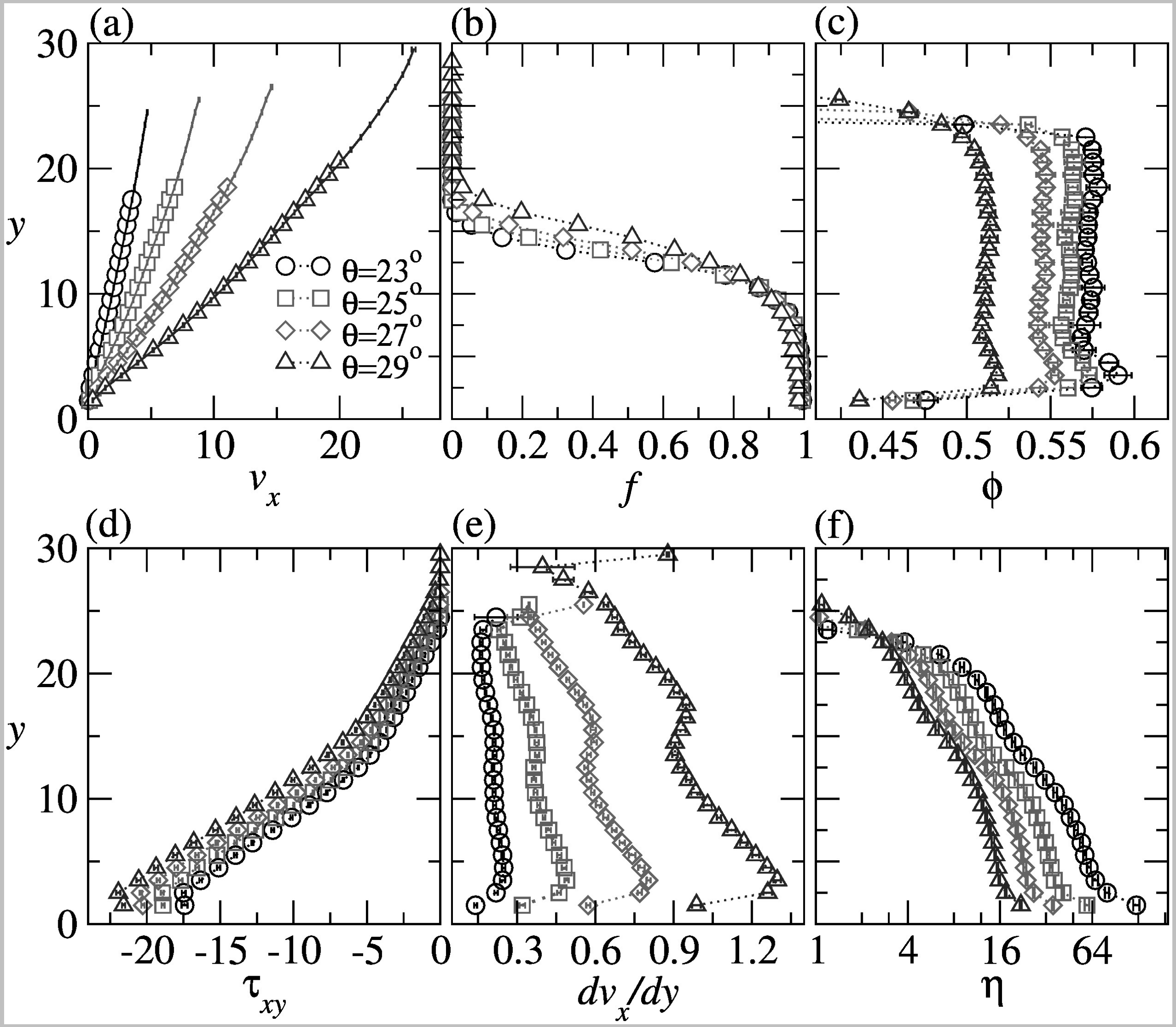
Self-diffusivity = Binary diffusivity
for equal size particles

Equilibrium profile

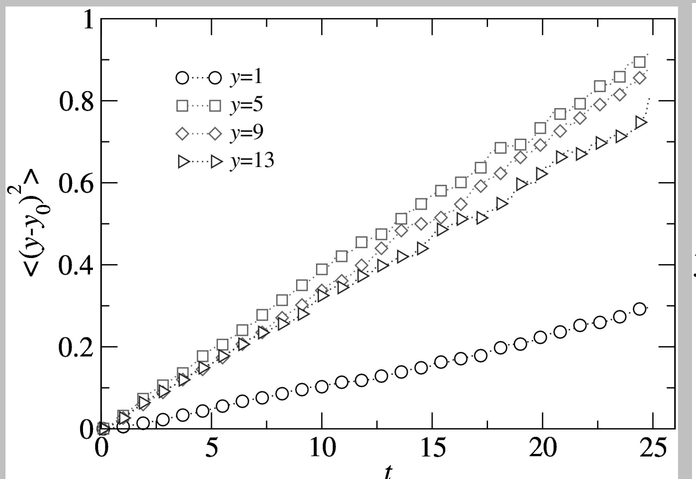
$$\frac{1}{f(1-f)}\frac{df}{dy} = -\frac{(m_H - m_L)g_y}{T_E}$$

Sarkar and Khakhar, *EPL*, 2008

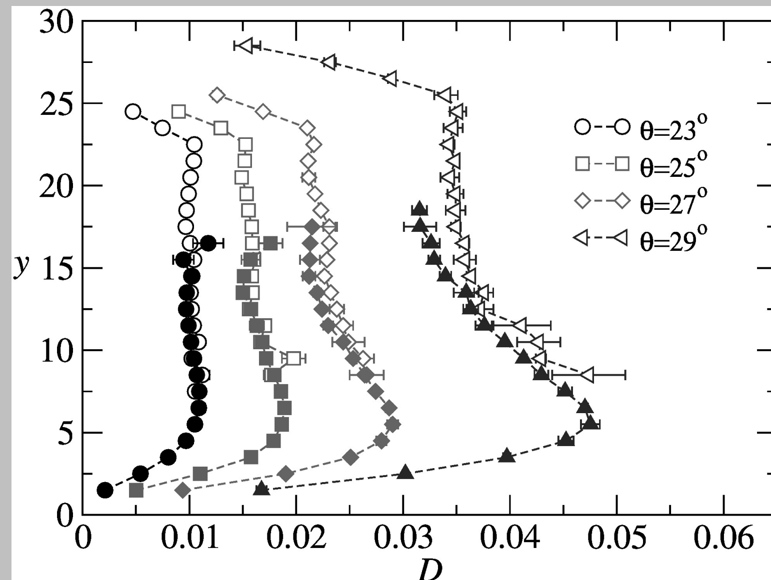
Mixture Profiles



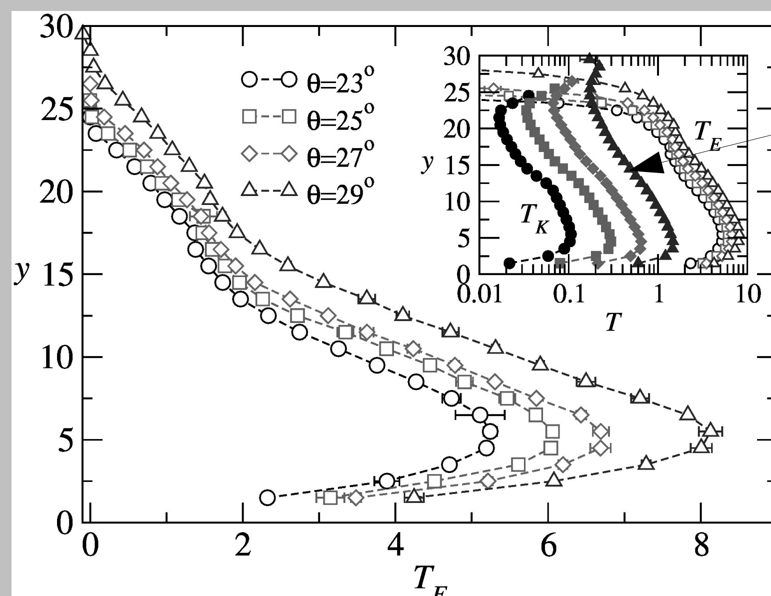
Results



Mean square displacement



Diffusivity
(D_{yy})

