Deconfined quantum critical points: Symmetries and dualities

T. Senthil (MIT)

Chong Wang, Adam Nahum, Max Metlitski, Cenke, Xu, TS, arXiv (March 2017)

Plan

I will discuss phase transitions beyond the Landau-Ginzburg-Wilson paradigm in quantum magnets.

Dualities will be useful (but with a perspective that is slightly different from what may be familiar in high energy physics).

In this context some of the recent conjectured dualities can be tested very concretely in numerics.

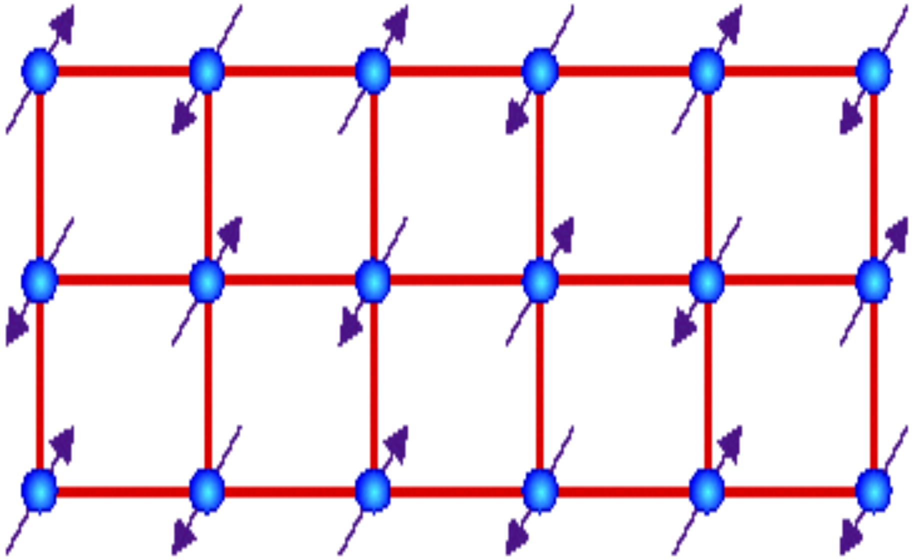
Phase transitions in quantum magnets

Spin-1/2 magnetic moments on a square lattice

Model Hamiltonian

$$H_0 = J \sum_{<rr'>} \vec{S}_r \cdot \vec{S}_{r'} + \cdots$$

$\cdots$ = additional interactions to tune quantum phase transitions



Usual fate: Neel antiferromagnetic order

Breaks SO(3) spin rotation symmetry.

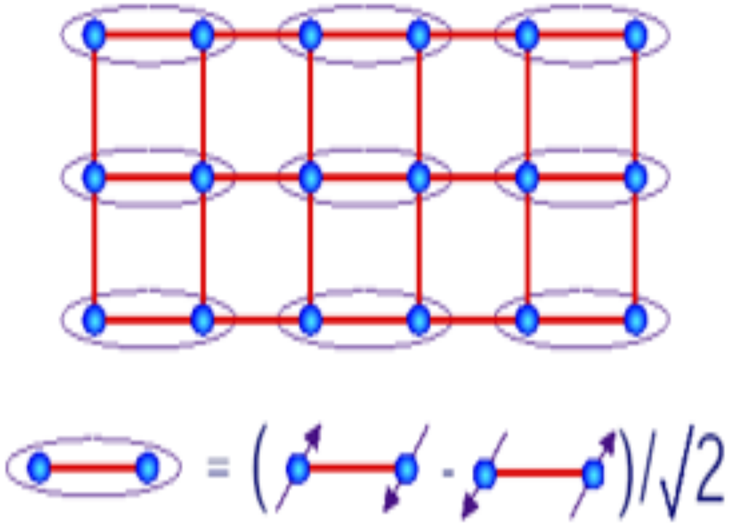
Neel order parameter: SO(3) vector

An SO(3) symmetry preserving phase (``quantum paramagnet")

With suitable additional interactions, obtain other phases that preserve spin rotation symmetry.

Here I will focus on a particular such phase called a Valence Bond Solid (VBS) that breaks lattice symmetries.

