Elementary Mechanisms of Deformation in Amorphous Solids



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Amorphous materials are, well... disordered

In crystals

defects = dislocations (Volterra, 1930; SEM, 1960)



Interaction and motion understoon (Peierls, Nabarro, Friedel, 1950's)

Dislocation dynamics in computer codes since the 1980's

In disordered materials

No topological order => defects?



What are the elementary mechanisms of deformation?

How can we up-scale the dynamics?

Length and energy scales in amorphous materials



Can we identify some mechanisms of deformation, at least for broad classes of materials, or time-, energy-, length-scales?

Deformation map for a metallic glass



Schuh et al, Acta Mat. 55, 4067 (2007)





Deformation map for a metallic glass



Deformation map for a metallic glass



What are the elementary mechanisms of deformation in amorphous solids?

Argon (1979):

local shear transformations



stress-induced hopping among inherent states

In real space



The AQS limit

Low temperature:

 $T < T_g$ $\dot{\gamma} \ll 1/\tau_\alpha$

Neglect any thermally activated process





The system resides at all times in local energy minima





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The system resides at all times in local energy minima



Occasionally the occupied minimum becomes unstable:

A plastic event then occurs leading to a new local minimum

Athermal, quasi-static protocol:

- Minimize energy
- Apply a small increment of strain (homogeneously)
- Repeat

The system resides at all times in local energy minima



Occasionally the occupied minimum becomes unstable:

A plastic event then occurs leading to a new local minimum



L=20 L = 40



L=20,40,80,160



AQS I: Saddle-node bifurcation



AQS II: Eshelby quadrupolar events







AQS III: Avalanches





Full plastic event = avalanche

AQS III: Avalanches





<u>In 2D</u>

C. Maloney and AL, PRL 93, 016001 (2004);

PRE 74, 016118 (2006) $\Delta\,E\,{\sim}\,L$

E. Lerner and I. Procaccia, PRE 79, 066109 (2009)

 $\Delta E \sim L^{\beta}$, $\beta = 0.74$

<u>In 3D</u>

N. Bailey et al PRL 98, 095501 (2007)

 $\Delta E \sim L^{1.4}$

Particle displacement distribution in AQS









Maloney & Robbins, J. Phys. Cond. Mat. 20, 244128 (2008)

Athermal, finite-strain rate





AL and C. Caroli, PRL 103, 065501 (2009)

Athermal, finite strain-rate simulations:T = 0 $\dot{y} \neq 0$ - Standard MD simulation- Damping forces $f_{ij} = \frac{m}{\tau} \Phi(r)(\vec{v}_j - \vec{v}_i)$



AL and C. Caroli, PRL 103, 065501 (2009)

Athermal, finite strain-rate simulations:T = 0 $\dot{y} \neq 0$ - Standard MD simulation- Damping forces $f_{ij} = \frac{m}{\tau} \Phi(r)(\vec{v}_j - \vec{v}_i)$

Athermal, finite strain-rate

Non-affine velocity

$$\vec{v}_i - \dot{\gamma} y_i \vec{e}_x$$

L = 160 $\dot{y} = 5.10^{-5}$

PRL 103, 065501 (2009)

 $T < 10^{-4}$



Deformation maps

 $\epsilon_{xy}(\vec{r})$



$$\dot{\gamma} = 10^{-4}$$

$$\dot{\gamma} = 10^{-2}$$



 $\Delta \gamma = 1\%$

How slow should we drive an athermal system to reach the AQS limit?



System size: LTotal flip rate: $R_{\text{flip}} = \frac{L^2 \dot{y}}{a^2 \Delta \epsilon_0}$









How to characterize avalanches?



