

# TESTING THE COSMOLOGICAL PRINCIPLE

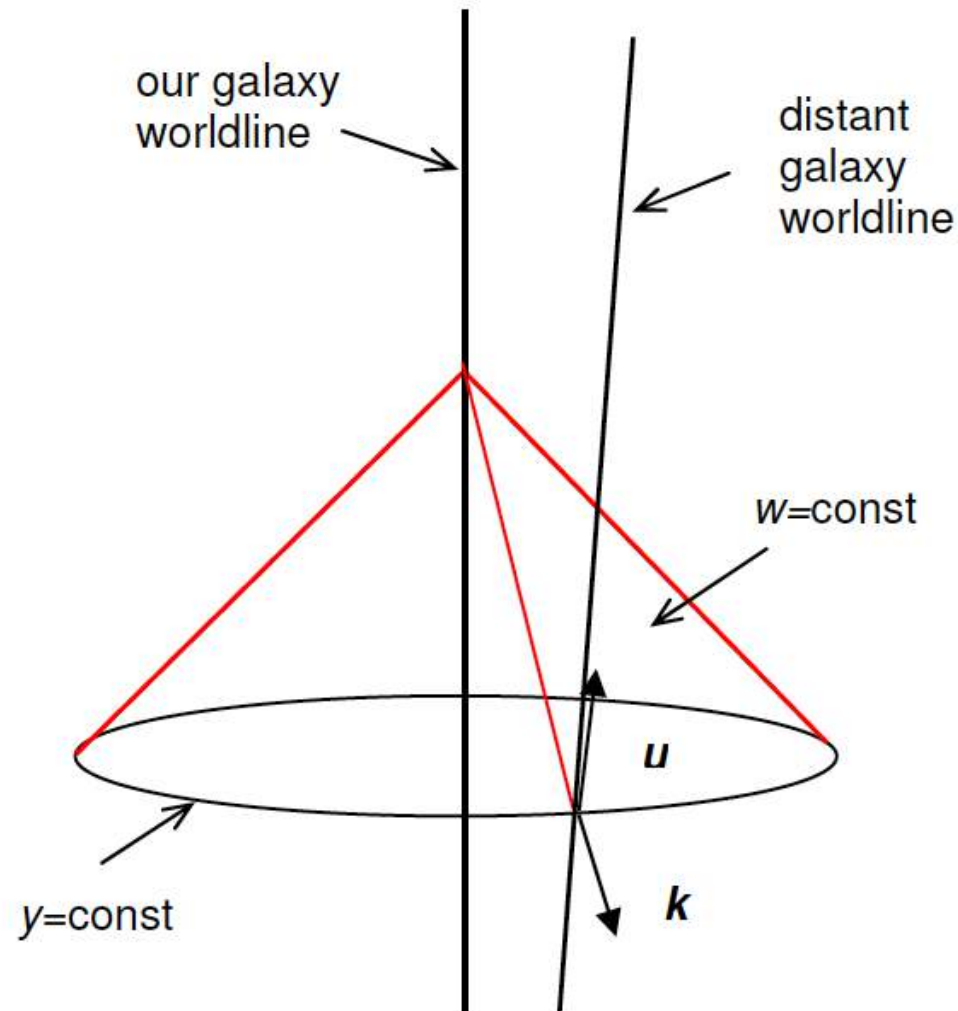
**Subir Sarkar**

*Rudolf Peierls Centre for Theoretical Physics  
University of Oxford*

*None of us can understand why there is a Universe at all, why anything should exist; that's the ultimate question. But while we cannot answer this question, we can at least make progress with the next simpler one, of what the Universe as a whole is like.*

**Dennis Sciama (1978)**

# ALL WE CAN *EVER* LEARN ABOUT THE UNIVERSE IS CONTAINED WITHIN OUR PAST LIGHT CONE



We *cannot* move over cosmological distances and check if the universe looks the same from 'over there' as it does from here ... so there are ***limits to what we can know*** ('cosmic variance')

# STANDARD COSMOLOGICAL MODEL

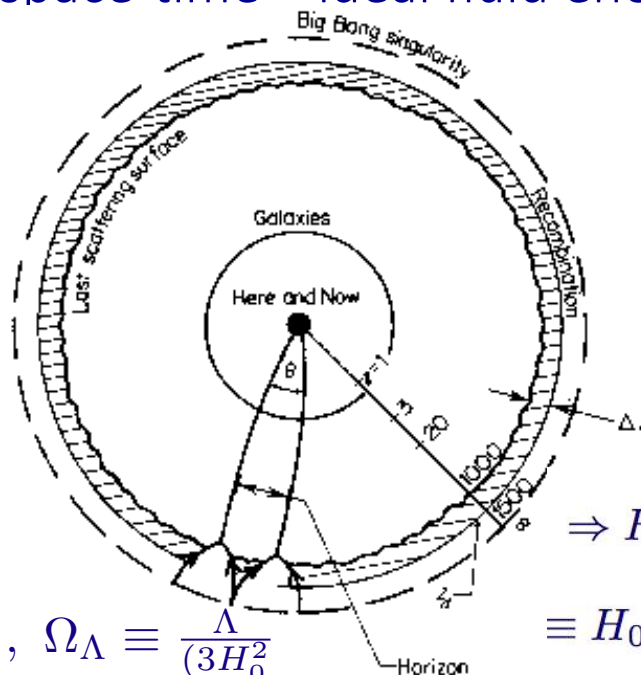
The universe is isotropic + homogeneous (when averaged on 'large' scales)  
 $\Rightarrow$  Maximally-symmetric space-time + ideal fluid energy-momentum tensor

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

$$= a^2(\eta) [d\eta^2 - d\vec{x}^2]$$

$$a^2(\eta) d\eta^2 \equiv dt^2$$

Robertson-Walker



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Einstein

$$\Omega_m \equiv \frac{\rho_m}{(3H_0^2/8\pi G_N)}, \quad \Omega_k \equiv \frac{k}{(3H_0^2 a_0^2)}, \quad \Omega_\Lambda \equiv \frac{\Lambda}{(3H_0^2)}$$

$$\Rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\equiv H_0^2 [\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda]$$

So the **Friedmann-Lemaître equation**  $\Rightarrow$  cosmic sum rule:  $\Omega_{\text{matter}} + \Omega_{\text{curvature}} + \Omega_\Lambda = 1$

We observe  $\sim$ zero curvature (CMB fluctuations) + insufficient matter to make up critical density

$\rightarrow$  infer universe is dominated by **dark energy** with:  $\Omega_\Lambda = 1 - \Omega_m - \Omega_k \sim 0.7 \Rightarrow \Lambda \sim 2H_0^2$

Since 1998 (Riess *et al.*<sup>1</sup>, Perlmutter *et al.*<sup>2</sup>), surveys of cosmologically distant Type Ia supernovae (SNe Ia) have indicated an acceleration of the expansion of the Universe, distant SNe Ia being dimmer than expected in a decelerating Universe. With the assumption that the Universe can be described on average as isotropic and homogeneous, this acceleration implies either the existence of a fluid with negative pressure usually called “Dark Energy”, a constant in the equations of general relativity or modifications of gravity on cosmological scales.

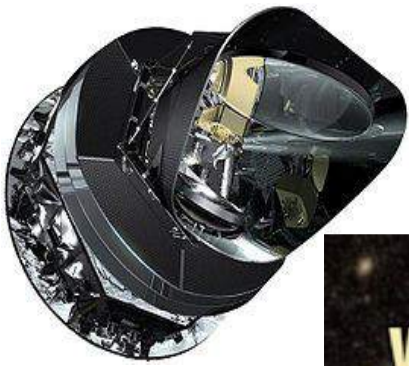
*... the Universe must appear to be the same to all observers wherever they are.  
This 'cosmological principle' ...*



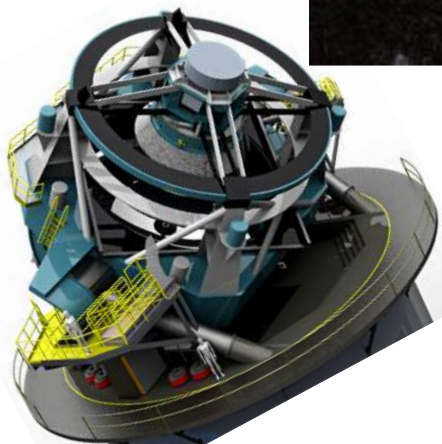
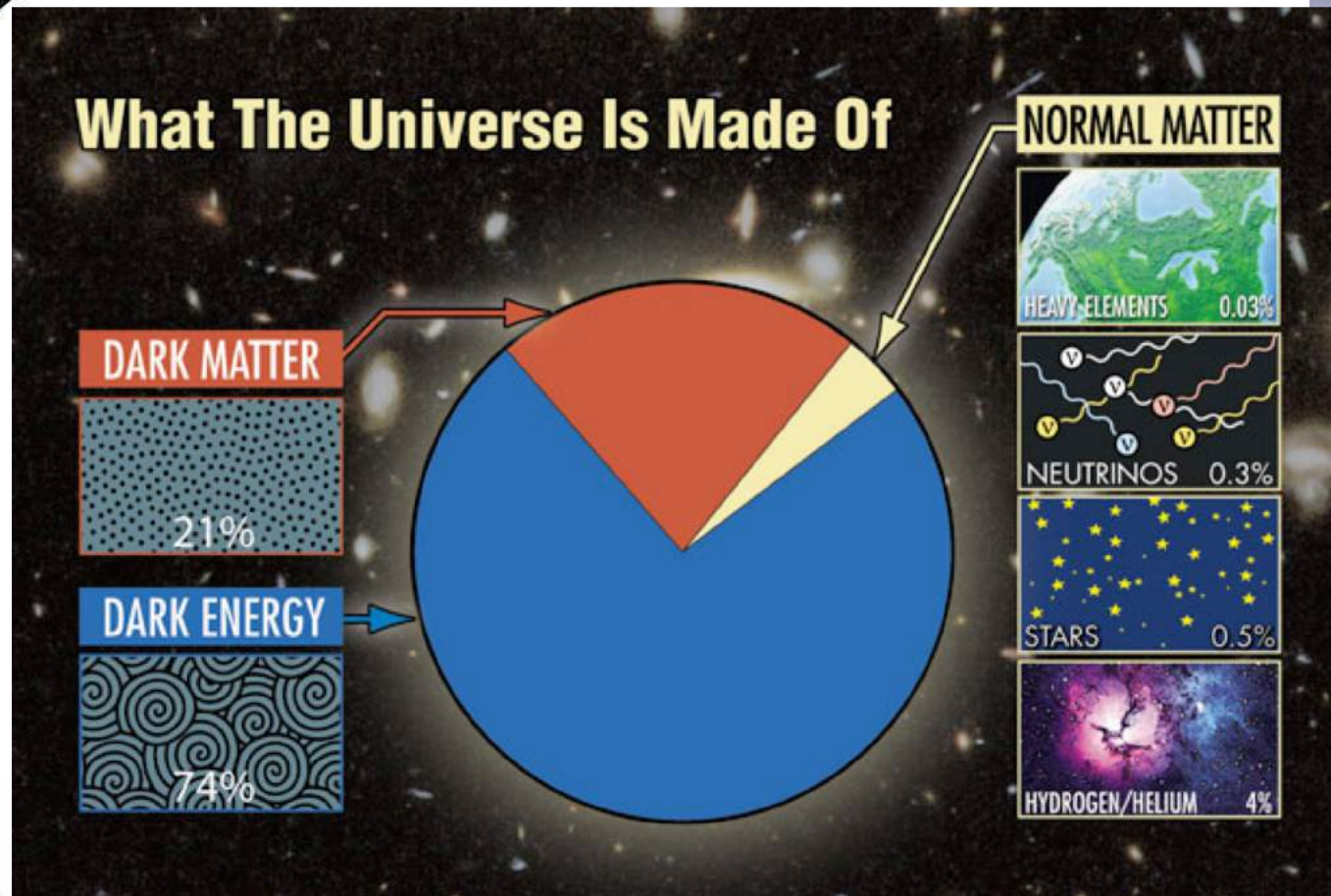
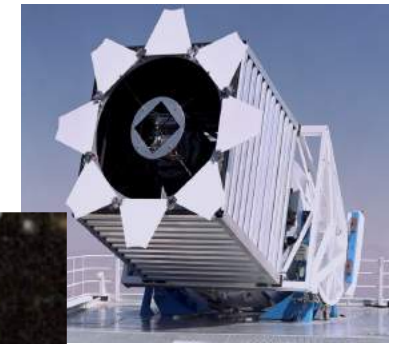
**Edward Arthur Milne (1896-1950)**

Rouse Ball Professor of Mathematics & Fellow of Wadham College, Oxford, 1928-

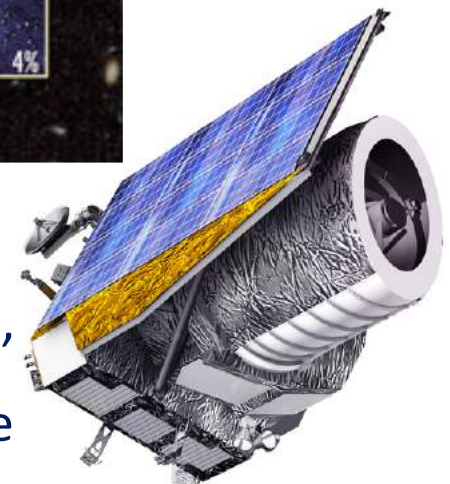




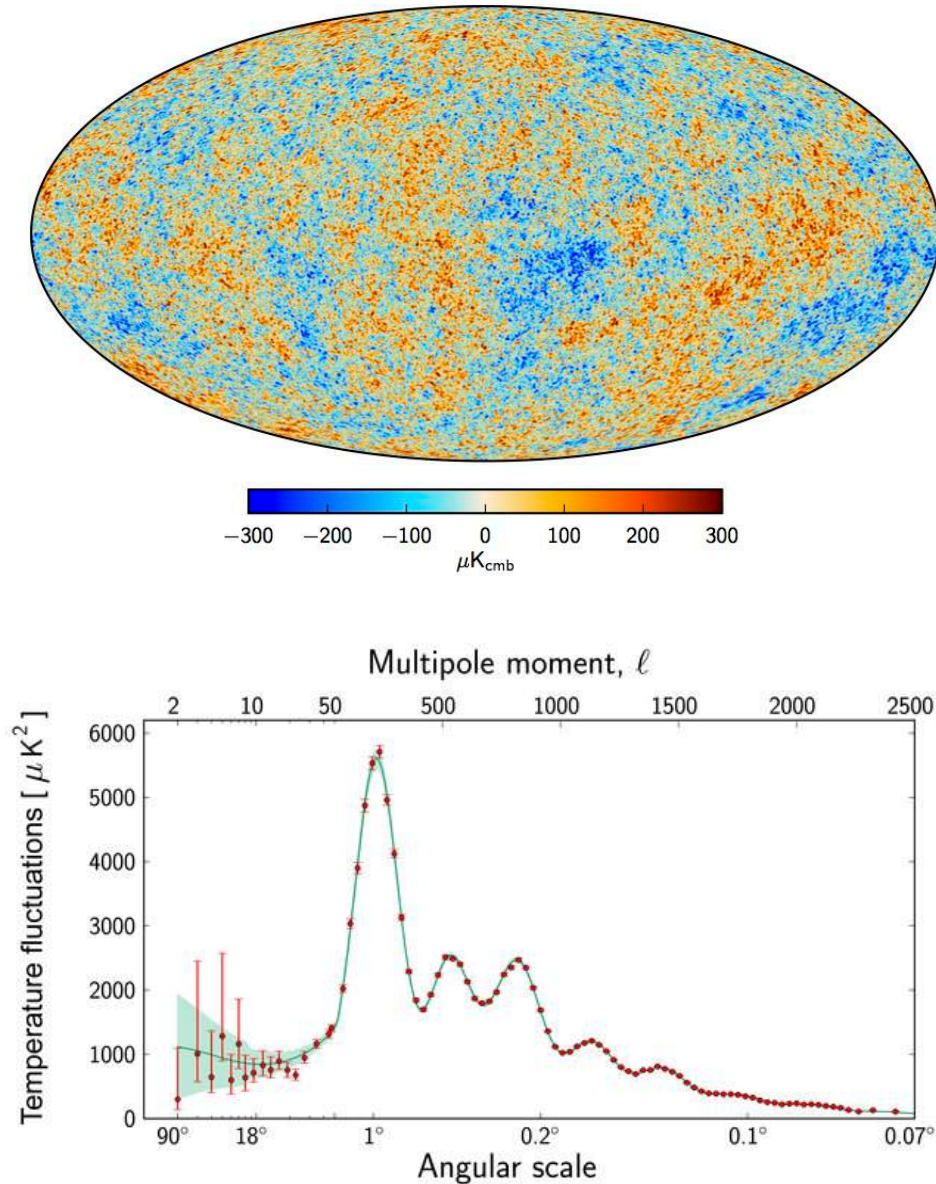
BUT IS THIS *SIMPLEST POSSIBLE* MODEL  
(STILL) FIT FOR PURPOSE?