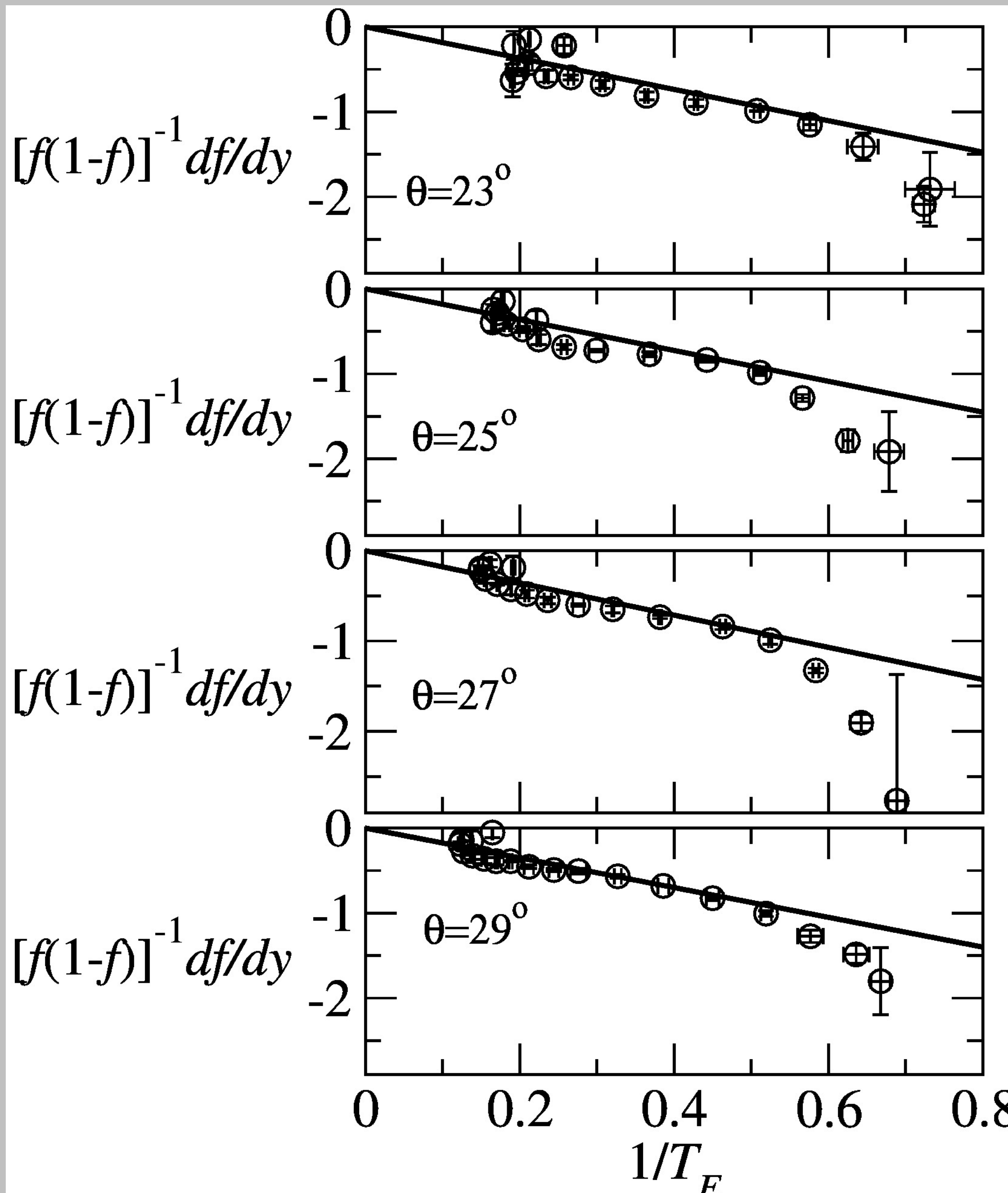


Granular temperature

Effective temperature

IIT Bombay

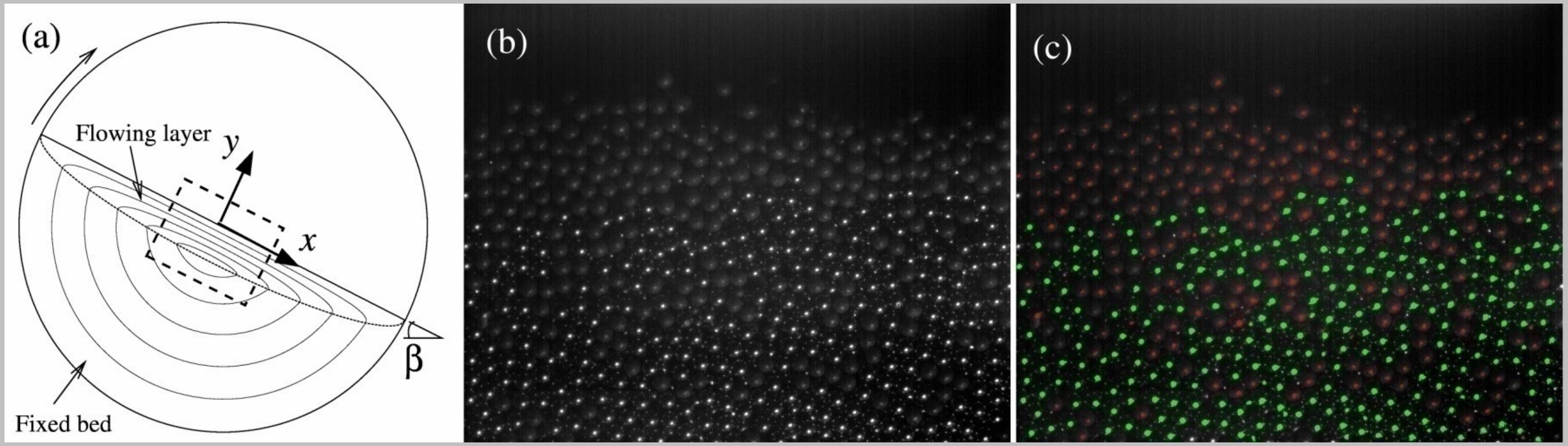
Results



$[f(1-f)]^{-1} df/dy$
$\theta=23^o$
$[f(1-f)]^{-1} df/dy$
$\theta=25^o$
$[f(1-f)]^{-1} df/dy$
$\theta=27^o$
$[f(1-f)]^{-1} df/dy$
$\theta=29^o$
0
-1
-2
0
0.2
0.4
0.6
0.8
$1/T_E$

