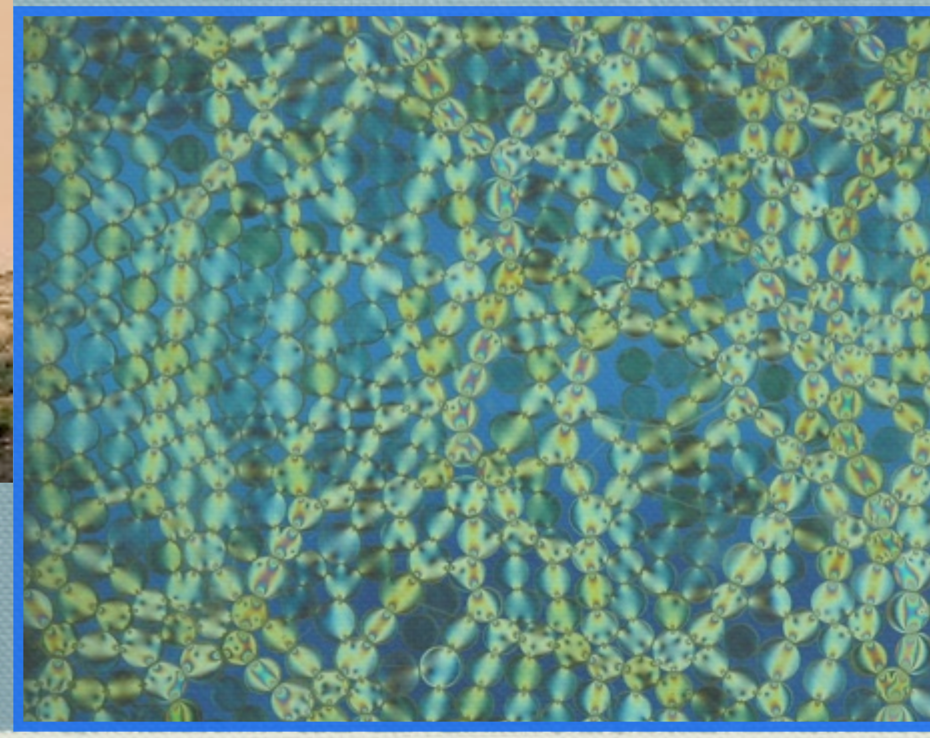


Statistical Mechanics of Jamming

of frictional grains

Dapeng (Max) Bi & Bulbul Chakraborty

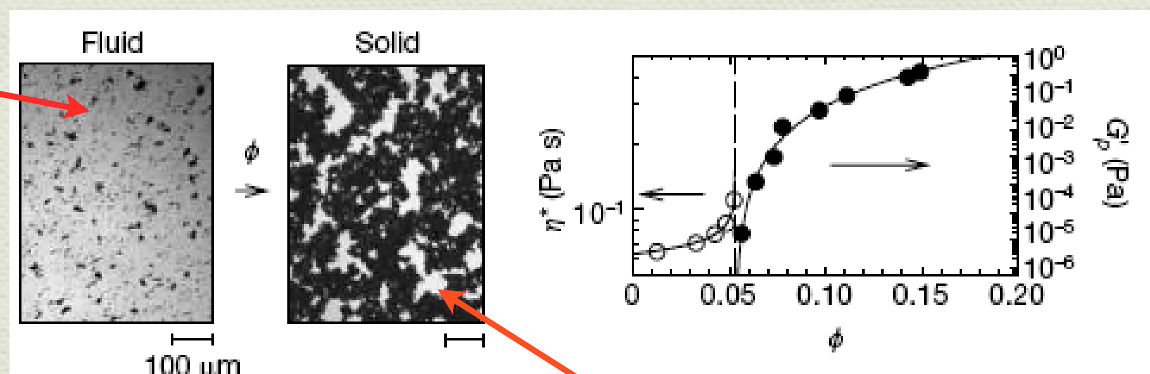
Bob Behringer & Jie Zhang, Duke University

Jamming

"A wide variety of systems, including granular media, colloidal suspensions and molecular systems, exhibit non-equilibrium transitions from a fluid-like to a solid-like state characterized solely by the sudden arrest of their dynamics."
(Trappe et al, Nature, 411, 772 (2001))

Dry Granular: A special class since a solid can only be produced through external stress

Unjammed phase flows like a liquid



Jamming is the transition between them: is this a phase transition?

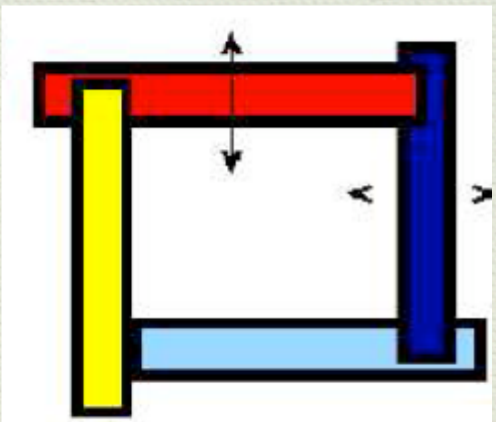
Increasing volume fraction

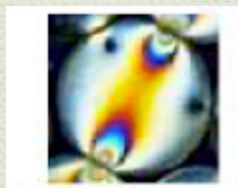
Jammed (resists shear)

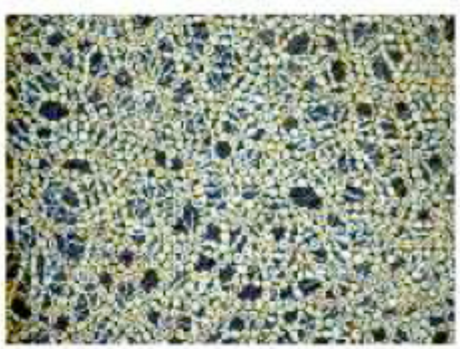
Granular: Shear can lead to jamming !

Experiments in 2D Packings of Grains with Friction

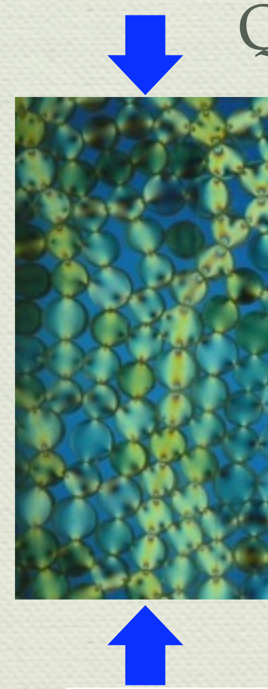
Solidity stems from applied stress itself

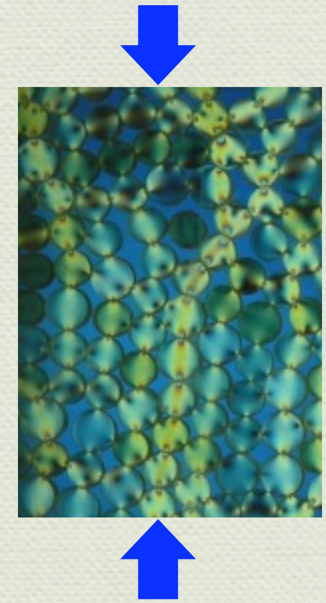






Quasistatic shear

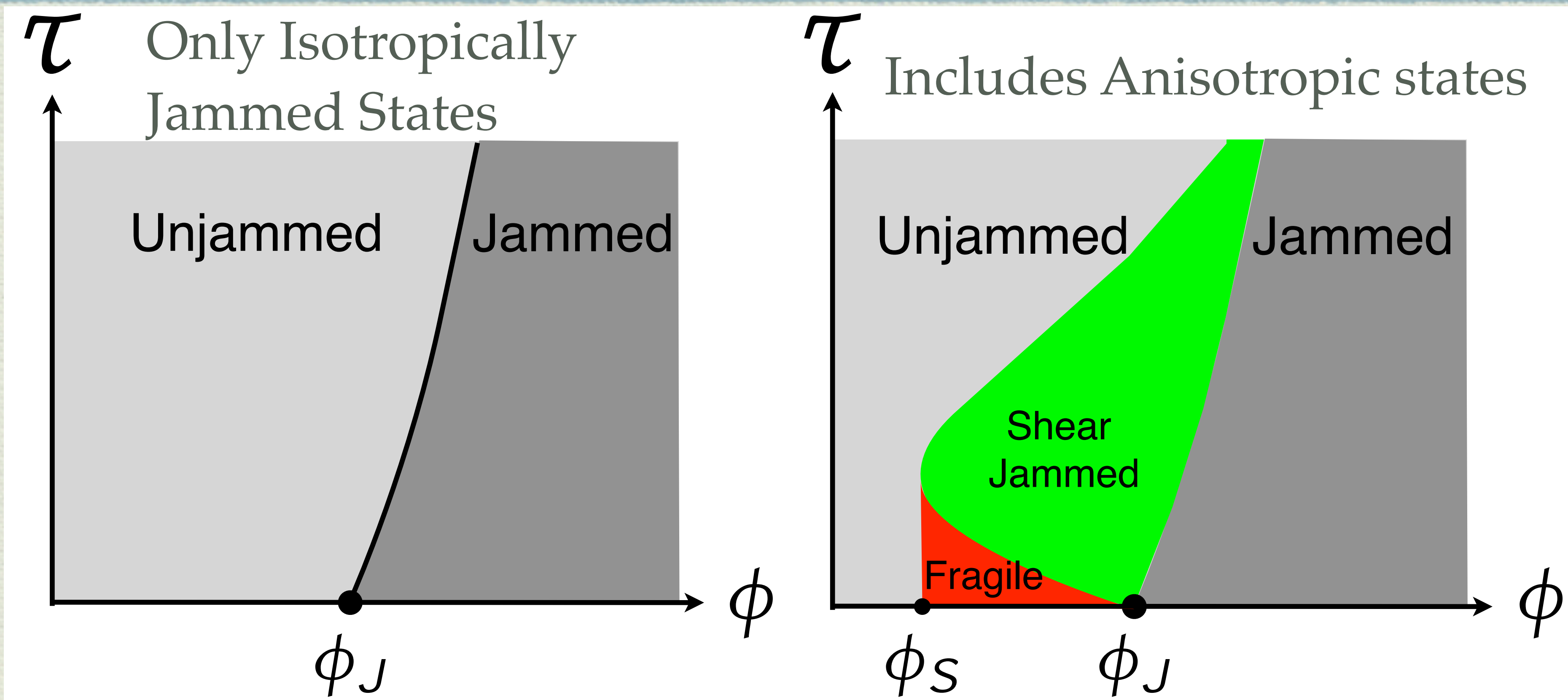




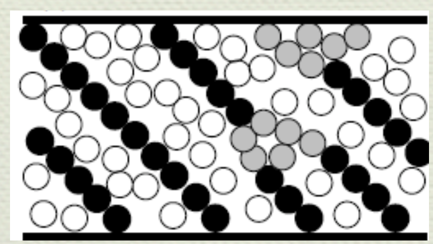
$$\hat{\sigma} = \frac{1}{V} \sum_{i \neq j} \vec{r}_{ij} \otimes \vec{f}_{ij};$$

$$\hat{R} = \frac{1}{N} \sum_{i \neq j} \frac{\vec{r}_{ij}}{||\vec{r}_{ij}||} \otimes \frac{\vec{r}_{ij}}{||\vec{r}_{ij}||}$$

