REDUCTION OF CRYSTALLINE REPRESENTATIONS AND LOCAL CONSTANCY IN THE WEIGHT SPACE

For any prime p, we denote the local Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}_p|\mathbb{Q}_p)$ by G_p. It follows from Fontaine's classification of p-adic representations of G_p that two-dimensional irreducible crystalline representations are parametrized (up to character twists) by the pairs (k, α) with $k \in Z_{\geq 2}$ and $\alpha \in \mathfrak{m}_{\bar{\mathbb{Q}}_p}$, the maximal ideal in the ring of integers of $\bar{\mathbb{Q}}_p$. Let $V_{k,\alpha}$ denote the standard crystalline representation with Hodge-Tate weights $(0, k-1)$ and trace α of the φ operator acting on the associated filtered φ-module. We are interested in the map $(k, \alpha) \mapsto \bar{V}_{k,\alpha}$, where $\bar{V}_{k,\alpha}$ is the semi-simplified mod p reduction of any $\mathbb{G}_p$-stable integral lattice in $V_{k,\alpha}$. Laurent Berger (2012) proved the local constancy of the above map with respect to both variables, under some extra hypotheses when k is varying. However, no estimate is known for the radius of local constancy in the weight space. We will study some examples to see why the radius above should be highly dependent on the slope of the crystalline representations. Then we will use the compatibility of p-adic and mod p Local Langlands correspondence to prove the local constancy around some special weight points together with an explicit lower bound on the radius of local constancy.