There has been substantial investment in major satellites and telescopes to *measure the parameters* of the 'standard cosmological model' with increasing 'precision'... but surprisingly little interest in ***testing its foundational assumptions***



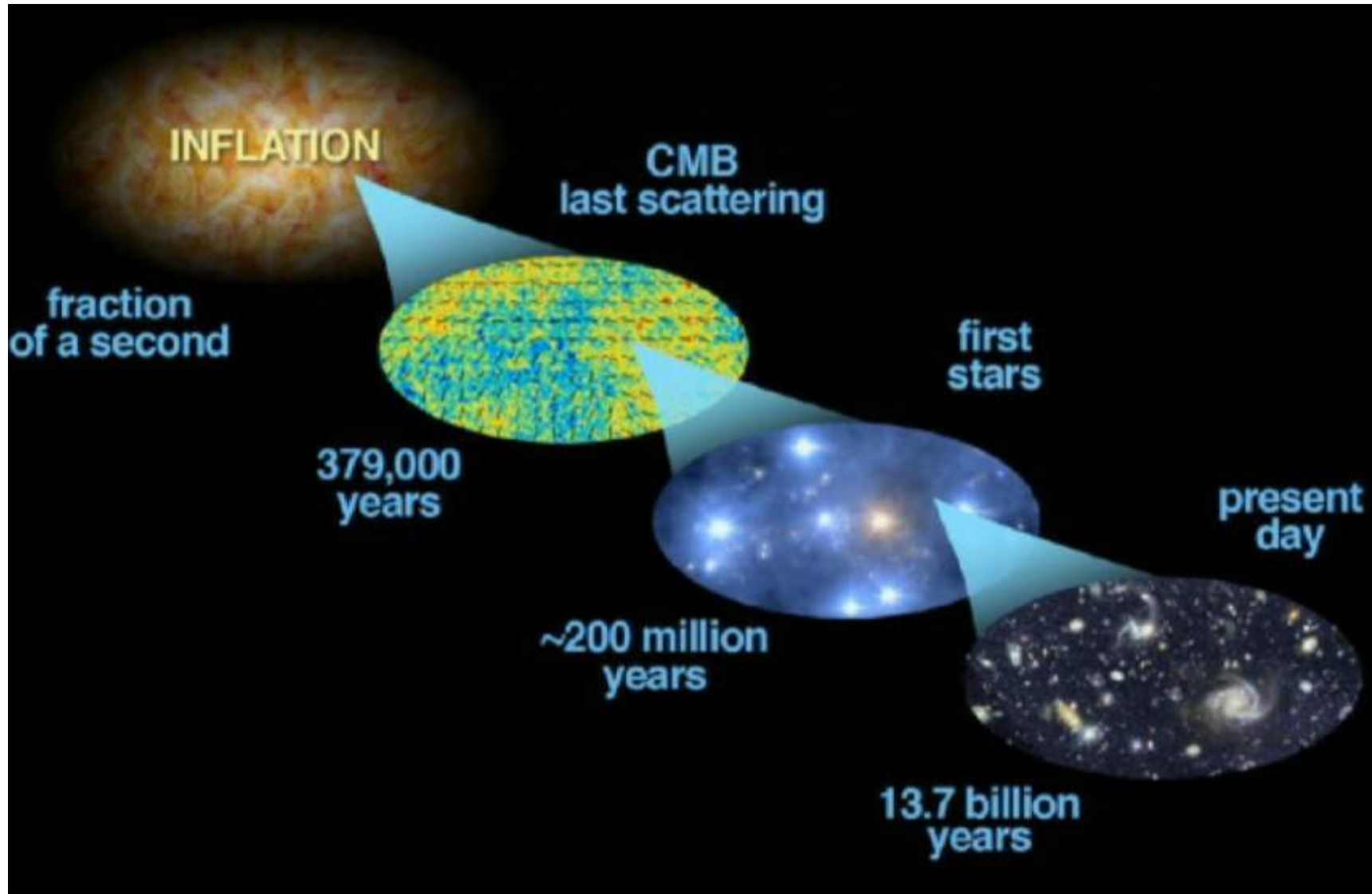
*“Data from the Planck satellite show the universe to be highly isotropic” (Wikipedia)*



We observe a statistically isotropic Gaussian random field of small temperature fluctuations (fully quantified by the 2-point correlations → angular power spectrum)



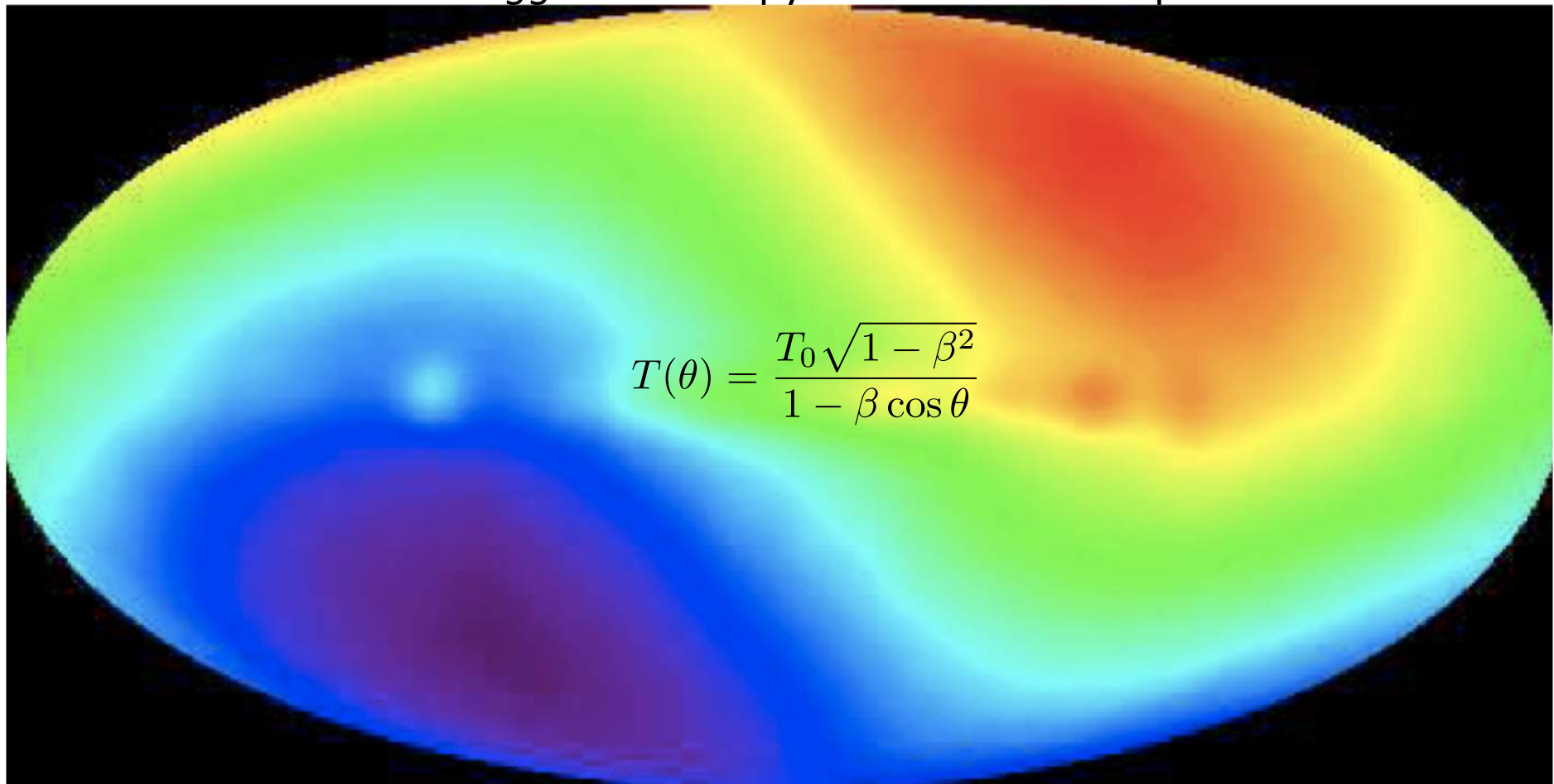
# STANDARD MODEL OF STRUCTURE FORMATION



The tiny **CMB temperature fluctuations** are understood as due to **scalar density perturbations** with an  $\sim$ scale-invariant spectrum which were generated during an early de Sitter phase of **inflationary expansion** ... these perturbations have subsequently grown into the **large-scale structure** of galaxies observed today through **gravitational instability** in a sea of **dark matter**

## BUT THE CMB SKY IS IN FACT QUITE ANISOTROPIC

There is a  $\sim 100$  times *bigger* anisotropy in the form of a dipole with  $\Delta T/T \sim 10^{-3}$



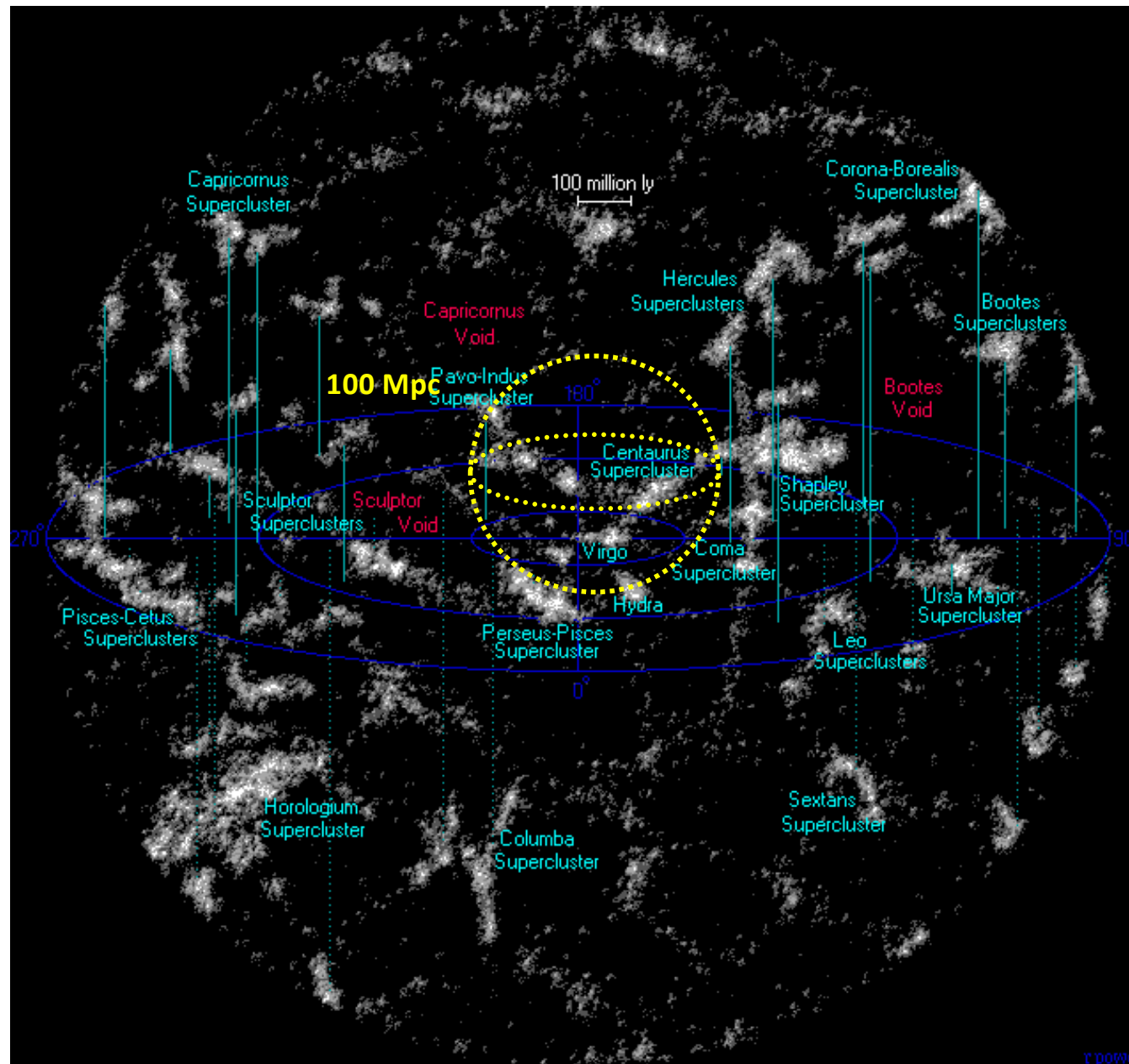
Stewart & Sciamia 1967, Peebles & Wilkinson 1968

This is *interpreted* as due to our motion at 370 km/s wrt the frame in which the CMB is truly isotropic  $\Rightarrow$  motion of the Local Group at 620 km/s towards  $l=271.9^\circ$ ,  $b=29.6^\circ$

**This motion is presumed to be due to local *inhomogeneity* in the matter distribution**  
Its scale – beyond which we converge to the CMB frame – is supposedly of  $O(100)$  Mpc  
(Counts of galaxies in the SDSS & WiggleZ surveys said to scale as  $\sim r^3$  on larger scales)



This is what our universe *actually* looks like locally (out to ~300 Mpc)  
We are moving towards the Shapley supercluster supposedly due to a 'Great Attractor'



We are *not* comoving ('Copernican') observers .. as is generally assumed

## THEORY OF PECULIAR VELOCITY FIELDS

In linear perturbation theory, the growth of the density contrast  $\delta(x) = [\rho(x) - \bar{\rho}]/\bar{\rho}$  as a function of comoving coordinates and time is governed by:

$$\frac{\partial^2 \delta}{\partial t^2} + 2H(t) \frac{\partial \delta}{\partial t} = 4\pi G_N \bar{\rho} \delta$$

We are interested in the ‘growing mode’ solution – the density contrast grows self-similarly and so does the perturbation potential and its gradient ... so the direction of the acceleration (and its integral – the peculiar velocity) remains *unchanged*.

The peculiar velocity field is related to the density contrast as:

$$v(\mathbf{x}) = \frac{2}{3H_0} \int d^3y \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^3} \delta(\mathbf{y}),$$

So the peculiar Hubble flow,  $\delta H(\mathbf{x}) = H_L(\mathbf{x}) - H_0$  ( $\Rightarrow$  trace of the shear tensor), is:

$$\delta H(\mathbf{x}) = \int d^3y \mathbf{v}(\mathbf{y}) \cdot \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^2} W(\mathbf{x} - \mathbf{y}),$$

where  $H_L(\mathbf{x})$  is the **local** value of the Hubble parameter and  $W(\mathbf{x} - \mathbf{y})$  is the ‘window function’ (e.g.  $\Theta(R - |\mathbf{x} - \mathbf{y}|) (4\pi R^3/3)^{-1}$  for a volume-limited survey, out to distance  $R$ )

# THEORY OF PECULIAR VELOCITY FIELDS (CONT.)

Rewrite in terms of the Fourier transform  $\delta(\mathbf{k}) \equiv (2\pi)^{3/2} \int d^3x \delta(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$  :

$$\frac{\delta H}{H_0} = \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k) \mathcal{W}_H(kR) e^{ik \cdot x}, \quad \mathcal{W}_H(x) = \frac{3}{x^3} \left( \sin x - \int_0^x dy \frac{\sin y}{y} \right)$$

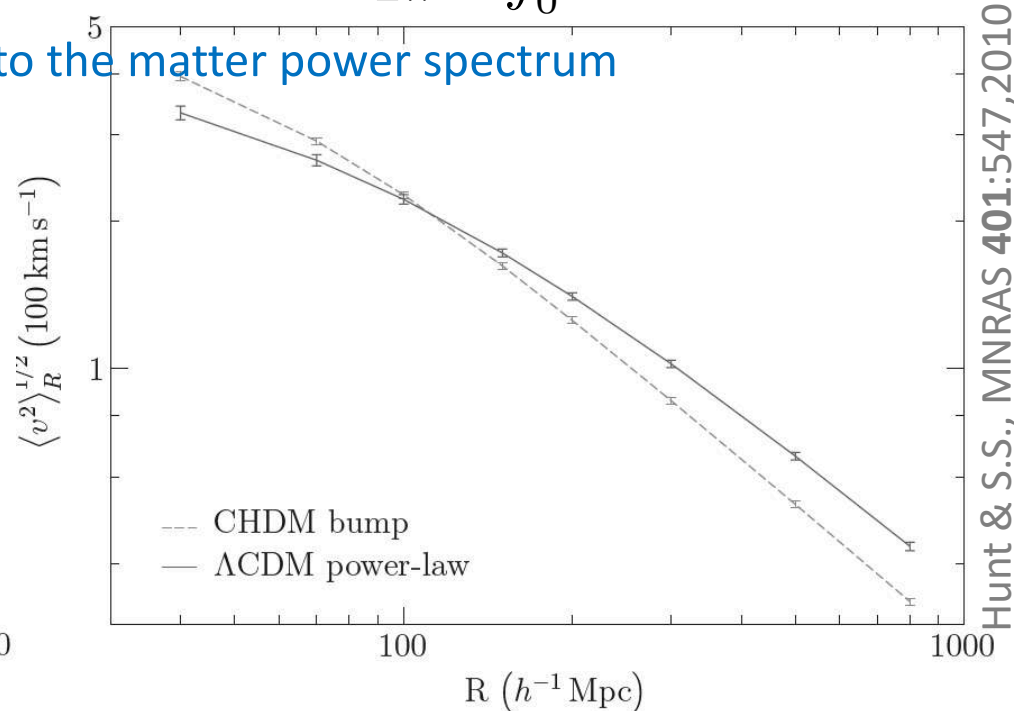
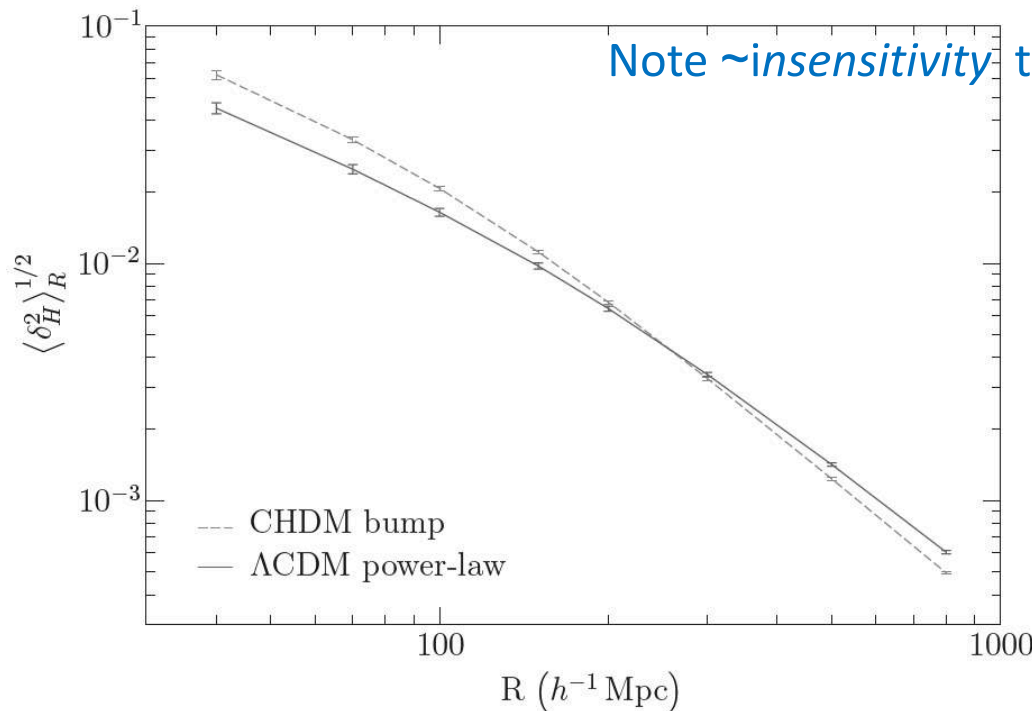
Window function

Then the RMS fluctuation in the local Hubble constant  $\delta_H \equiv \langle (\delta H/H_0)^2 \rangle^{1/2}$  is:

$$\delta_H^2 = \frac{f^2}{2\pi^2} \int_0^\infty k^2 dk P(k) \mathcal{W}^2(kR), \quad P(k) \equiv |\delta(k)|^2, \quad f \simeq \Omega_m^{4/7} + \frac{\Omega_\Lambda}{70} \left( 1 + \frac{\Omega_m}{2} \right)$$

Power spectrum of matter fluctuations      Growth rate

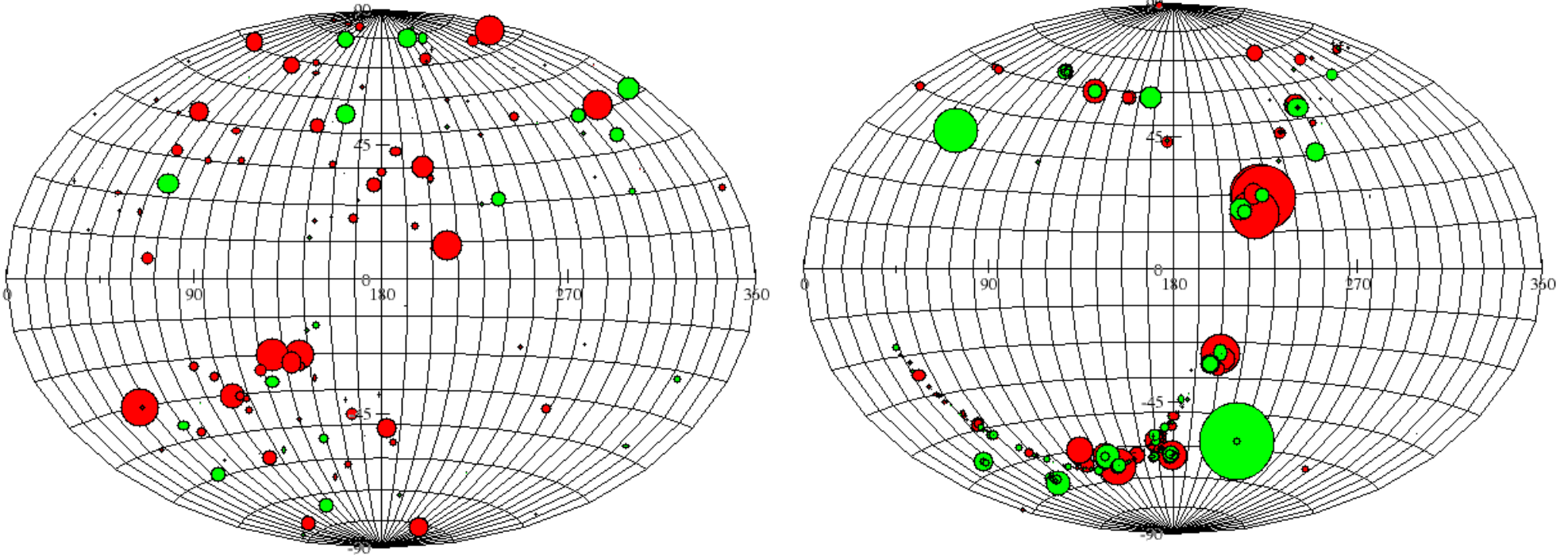
Similarly the variance of the peculiar velocity is:  $\langle v^2 \rangle_R = \frac{f^2 H_0^2}{2\pi^2} \int_0^\infty dk P(k) \mathcal{W}^2(kR)$





# UNION 2 COMPILATION OF 557 SNE IA

Aitoff-Hammer plot, Galactic coordinates



**Left panel:** The red spots represent the data points for  $z < 0.06$  with distance moduli  $\mu_{\text{data}}$  bigger than the values  $\mu_{\text{CDM}}$  predicted by  $\Lambda\text{CDM}$ , and the green spots are those with  $\mu_{\text{data}}$  less than  $\mu_{\text{CDM}}$ ; the spot size is a relative measure of the discrepancy. A dipole anisotropy is visible around the direction  $b = -30^\circ$ ,  $l = 96^\circ$  (red points) and its opposite direction  $b = 30^\circ$ ,  $l = 276^\circ$  (small green points), which is the direction of the CMB dipole.