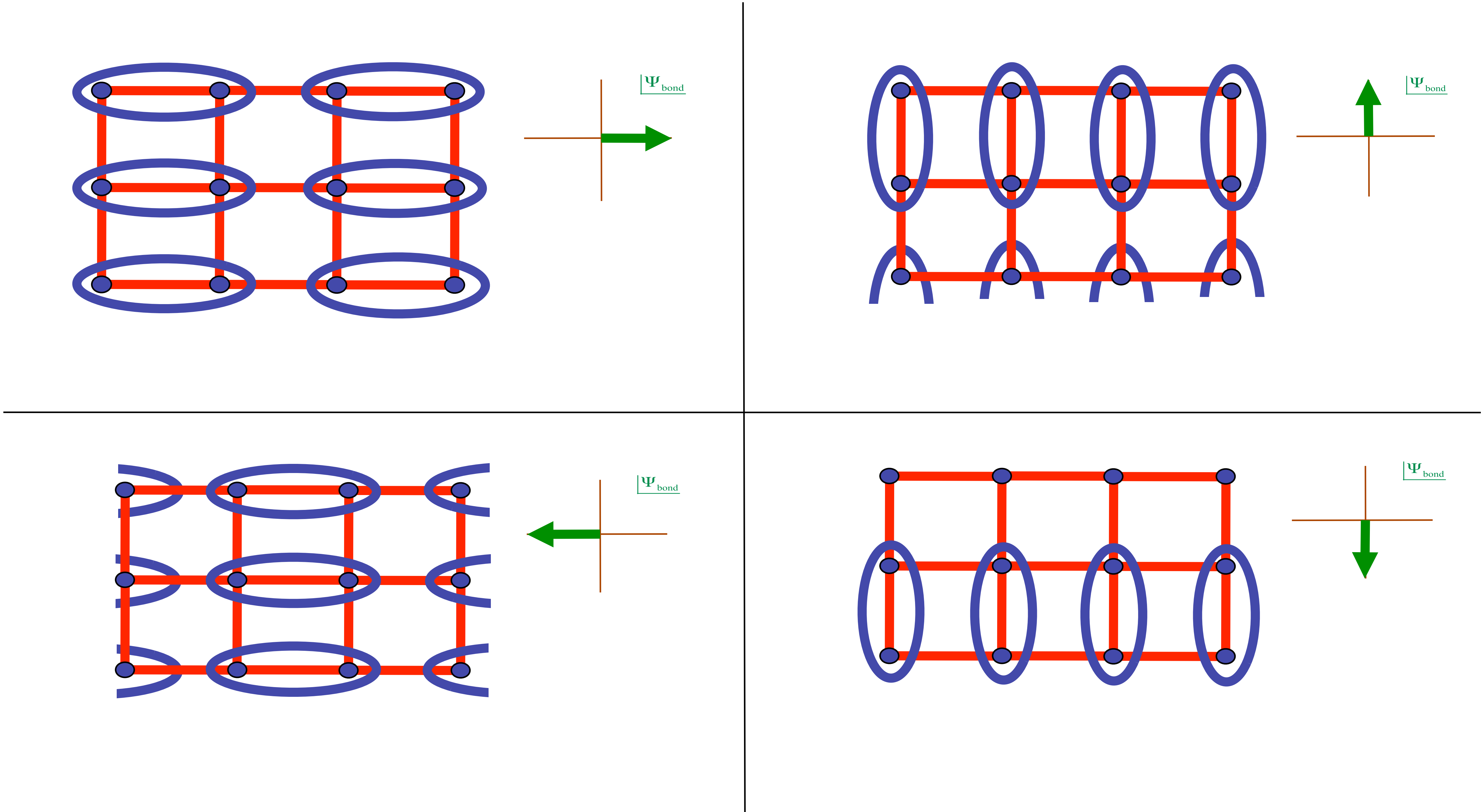
Z_4 order parameter associated with four patterns of VBS ordering.