Jamming Diagram
(Old) and (New)

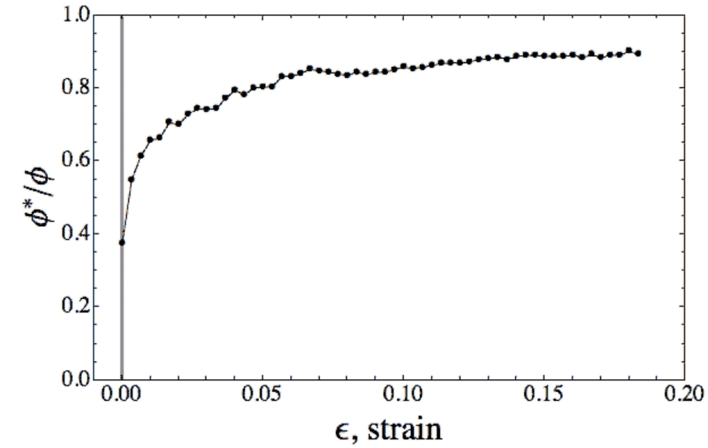


(Cates et al, PRL, 81, 1841 (1998))

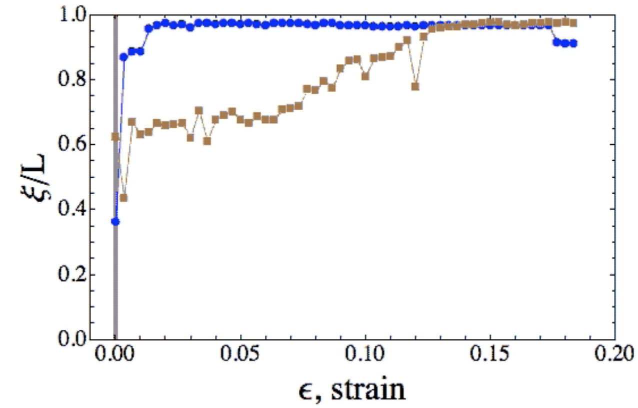


Experimental run at fixed packing fraction, ϕ=0.80

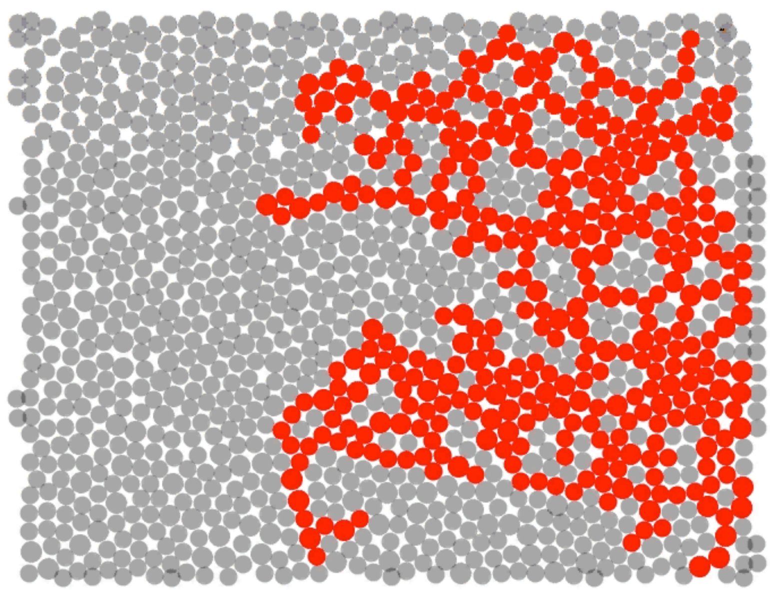
Evolution of the non−rattler fraction, ϕ^*/ϕ



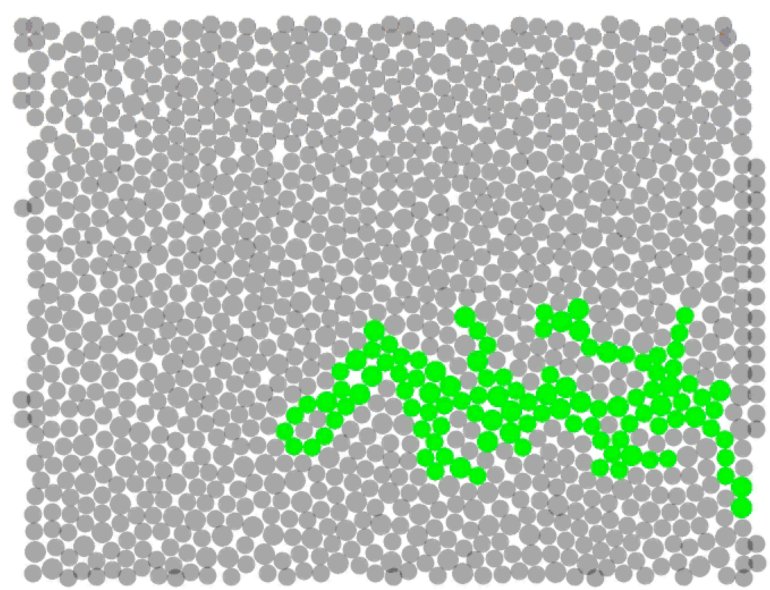
Blue ■ : Cluster Size ratio in the compressive direction
Brown ●: Cluster Size ratio in the expanding direction