**Right panel:** Same plot for  $z > 0.06$

Colin, Mohayaee, S.S. & Shafieloo, MNRAS **414**:264,2011

Can do *tomography* of the Hubble flow by testing if the supernovae are at the expected Hubble distances: **Residuals**  $\Rightarrow$  'peculiar velocity' flow in local universe

# METHOD OF RESIDUALS AND SMOOTHING

Colin, Mohayaee, S.S. & Shafieloo, MNRAS 414:264,2011

$$q_i(z_i, \theta_i, \phi_i) = \frac{\mu_i(z_i, \theta_i, \phi_i) - \tilde{\mu}_i(z_i, \theta_i, \phi_i)}{\sigma_i(z_i, \theta_i, \phi_i)} \quad \text{Calculation of Residuals}$$

$$Q(\theta, \phi) = \sum_{i=1}^N q_i(z_i, \theta_i, \phi_i) W(\theta, \phi, \theta_i, \phi_i) \quad \text{2D smoothing on unit sphere}$$

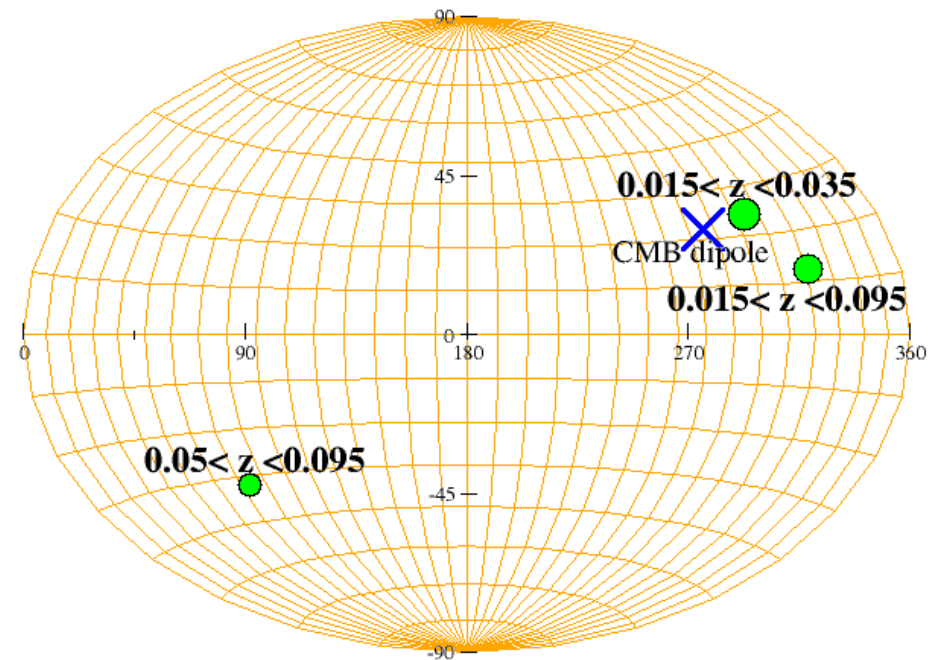
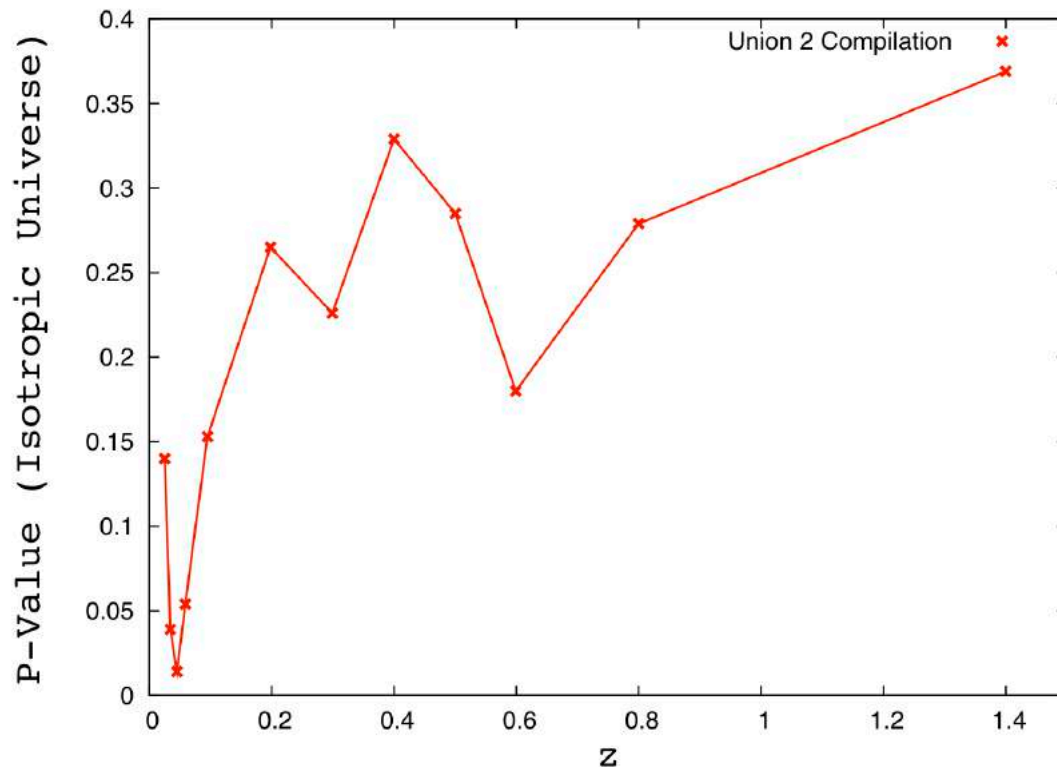
$$W(\theta, \phi, \theta_i, \phi_i) = \frac{1}{\sqrt{2\pi}\delta} \exp \left[ -\frac{L(\theta, \phi, \theta_i, \phi_i)^2}{2\delta^2} \right] \quad \text{Window function}$$

$$L(\theta, \phi, \theta_i, \phi_i) = 2 \arcsin \frac{R}{2}, \quad R = \left( [\sin(\theta_i) \cos(\phi_i) - \sin(\theta) \cos(\phi)]^2 + [\sin(\theta_i) \sin(\phi_i) - \sin(\theta) \sin(\phi)]^2 + [\cos(\theta_i) - \cos(\theta)]^2 \right)^{1/2}$$

$$\Delta Q_{\text{data}} = Q(\theta_{\text{max}}, \phi_{\text{max}}) - Q(\theta_{\text{min}}, \phi_{\text{min}}) \quad \text{Statistical measure}$$

Calculate for the data (as well as for Monte Carlo simulations of isotropic distribution, in order to obtain p-value) using a ratio method to minimise systematic uncertainties

# IS THE UNIVERSE ISOTROPIC?

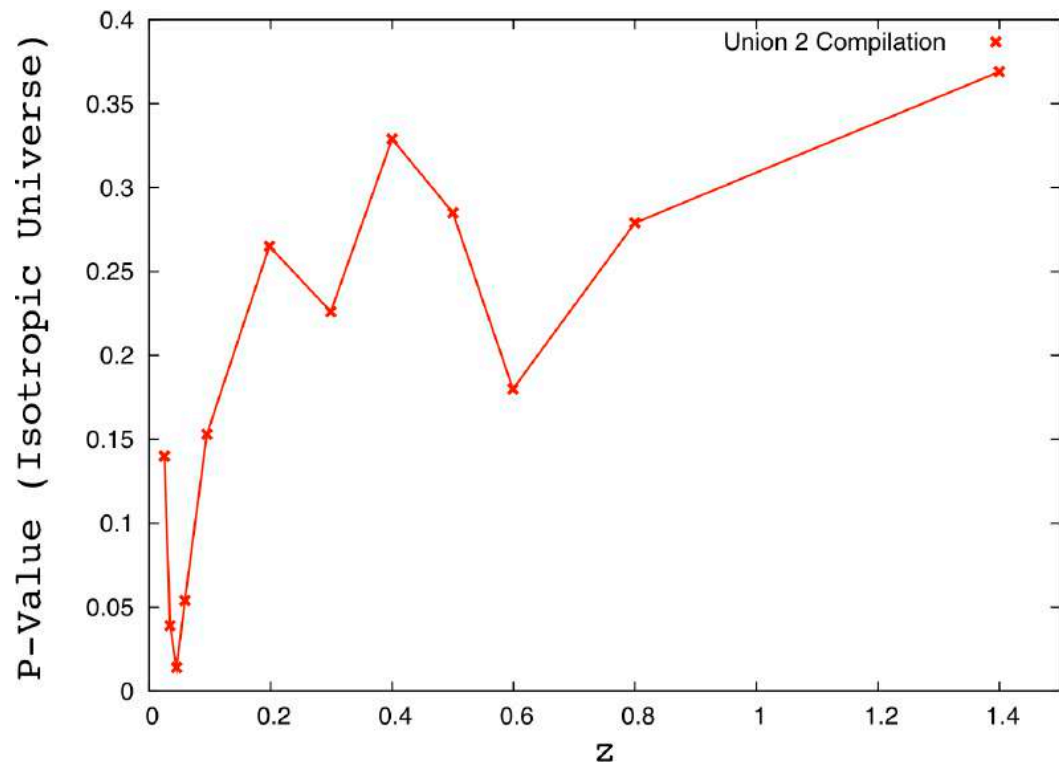


**Left panel:** P-value for the consistency of the isotropic universe with the data versus redshift. At  $z \approx 0.05$  ( $\sim 200$  Mpc) the P-value drops to 0.014 showing that isotropy is *excluded* at  $3\sigma$  ... i.e. we have *not* converged to the CMB rest frame.

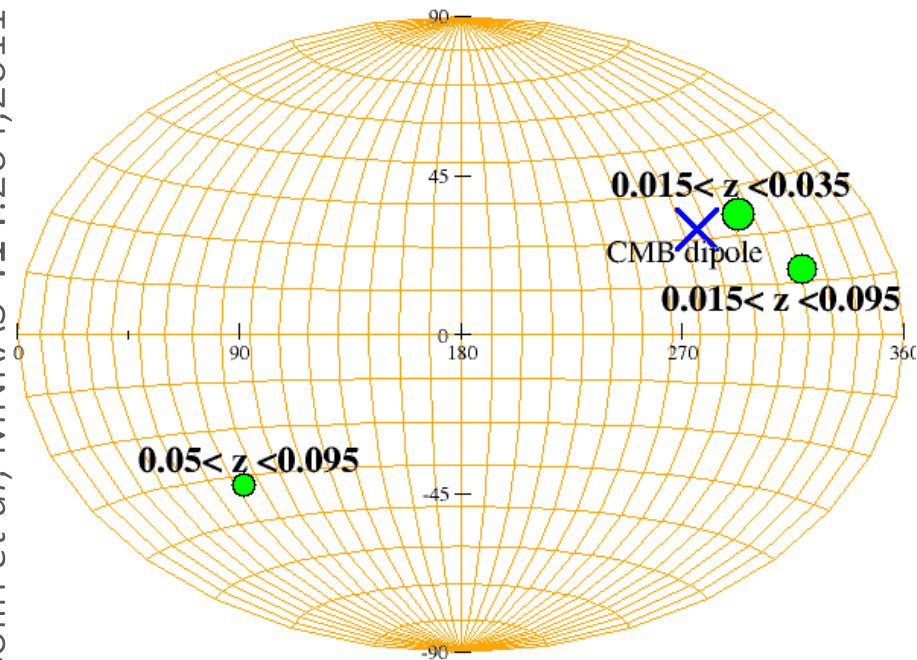
**Right panel:** Cumulative analysis shows that at low redshift,  $0.015 < z < 0.06$ , isotropy is excluded at  $2-3 \sigma$  with  $P = 0.054$ ; at higher redshift,  $0.15 < z < 1.4$  the data is consistent with isotropy within  $1\sigma$  ( $P = 0.594$ ).



# IS THE UNIVERSE ISOTROPIC?



Colin et al, MNRAS 414:264,2011



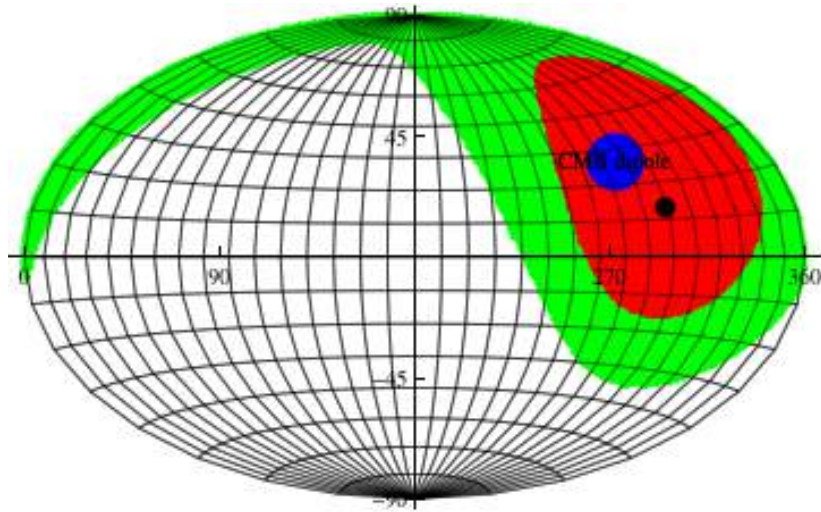
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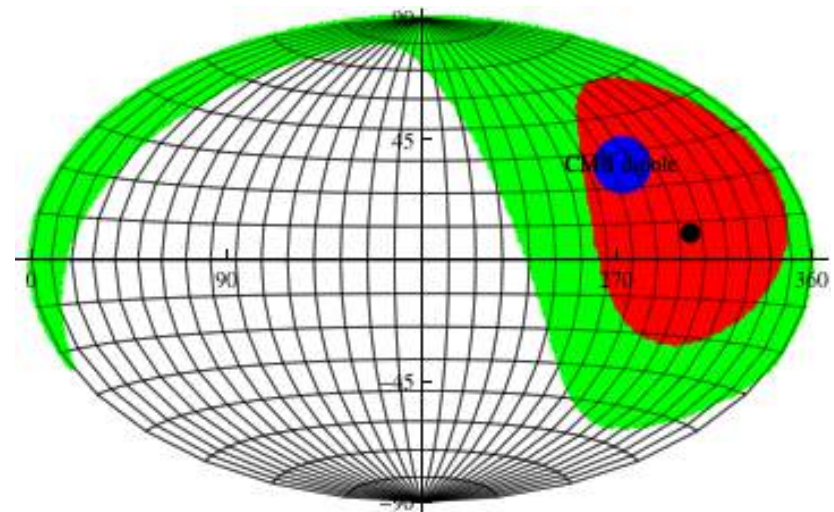
Maximum likelihood analysis can now be used to estimate the bulk flow at low redshifts where the velocities are not yet dominated by the cosmic expansion

# DIPOLE IN THE SN IA VELOCITY FIELD ALIGNED WITH THE CMB DIPOLE

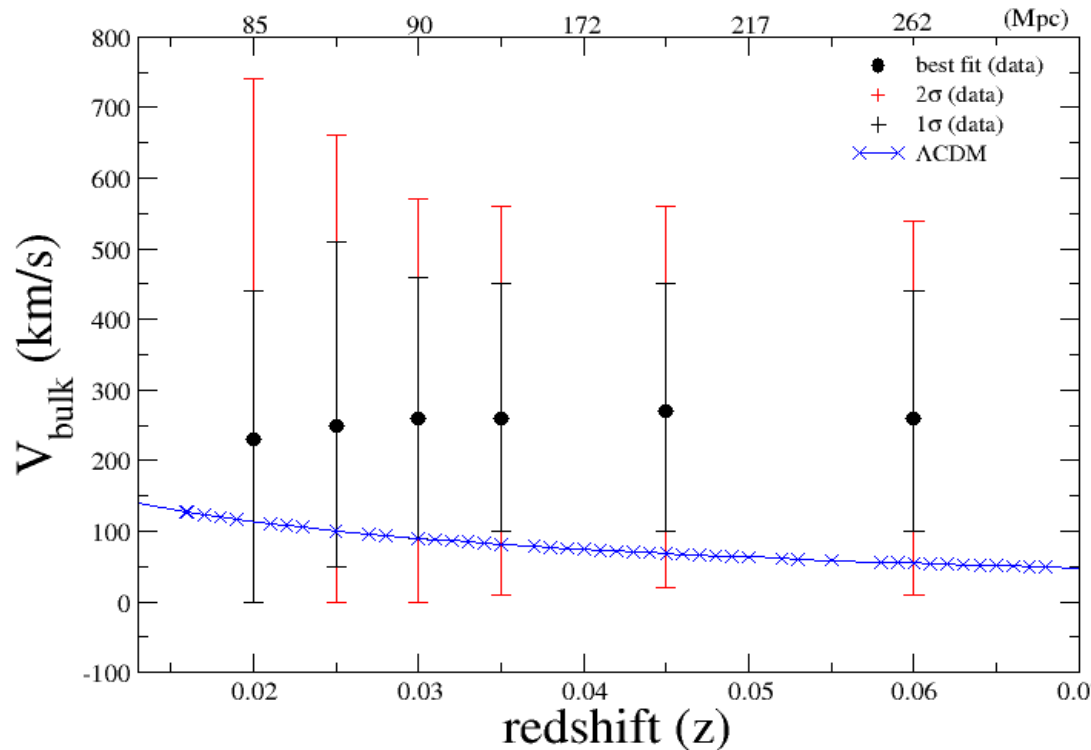
$0.015 < z < 0.045, v = 270 \text{ km/s}, l = 291, b = 15$