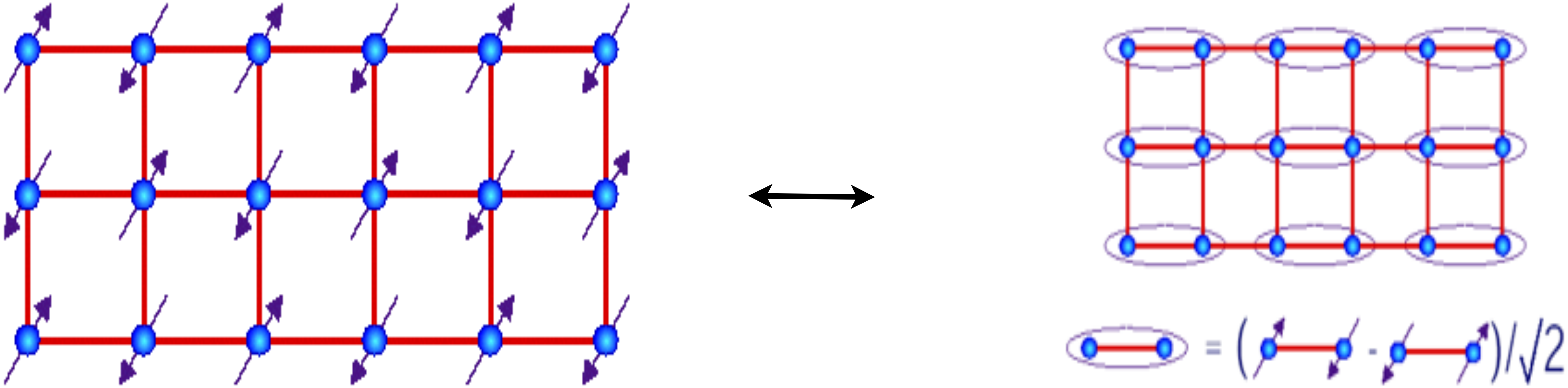
(Read, Sachdev, 89-91; many lattice models, eg, Sandvik 06)
* (VBS a.k. a ``Spin-Peierls")

VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



The Neel-VBS quantum phase transition: A lightning review



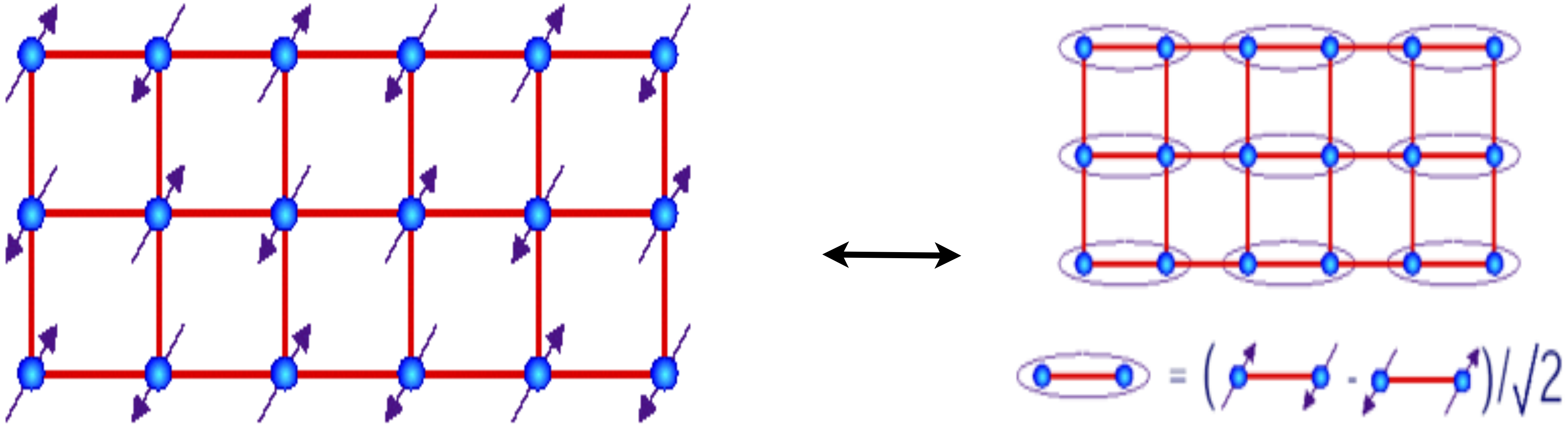
Naive Landau expectation: Two independent order parameters - no generic direct second order transition.

Naive expectation is incorrect: Possibility of a continuous Landau-forbidden phase transition between Landau allowed phases

The Neel-VBS transition: A lightning review

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Possible Landau-forbidden continuous transitions between Landau allowed phases



Field theoretic framework:

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

z_α: $SU(2)$ doublet (“spinon”)

b: dynamical $U(1)$ gauge field.