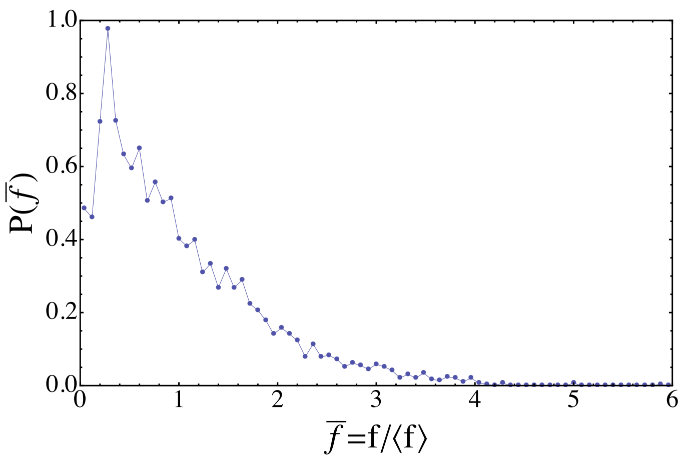
f>0 (fabric) cluster



f> f_{avg} cluster

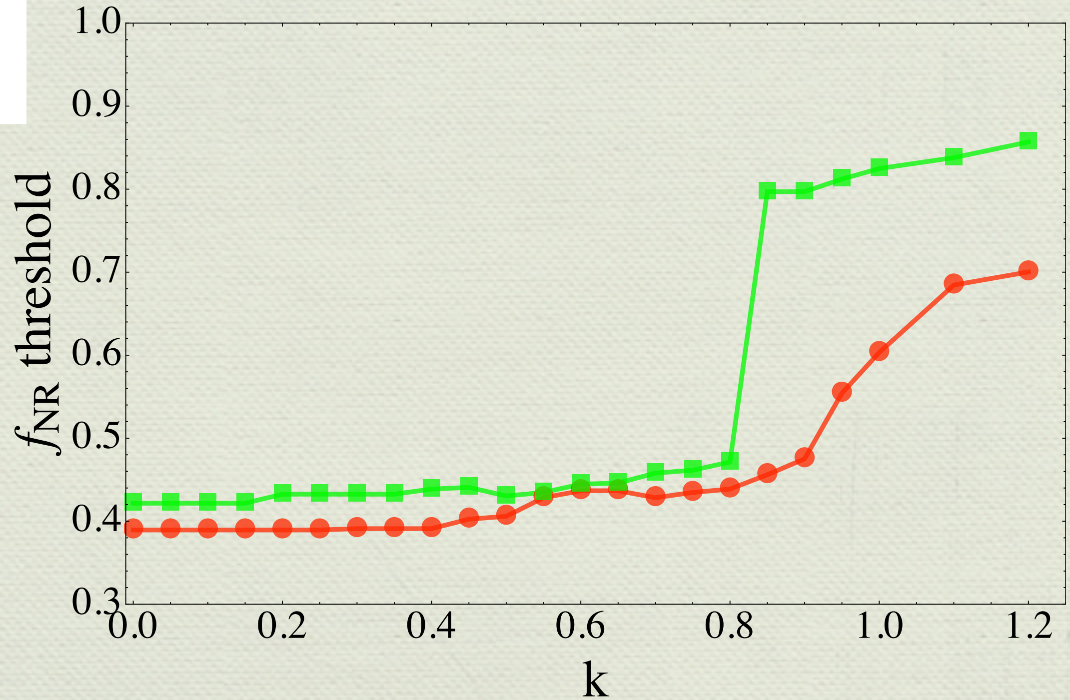


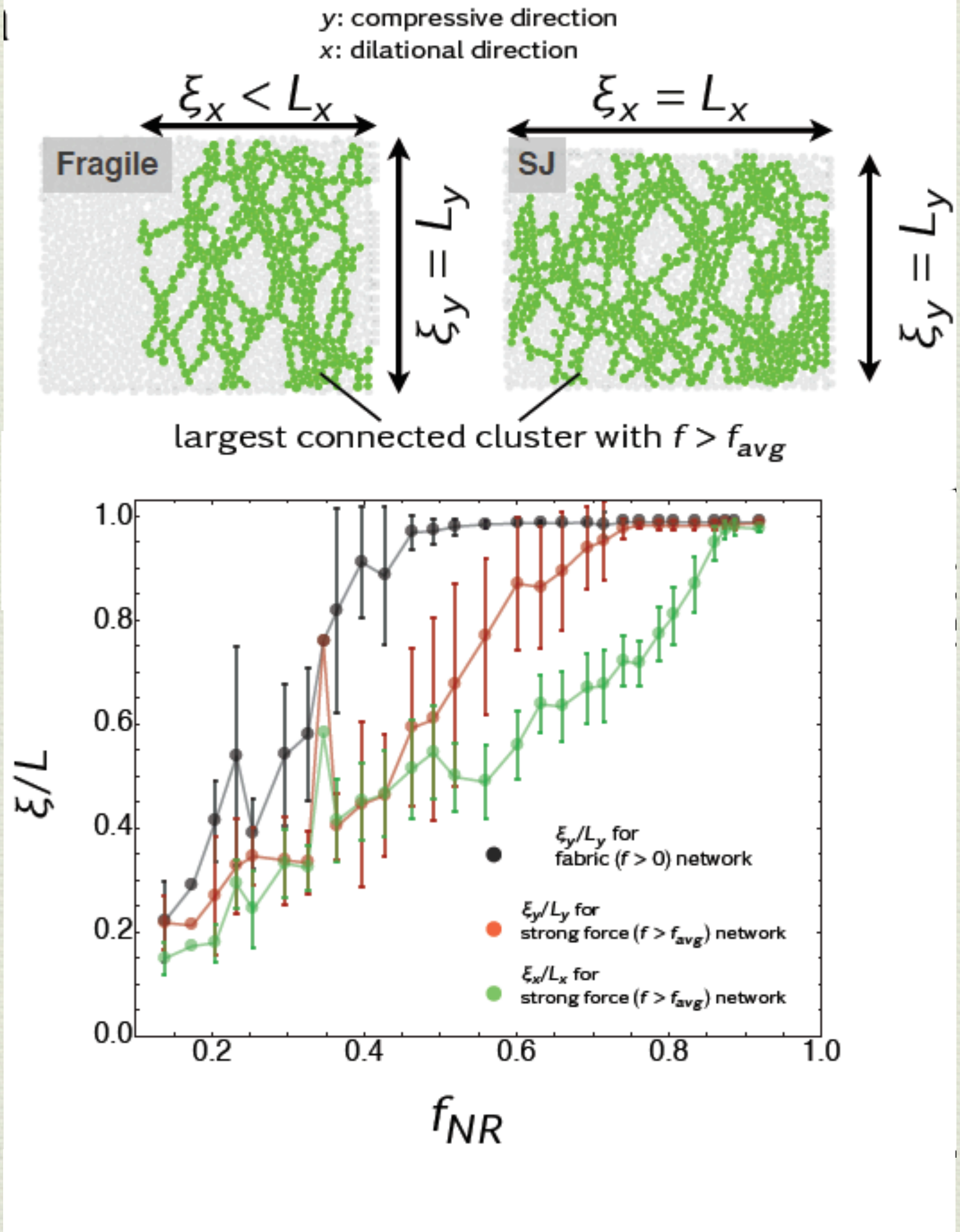
Force Networks



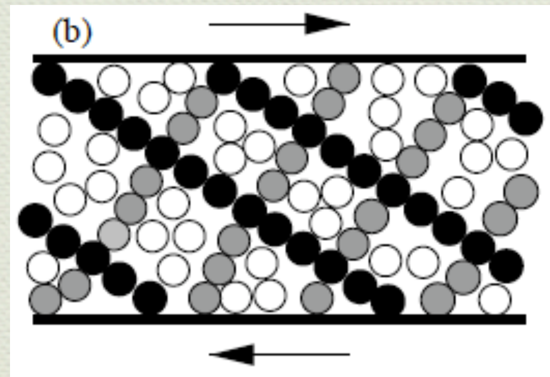
Define k-networks

$f > k\ f_{avg}$

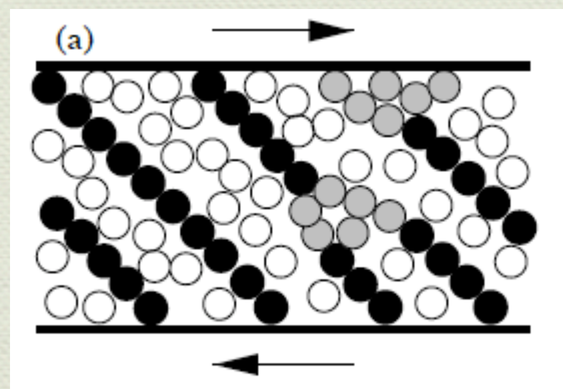




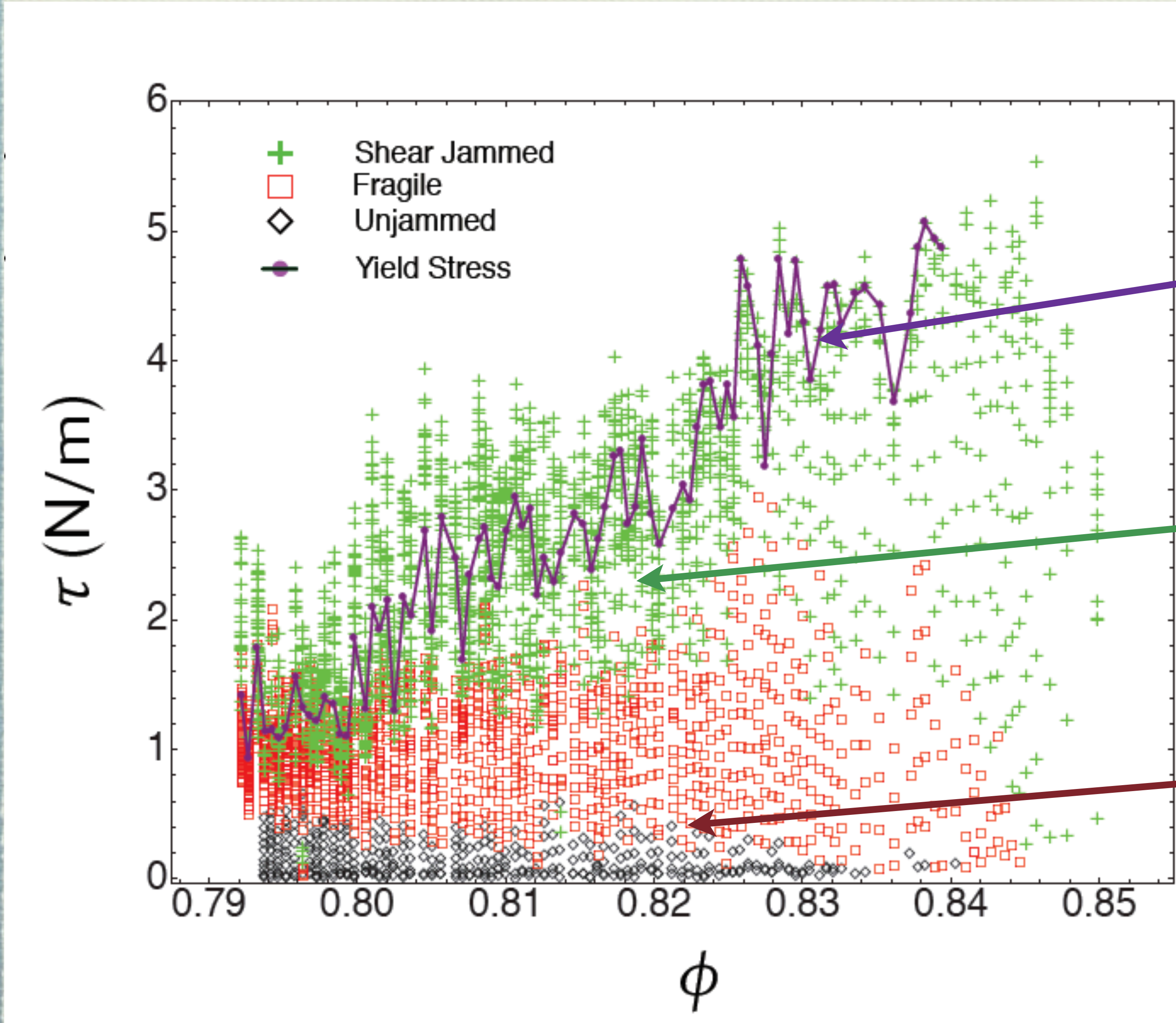
The fraction of non-rattlers can be used to distinguish between two types of states



(Cates et al, PRL, 81, 1841 (1998))



Phase Diagram



Yield-Stress line

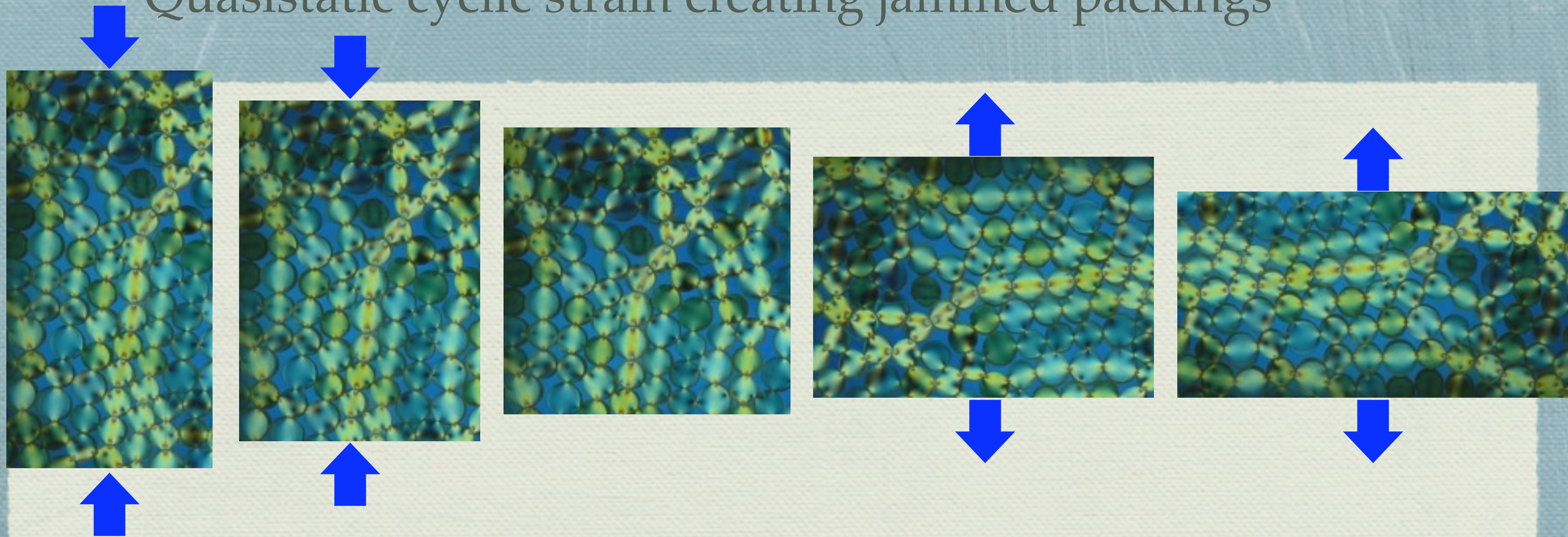
SJ states have finite yield stress at unjamming (first order transition)