$0.015 < z < 0.06, v = 260 \text{ km/s}, l = 298, b = 8$



Colin et al, MNRAS 414:264,2011

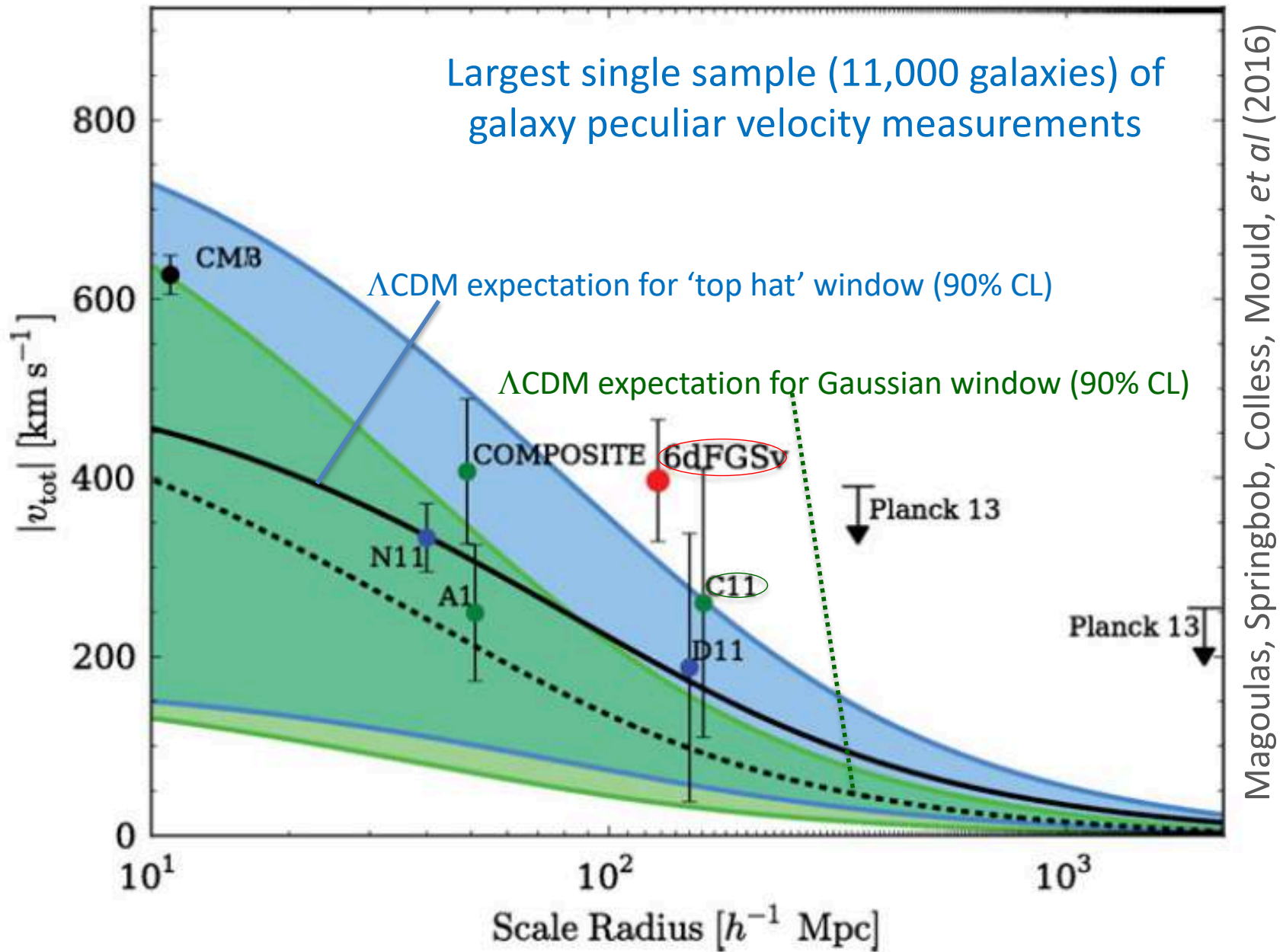


This is  $\gtrsim 1\sigma$  higher than expected for the standard  $\Lambda$ CDM model ... and extends *beyond* Shapley at 260 Mpc

(... consistent with Watkins *et al* (2009) who found a bulk flow of  $416 \pm 78 \text{ km/s}$  towards  $b = 60 \pm 6^\circ, l = 282 \pm 11^\circ$  extending up to  $\sim 100 h^{-1} \text{ Mpc}$ )

No convergence to CMB frame, even well beyond 'scale of homogeneity'

OUR RESULT IS **CONFIRMED** BY THE 6-DEGREE FIELD GALAXY SURVEY (6DFGSV)



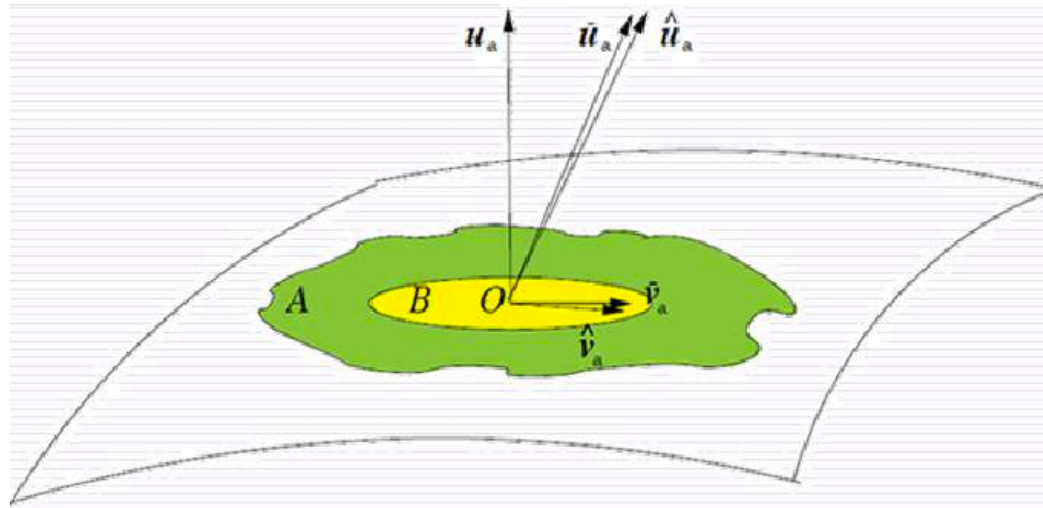
According to the 'Dark Sky'  $\Lambda\text{CDM}$  Hubble Volume simulations, *less than 1%* of Milky Way-like observers should experience a bulk flow as large as is observed, extending out as far as is seen



Do we infer acceleration even though the expansion is actually decelerating ... because we are inside a local ‘bulk flow’?

(Tsagas 2010, 2011, 2012; Tsagas & Kadiltzoglou 2015)

... if so there should be a dipole asymmetry in the inferred deceleration parameter in the same direction – i.e. *towards* the CMB dipole



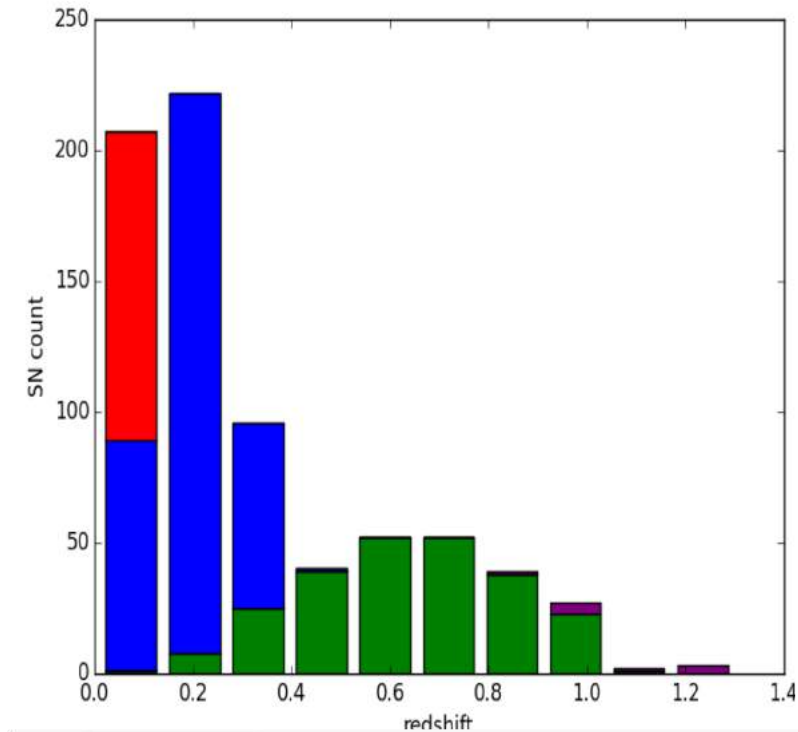
The patch A has mean peculiar velocity  $\tilde{v}_a$  with  $\vartheta = \tilde{D}^a v_a \gtrless 0$  and  $\dot{\vartheta} \gtrless 0$  (the sign depending on whether the bulk flow is faster or slower than the surroundings)

Inside region B, the r.h.s. of the expression

$$1 + \tilde{q} = (1 + q) \left( 1 + \frac{\vartheta}{\Theta} \right)^{-2} - \frac{3\dot{\vartheta}}{\Theta^2} \left( 1 + \frac{\vartheta}{\Theta} \right)^{-2}, \quad \tilde{\Theta} = \Theta + \vartheta,$$

drops below 1 and the comoving observer ‘measures’ *negative* deceleration parameter

# JOINT LIGHTCURVE ANALYSIS DATA (740 SNE IA)



This page contains links to data associated with the SDSS-II/SNLS3 Joint Light-Curve Analysis (Betoule et al. 2014, submitted to A&A).

The release consists in:

1. The end products of the analysis and a C++ code to compute the likelihood of this data associated to a cosmological model. The code enables both evaluations of the *complete* likelihood, and fast evaluations of an *approximate* likelihood (see Betoule et al. 2014, Appendix E).
2. The version 2.4 of the SALT2 light-curve model used for the analysis plus 200 random realizations usable for the propagation of model uncertainties.
3. The exact set of Supernovae light-curves used in the analysis.

We also deliver presentation material.

Since March 2014, the JLA likelihood plugin is included in the official release of *cosmomc*. For older versions, the plugin is still available (see below: *Installation of the cosmomc plugin*).

To analyze the JLA sample with *SNANA*, see `$SNDATA_ROOT/sample_input_files/JLA2014/AAA_README`.

## 1 Release history

### 1. Release history

- V1 (January 2014, paper submitted):
- V2 (March 2014):
- V3 (April 2014, paper accepted):
- V4 (June 2014):
- V5 (March 2015):
- V6 (March 2015):

### 2. Installation of the C++ likelihood code

#### Installation of the cosmomc plugin

#### 3. SALT2 model

#### 4. Error propagation

Error decomposition:  
SALT2 light-curve model  
uncertainties

### V1 (January 2014, paper submitted):

First arxiv version.

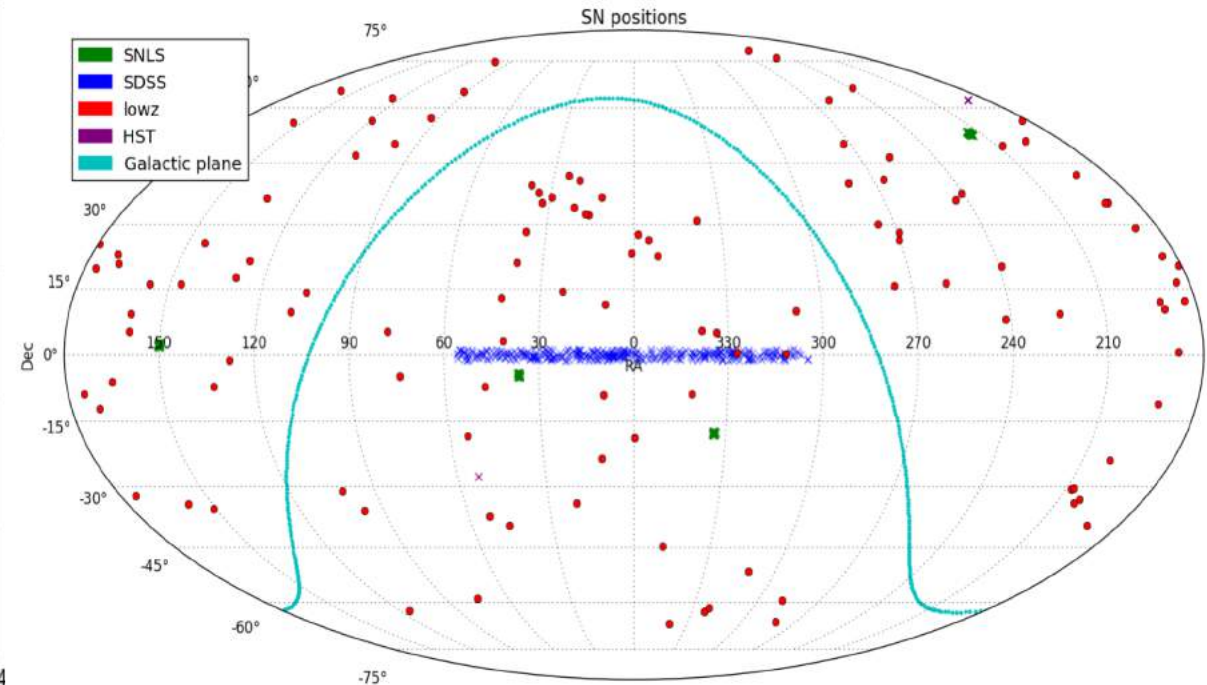
### V2 (March 2014):

Same as v1 with additional information (R.A., Dec. and bias correction) in the file of light-curve parameters.

### V3 (April 2014, paper accepted):

Same as v2 with the addition of a C++ likelihood code in an independent archive ([jla\\_likelihood\\_v3.tgz](#)).

### V4 (June 2014):



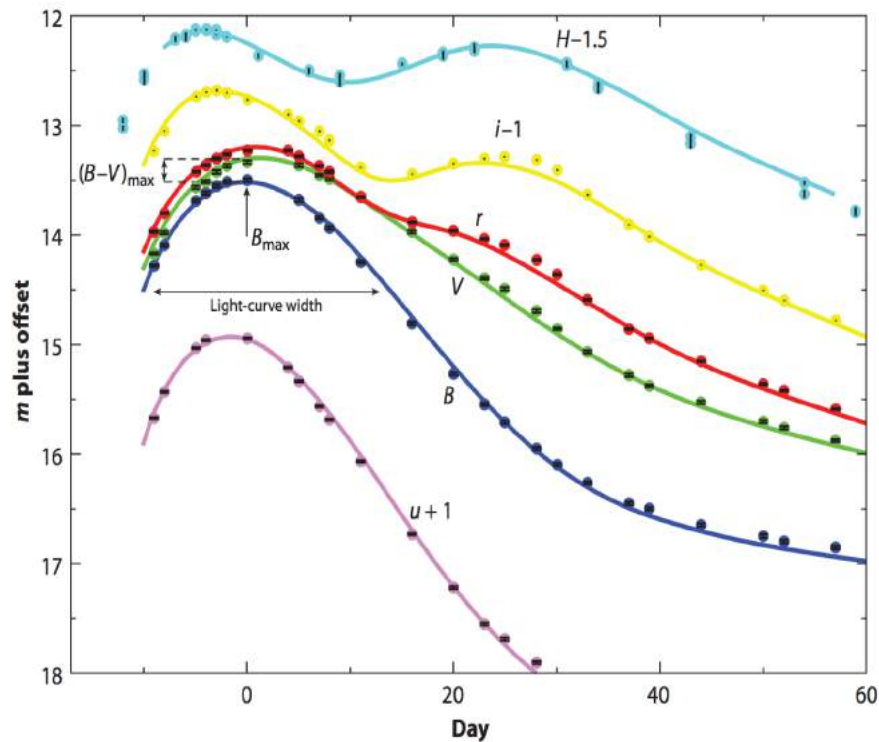
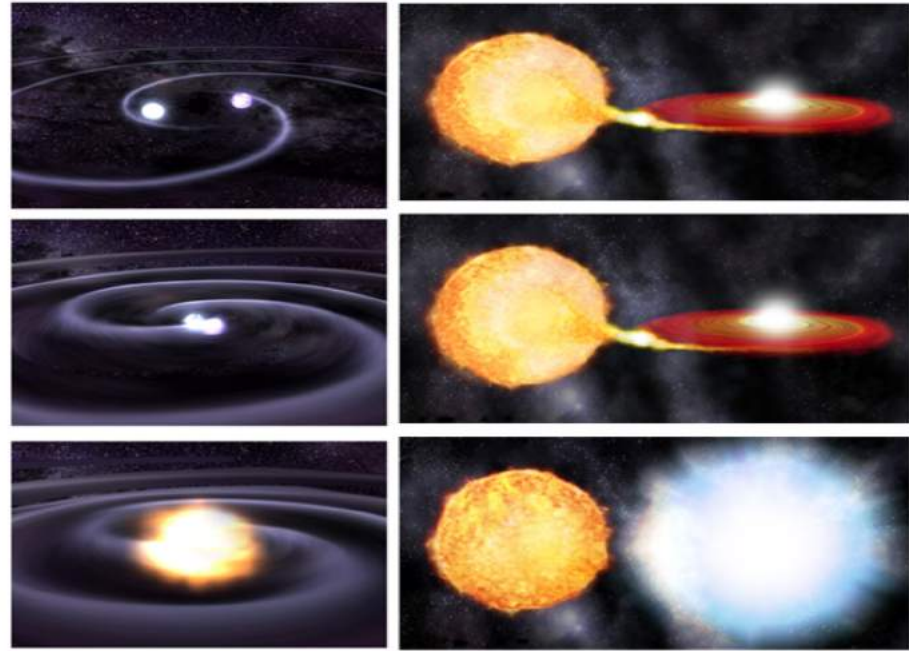
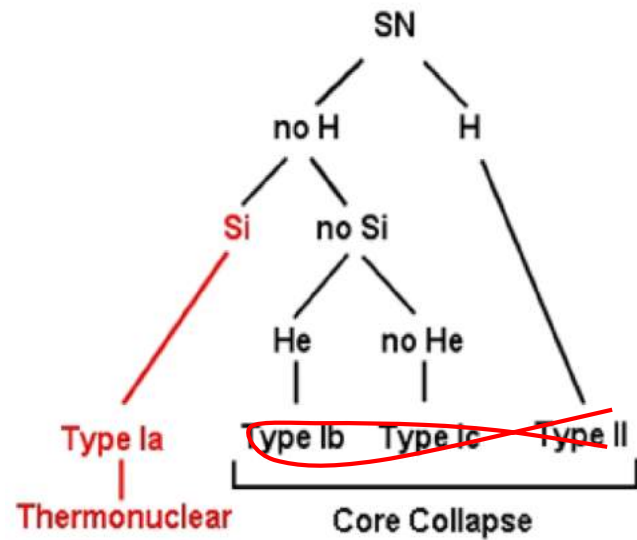
Betoule et al, A&A 568:A22,2014