Experimental Study

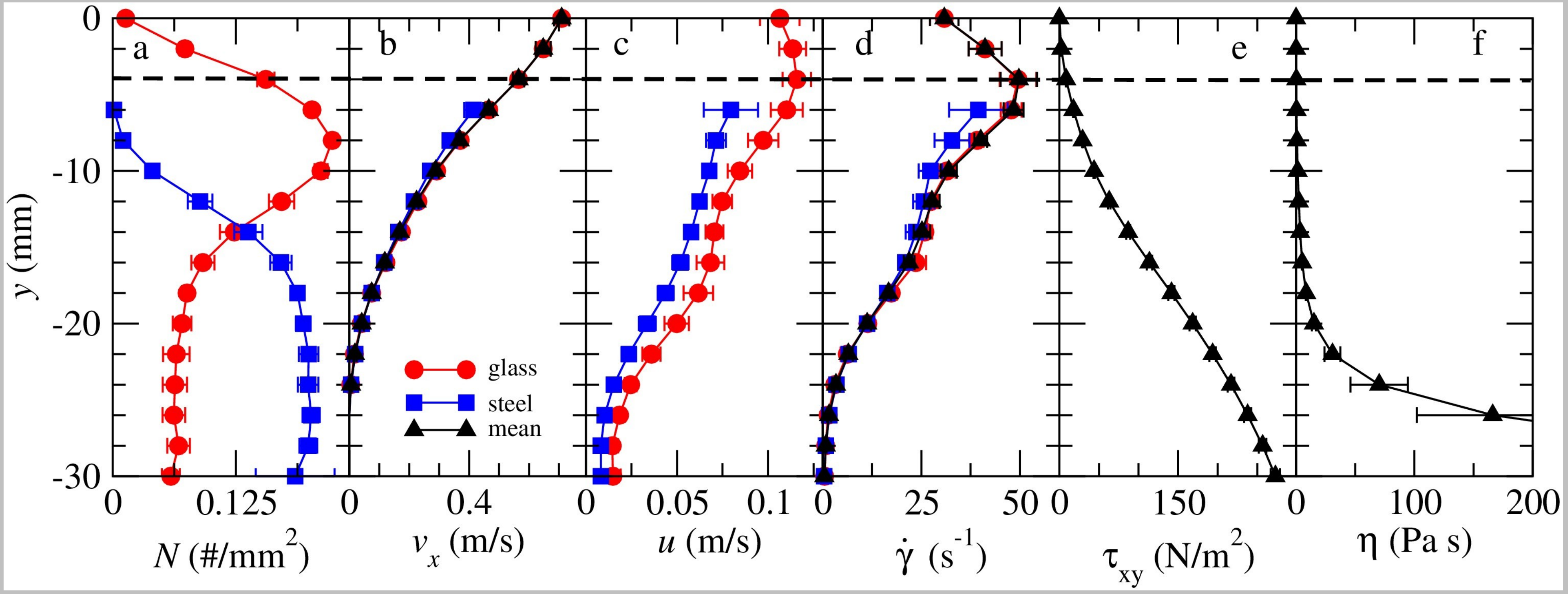
Equal size particles – different density



2 mm steel balls + 2 mm glass beads
High speed video (500 frames/s), Image analysis

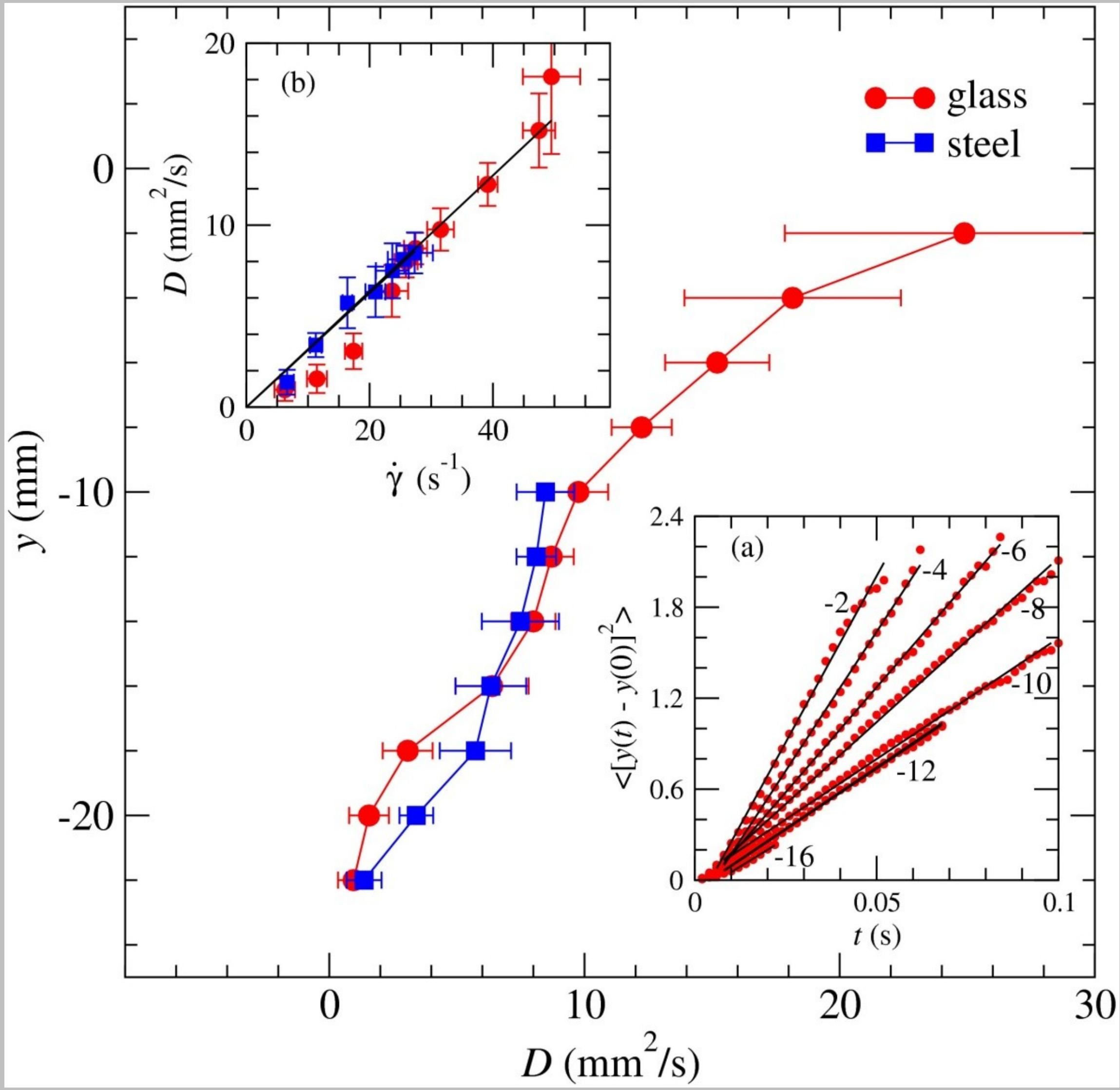
Sarkar and Khakhar, *EPL*, 2008

Profiles



0
-10
-20
-30
y (mm)
a
b
c
d
e
f
0
0.125
N (#/mm²)
0
0.4
v_x (m/s)
0
0.05
0.1
u (m/s)
0
25
50
γ̇ (s⁻¹)
0
150
τ_xy (N/m²)
0
100
200
η (Pa s)
glass
steel
mean