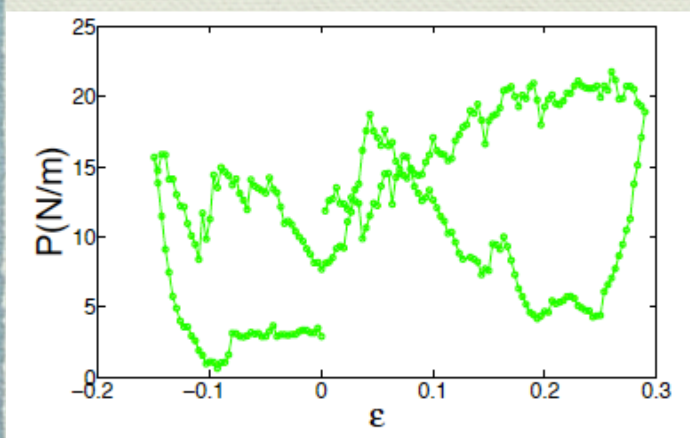
Shear-Jammed

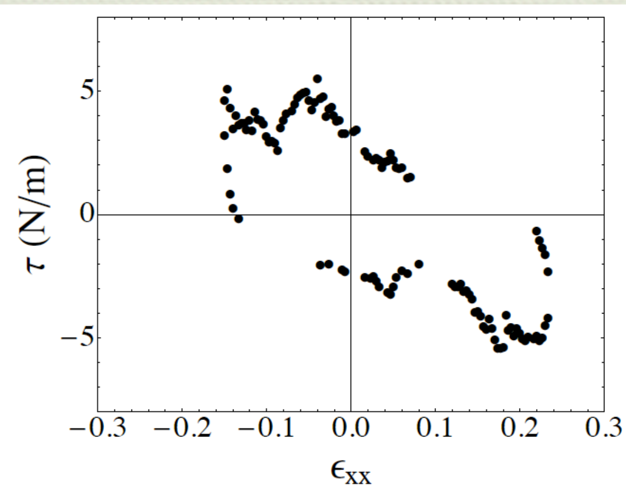
Fragile

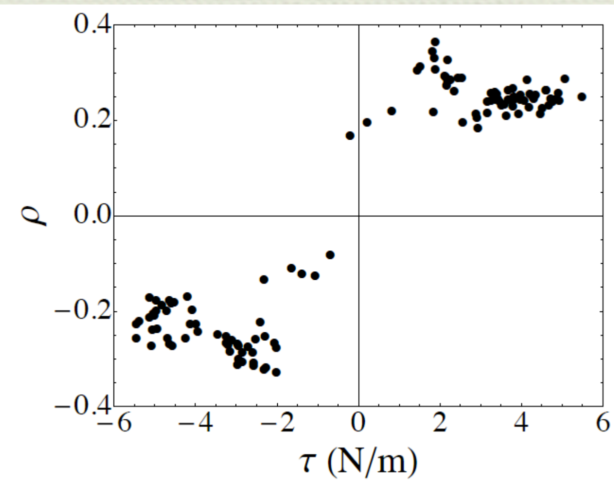
Cyclic Shear

Quasistatic cyclic strain creating jammed packings

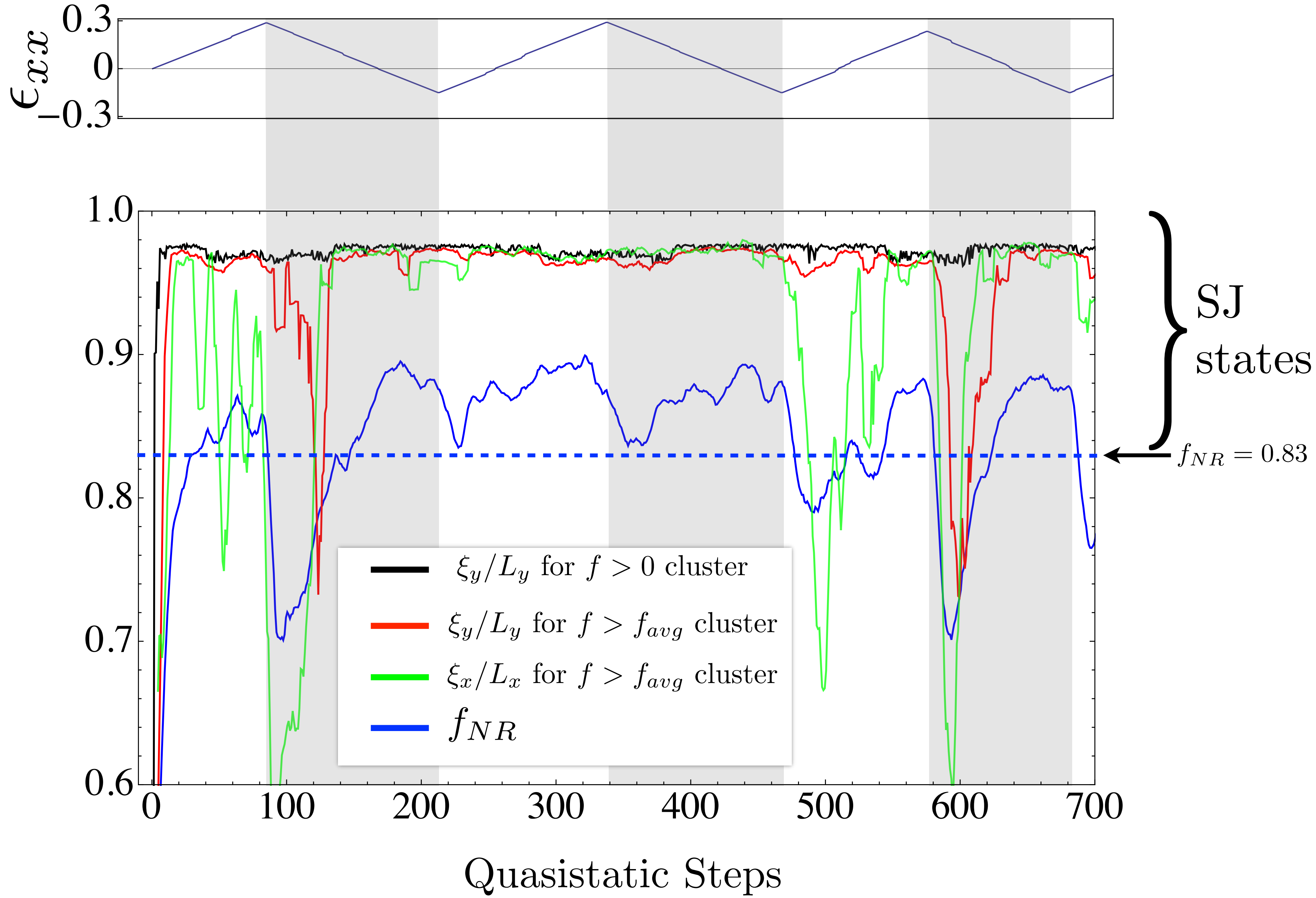








Percolation (Cyclic Shear)



What types of Solids are these?

- Response of shear -jammed states
- Is Point J a special point?
- Fabric and Stress Relationship
- Statistical Framework: Force and torque balance are of paramount importance
- Concept of a Stress Temperature (Angoricity)

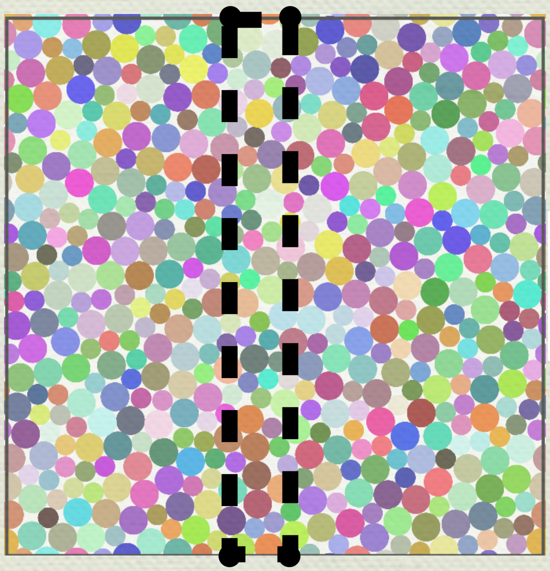
Topological Invariants of Mechanically Stable Packings

Force and torque balance: Take a continuum approach

$$\hat{\sigma}(\vec{r}) = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$$

$$\vec{\nabla} \cdot \hat{\sigma}(\vec{r}) = \vec{0}$$

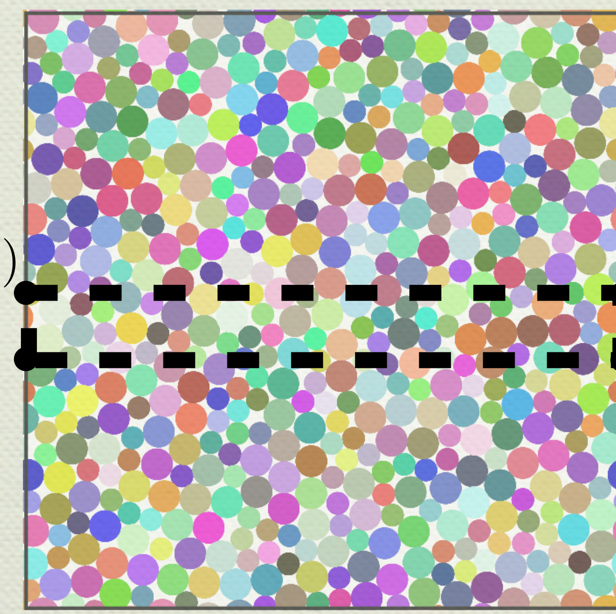
Cleanest with periodic boundary conditions



$$\frac{d\vec{F}_x}{dx} = 0$$

$$\vec{F}_x = \int_0^L dy \begin{bmatrix} \sigma_{11}(x,y) \\ \sigma_{12}(x,y) \end{bmatrix}$$

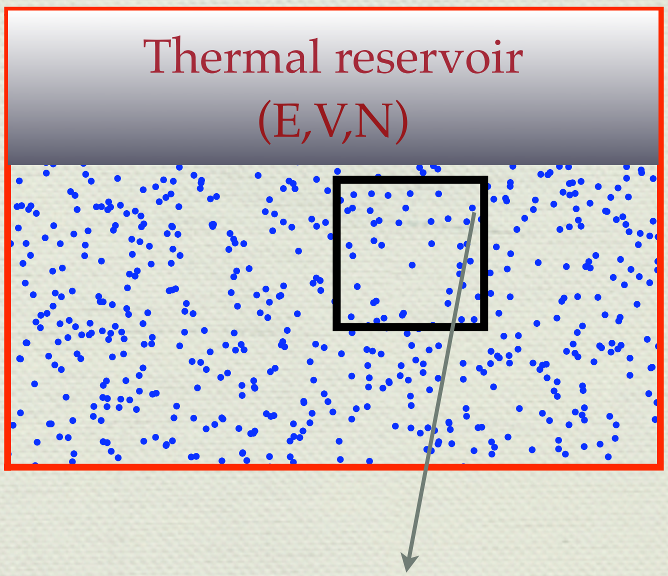
$$\vec{F}_y = \int_0^L dx \begin{bmatrix} \sigma_{12}(x,y) \\ \sigma_{22}(x,y) \end{bmatrix}$$