[http://supernovae.in2p3.fr/sdss\\_snls\\_jla/](http://supernovae.in2p3.fr/sdss_snls_jla/)

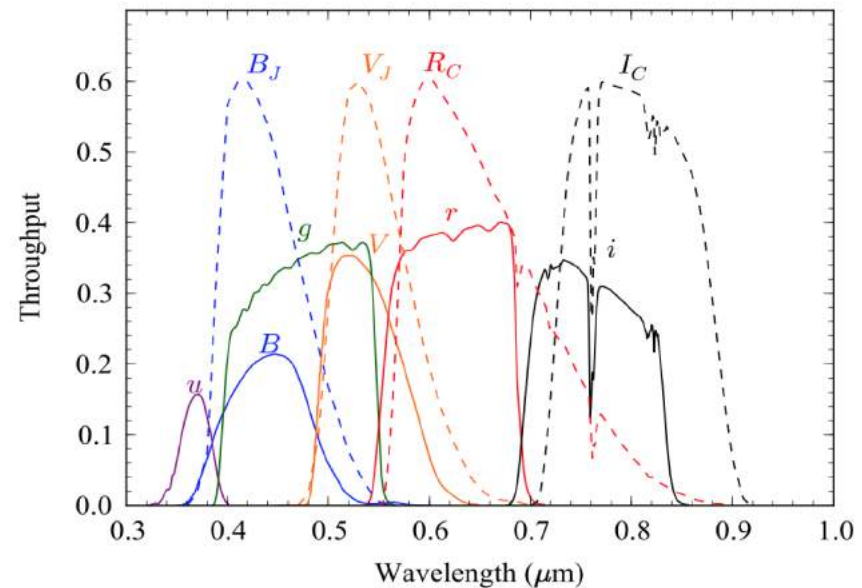
In contrast to previous analyses (which assumed  $\Lambda$ CDM and adjusted the errors to get a good fit) we apply a *principled* statistical analysis: **Maximal Likelihood**

Nielsen, Guffanti & S.S., Sci.Rep. 6:35596,2016

# WHAT ARE TYPE IA SUPERNOVAE?

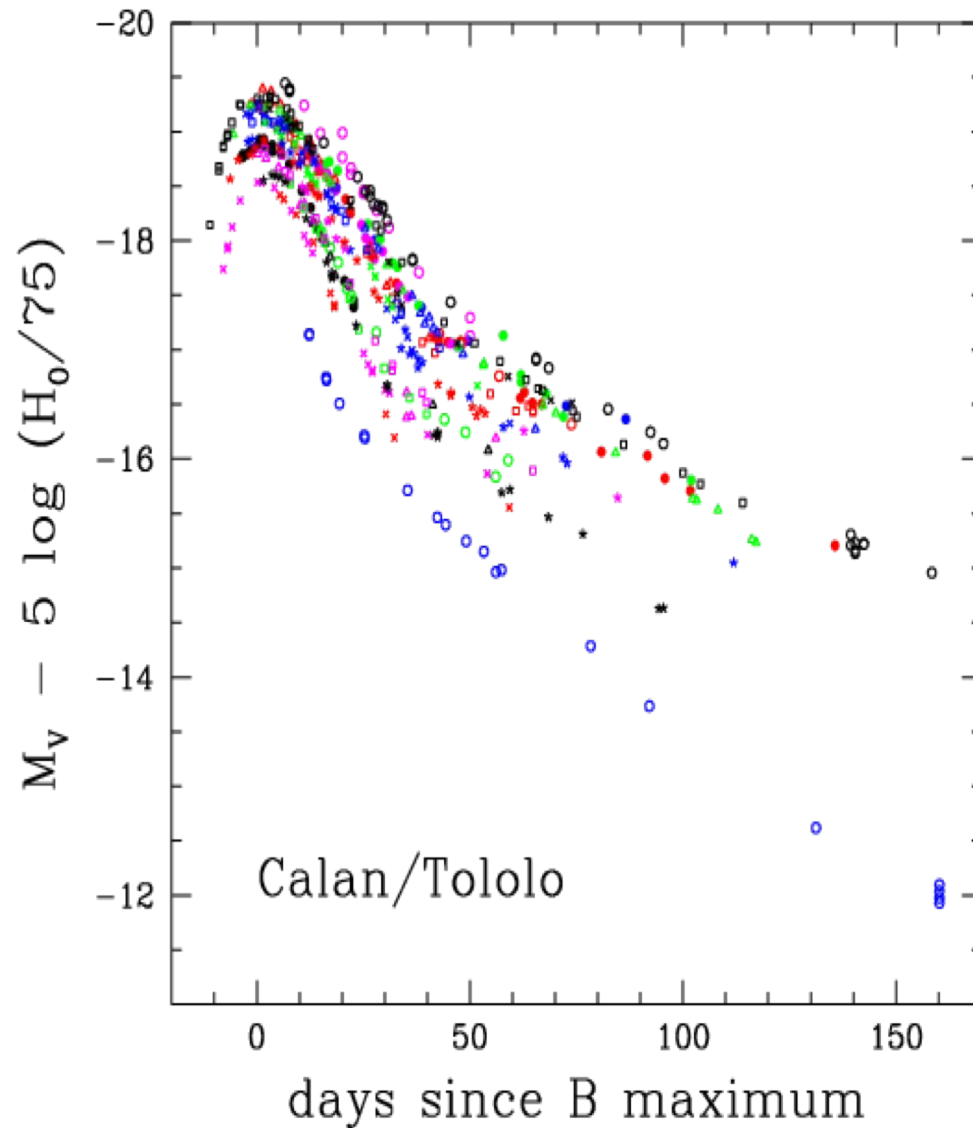


$$m = -2.5 \log(F/F_{\text{ref}})$$

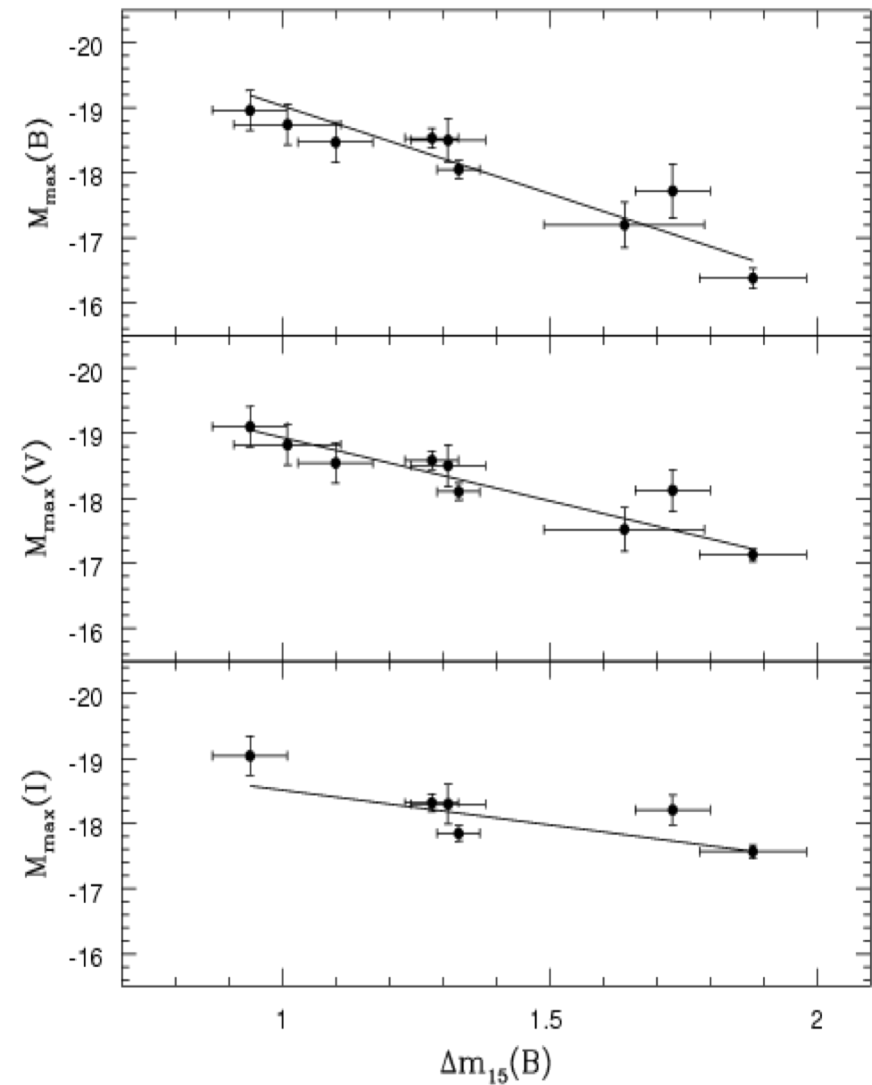




# THEY ARE CERTAINLY *NOT* ‘STANDARD CANDLES’



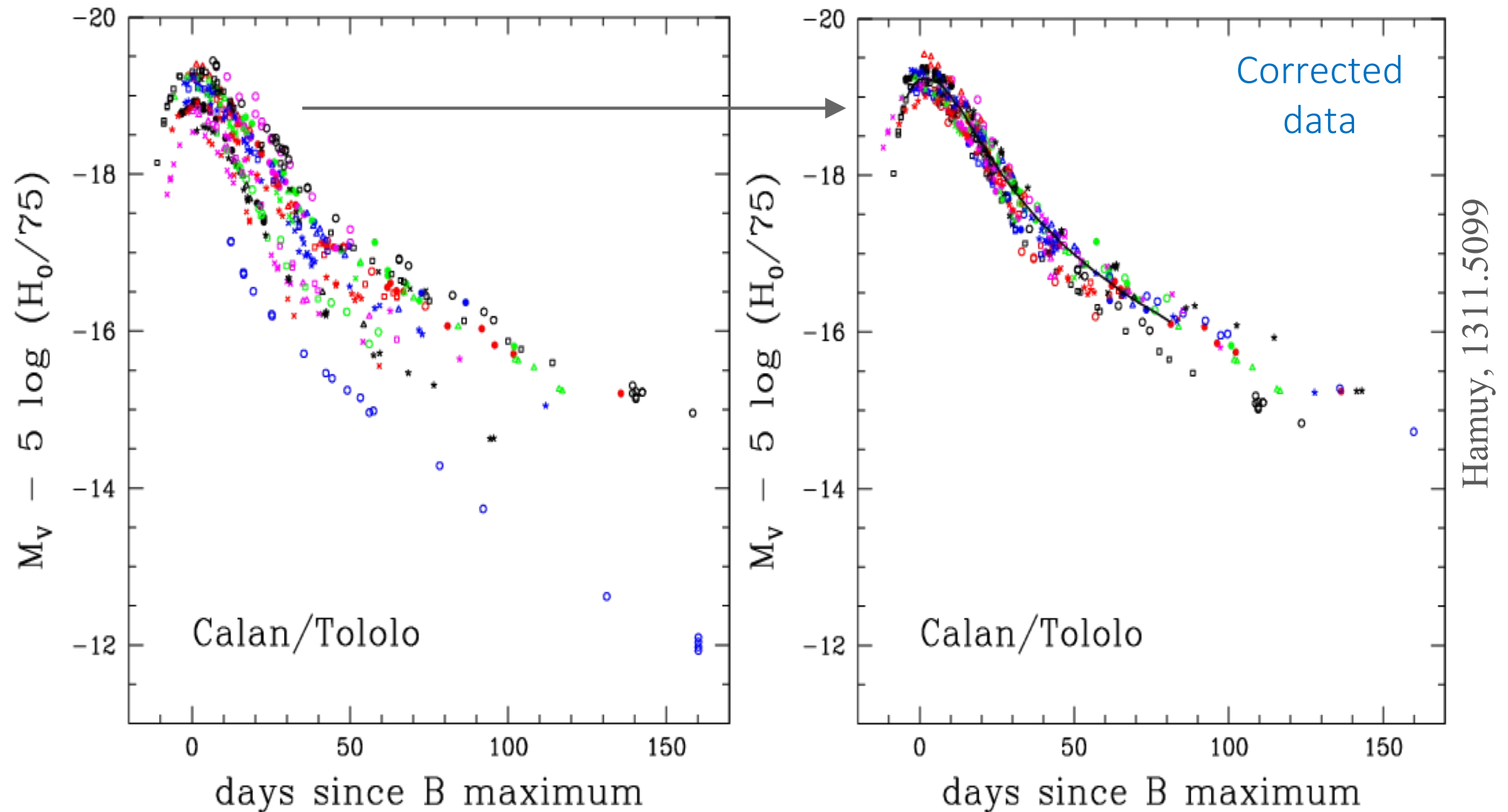
Hamuy, [arXiv:311.5099](https://arxiv.org/abs/311.5099)



Phillips, ApJ **413**:L105, 1993

But they can be ‘standardised’ using the observed correlation between their peak magnitude and light-curve width (NB: this correlation is *not* understood theoretically)

# TYPE IA SUPERNOVAE AS ‘STANDARDISABLE CANDLES’



$$\mu_B = m_B^* - M + \alpha X_1 - \beta C$$

Use a standard template (e.g. SALT 2) to make ‘stretch’ and ‘colour’ corrections ...

# SPECTRAL ADAPTIVE LIGHTCURVE TEMPLATE

(For making 'stretch' and 'colour' corrections to the observed lightcurves)

$$\mu_B = m_B^* - M + \alpha X_1 - \beta C$$

B-band

SALT 2 parameters

Betoule *et al.*, A&A **568**:A22,2014

Name	$z_{\text{cmb}}$	$m_B^*$	$X_1$	$C$	$M_{\text{stellar}}$	?
03D1ar	0.002	$23.941 \pm 0.033$	$-0.945 \pm 0.209$	$0.266 \pm 0.035$	$10.1 \pm 0.5$	?
03D1au	0.503	$23.002 \pm 0.088$	$1.273 \pm 0.150$	$-0.012 \pm 0.030$	$9.5 \pm 0.1$	?
03D1aw	0.581	$23.574 \pm 0.090$	$0.974 \pm 0.274$	$-0.025 \pm 0.037$	$9.2 \pm 0.1$	?
03D1ax	0.495	$22.960 \pm 0.088$	$-0.729 \pm 0.102$	$-0.100 \pm 0.030$	$11.6 \pm 0.1$	?
03D1bp	0.346	$22.398 \pm 0.087$	$-1.155 \pm 0.113$	$-0.041 \pm 0.027$	$10.8 \pm 0.1$	?
03D1co	0.678	$24.078 \pm 0.098$	$0.619 \pm 0.404$	$-0.039 \pm 0.067$	$8.6 \pm 0.3$	?
03D1dt	0.611	$23.285 \pm 0.093$	$-1.162 \pm 1.641$	$-0.095 \pm 0.050$	$9.7 \pm 0.1$	
03D1ew	0.866	$24.354 \pm 0.106$	$0.376 \pm 0.348$	$-0.063 \pm 0.068$	$8.5 \pm 0.8$	
03D1fc	0.331	$21.861 \pm 0.086$	$0.650 \pm 0.119$	$-0.018 \pm 0.024$	$10.4 \pm 0.0$	
03D1fq	0.799	$24.510 \pm 0.102$	$-1.057 \pm 0.407$	$-0.056 \pm 0.065$	$10.7 \pm 0.1$	
03D3aw	0.450	$22.667 \pm 0.092$	$0.810 \pm 0.232$	$-0.086 \pm 0.038$	$10.7 \pm 0.0$	
03D3ay	0.371	$22.273 \pm 0.091$	$0.570 \pm 0.198$	$-0.054 \pm 0.033$	$10.2 \pm 0.1$	
03D3ba	0.292	$21.961 \pm 0.093$	$0.761 \pm 0.173$	$0.116 \pm 0.035$	$10.2 \pm 0.1$	
03D3bl	0.356	$22.927 \pm 0.087$	$0.056 \pm 0.193$	$0.205 \pm 0.030$	$10.8 \pm 0.1$	

There may well be other variables that the magnitude correlates with ...

# COSMOLOGY

$$\mu \equiv 25 + 5 \log_{10}(d_L/\text{Mpc}), \quad \text{where:}$$

$$d_L = (1+z) \frac{d_H}{\sqrt{\Omega_k}} \text{sinn} \left( \sqrt{\Omega_k} \int_0^z \frac{H_0 dz'}{H(z')} \right),$$

$$d_H = c/H_0, \quad H_0 \equiv 100h \text{ km s}^{-1} \text{Mpc}^{-1},$$

$$H = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda},$$

$\text{sinn} \rightarrow \sinh$  for  $\Omega_k > 0$  and  $\text{sinn} \rightarrow \sin$  for  $\Omega_k < 0$

Distance  
modulus

$$\mu_C = m - M = -2.5 \log \frac{F/F_{\text{ref}}}{L/L_{\text{ref}}} = 5 \log \frac{d_L}{10 \text{ pc}}$$

Acceleration is a *kinematic* quantity so the data can be analysed without assuming any dynamical model, by expanding the time variation of the scale factor in a Taylor series  
(e.g. Visser, CQG **21**:2603,2004)

$$d_L(z) = \frac{c}{H_0} \left\{ 1 + \frac{1}{2} [1 - q_0] z - \frac{1}{6} \left[ 1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2} \right] z^2 + O(z^3) \right\}$$

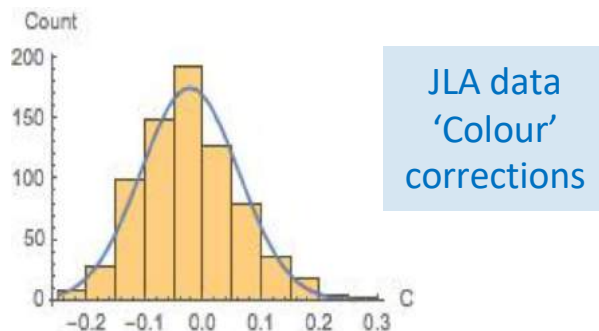
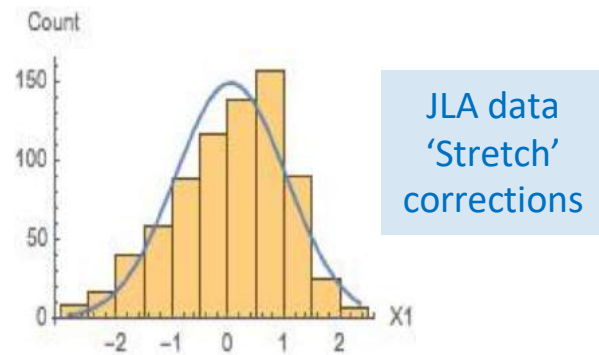


# CONSTRUCT A MAXIMUM LIKELIHOOD ESTIMATOR

$\mathcal{L}$  = probability density(data|model)

$$\begin{aligned}\mathcal{L} &= p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|\theta] \\ &= \int p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta_{\text{cosmo}}] \\ &\quad \times p[(M, x_1, c)|\theta_{\text{SN}}] dM dx_1 dc\end{aligned}$$

Well-approximated as Gaussian



$$p[(M, x_1, c)|\theta] = p(M|\theta)p(x_1|\theta)p(c|\theta),$$

$$p(M|\theta) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left(-\left[\frac{M - M_0}{\sigma_{M0}}\right]^2 / 2\right)$$

$$p(x_1|\theta) = \frac{1}{\sqrt{2\pi\sigma_{x0}^2}} \exp\left(-\left[\frac{x_1 - x_{10}}{\sigma_{x0}}\right]^2 / 2\right)$$

$$p(c|\theta) = \frac{1}{\sqrt{2\pi\sigma_{c0}^2}} \exp\left(-\left[\frac{c - c_0}{\sigma_{c0}}\right]^2 / 2\right)$$