$\cdots$: all allowed local operators consistent with symmetries of lattice magnet.

Comments

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

- Theory known as "Non-compact CP^1 model" $(NCCP^1)$

  Monopole operators in b not added to action

- Neel order parameter $\vec{N} = z^\dagger \vec{\sigma} z$

  VBS order parameter $\psi_{VBS} = \mathcal{M}_b$ (monopole operator)

Read, Sachdev, 89; Haldane 88

Theory not in terms of natural order parameters but in terms of `fractional spin' fields z + gauge fields.

``Deconfined quantum critical point"

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Physical mechanism for non-Landau transition

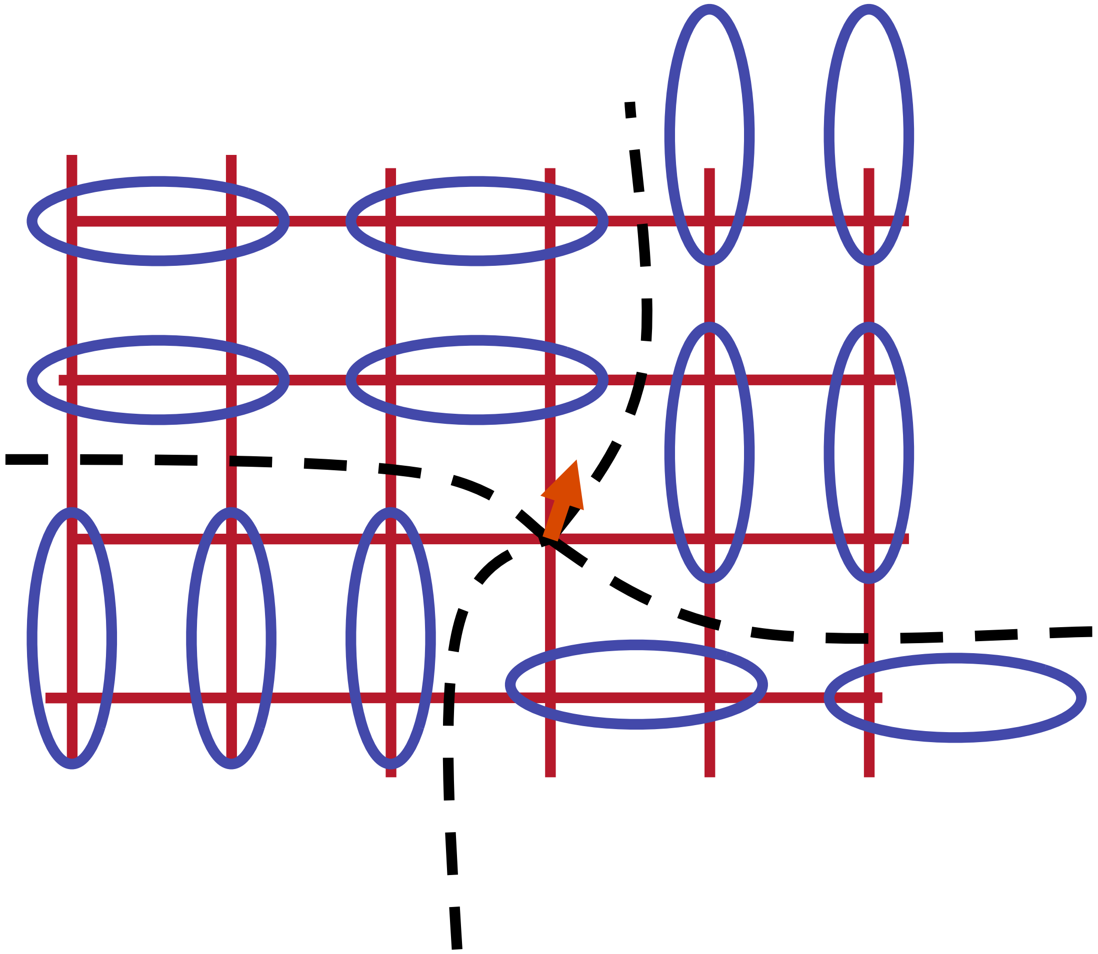
Topological defects carry non-trivial quantum numbers.