$$\frac{d\vec{F}_y}{dy} = 0$$

Stress Ensemble

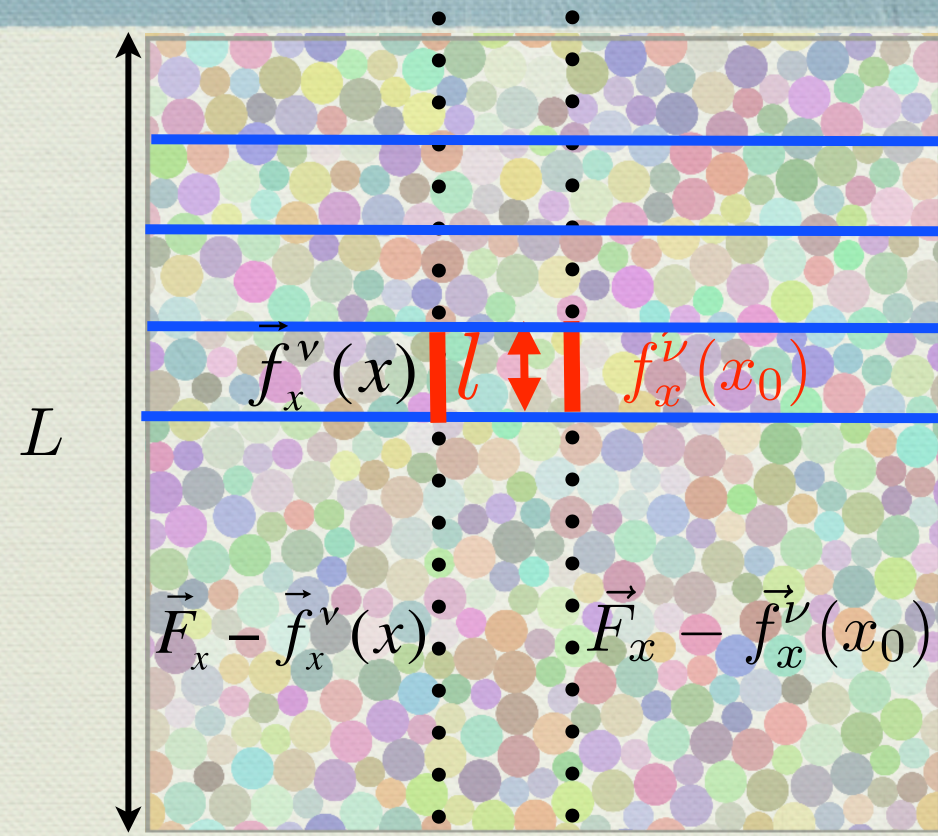
Thermal System



The distribution of energy is then given by:

$$P(E_m) = \frac{1}{Z}\Omega(E_m, V_m, m)e^{-\beta E_m}$$

Granular materials (athermal)

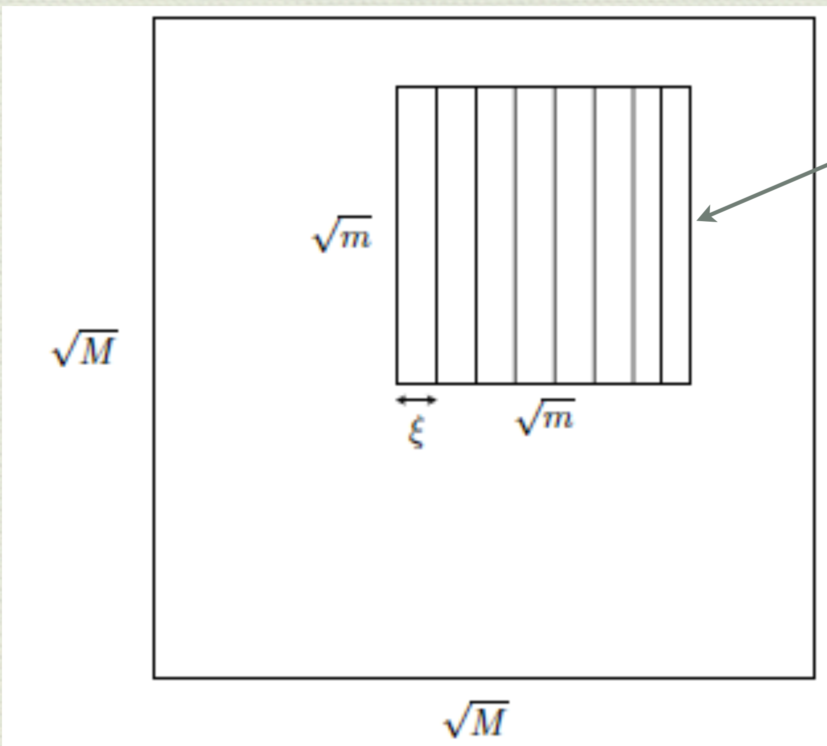


All blue lines have same $\vec{F}_y$

All blue strips have the same “temperature” wrt to $\vec{F}_x$ fluctuations

Need to consider a collection of 1d systems

Distribution of Local Stresses



The probability distribution of stresses in this subsystem

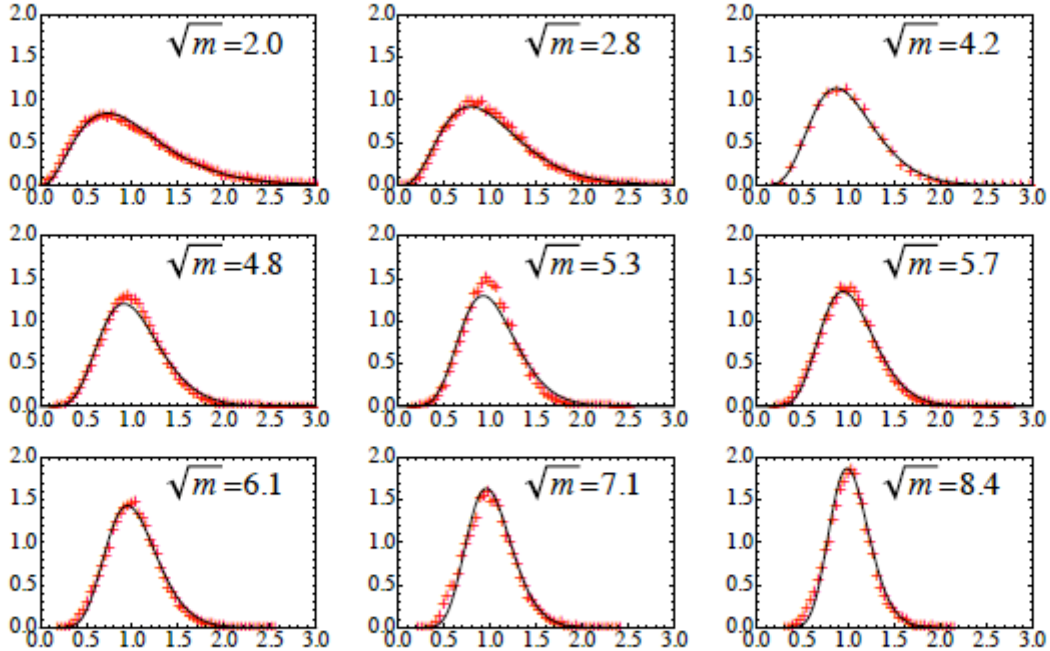
$$f_x(u_{xx}) = \frac{(m/\xi_x)^{2m+1}}{\Gamma(2m+1)} u_{xx}{}^{m+\sqrt{m}/\xi_x-1} e^{-(m/\xi_y) u_{xx}}$$

$$f_y(u_{yy}) = \frac{(m/\xi_y)^{2m+1}}{\Gamma(2m+1)} u_{yy}{}^{m+\sqrt{m}/\xi_y-1} e^{-(m/\xi_y) u_{yy}}$$

$$u_{xx} = \sigma_{xx}^{local} / \sigma_{xx}^{global}$$

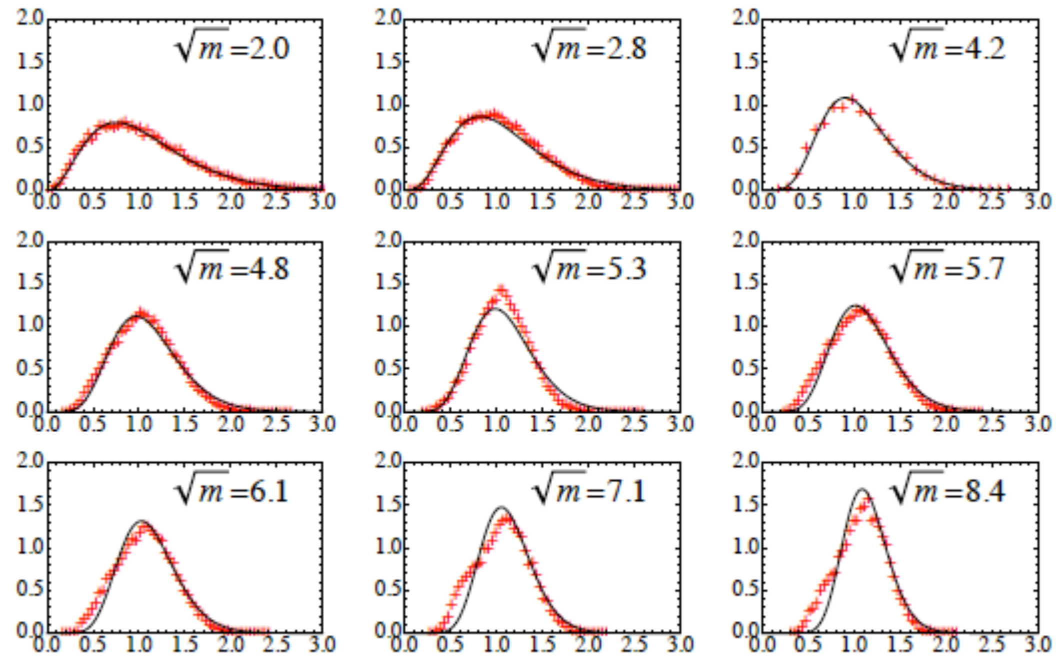
$$u_{yy} = \sigma_{yy}^{local} / \sigma_{yy}^{global}$$