# Likelihood

$$p(Y|\theta) = \frac{1}{\sqrt{|2\pi\Sigma_l|}} \exp \left[ -\frac{1}{2}(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^T \right]$$

$$p(\hat{X}|X, \theta) = \frac{1}{\sqrt{|2\pi\Sigma_d|}} \exp \left[ -\frac{1}{2}(\hat{X} - X)\Sigma_d^{-1}(\hat{X} - X)^T \right]$$

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi(\Sigma_d + A^T\Sigma_l A)|}} \times \exp \left( -\frac{1}{2}(\hat{Z} - Y_0 A)(\Sigma_d + A^T\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^T \right)$$

intrinsic distributions

cosmology
SALT2

# Confidence regions

Nielsen, Guffanti & S.S., Sci.Rep. 6:35596,2016

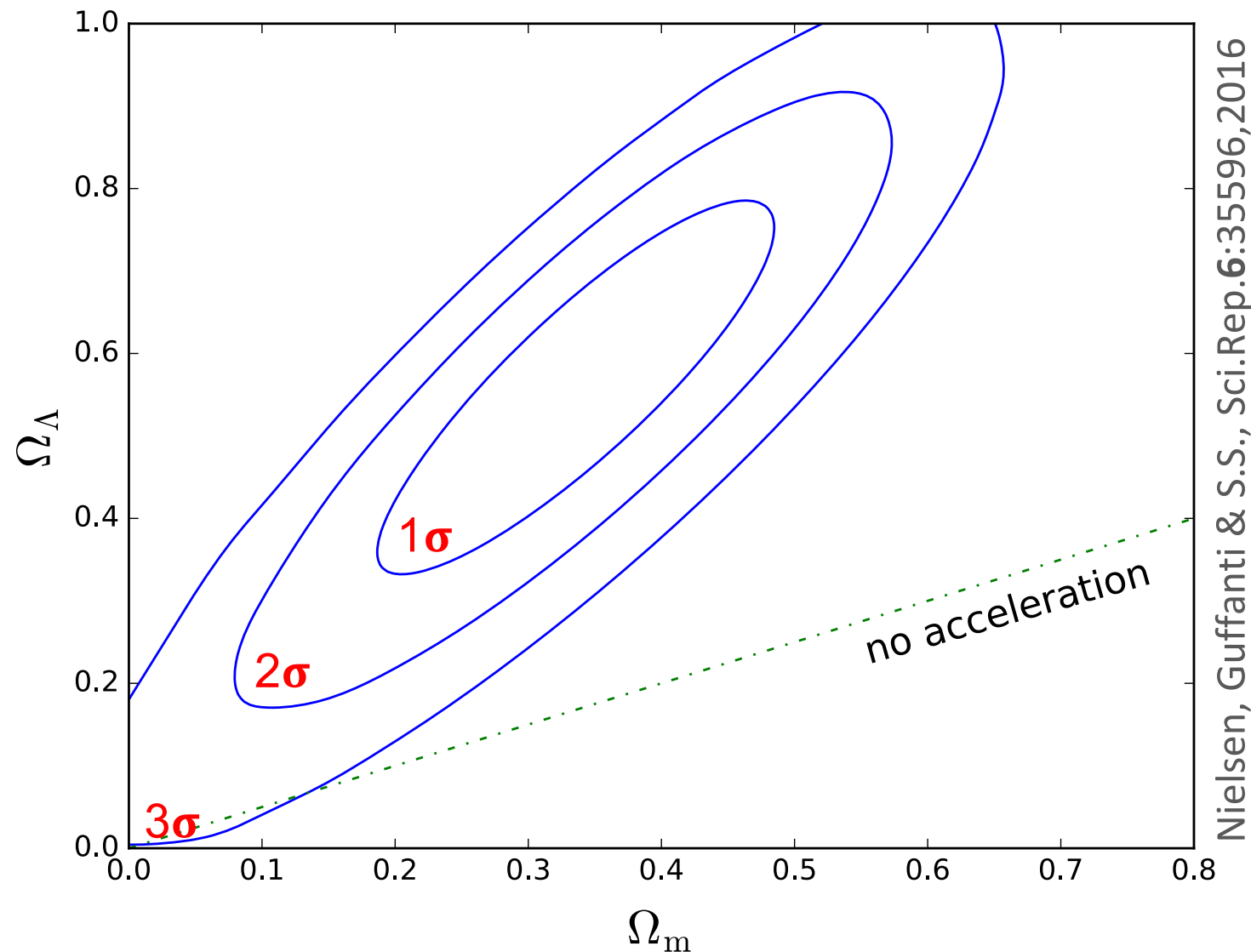
$$p_{\text{cov}} = \int_0^{-2 \log \mathcal{L} / \mathcal{L}_{\text{max}}} \chi^2(x; \nu) dx$$

$$\mathcal{L}_p(\theta) = \max_{\phi} \mathcal{L}(\theta, \phi)$$

1,2,3-sigma

solve for Likelihood value

Data consistent with *uniform* rate of expansion @ $3\sigma$  ( $\Rightarrow \rho+3p=0$ )!



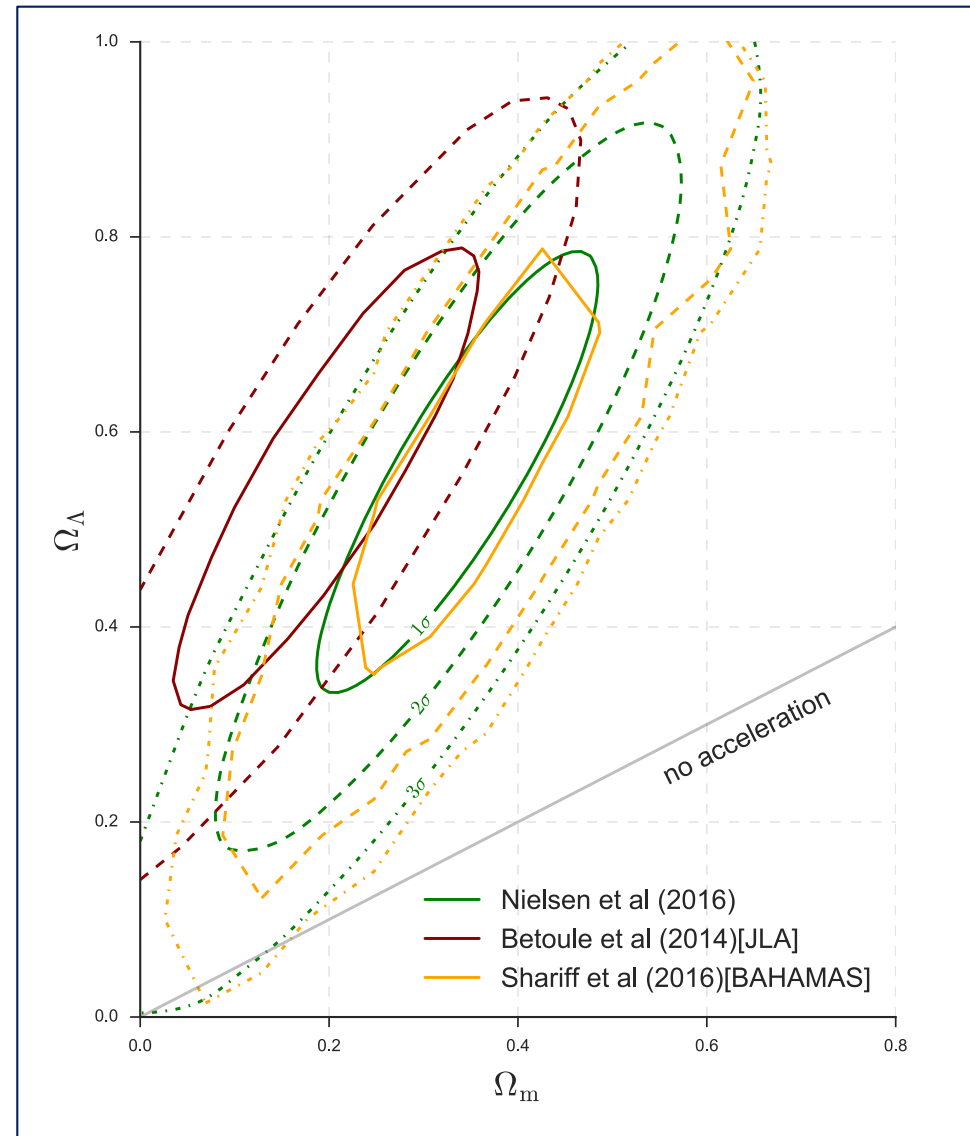
### Profile Likelihood

#### MLE, best fit

$\Omega_M$	0.341
$\Omega_\Lambda$	0.569
$\alpha$	0.134
$x_0$	0.038
$\sigma_{x0}^2$	0.931
$\beta$	3.058
$c_0$	-0.016
$\sigma_{c0}^2$	0.071
$M_0$	-19.05
$\sigma_{M0}^2$	0.108

NB: We show the result in the  $\Omega_m$ - $\Omega_\Lambda$  plane for comparison with the usual result ... simply to emphasise that the statistical analysis has previously *not* been done correctly (Other constraints e.g.  $\Omega_M \gtrsim 0.2$  or  $\Omega_M + \Omega_\Lambda \simeq 1$  are relevant *only* to the  $\Lambda$ CDM model)

Our result has been confirmed by a subsequent *Bayesian* analysis  
(Contours are ragged because of numerical issues in MCMC scan)

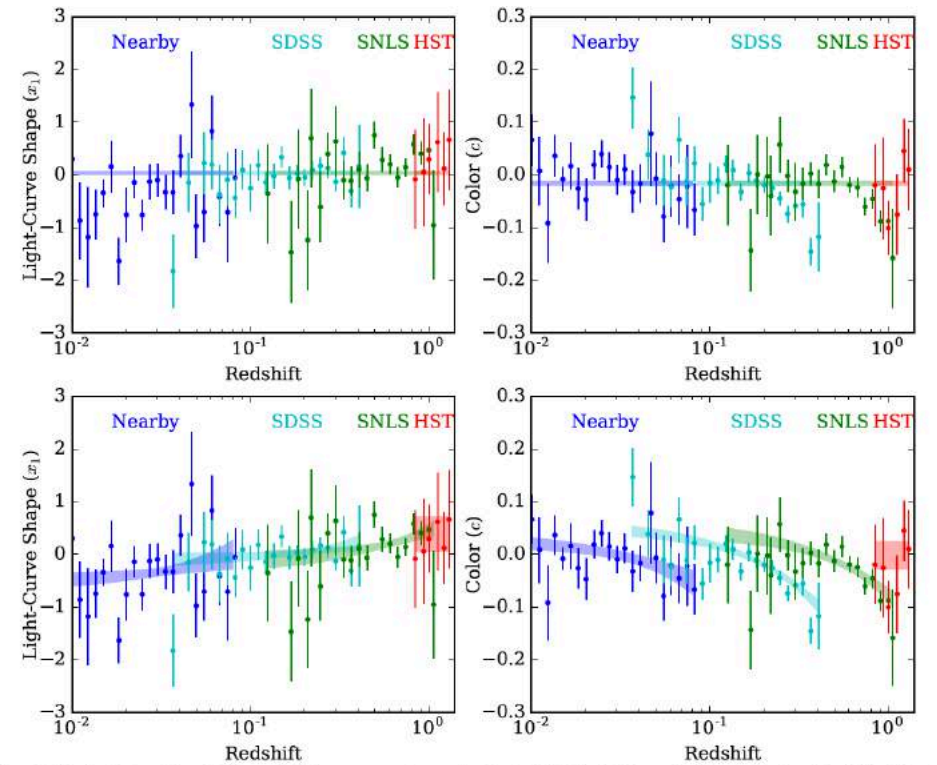


Shariff, Jiao, Trotta & van Dyk, ApJ **827**, 1, 2015

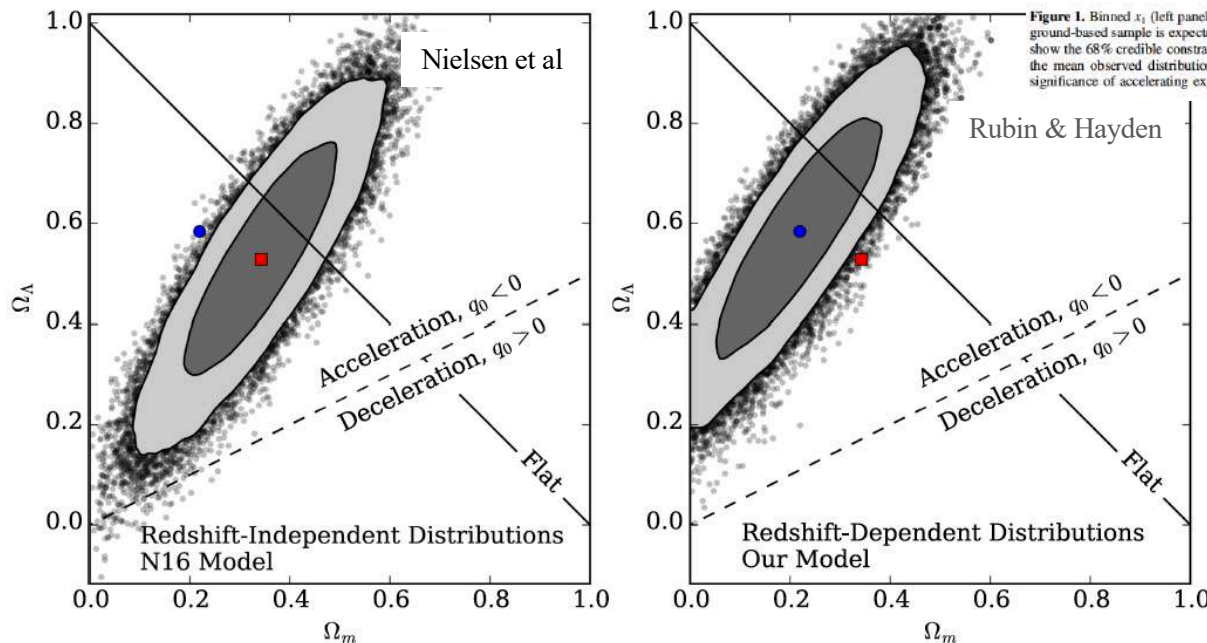
... it is clear that previous analyses have overestimated the significance of acceleration



Rubin & Hayden (ApJ 833:L30,2016) say that our model for the distribution of the JLA light curve fit parameters should have included a dependence on redshift (to allow for ‘Malmqvist bias’ which in fact the JLA collab. had already corrected for) ... they add 12 more parameters to our (10 parameter) model to describe this!



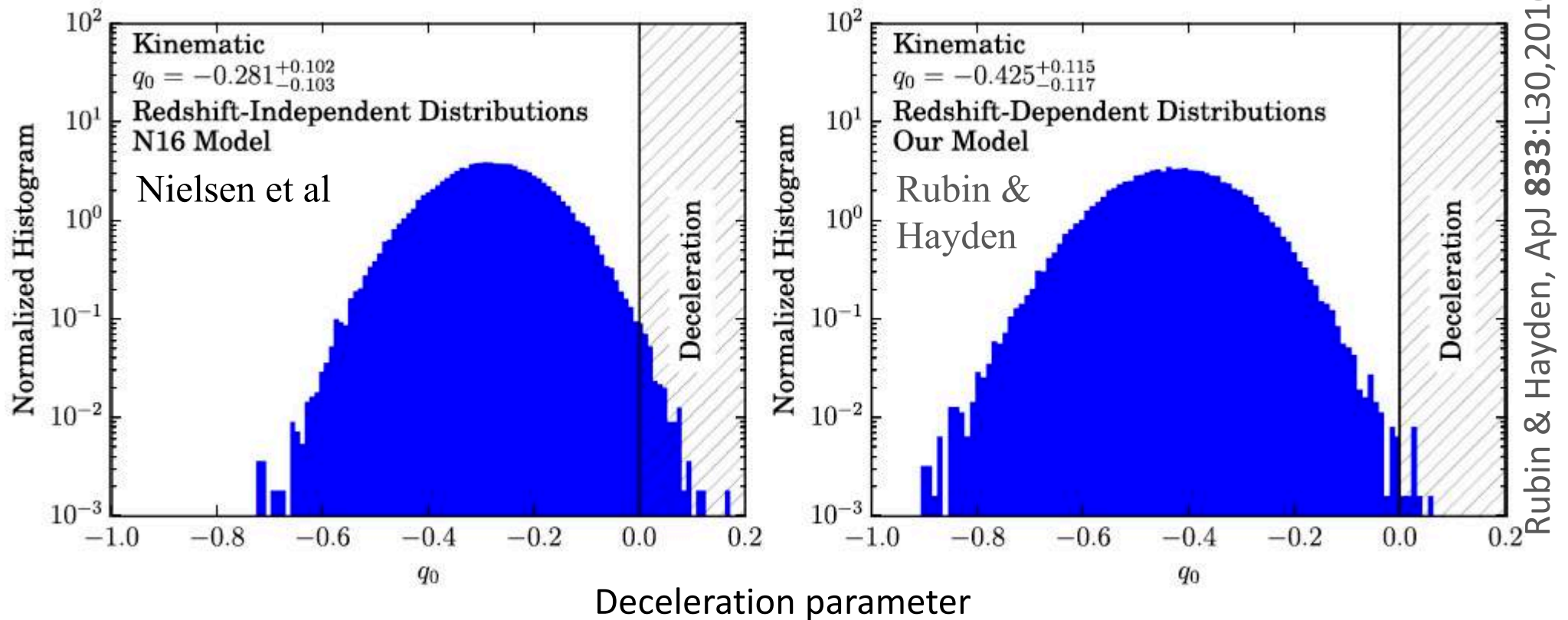
**Figure 1.** Binned  $x_1$  (left panels) and  $c$  (right panels) light curve parameters as a function of redshift for the JLA sample. The trend of color with redshift within each ground-based sample is expected due to the combination of the color-luminosity relation combined with redshift-dependent luminosity detection limits. The top panels show the 68% credible constraints on a constant-in-redshift model, as was used in N16. The bottom panels show our proposed revision. Failing to model the drift in the mean observed distributions demonstrated by the bottom panels will tend to cause high-redshift SNe to appear brighter on average, therefore reducing the significance of accelerating expansion.



**Figure 2.**  $\Omega_m$ - $\Omega_\Lambda$  constraints enclosing 68.3% and 95.4% of the samples from the posterior. Underneath, we plot all samples. The left panel shows the constraints obtained with  $x_1$  and  $c$  distributions that are constant in redshift, as in the N16 analysis; the right panel shows the constraints from our model. The red square and blue circle show the location of the median of the samples from the respective posteriors.