Eg: VBS phase with a Z_4 order parameter:

Z_4 vortices where four elementary domain walls meet hosts spin-1/2 moment.

Transition to Neel phase:

Proliferate these vortices.



Levin, TS 04

Putative critical theory (using usual charge-vortex duality):
$NCCP^1$ (+ anisotropies)

Identify z_α with the VBS vortex.

A formal description of Neel/VBS competition

Tanaka, Hu 06
TS, Fisher, 06

Form 5-component unit vector $\in S^4$

$$\hat{n} = (n^1, n^2, n^3, n^4, n^5), \quad \hat{n}^2 = 1$$

$\psi_{VBS} = n_1 + in_2$, Neel vector $= n^{3,4,5}$
Neel-VBS competition:

$$S = \frac{1}{2g}\int d^3x \, (\partial n^a)^2 + 2\pi\Gamma_{WZW}\,[n^a] + \cdots .$$

$\cdots =$ anisotropies demanded by microscopic symmetries.

Wess-Zumino-Witten term Γ_{WZW} crucial to capture non-trivial structure of topological defects.

Vortex in any 2 components (breaks $SO(5)$ to $SO(2) \times SO(3)$):
$SO(2)$ charge 0 but $SO(3)$ spinor.

Is the Neel/VBS transition continuous?

Good evidence it is described by NCCP^1 but does NCCP^1 have a second order transition?

Apparently yes but ultimate fate not yet clear.

(i) Indirect support: NCCP^N in 1/N flows to CFT.

(ii) Direct support: Numerics (Sandvik, Kaul, Melko, Damle, Alet,……., Nahum et al, 06- present)

Simulations of variety of lattice models in same expected universality class: apparent continuous transition with roughly consistent properties and with theory.

Caveat: Drift of critical exponents with system size

Recent numerics: emergence of SO(5) rotating Neel into VBS (Nahum et al, 15)

(But putative SO(5) CFT in tension with bootstrap (Nakayama 16; Simmons-Duffin, unpublished))

Some questions

1. How to think about emergence of SO(5)?

2. Field theoretic formulation with manifest SO(5) symmetry?

Some questions

1. *How to think about emergence of SO(5)?*

Proposal: duality web for NCCP^1 which implies SO(5) as a ``quantum symmetry".

2. Field theoretic formulation with manifest SO(5) symmetry?

Wang, Nahum, Metlitski, Xu, TS, 17

Duality web for NCCP^1: A proposal

A first member: Self-duality

$$NCCP^1 \longleftrightarrow \widehat{NCCP}^1$$

$$\mathcal{L} = |(\partial_\mu - ib_\mu)z_\alpha|^2 + \cdots \qquad \hat{\mathcal{L}} = |(\partial_\mu - i\hat{b}_\mu)w_\alpha|^2 + \cdots$$

$$z_1^* z_2 \longleftrightarrow \mathcal{M}_{\hat{b}} \text{ (dual monopole)}$$

$$\text{(Monopole) } \mathcal{M}_b \longleftrightarrow w_1^* w_2$$

$$z^\dagger \sigma^z z \longleftrightarrow \text{-}w^\dagger \sigma^z w$$

Can match other local operators.

Consequences of self-duality - I

Emergent SO(5) symmetry:
Either theory only has manifest SO(3) x U(1) continuous symmetry.

For symmetries to agree, must be that either theory has hidden SO(5) symmetry

$$\begin{aligned}(n_1, n_2, n_3, n_4, n_5) \quad &\sim \quad (2\,\mathrm{Re}\mathcal{M}_b, 2\,\mathrm{Im}\mathcal{M}_b, z^\dagger\sigma_x z, z^\dagger\sigma_y z, z^\dagger\sigma_z z) \\ &\sim \quad (w^\dagger\sigma_x w, -w^\dagger\sigma_y w, 2\,\mathrm{Re}\mathcal{M}_{\tilde{b}}, -2\,\mathrm{Im}\mathcal{M}_{\tilde{b}}, w^\dagger\sigma_z w)\end{aligned}$$

Observed SO(5) in numerics explained if self-duality is correct.

Conversely, the observed SO(5) supports (but not prove) the self-duality.