Transverse diffusion coefficient

Plasticity-induced diffusion

Over a large strain interval:

$$\Delta y_i = \sum_f u_y^e(\vec{r}_i - \vec{r}_f) \Rightarrow \langle \Delta y^2 \rangle = N_e(\Delta \gamma) \langle u_y^2 \rangle_e$$

<u>Events = single flips</u>

$$N_f(\Delta \gamma) = \frac{L^2 \Delta \gamma}{a^2 \Delta \epsilon_0}$$

$$\vec{u} = \frac{2a^2 \Delta \epsilon_0}{\pi} \frac{x y}{r^4} \vec{r}$$

$$\langle u_y^2 \rangle_f = \frac{a^4 \Delta \epsilon_0^2}{4\pi} \ln(L/a)$$

$$\frac{\langle \Delta y^2 \rangle}{\Delta \gamma} = \frac{a^2 \Delta \epsilon_0}{4 \pi} \ln(L/a)$$

Events = linear avalanches



$$N_{a}(\Delta \boldsymbol{\gamma}) = N_{f}(\Delta \boldsymbol{\gamma}) / \boldsymbol{\nu} l$$

$$\langle u_{y}^{2} \rangle_{a} = \frac{a^{4} \Delta \epsilon_{0}^{2} v^{2}}{2 \pi} \left(\frac{l}{L}\right)^{2} \ln(L/l)$$

$$\frac{\langle \Delta y^2 \rangle}{\Delta \gamma} = \frac{a^2 \Delta \epsilon_0}{4\pi} v l \ln(L/l)$$

AL and C. Caroli, PRL 103, 065501 (2009) Chattoraj *et al*, PRE 011501 (2011)

Athermal, finite strain rate: transverse diffusion

$$\hat{D} \equiv \frac{\langle \Delta y^2 \rangle}{\Delta y} = \frac{a^2 \Delta \epsilon_0}{4\pi} v l \ln (L/l)$$

$$\hat{D} = \frac{\langle \Delta y^2 \rangle}{\Delta y} = \frac{a^2 \Delta \epsilon_0}{4\pi} v l \ln (L/l)$$

$$\hat{D} = \frac{\hat{D}}{\frac{1}{2}} \frac{\hat{D}}{\frac{1}$$

Athermal, finite strain rate: transverse diffusion

$$\widehat{D} = \frac{\langle \Delta y^2 \rangle}{\Delta y} = \frac{a^2 \Delta \epsilon_0}{4\pi} \vee l \ln (L/l)$$
Large $\dot{y} \Rightarrow l \sim a$ $\widehat{D} \sim \ln L$
 $\dot{y} \rightarrow 0 \Rightarrow l \sim L$ $\widehat{D} \sim L$
QS regime
 $U \sin l(\dot{y}) \propto 1/\sqrt{\dot{y}} \Rightarrow \widehat{D}/L = f(L\sqrt{\dot{y}})$

Relevance of avalanche size

• Extension to 3D $l(\dot{y}) \sim a (\Delta \epsilon_0 / \dot{y} \tau_{flip})^{1/3}$

⇒ For atomic glass, with $\tau_{LJ} \sim 10^{-13} \sec a \sim 1 \text{ nm} \Delta \epsilon_0 \sim 5\%$

For $\dot{y} \le 10^{-3} \, \text{s}^{-1}$ $l \ge 1 \, \mu \, \text{m}$ (see: Ni

(see: Nieh et al (2002))

• 2D flow curve $\sigma(\dot{y})$

guess: $\sigma - \sigma_y \approx \mu \dot{\gamma} \tau_{av}$

event duration: $\tau_{av} \sim l/c_s$ (domino-like avalanches)

$$\Rightarrow \sigma = \sigma_y + C \sqrt{\dot{y}}$$
$$C = \frac{\mu}{c_s} a^2 \frac{\Delta \epsilon_0}{\tau} \approx 1$$

3





At finite temperature







Chattoraj et al PRL 105, 266001 (2010)

Finite T, finite strain-rate simulations:

- Standard MD simulation
- Velocity rescaling

$$T \neq 0 \quad \dot{y} \neq 0$$







Finite T, finite strain-rate simulations: $T \neq 0$ $\dot{y} \neq 0$ - Standard MD simulation- Velocity rescalingChattoraj et al, PRE 2011





Furukawa et al, PRL (2009)

Finite T, finite strain-rate simulations:

- Standard MD simulation
- Velocity rescaling

Chattoraj et al, PRE 2011

 $T \neq 0 \quad \dot{y} \neq 0$







Schuh *et al*, Acta Mat. 55, 4067 (2007)





Stress data



- $\sigma(\dot{\mathbf{y}})$
- Decreases strongly with T
- No longer fits Hershel Bulkley law

Activation and driven zones T = 0 $T \neq 0$ $P(\gamma)$ \uparrow Probability that the zone does not flip before $\gamma < \gamma_c$ $\delta \gamma^*$ γ_c γ_0 $\frac{\Upsilon}{R = \omega \exp\left(-\frac{\Delta E}{T}\right)}$ $\frac{\partial P}{\partial \gamma} = -\frac{1}{\dot{\gamma}} P R(\gamma)$ rate of activated jumps: $\Rightarrow P(\gamma; \gamma_0) = \exp\left(-\frac{1}{\dot{\gamma}} \int_{\gamma_0}^{\gamma} R(\gamma') \, \mathrm{d} \gamma'\right) \qquad \text{with:} \quad \left\{ \begin{array}{c} \omega \propto (\gamma_c - \gamma)^{1/4} \\ \Delta E \propto (\gamma_c - \gamma)^{3/2} \end{array} \right.$ $P(\gamma) = \exp\left(-\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6} \left(Q(\delta\gamma) - Q(\delta\gamma_0)\right)\right) \qquad Q(\delta\gamma) = \Gamma\left(\frac{5}{6};\frac{B}{T}\delta\gamma^{3/2}\right)$

Chattoraj et al, PRL 105, 26601 (2010)



 Yields: Average shift of occurrence of plastic events

$$\sigma(\dot{\mathbf{y}};T) = \sigma(\dot{\mathbf{y}};T=0) - 2\,\mu\,\overline{\delta\,\mathbf{y}^*}$$







Metallic glass yield stress

Johnson & Samwer 95, 195501 (2005)