Stress Ensemble Correctly Predicts Stress Distributions



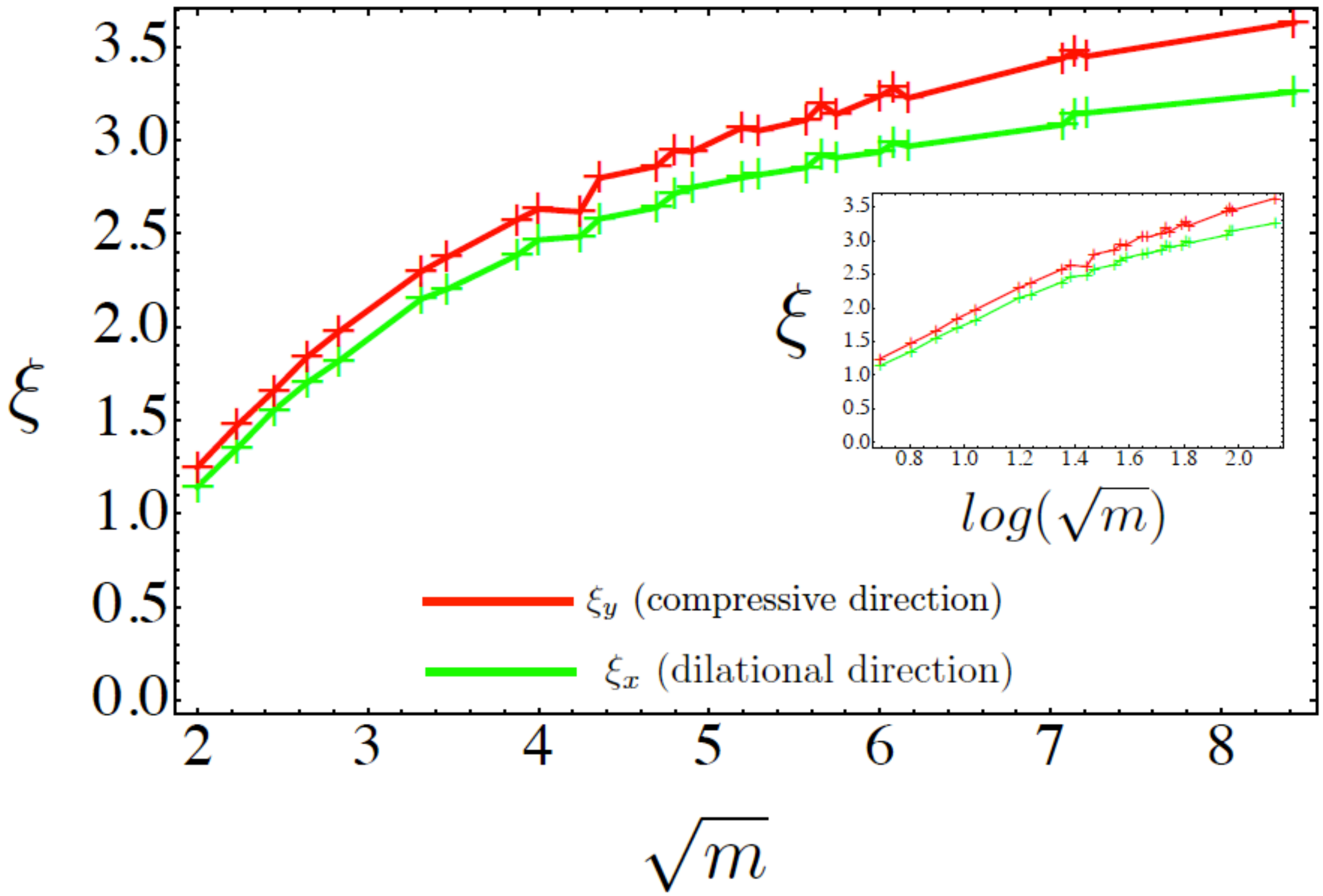
$f_x(u_{xx})$

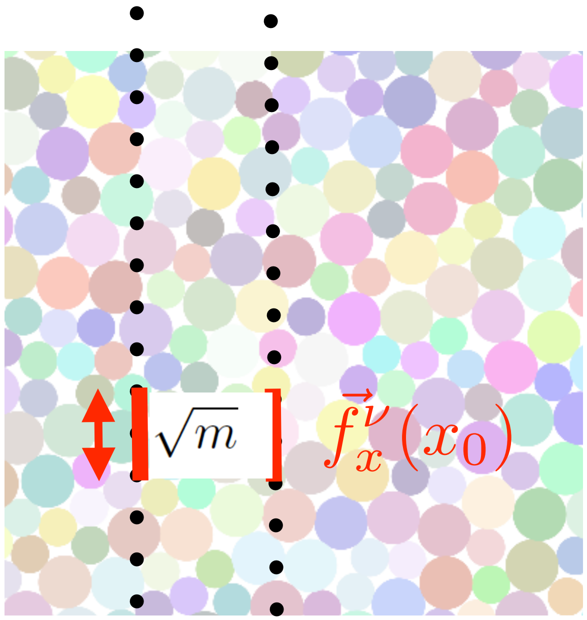
Shear-Jammed states have significant correlations
These are less evident for the states above Point J