Consequences of self-duality - II

Easy plane deformation:

Perturb both sides by the same operator $\mathcal{O} \sim (z^\dagger \sigma^z z)^2 \sim (w^\dagger \sigma^z w)^2$

Symmetry of either theory is reduced to O(2) x O(2):

Self-duality of original theory => self-duality of the perturbed theories.

Can be derived directly using the standard charge-vortex duality (Peskin-Dasgupta-Halperin) and has long been known to be true (Motrunich, Vishwanath 04).

Useful consistency check ✓

Other members of the duality web: Fermionic QED3-Gross Neveau

$$\begin{tikzcd}
NCCP^1 \arrow[r, leftrightarrow, "\text{self-dual}"] \arrow[d, leftrightarrow] & \widehat{NCCP}^1 \arrow[d, leftrightarrow] \\
QED_3 - GN \arrow[r, leftrightarrow, "\text{self-dual}"] & \widehat{QED_3 - GN}
\end{tikzcd}$$

$\mathcal{L} = \sum_{j=1,2} \bar{\psi}_j \gamma^\mu (-i\partial_\mu - a_\mu)\psi_j + \phi \sum_{j=1,2} \bar{\psi}_j \psi_j + V(\phi)$

$\mathcal{L} = \sum_{j=1,2} \bar{\chi}_j \gamma^\mu (-i\partial_\mu - \hat{a}_\mu)\chi_j + \hat{\phi} \sum_{j=1,2} \bar{\chi}_j \chi_j + \hat{V}(\hat{\phi})$

Passes simple consistency checks.
Many predictions for numerics that can tested!

Conversely fermionic formulation allows new numerical (and analytical?) handle on deconfined criticality theory.

Easy plane deformation

Interesting class of theories:

Break manifest continuous symmetry to O(2) x O(2)

Describes

(i) Neel-VBS phase transitions in S = 1/2 square lattice quantum magnets with XY symmetry.
(ii) Integer quantum Hall phase transition of bosons in two space dimensions

Fermionic duals? Consequences?

Duality web for the O(2) x O(2) theory

$$\mathcal{L}_{EP-NCCP^1} = |D_b z_\alpha|^2 - V(|z_1|^2, |z_2|^2)$$

$$\begin{tikzcd}
EP - NCCP^1 \arrow[r, leftrightarrow, "\text{self-dual}"] \arrow[d, leftrightarrow] & EP - \widehat{NCCP}^1 \arrow[d, leftrightarrow] \\
QED_3 \arrow[r, leftrightarrow, "\text{self-dual}"] & \widehat{QED}_3
\end{tikzcd}$$

QED_3 with 2 massless 2-component fermions (Nf = 2).

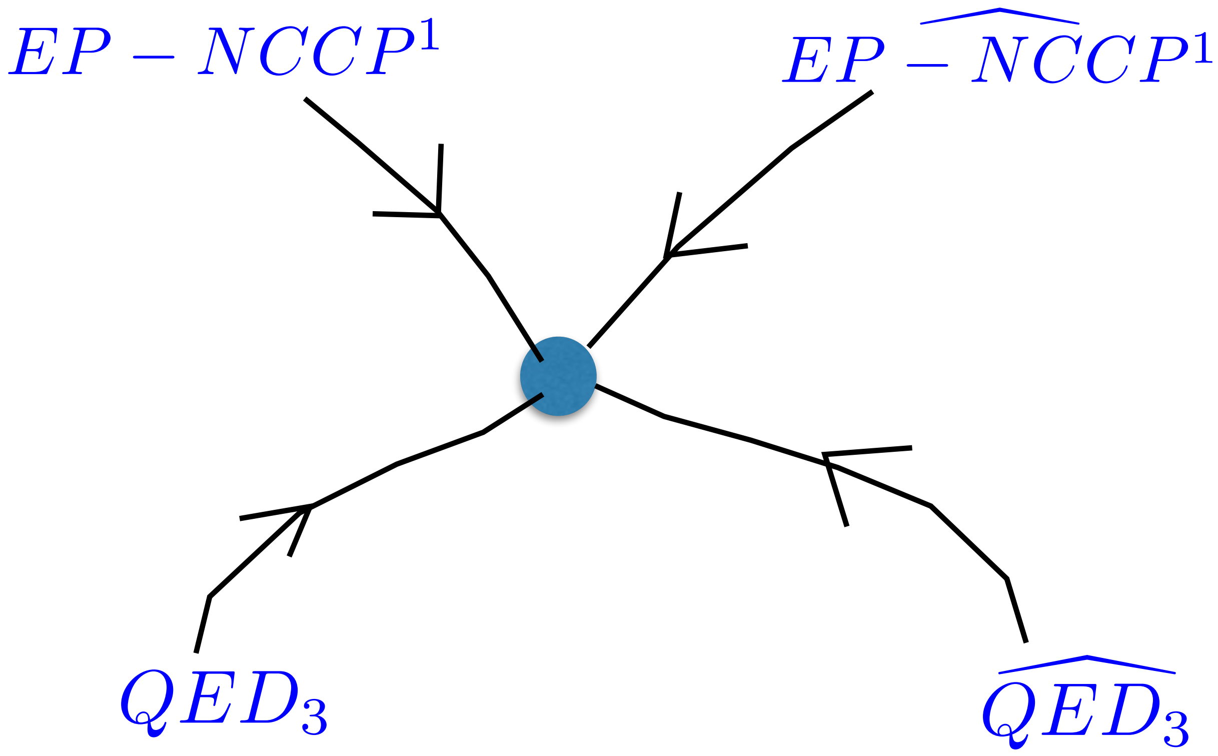
Related to several statements in old and recent literature (Motrunich, Vishwanath 04; Yu, Xu, 16; Karch, Tong, 16; Hsin, Seiberg 16)