					Property				
Alloy	ρ	Y)	G	B		σ_y	T_g		
	(g/cc)	(GPa)	(GPa)	(GPa)	ν	(GPa)	(K)	σ_y/Y	Ref.
1. Zr41 2Ti 13 8Ni 10Cu 12 5Be22 5	5.9	95	34.1	114.1	0.352	1.86	618	0.0196	[13-15]
		97.2	35.9	111.2	0.354	1.85	613	0.0190	. ,
2. Zr ₄₈ Nb ₈ Ni ₁₂ Cu ₁₄ Be ₁₈	6.7	93.9	34.3	118	0.367	1.95	620	0.0208	[15]
3. Zr ₅₅ Ti ₅ Cu ₂₀ Ni ₁₀ Al ₁₀	6.62	85	31	118	0.375	1.63	625	0.0192	[15]
4. Zr _{57,5} Nb ₅ Cu _{15,4} Ni ₁₂ Al ₁₀	6.5	84.7	30.8	117.6	0.379	1.58	663	0.0187	[15]
5. Zr ₅₅ Al ₁₉ Co ₁₉ Cu ₇	6.2	101.7	37.6	114.9	0.352	2.2	733	0.0216	[16]
6. Pd ₄₀ Cu ₃₀ Ni ₁₀ P ₂₀	9.28	92	34.5	151.8	0.399	1.72	593	0.0187	[17]
 Pd₄₀Cu₃₀Ni₁₀P₂₀ 	9.28	92	33	146	0.394	1.72	593	0.0187	[18]
 Pd₄₀Cu₃₀Ni₁₀P₂₀ 	9.30	92	35.8	144.7	0.394	1.75	595	0.0190	[17]
9. Pd ₆₀ Cu ₂₀ P ₂₀	9.78	91	32.3	167	0.409	1.70	604	0.0187	[15]
10. Pd ₄₀ Cu ₄₀ P ₂₀	9.30	93	33.2	158	0.402	1.75	548	0.0188	[15]
11. Ni45Ti20Zr25Al10	6.4	109.3	40.2	129.6	0.359	2.37	791	0.0217	[19]
12. Ni ₄₀ Ti ₁₇ Zr ₂₈ Al ₁₀ Cu ₅	6.48	127.6	47.3	140.7	0.349	2.59	862	0.0203	[19]
13. Ni ₆₀ Nb ₃₅ Sn ₅	8.64	183.7	66.32	267	0.385	3.85	885	0.0210	[20]
14. Ni ₆₀ Sn ₆ (Nb _{0.8} Ta _{0.2}) ₃₄	9.24	161.3	59.41	189	0.357	3.50	875	0.0217	[16]
15. Ni ₆₀ Sn ₆ (Nb _{0.6} Ta _{0.4}) ₃₄	9.80	163.7	60.1	197.6	0.361	3.58	882	0.0219	[16]
16. Cu ₆₄ Zr ₃₆	8.07	92	34	104.3	0.352	2.0	787	0.0217	[21]
17. Cu ₄₆ Zr ₅₄	7.62	83.5	30.0	128.5	0.391	1.40	696	0.0168	[22]
18. Cu ₄₆ Zr ₄₂ Al ₇ Y ₅	7.23	84.6	31	104.1	0.364	1.60	713	0.0189	[23]
19. Pd _{77.5} Cu ₆ Si _{16.5}	10.4	89.7	31.8	166	0.409	1.5	550	0.0167	[24]
20. Pt ₆₀ Ni ₁₅ P ₂₅	15.7	96.1	33.8	202	0.420	1.4	488	0.0146	[25]
21. Pt _{57.5} Cu _{14.7} Ni ₅ P _{22.8}	15.2	95.7	33.4	243.2	0.434	1.45	490	0.0151	[26]
22. Pd ₆₄ Ni ₁₆ P ₂₀	10.1	91.9	32.7	166	0.405	1.55	452	0.0169	[24]
23. MgGd ₁₀ Cu ₂₅	4.04	49.1	18.6	46.3	0.32	0.98	428	0.020	[16]
24. La ₅₅ Al ₂₅ Cu ₁₀ Ni ₅ Co ₅	6.0	41.9	15.6	44.2	0.342	0.85	430	0.0203	[15]
25. Ce ₇₀ Al ₁₀ Ni ₁₀ Cu ₁₀	6.67	30.3	11.5	27	0.313	0.65	359	0.0215	[27]
26. Cu ₅₀ Hf ₄₃ Al ₇	11.0	113	42	132.8	0.358	2.2	774	0.0195	[16]
27. Cu _{57.5} Hf _{27.5} Ti ₁₅	9.91	103	37.3	117.5	0.356	1.94	729	0.0188	[16]
28. Fe ₆₁ Mn ₁₀ Cr ₄ Mo ₆ Er ₁ C ₁₅ B ₆	6.89	193	75	146	0.280	4.16	870	0.0216	[28]
29. Fe ₅₃ Cr ₁₅ Mo ₁₄ Er ₁ C ₁₅ B ₆	6.92	195	75	180	0.32	4.2	860	0.0215	[28]
30. Au _{49.5} Ag _{5.5} Pd _{2.3} Cu _{26.9} Si _{16.3}	11.6	74.4	26.5	132.3	0.406	1.20	405	0.0141	[29]
31. Au ₅₅ Cu ₂₅ Si ₂₀	12.2	69.8	24.6	139.8	0.417	1.00	348	0.0143	[29]

TABLE I. Summary of data on alloy compositions and properties used in this Letter.

Metallic glass yield stress

Johnson & Samwer 95, 195501 (2005)



Conclusion

-ow T

- Activation over driven barriers
 - thermal fluctuations primarily trigger activation above driven barriers
 - the avalanche dynamics is unchanged: mere shift of the occurrence of plastic events
 - permits to predict $\sigma(\dot{m{y}}$, T)
- Diffusion measurements
 - particle displacements dominated by shearing effect when with $\dot{y}^* \tau_{\alpha} \sim 10^{-2}, 10^{-3}$ - in this region and for $\dot{y} > \dot{y}^*$ $\dot{y} > \dot{y}_c(L)$ $l(\dot{y}) \propto 1/\sqrt{\dot{y}}$