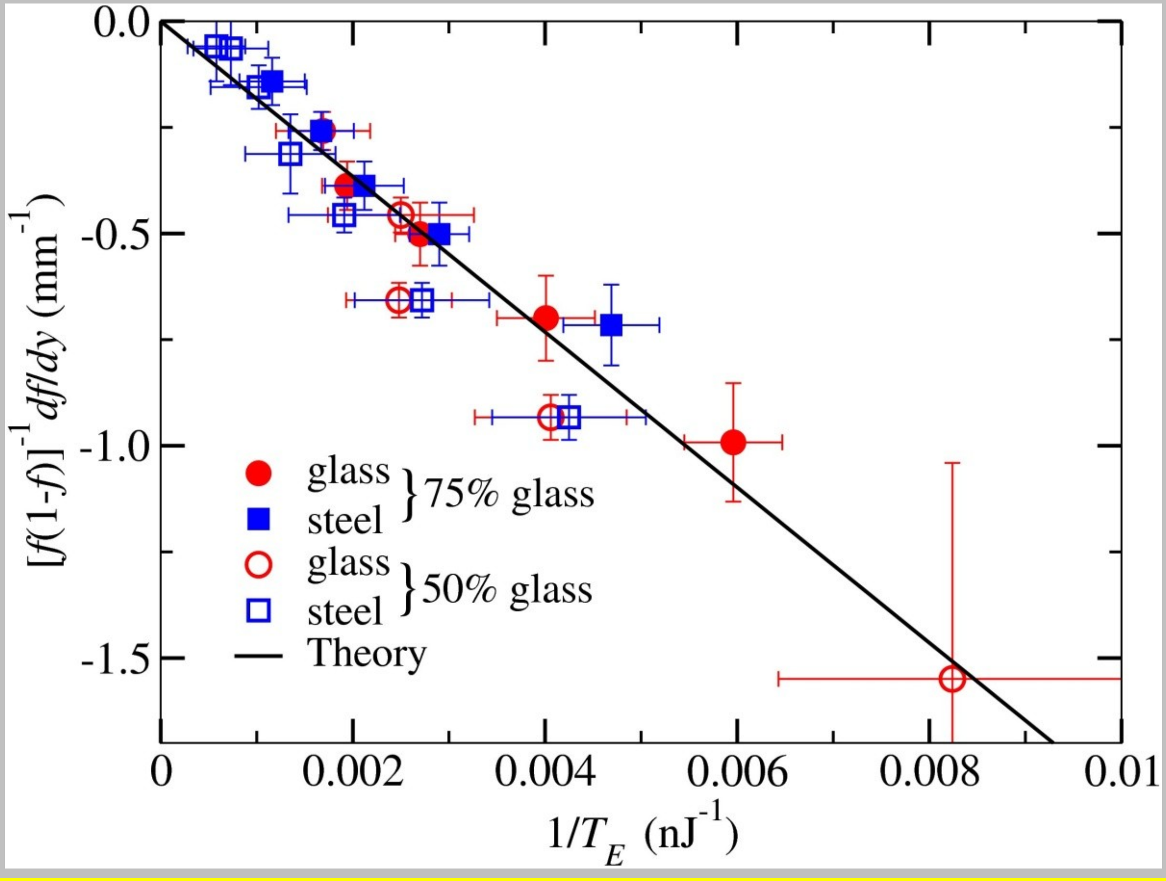
Diffusivity



Diffusivity of glass beads and steel balls is nearly the same

Diffusivity scales with shear rate

Comparison to Theory



Good agreement between theory and experiment – no fitted parameters; two compositions