Even if this is justified, the significance with which a non-accelerating universe is rejected rises only to  $\lesssim 4\sigma$  ... still inadequate to claim a ‘discovery’ (even though the dataset has increased from 50 to 740 SNe Ia in 20 yrs)

The data can be analysed by expanding the time variation of the scale factor in a Taylor series, *without* reference to the  $\Lambda$ CDM model

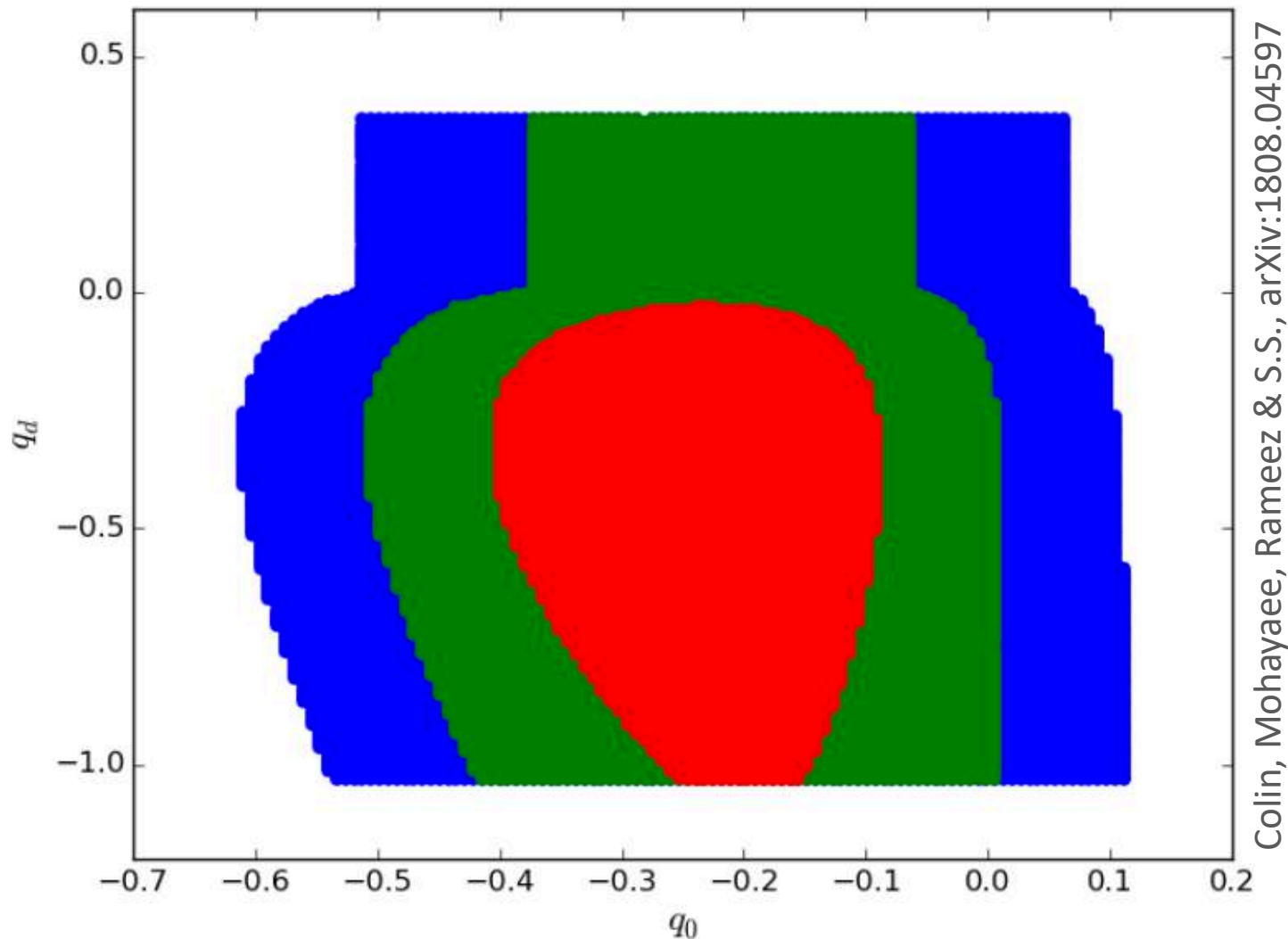


$$q_0 \equiv -(\ddot{a}a)/\dot{a}^2$$

This yields  $2.8\sigma$  evidence for acceleration in our approach ... increasing to  $3.7\sigma$  when an *ad-hoc* redshift-dependence is allowed in the light-curve parameters

Moreover allowing  $z$ -dependence in the lightcurve fitting parameters raises the spectre of whether the absolute magnitude of SNe Ia might also be  $z$ -dependent?  
 ... such luminosity evolution would totally undermine their use as 'standard candles'!

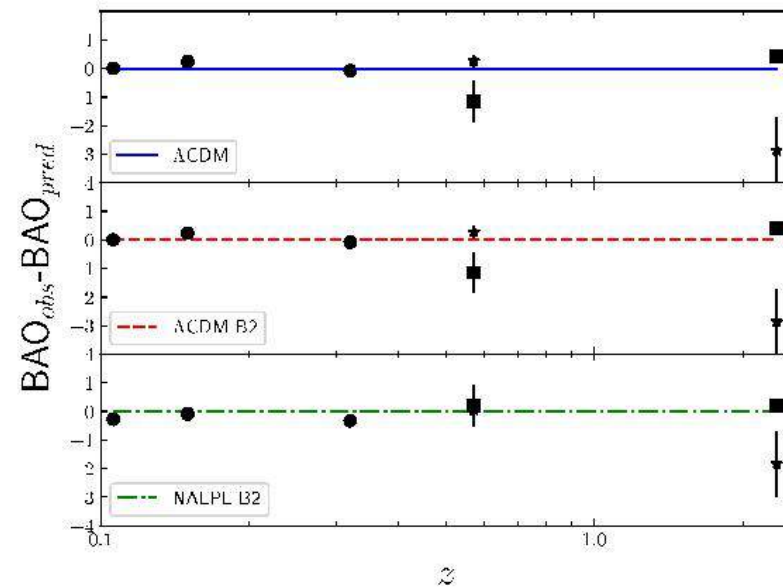
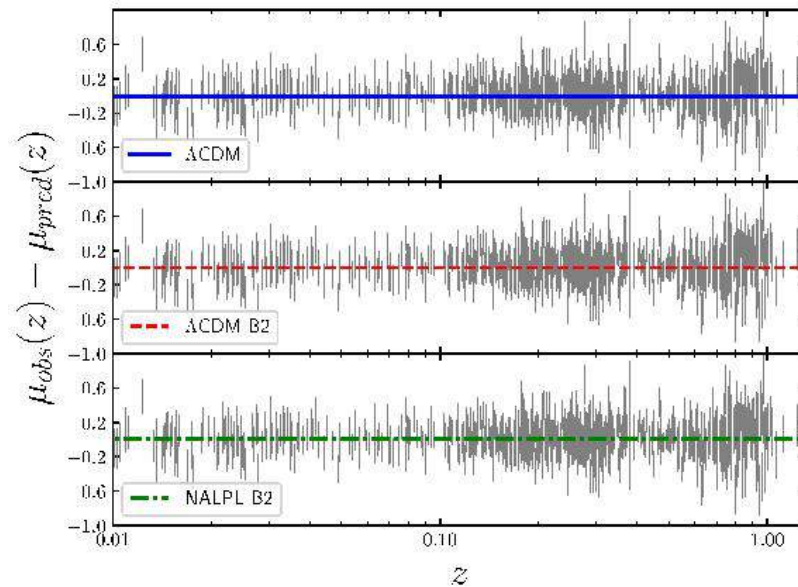
When we analyse the JLA catalogue allowing for a dipole, we find that there *is* one ... of comparable magnitude to the monopole (albeit with smaller significance)



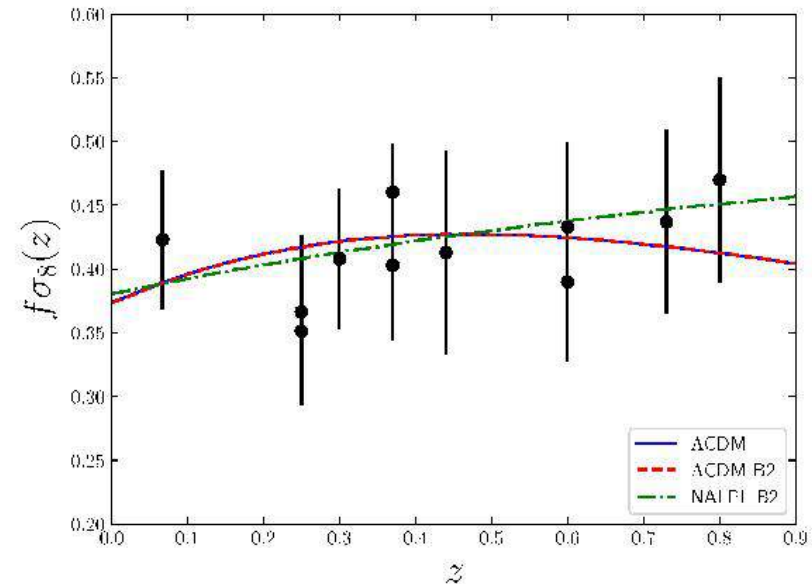
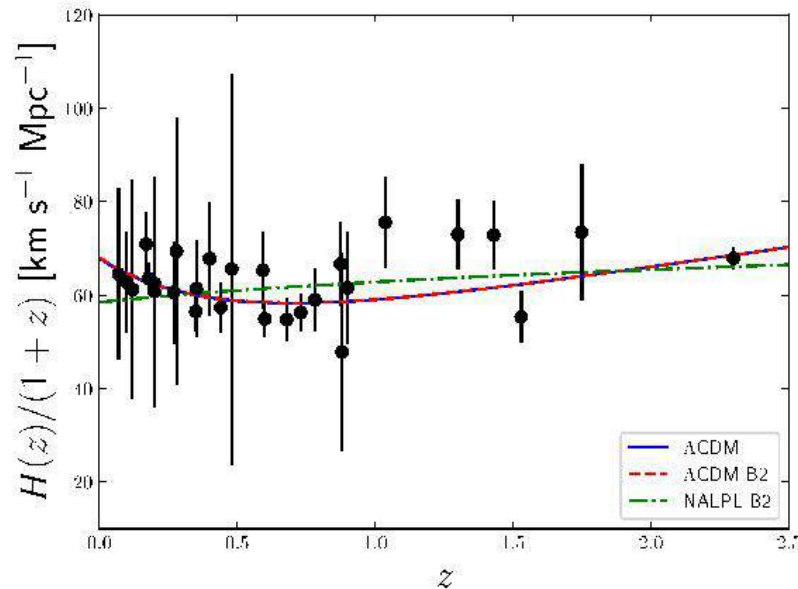
**The significance of  $q_0$  being negative has now *decreased* to only  $2\sigma$**

This implies that cosmic acceleration may simply be an artefact of our being located inside a 'bulk flow' which includes  $\sim 3/4$  of the observed SNe Ia

# What about the evidence from BAO, $H(z)$ , growth of structure, ...?



The 'independent' lines of evidence are usually obtained using  $\Lambda$ CDM templates!



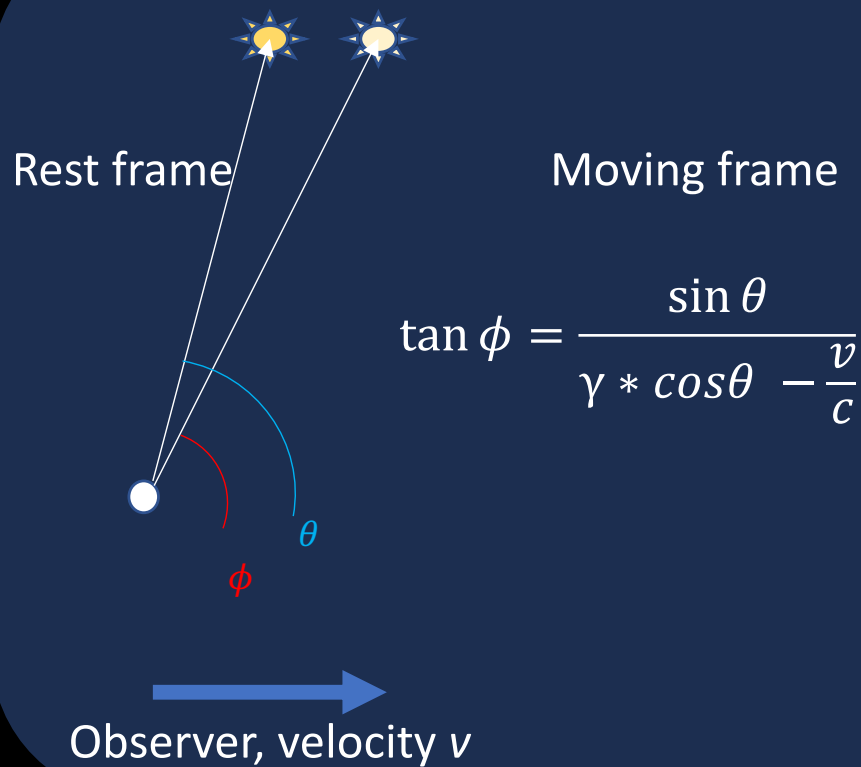
All data are *equally consistent* with *non-accelerated* expansion



# A MOVING OBSERVER $\rightarrow$ KINEMATIC DIPOLE

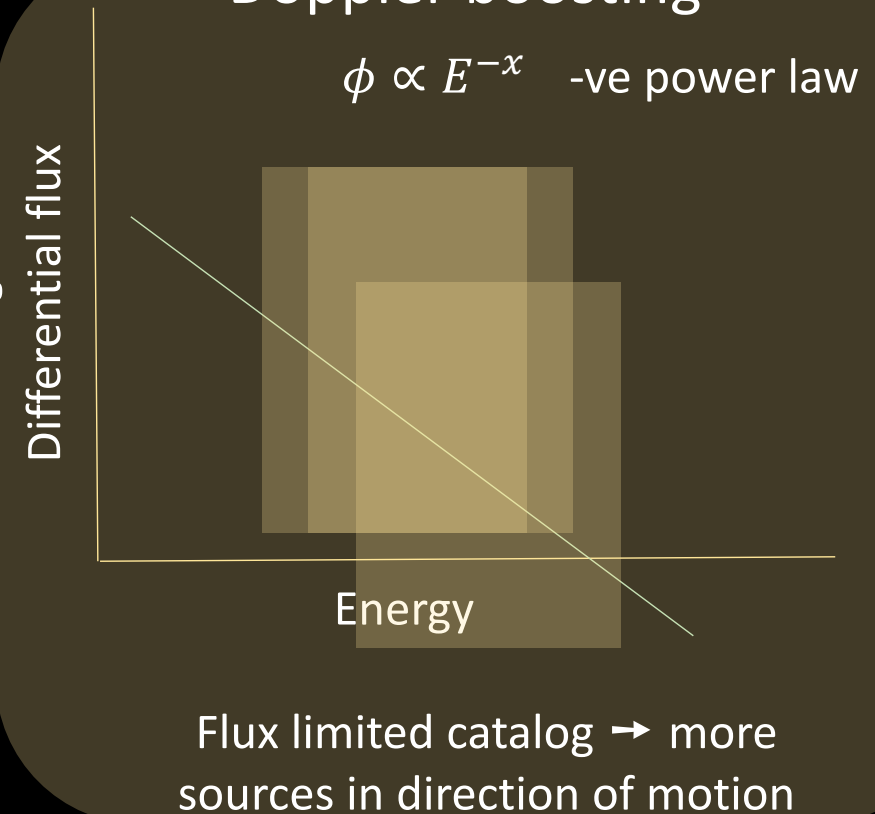
$$\sigma(\theta)_{obs} = \sigma_{rest} \left[ 1 + \left[ 2 + x(1 + \alpha) \right] \frac{v}{c} \cos(\theta) \right]$$

## Aberration



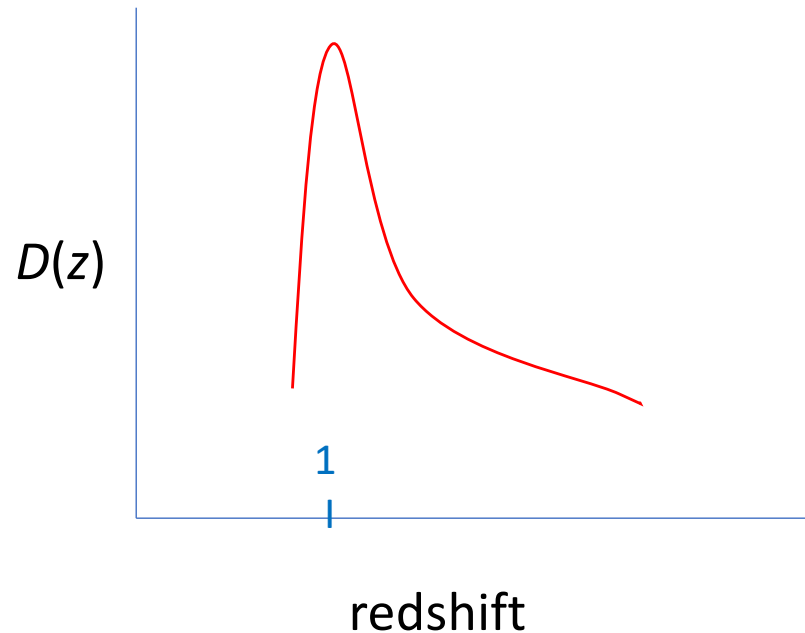
+

## Doppler boosting



# DIPOLES IN A CATALOGUE OF GALAXIES

All-sky catalogue with  $N$  sources  
with redshift distribution  $D(z)$  from  
a directionally unbiased survey



$$\vec{\delta} = \vec{\mathcal{K}}(\vec{v}_{obs}, x, \alpha) + \vec{\mathcal{R}}(N) + \vec{\mathcal{S}}(D(z))$$

$\vec{\mathcal{K}} \rightarrow$  The kinematic dipole: *independent*  
of source distance, but depends on  
source spectrum, source flux  
function, observer velocity

$\vec{\mathcal{R}} \rightarrow$  The random dipole:  $\propto 1/\sqrt{N}$   
isotropically distributed

$\vec{\mathcal{S}} \rightarrow$  The dipole component of an actual  
anisotropy in the distribution of  
sources in the cosmic rest frame  
(significant for shallow surveys)

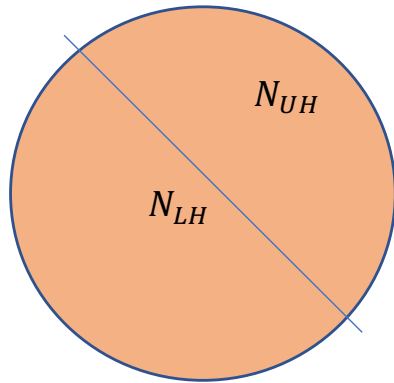
**Radio sources: NVSS + SUMSS**, 600,000 galaxies  $z \sim 1$ ,  $\vec{\mathcal{S}}(D(z)) \rightarrow 0$   
Colin, Mohayaee, Rameez & Sarkar, MNRAS **471**:1045,2017

**Wide Field Infrared Survey Explorer**, 2,400,000 galaxies,  $z \sim 0.14$ ,  $\vec{\mathcal{S}}(D(z))$  significant  
Rameez, Mohayaee, Sarkar & Colin MNRAS **477**:1722,2018