But care needed in interpretation!

Duality and symmetries of easy plane theory

With O(2) x O(2) symmetry: can show all theories have same local operators, symmetries and can match phase diagrams. (``weak duality'')

Very Strong Conjecture:
All theories flow to a common conformal fixed point along Neel-VBS phase boundary with enlarged O(4) symmetry(*).



(*)Rotates XY Neel to VBS.

Concrete predictions for numerics on QED3;
Only a less strong version may hold but these can also be tested through QED3 numerics.

Comments

IR fate of these theories not very well known (many controversies).

1. Neel-VBS phase transitions in S = 1/2 square lattice quantum magnets with XY symmetry
   - numerics on many models showing first order transition (Kuklov, Prokofiev et al, 08; Jiang, Nyfeler, Chandrasekharan, Wiese 08; ………; Emidio, Kaul 16).

Two very recent calculations on different lattice models find a second order transition (Qin,….Meng, 17; Zhang, He,…..Pollman, 17)

2. $N_f = 2$ QED3:

Early numerics - symmetry broken phase in IR (not CFT) (S. J. Hands, J. B. Kogut, L. Scorzato, and C. G. Strouthos, 04)

However more recent numerics (Karthik, Narayanan 2016) finds support for an IR CFT.

Dualities suggest these old controversies may be related and leads to new numerical ways to address these issues.

Some questions

1. How to think about emergence of SO(5)?

2. *Field theoretic formulation with manifest SO(5) symmetry?*

SO(5) is realized anomalously: can be viewed as surface theory of 3+1-D bosonic Symmetry Protected Topological paramagnet.

$N_f = 2$ QCD3 has same anomalous symmetry, and could have same IR physics.

Wang, Nahum, Metlitski, Xu, TS, 17

Anomalous SO(5) symmetry

Couple in a background $SO(5)$ gauge field A^5, and study instanton associated with $\Pi_1(SO(5)) = Z_2$.

Example $A^5 = A^{mon}(x)T^1$

Usual monopole potential

SO(5) generator

(Breaks continuous symmetry from SO(5) to SO(3) x SO(2))

Instanton creates a vortex in 2 components of the SO(5)-vector.

=> has zero SO(2) charge but **is an SO(3) spinor.**

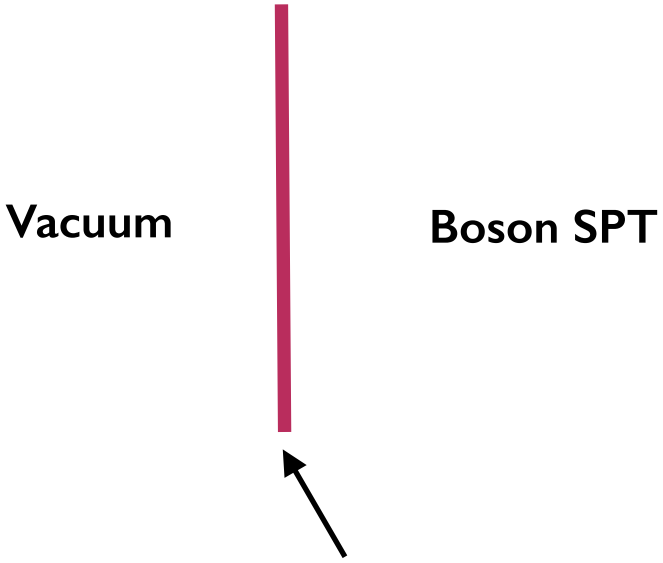
=> anomalous SO(5) symmetry.