Sarkar and Khakhar, *EPL*, 2008

Rheology of Dense Flows

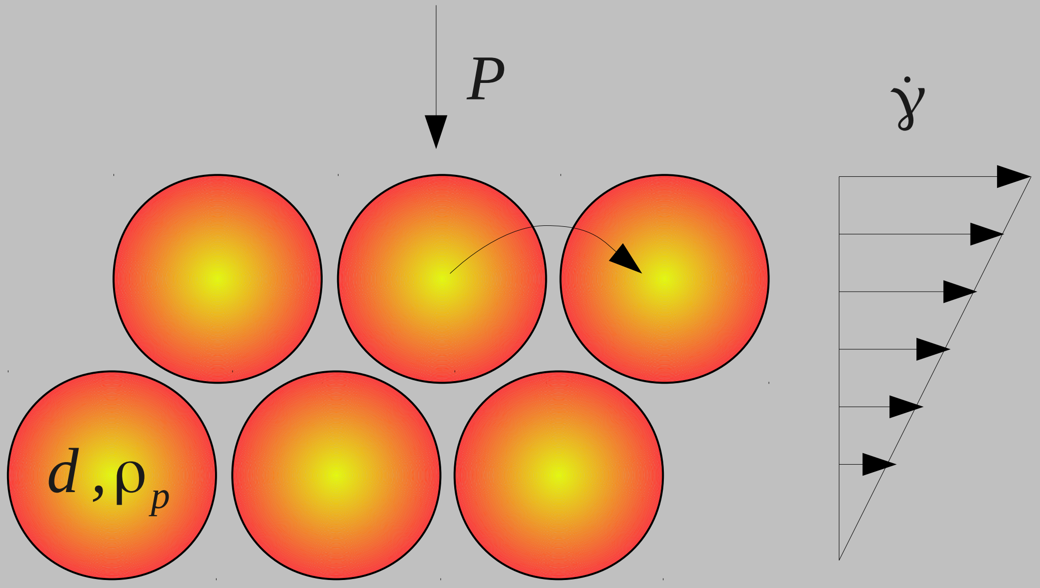
Macroscopic time scale

$$1/\dot{\gamma}$$

Microscopic time scale

$$d\left(\rho_p/P\right)^{1/2}$$

P : pressure
$\dot{\gamma}$: shear rate
ρ_p, d : particle density, diameter



Inertial number

$$I = \frac{\dot{\gamma}\, d}{\left(P/\rho_p\right)^{1/2}}$$

Dense flows: Low I

Rheology of Dense Flows

Friction coefficient

$$\mu = \frac{\tau_{xy}}{P} = \mu(I)$$

Viscosity

$$\eta = \frac{\tau_{xy}}{\dot{\gamma}} = \frac{\mu P}{\dot{\gamma}}$$

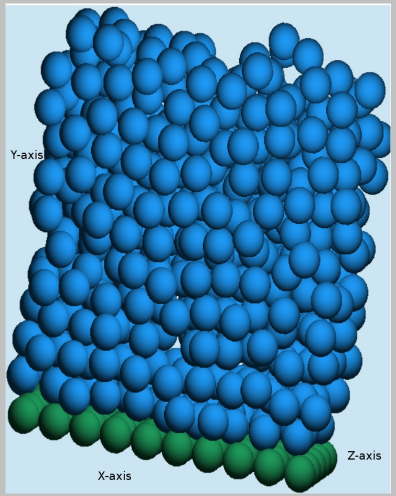
Solid volume fraction

$$\phi = \phi(I)$$

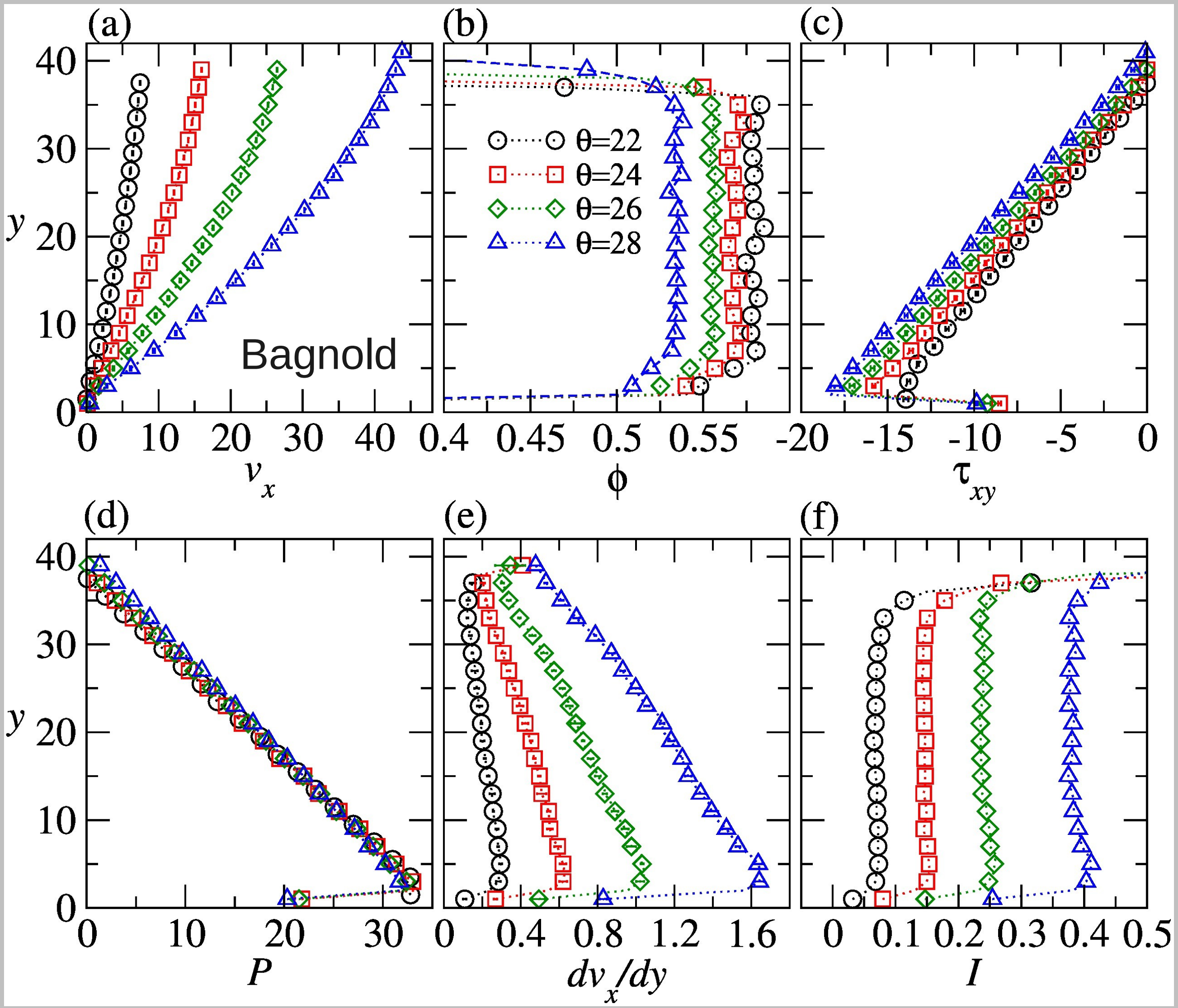
empirical functions

Pouliquen et al. 2004
Da Cruz et al. 2005