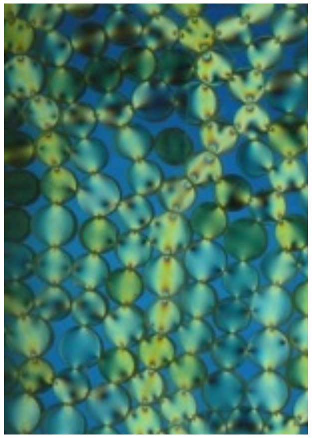


$f_y(u_{yy})$

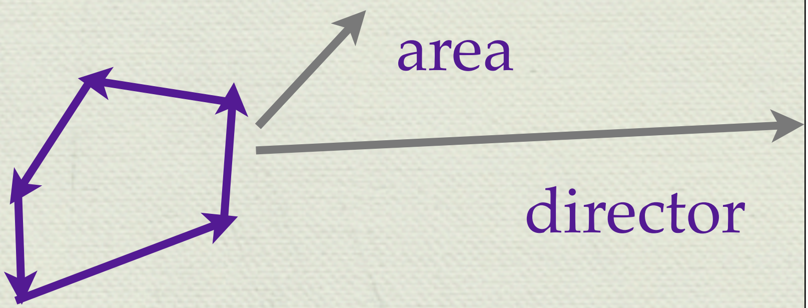
Correlation Length



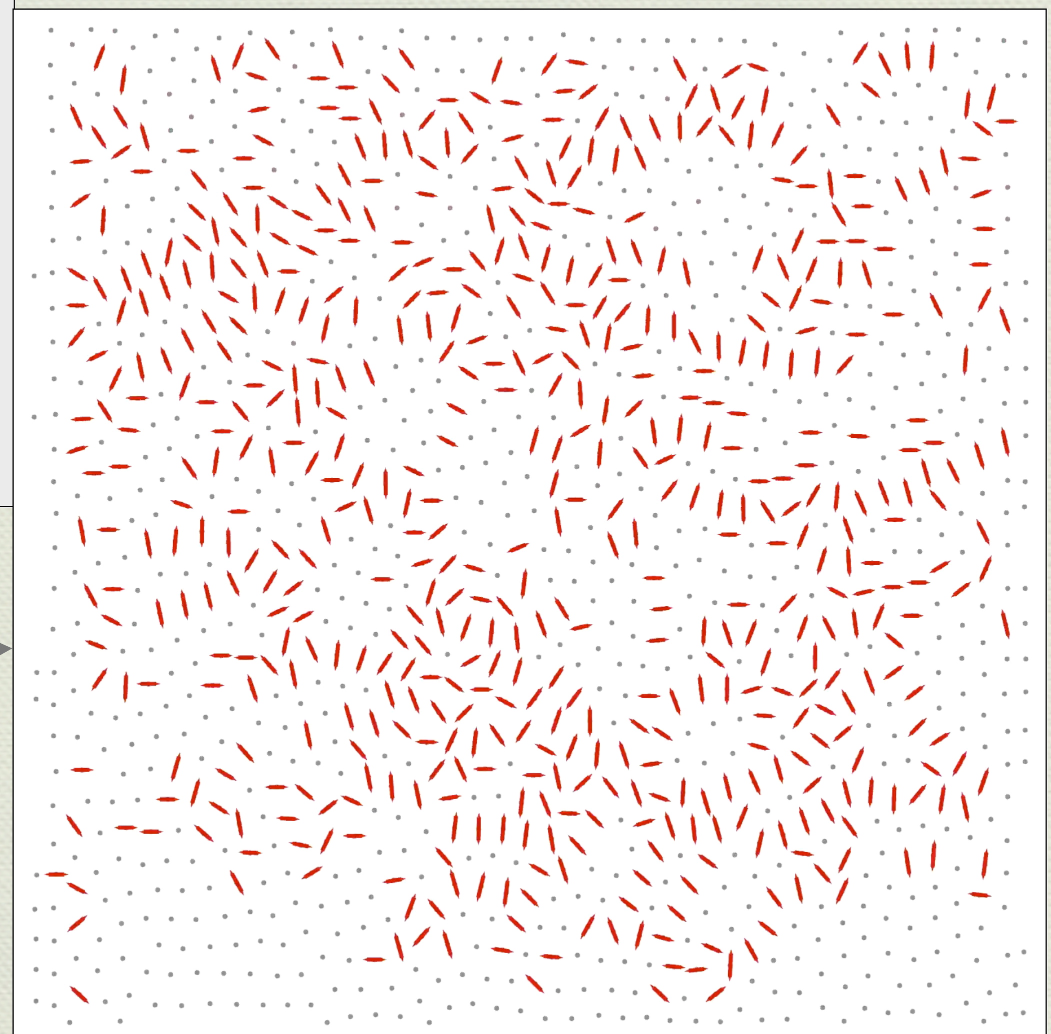




Increasing density: tiles become more isotropic

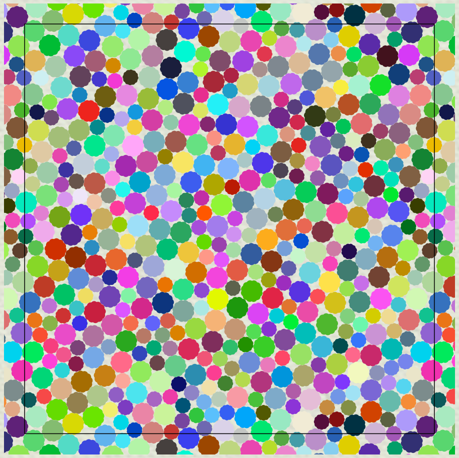


Force tile for a grain with 5 contacts

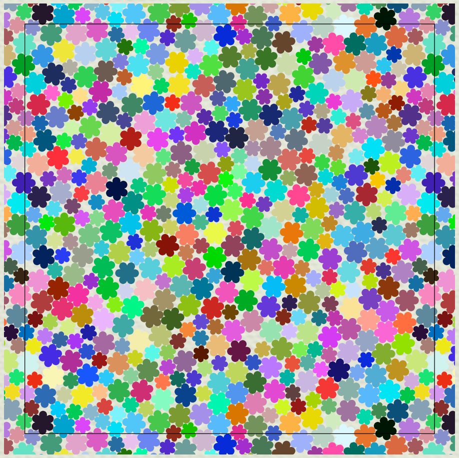


Angoricity

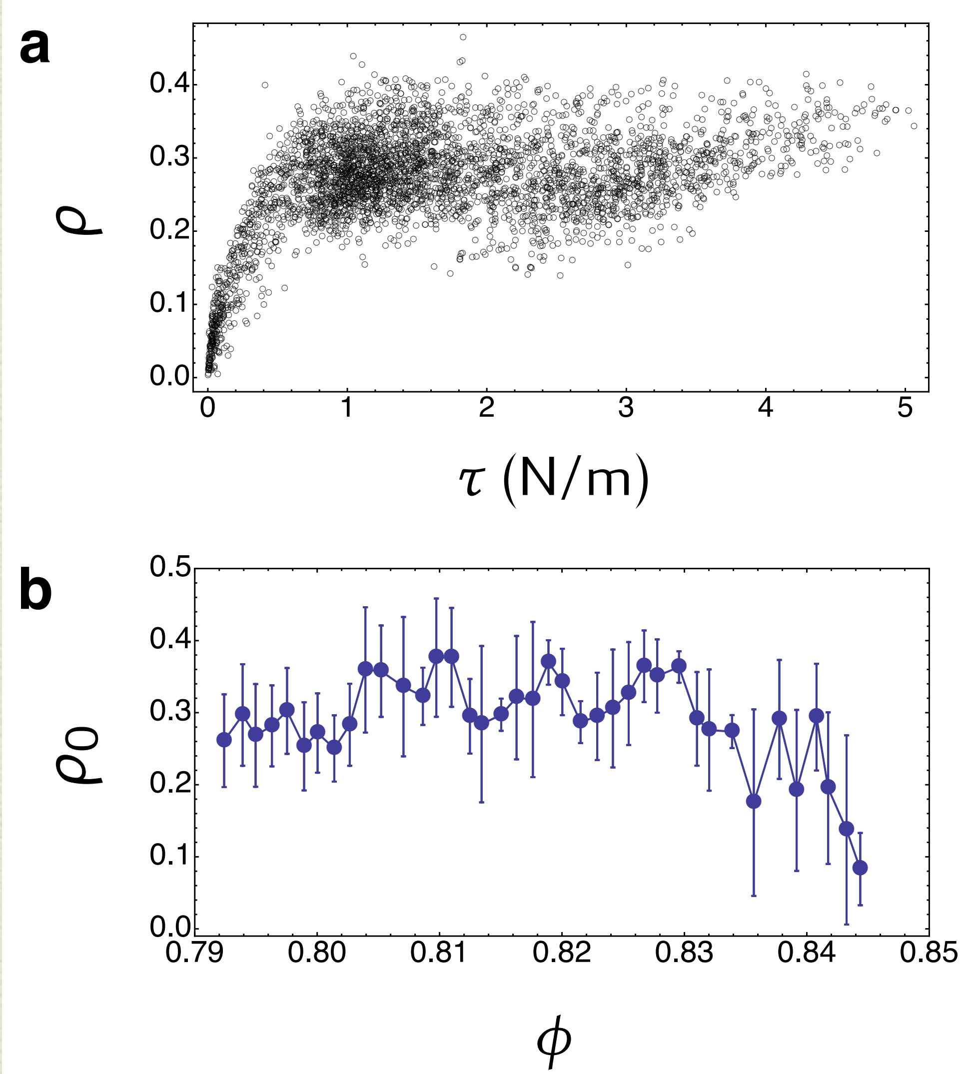
- The stress distributions of the packings are characterized by a temperature-like quantity, named angoricity
- Boltzman-like distribution
- Angoricity is a tensor that reflects the intrinsic 1d conservation principle
- Correlation length appear is distribution
- Fragile/SJ: Phase Transition ? Nematic Model, Simulations



Bumpy Particles (with Corey O'Hern)



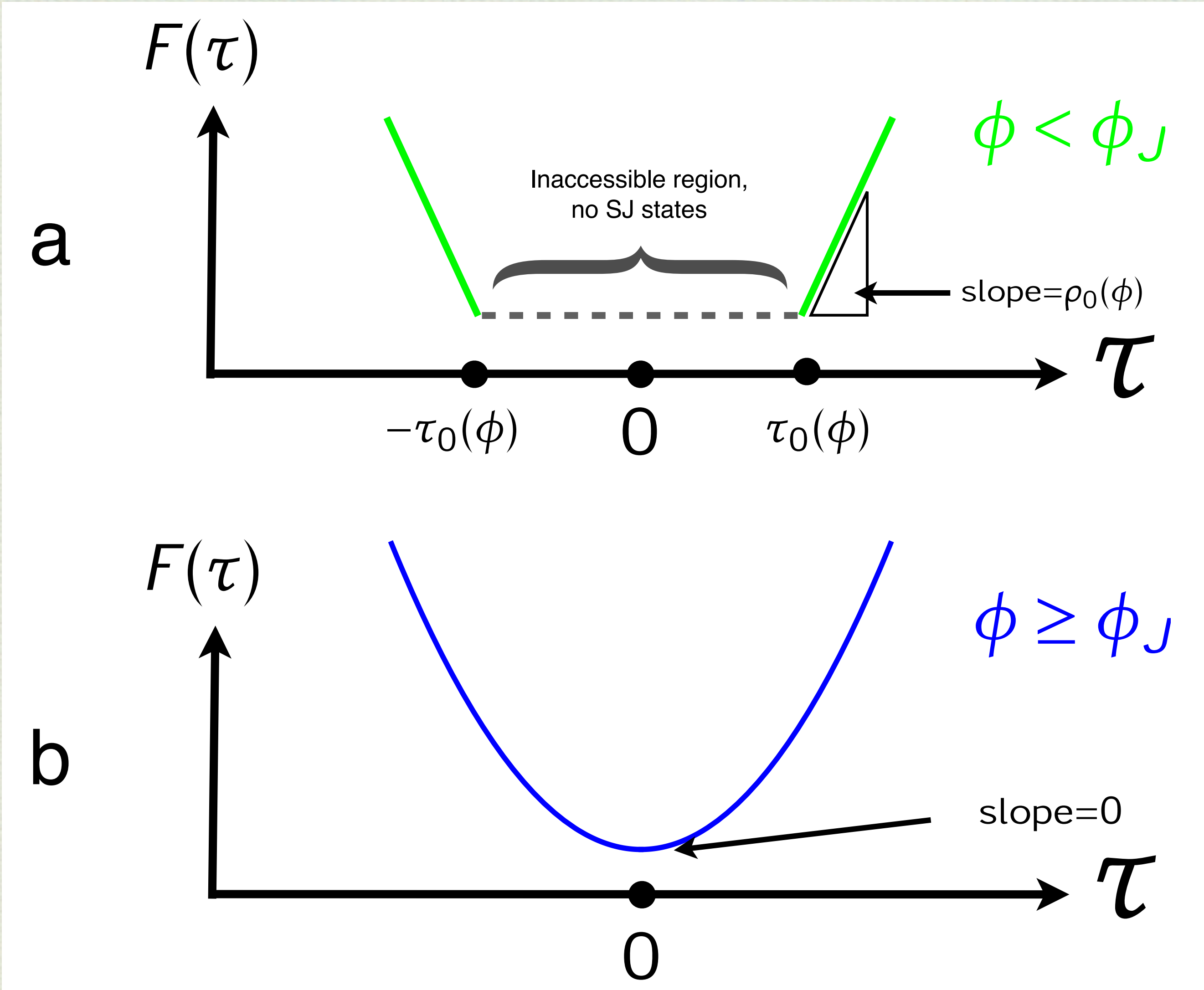
Order Parameter



Anisotropy of fabric vs Shear Stress

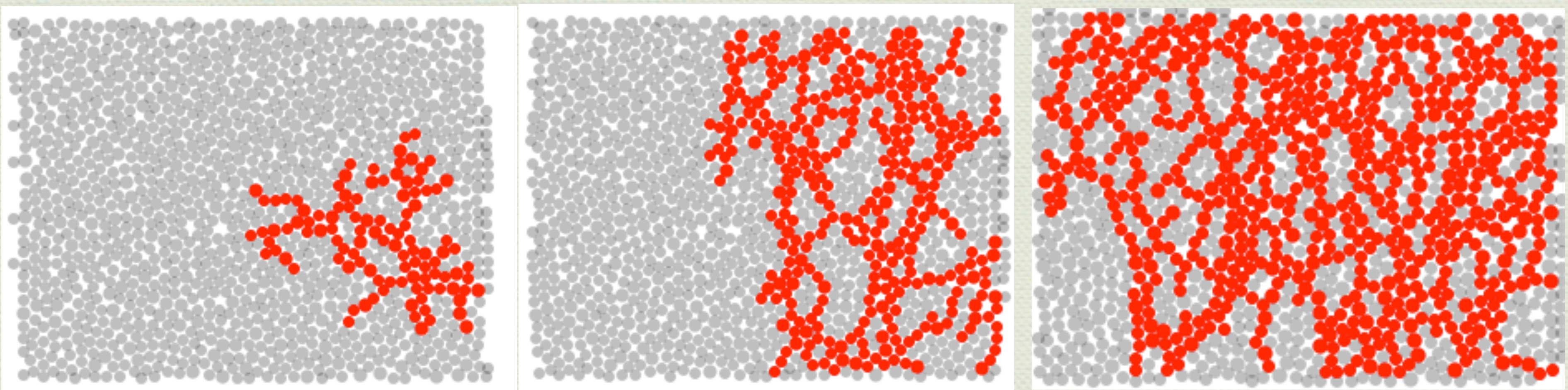
Minimum anisotropy of SJ states

"Free energy"

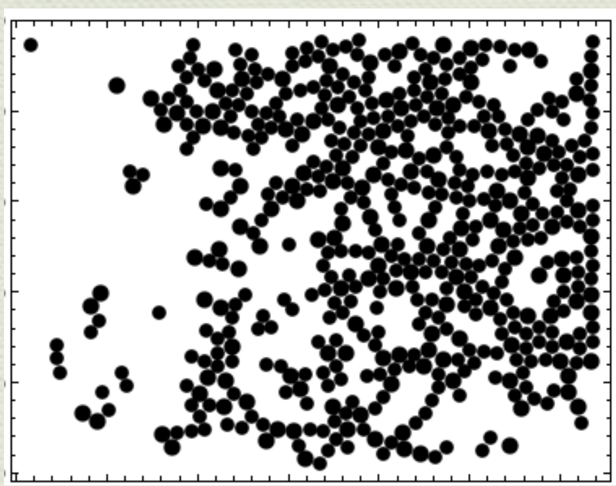


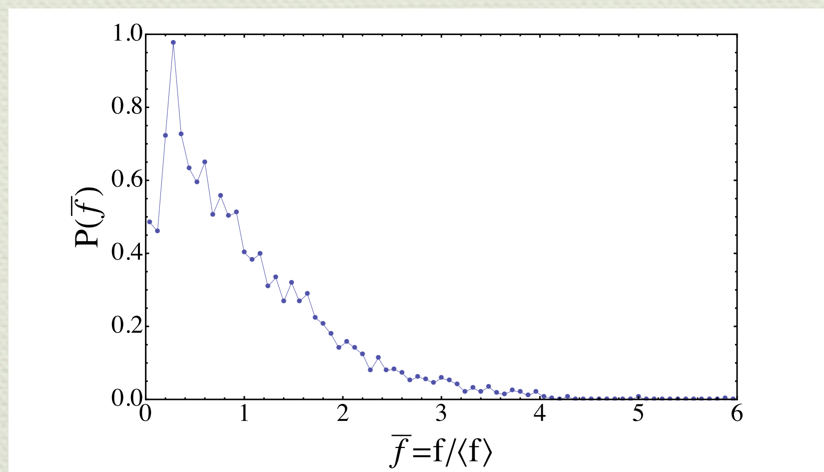
a
$F(\tau)$
$\phi < \phi_J$
Inaccessible region,
no SJ states
slope=$\rho_0(\phi)$
τ
$-\tau_0(\phi)$
0
$\tau_0(\phi)$
b
$F(\tau)$
$\phi \geq \phi_J$
slope=0
τ
0