Surface state of a a 3+1-D bosonic SPT state

A 3+1-D boson SPT with SO(5) x T can be constructed such that

Response to background SO(5) gauge field characterized by a **``discrete theta term''** (Aharony, Seiberg, Tachikawa, 13)

Modifies SO(5) monopole in precisely the right way to match the known instanton structure of the boundary theory.



putative 2+1-D theory with anomalous SO(5) x T symmetry.

A manifestly SO(5) x T invariant boundary theory

Massless fermionic $N_f = 2$ QCD_3 with an SU(2) gauge field(*)

(Start with 8 massless Majorana fermions with SO(8) symmetry and gauge an SU(2) subgroup).

Alternate formulation of Neel-VBS competition that can be tuned to have manifest anomalous SO(5) x T.

(*) In condensed matter this is familiar as the theory of the ``pi-flux" state of the antiferromagnet.

Comments

1. IR fate of $N_f = 2$ QCD$_3$?

Can show anomaly implies either SO(5) x T symmetry is broken, or theory is a CFT.
``Symmetry enforced gapless”

2. Same local operators, and anomaly as putative Neel/VBS critical point.

Useful to compare numerics on QCD3 with Neel/VBS.

3. Alternate to sigma model + WZW formulation which also has manifest symmetry (better suited for calculations/formal manipulations; a renormalizeable field theory).

Comments (cont'd)

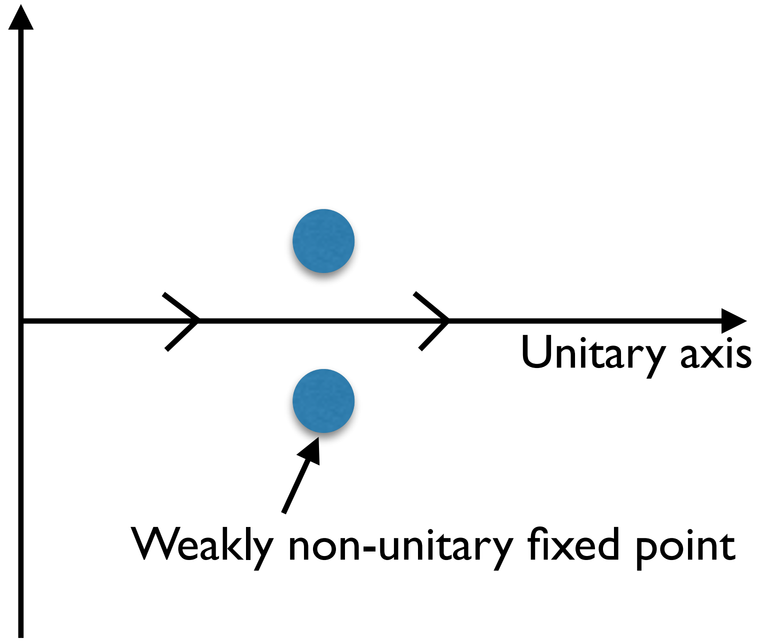
What about tension between SO(5) and bootstrap?

A possible resolution: Current numerics not really approaching the true asymptotic physics (even though lattice size is 512 x 512)

Observed SO(5) symmetry and scaling are properties of proximity of RG flow to a slightly non-unitary fixed point of QCD3.

``Quantum pseudocriticality"

Similar phenomena known in 2d 5-state Potts models.



Summary

New progress on old problem of deconfined critical points;
better understanding of possible emergent symmetries and dualities.

Many new predictions for tests of the dualities, etc in numerics.

Combined input from field theory + numerics + bootstrap will be great!

Quantum pseudocriticality - new paradigm for nearly scale invariant fluctuations in condensed matter?