Lois et al. 2005

Shear Flow Results

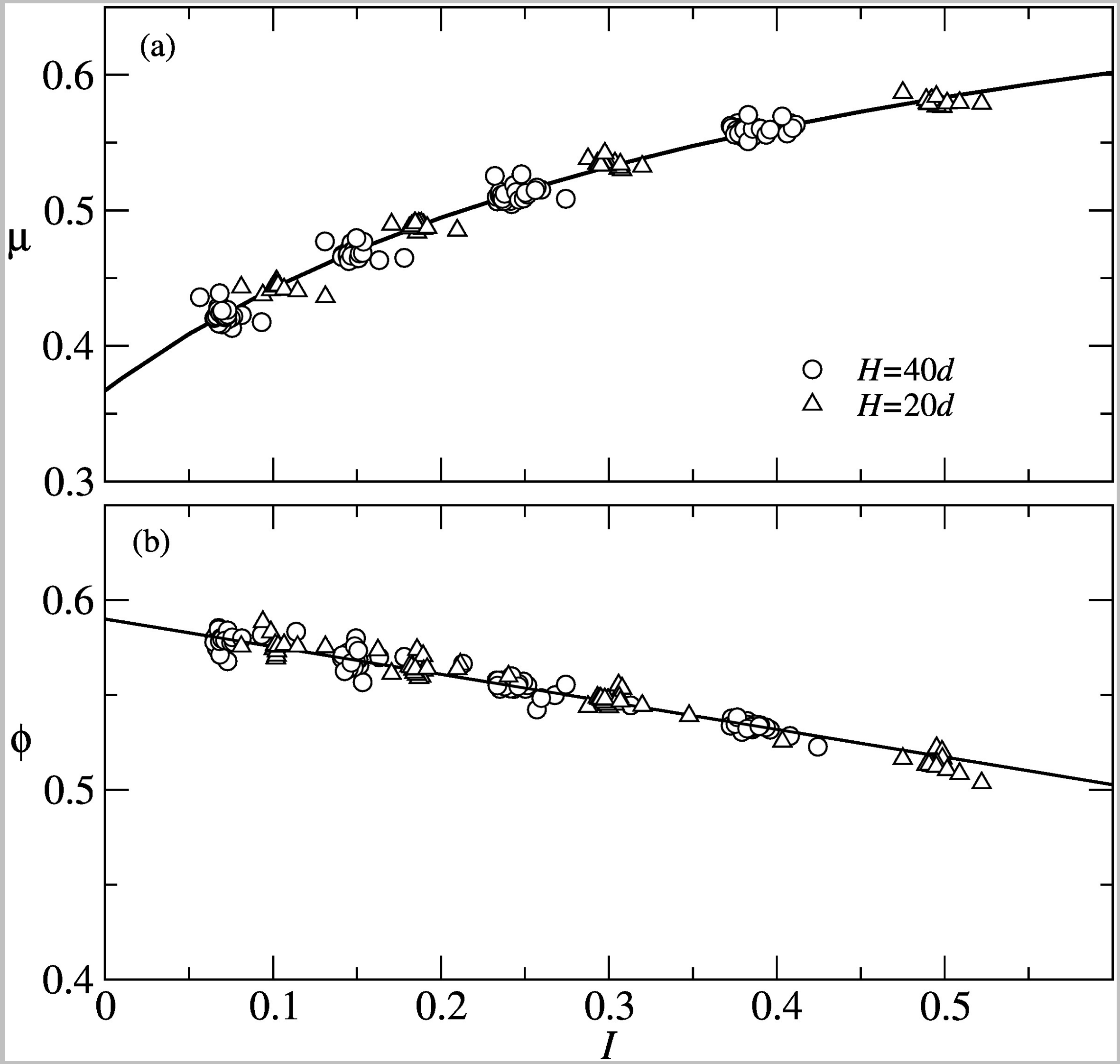


Frictional, inelastic particles



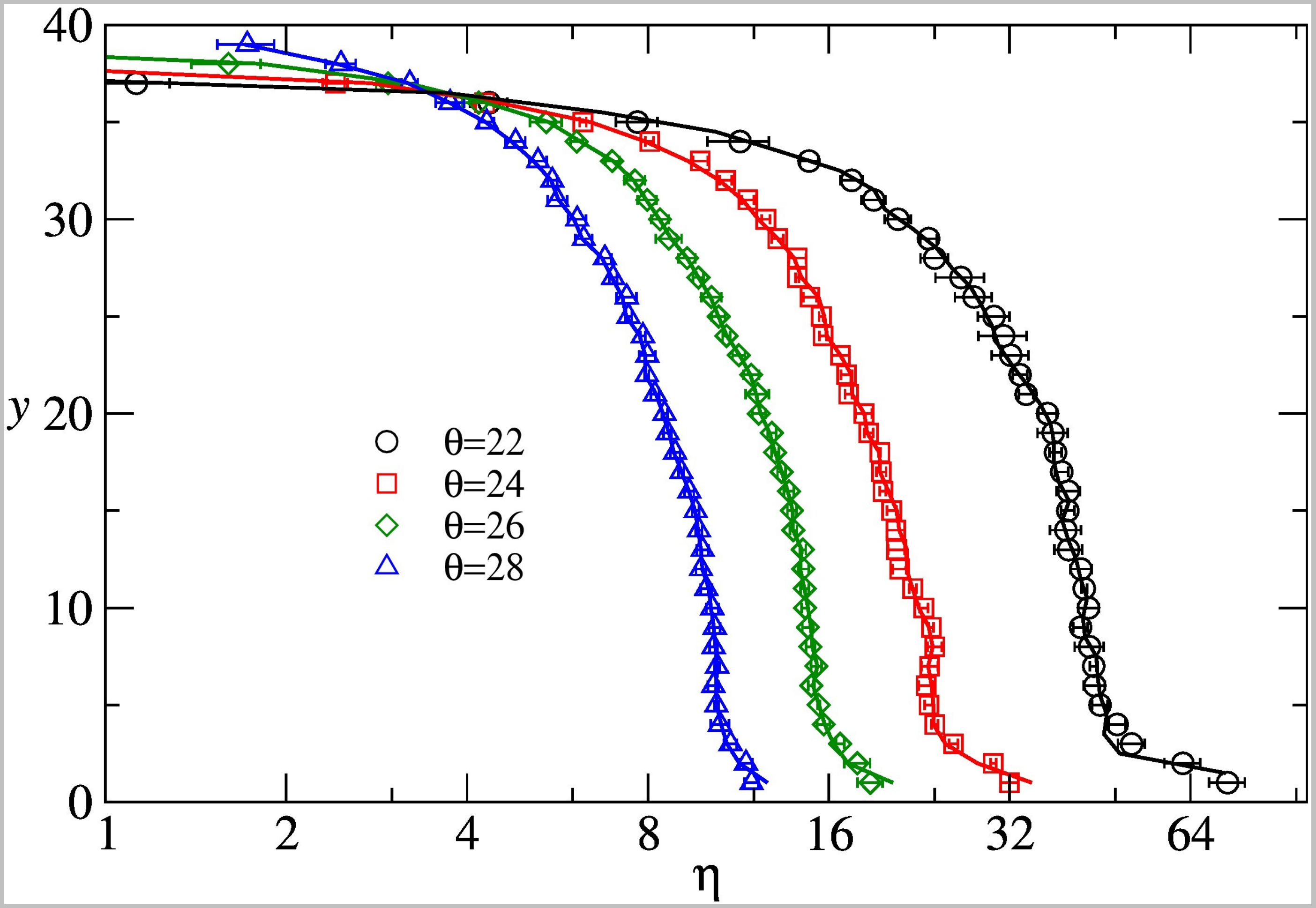
Tripathi and Khakhar, *Phys. Fluids*, 2011

$\mu(I)$, $\phi(I)$



Tripathi and Khakhar, *Phys. Fluids,* 2011

Viscosity



Symbols: DEM simulation results. **Lines:** Theory

Tripathi and Khakhar, *Phys. Fluids*, 2011

Extension to Mixtures

Inertial number

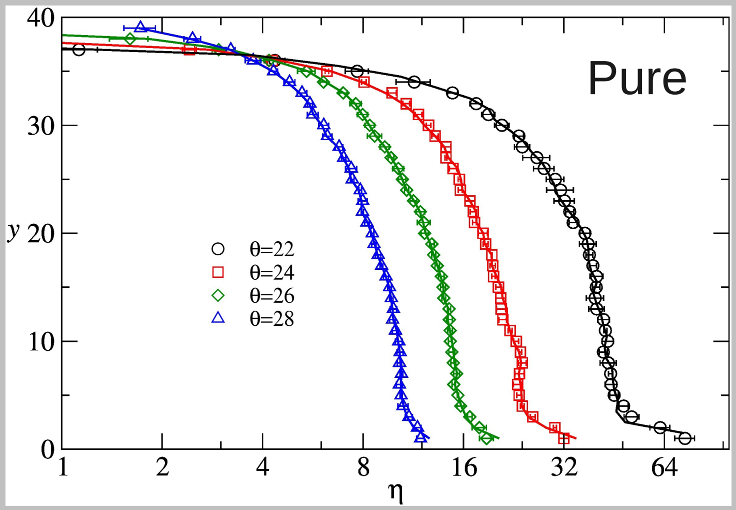
$$I = \frac{\dot{\gamma}\, d_{mix}}{\sqrt{P/\rho_{p,mix}}}$$

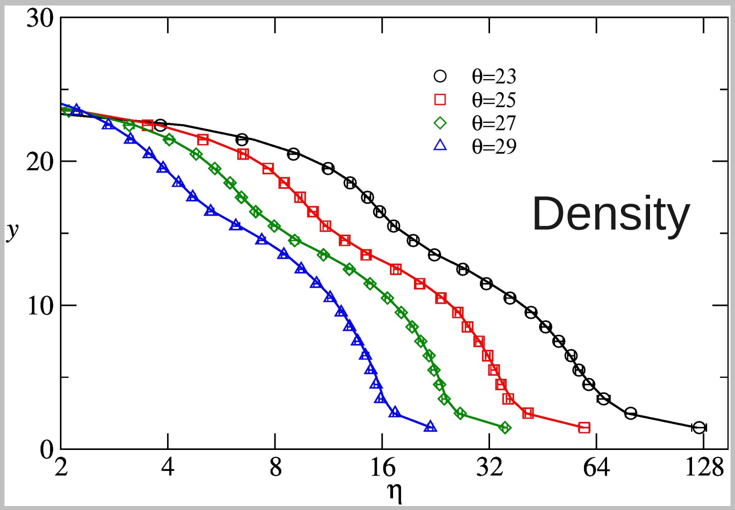
$$d_{mix} = \frac{d_1 \phi_1 + d_2 \phi_2}{\phi_1 + \phi_2}$$

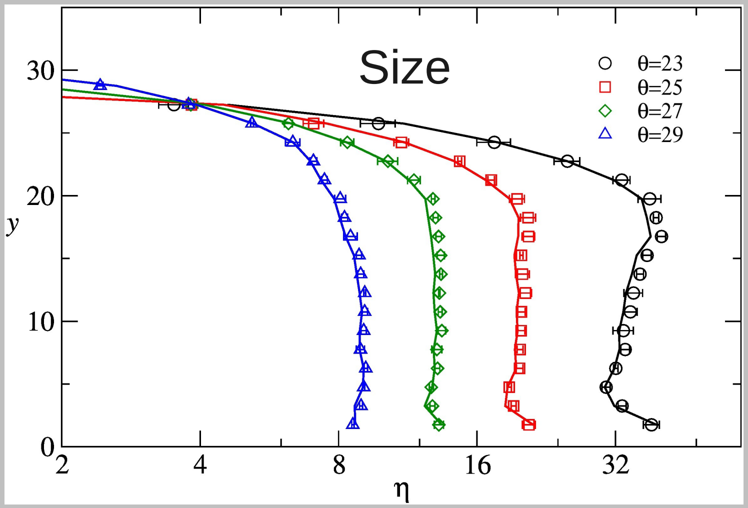
$$\rho_{p,mix} = \frac{\rho_{p,1} \phi_1 + \rho_{p,2} \phi_2}{\phi_1 + \phi_2}$$