Force Networks

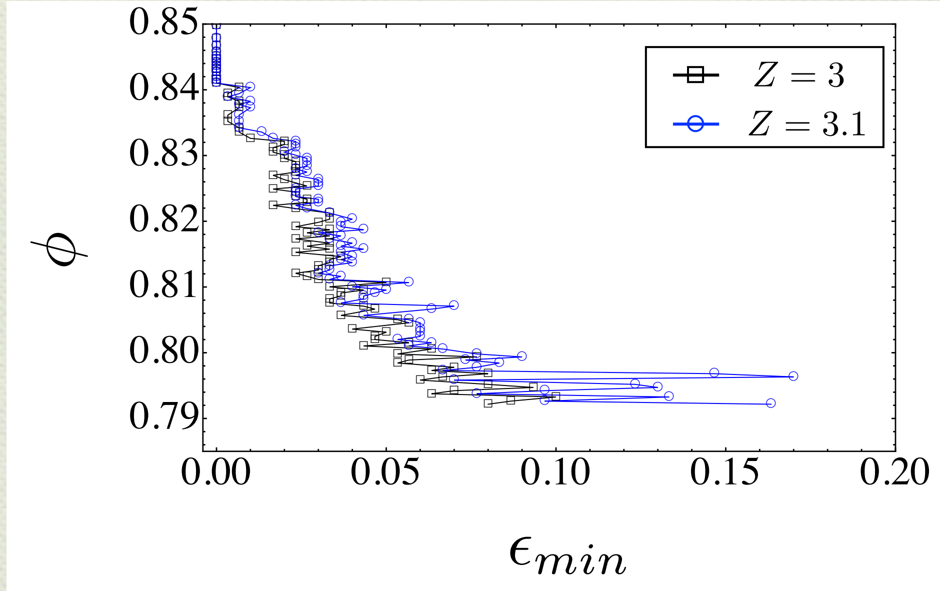


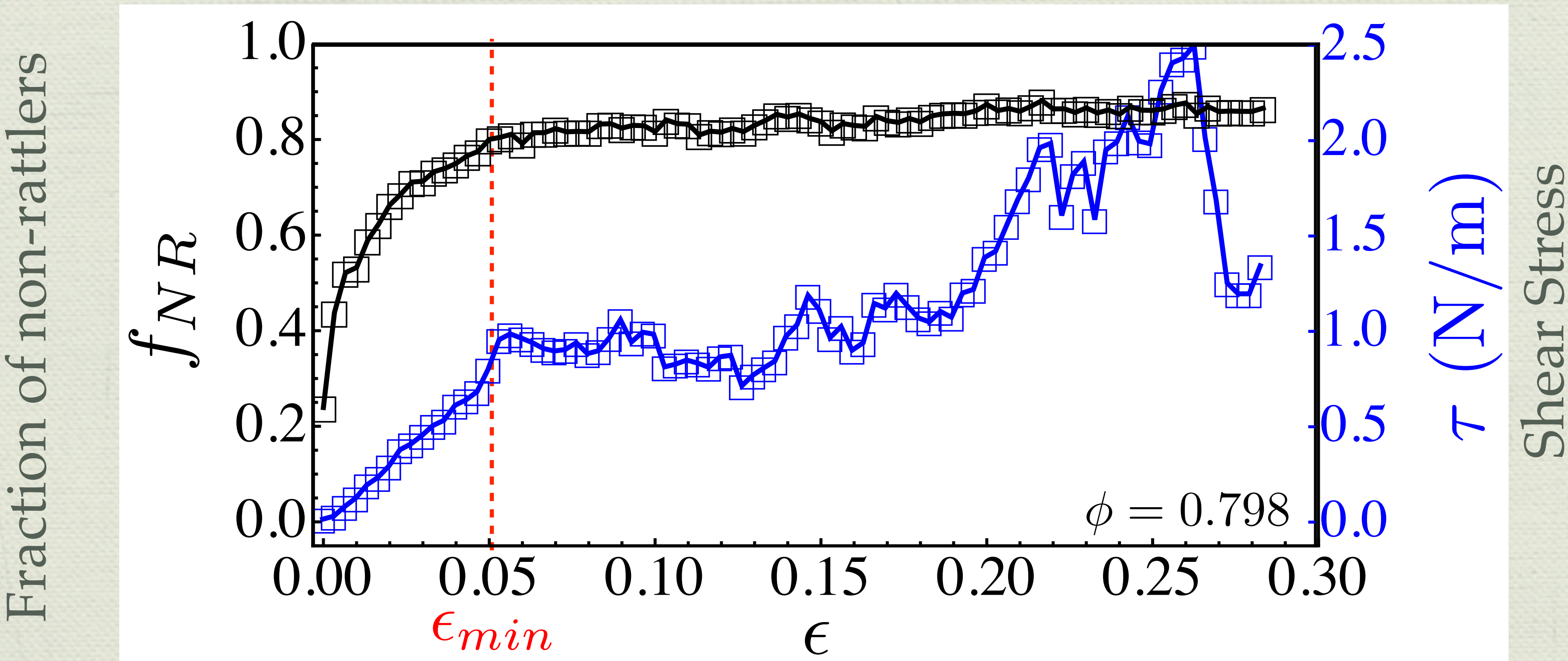
Increasing non-rattler fraction →

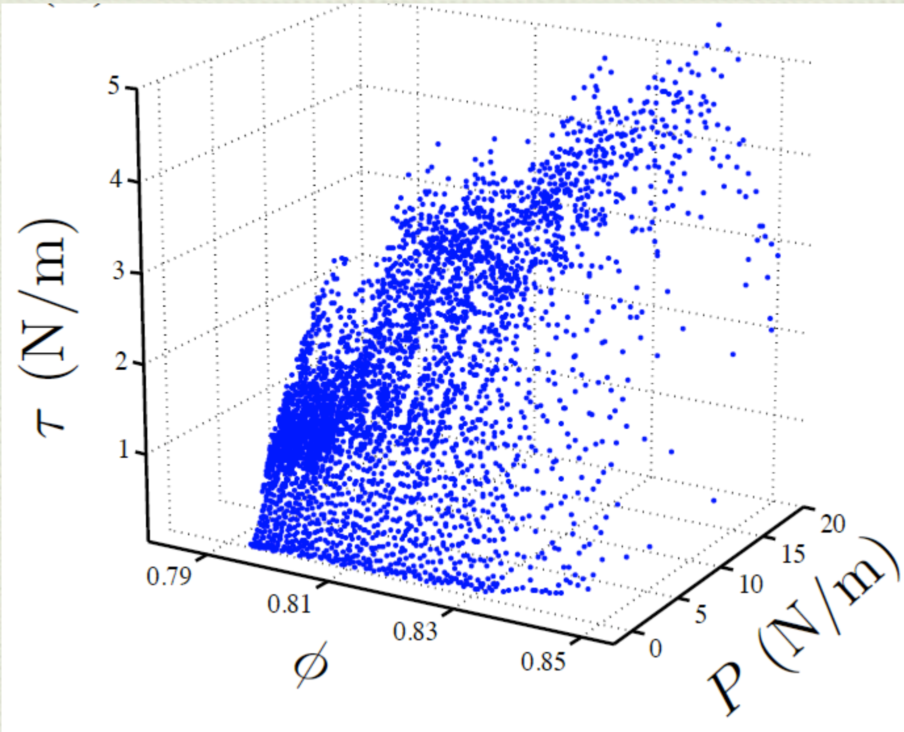




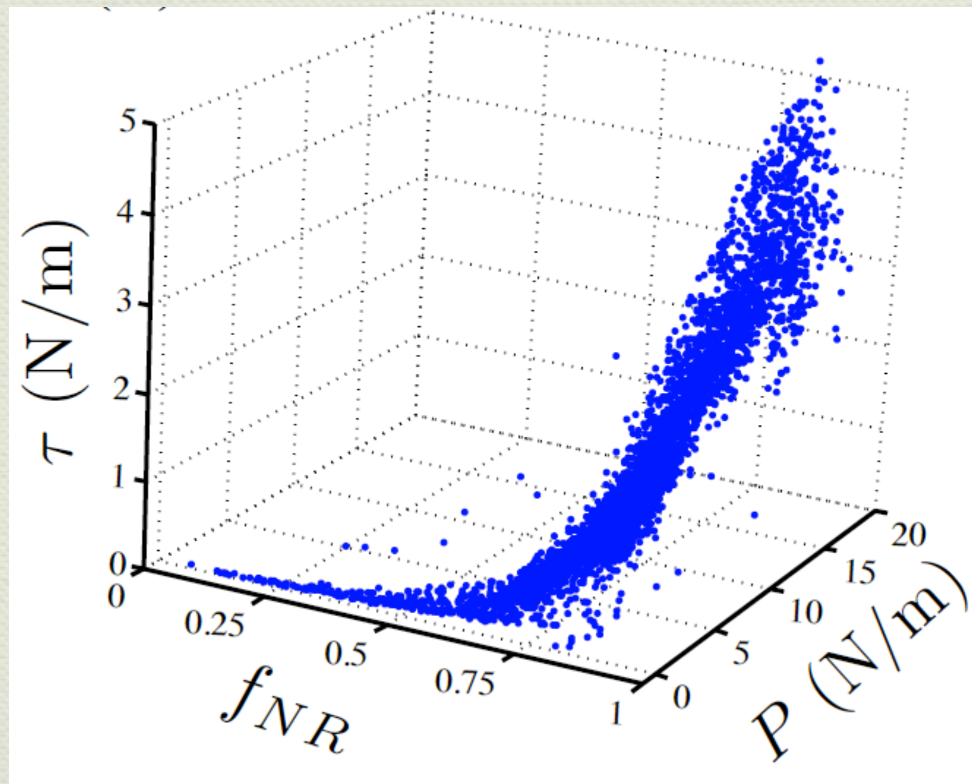
Forward Shear







Pressure is not just a function of packing fraction



Stresses are determined by fraction of non-rattlers

Summary so far

- Anisotropic jammed states occur in frictional systems at packing fractions well below the isotropic jamming threshold
- The fraction of non-rattlers emerges as parameter that controls the behavior of force networks, and therefore the rigidity
- Two qualitatively different classes of states: For infinitely rigid grains, fragile states cannot sustain any load that is not along there force-network axis. Both are fragile jammed states.
- Yield stress of SJ states vanishes discontinuously (Hayakawa, simulations)
- The observed anisotropic jamming is a reflection of dilatancy: under constant area conditions, shearing leads to jamming