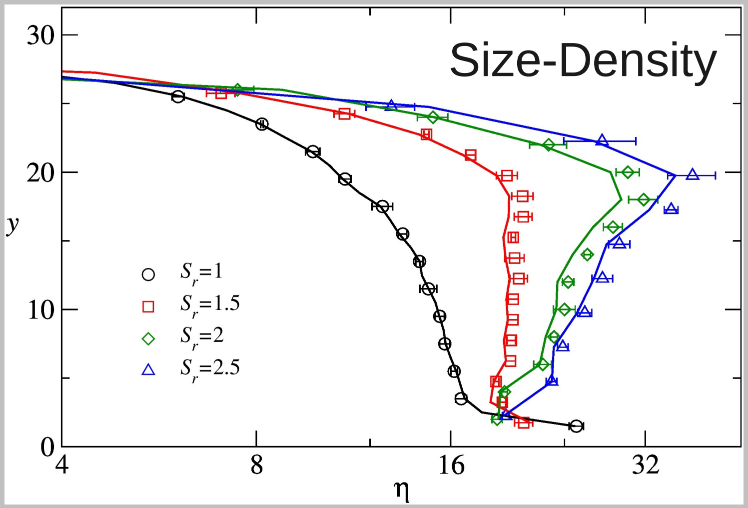
Tripathi and Khakhar, *Phys Fluids,* 2011

Results



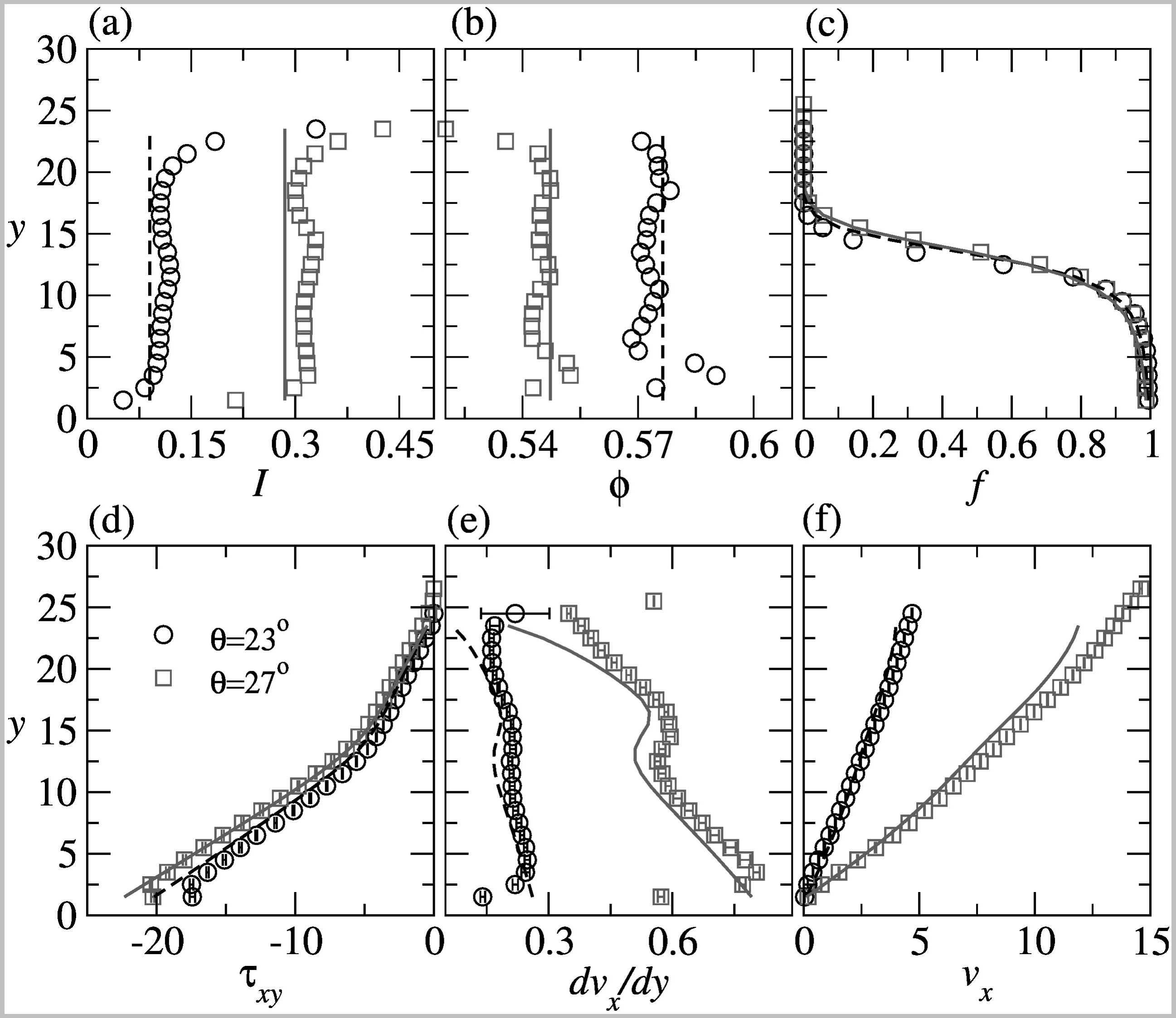






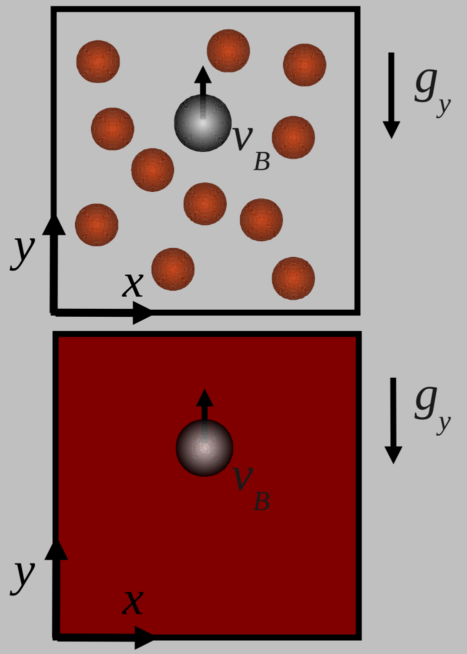
Symbols: DEM simulation results. **Lines:** Theory

Full Model Predictions



Theory: Single Particle Motion

Different size particles



m: mass, V: volume of particles
B: Big, S: Small

Net force on big particle

$$F_B = m_B g_y - \rho V_E g_y$$

weight buoyancy

Density: $\rho = (m_S / V_S)\phi$

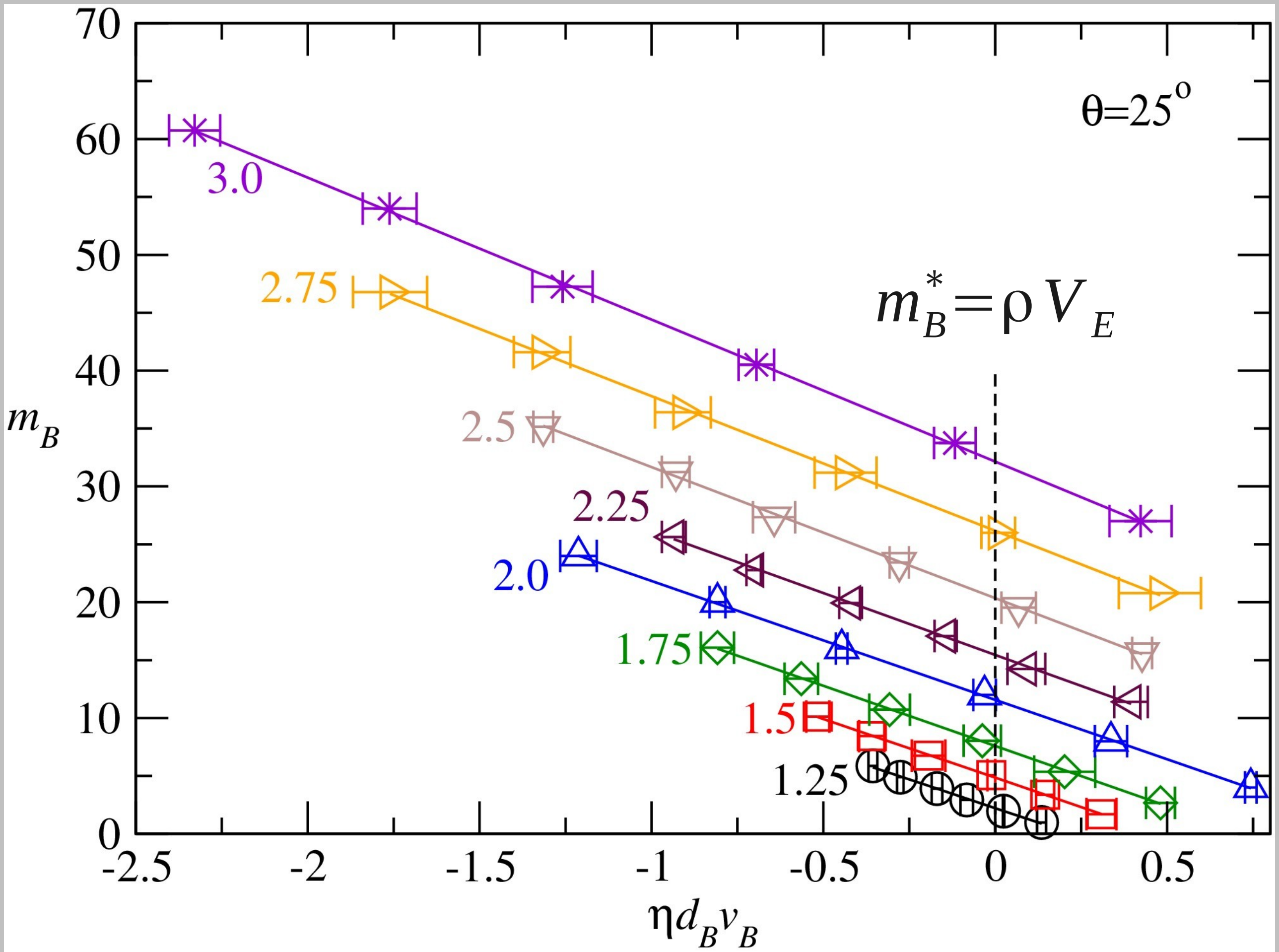
Effective volume: $V_E = ?$

Drag force: Modified Stokes Law

$$F_d = c \pi \eta v_B d_B$$

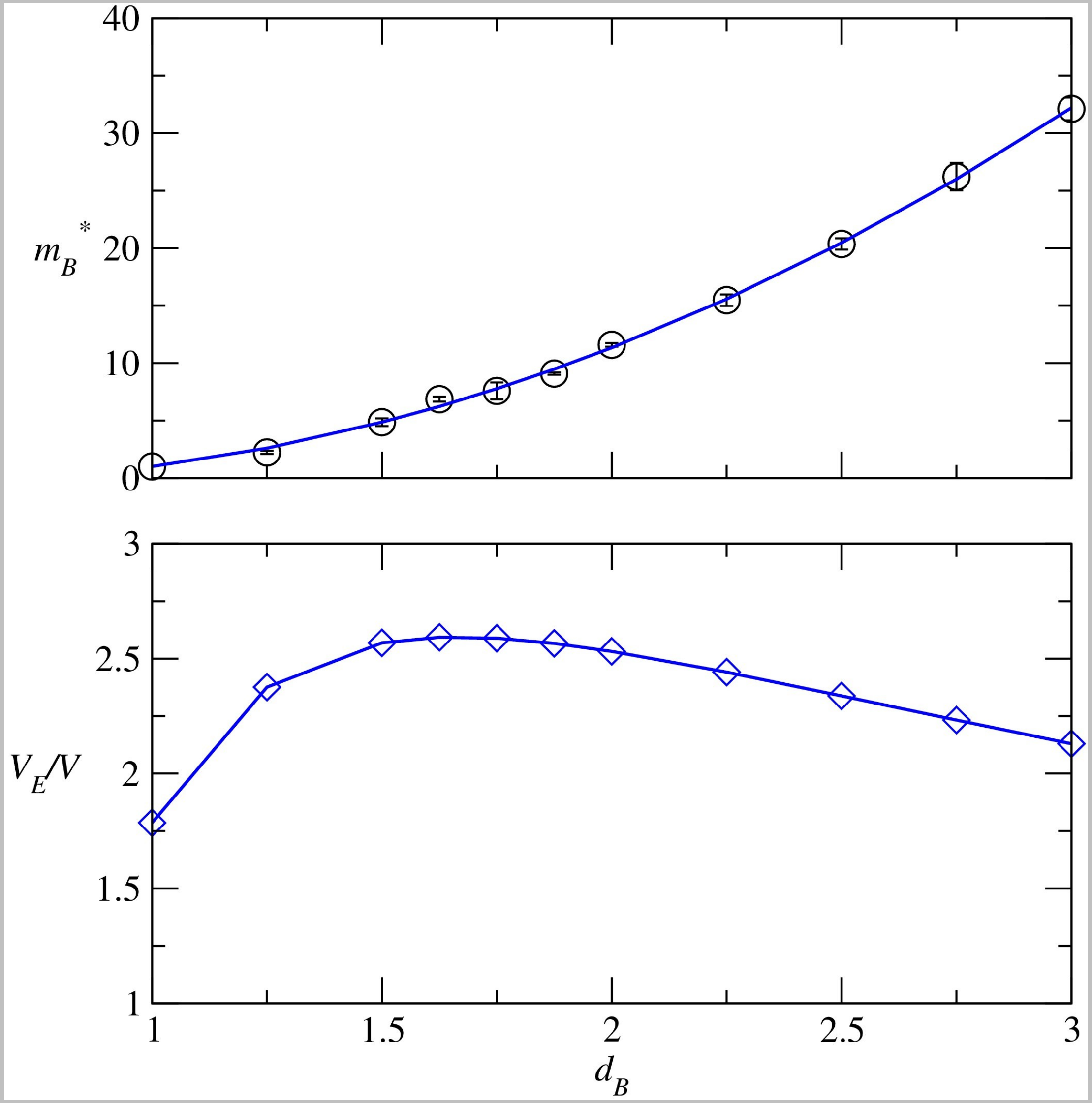
$$m_B g_y = \rho V_E g_y - c \pi \eta v_B d_B$$

Results



$$m_B = \rho V_E - c \pi \eta v_B d_B / g_y$$

Results



Conclusions

- Single particle motion: Buoyancy given by modified Archimedes principle and drag force by modified Stokes Law.
- Generalization to density segregation in mixtures: Role of the effective temperature.
- Model for rheology of mixtures (size and density).
- Predictions for combined model for rheology and density segregation.
- *Size segregation model: Effective volume*