# ESTIMATORS FOR THE DIPOLE

$$\vec{D}_H = \hat{z} * \frac{N_{UH} - N_{LH}}{N_{UH} + N_{LH}}$$

$$\vec{D}_C = \frac{1}{N} \sum_{i=1}^N \hat{r}_i$$



Vary the direction of the hemispheres until maximum asymmetry is observed

Easy visualisation

High bias and statistical error  $\sim 1/\sqrt{N}$

Add up unit vectors corresponding to directions in the sky for every source

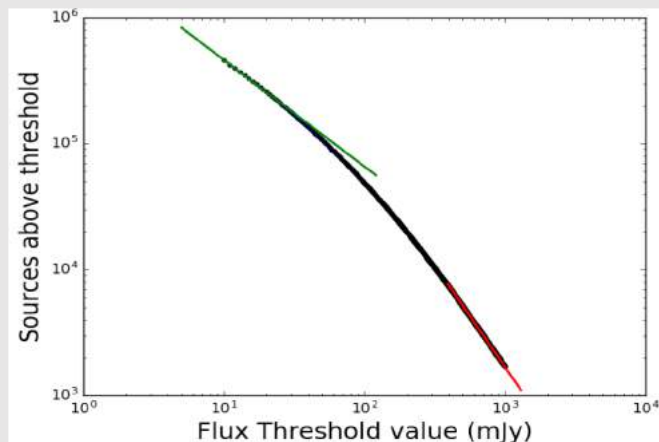
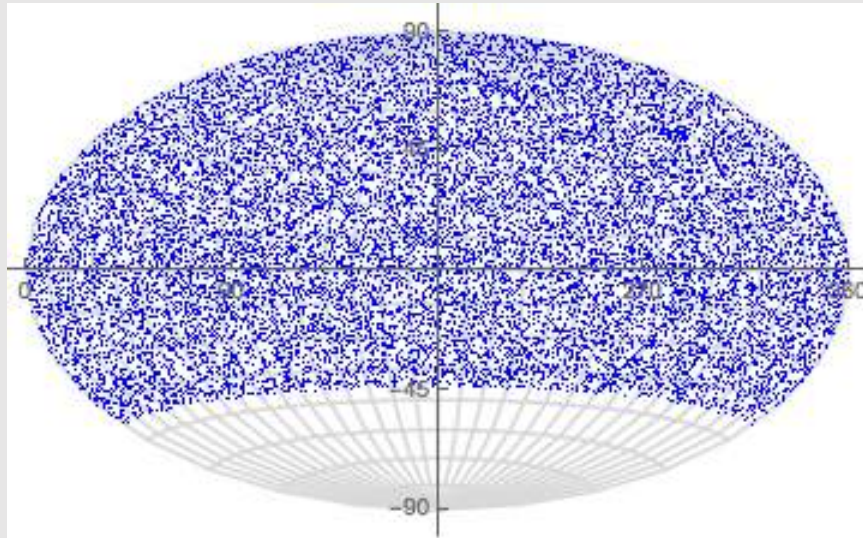
Relatively lower bias and statistical error  $1/\sqrt{N}$

$$\vec{D}_C = \frac{\hat{z}}{N} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \sigma(\theta) \cos\theta \sin\theta d\theta d\phi$$

$$\vec{D}_H = \frac{\hat{z}}{N} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \sigma(\theta) \frac{|\cos\theta|}{\cos\theta} \sin\theta d\theta d\phi$$

(Rubart & Schwarz 2013)

## THE NRAO VLA SKY SURVEY (NVSS)

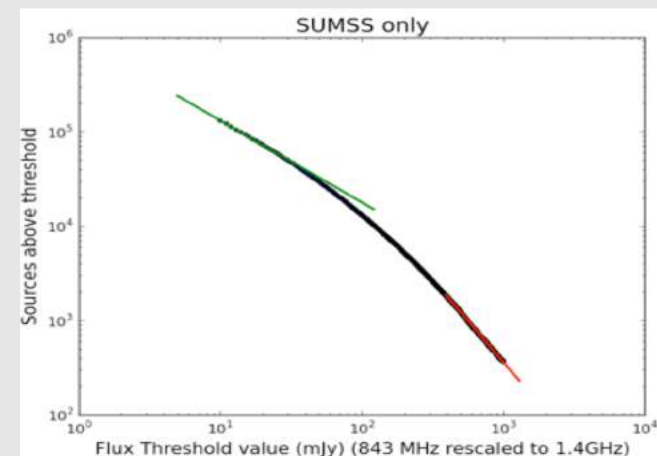
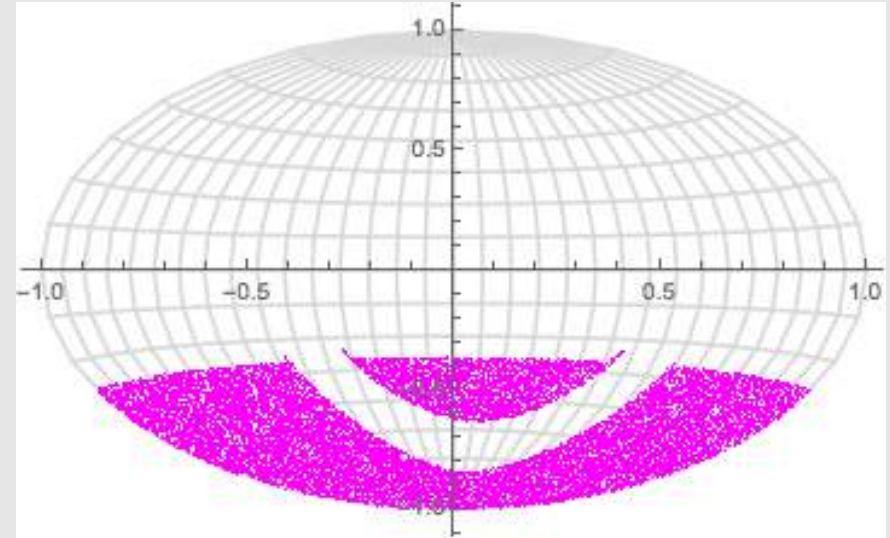


1.4 GHz survey (down to Dec =  $-40.4^\circ$ )  
National Radio Astronomy Observatory

1,773,488 sources  $>2.5$  mJy  
(complete above 10 mJy)

Most are believed to be at  $z \gtrsim 1$

## SYDNEY UNIVERSITY MOLONGLO SKY SURVEY (SUMSS)



843 MHz survey (Dec  $< -30.0^\circ$ )  
Molonglo Observatory Synthesis telescope

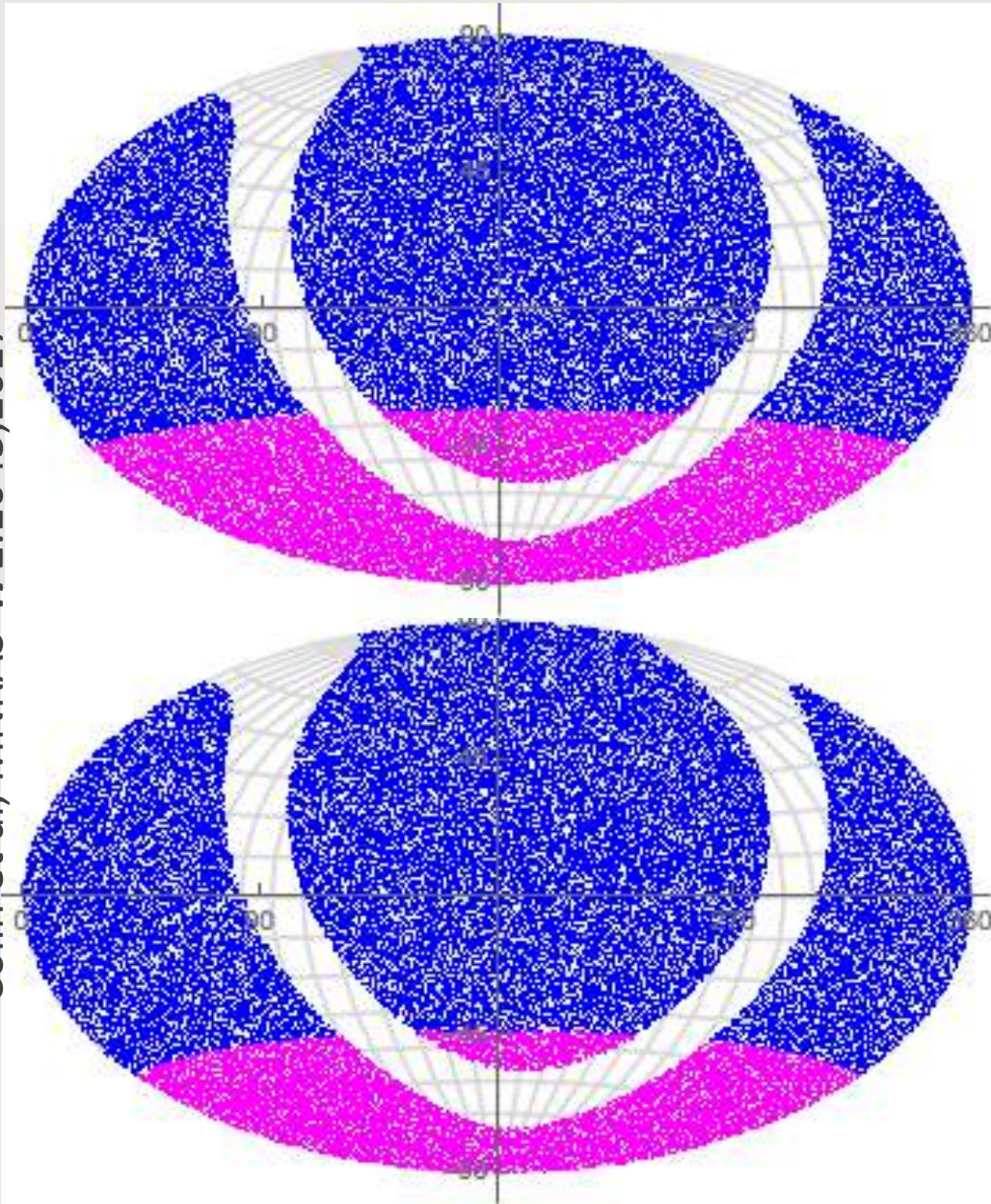
211,050 sources (with similar sensitivity and  
resolution to NVSS catalogue)

... Similar expected redshift distribution



# THE NVSUMSS-COMBINED ALL SKY CATALOG

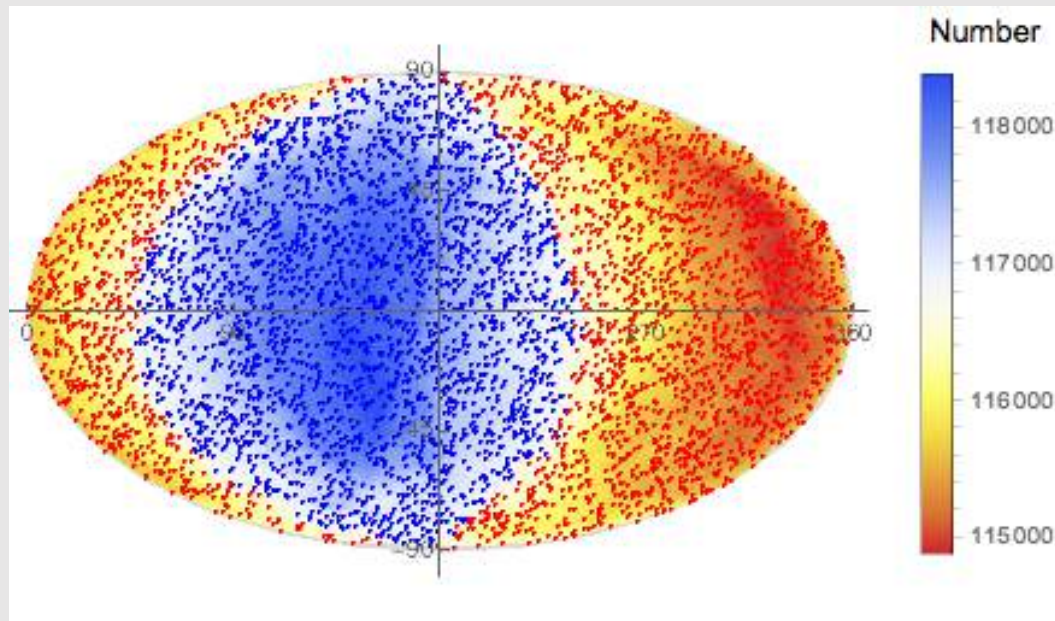
Colin et al, MNRAS 471:1045,2017



- Rescale SUMSS fluxes by  $(843/1400)^{-0.75} \sim 1.46$  to match with NVSS (works within  $\sim 1\%$ )
- Remove Galactic Plane at  $\pm 10^\circ$  (also Supergalactic plane)
- Remove NVSS sources below, and SUMSS sources above, dec -30 (or -40)
- Apply common threshold flux cut to both samples
- Remove *any* nearby sources (common with 2MRS & LRS)

# OUR PECULIAR VELOCITY WRT RADIO SOURCES

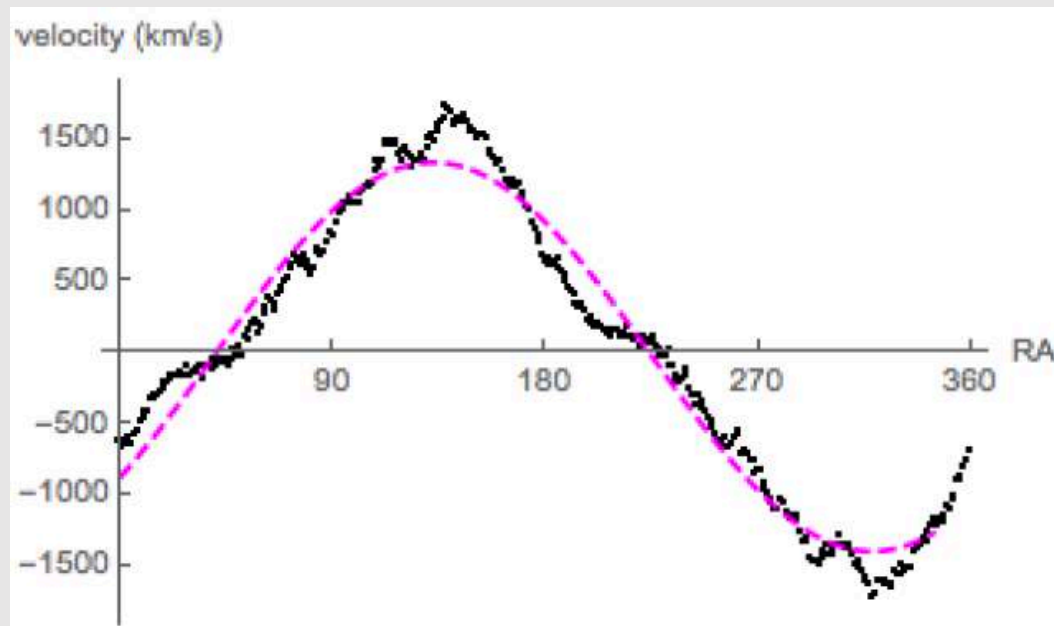
Colin et al, MNRAS **471**:1045,2017



Velocity  $\sim 1355 \pm 174$  km/s  
(with the 3D linear estimator)

Direction within  $10^\circ$  of CMB  
dipole (but **4 times faster**)!

Statistical significance: 99.75%  
 $\Rightarrow 2.81\sigma$  (by Monte Carlo)

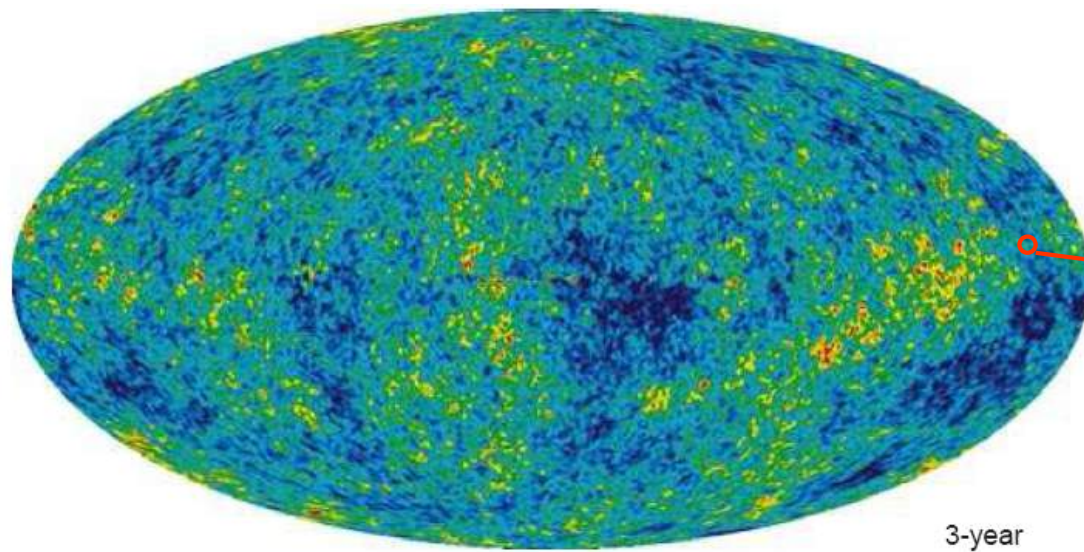


*Confirms* claim by Singal (2011)  
which was criticized subsequently  
(Gibelyou & Huterer 2012, Rubart &  
Schwarz 2013, Nusser & Tiwari 2015)

We have addressed *all* the concerns  
but this strange anomaly remains!



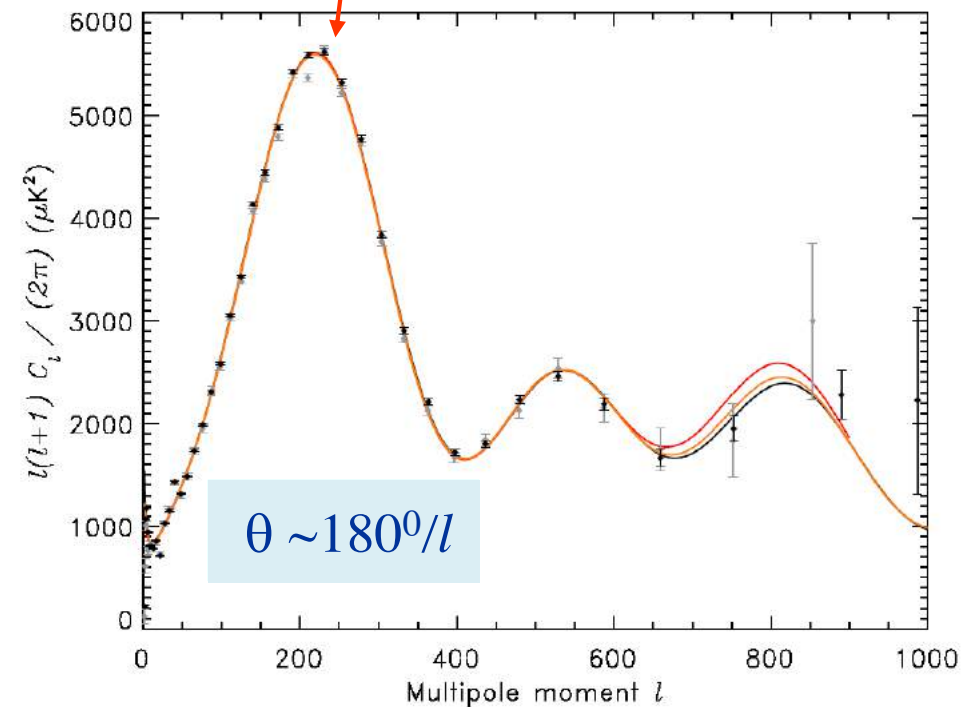
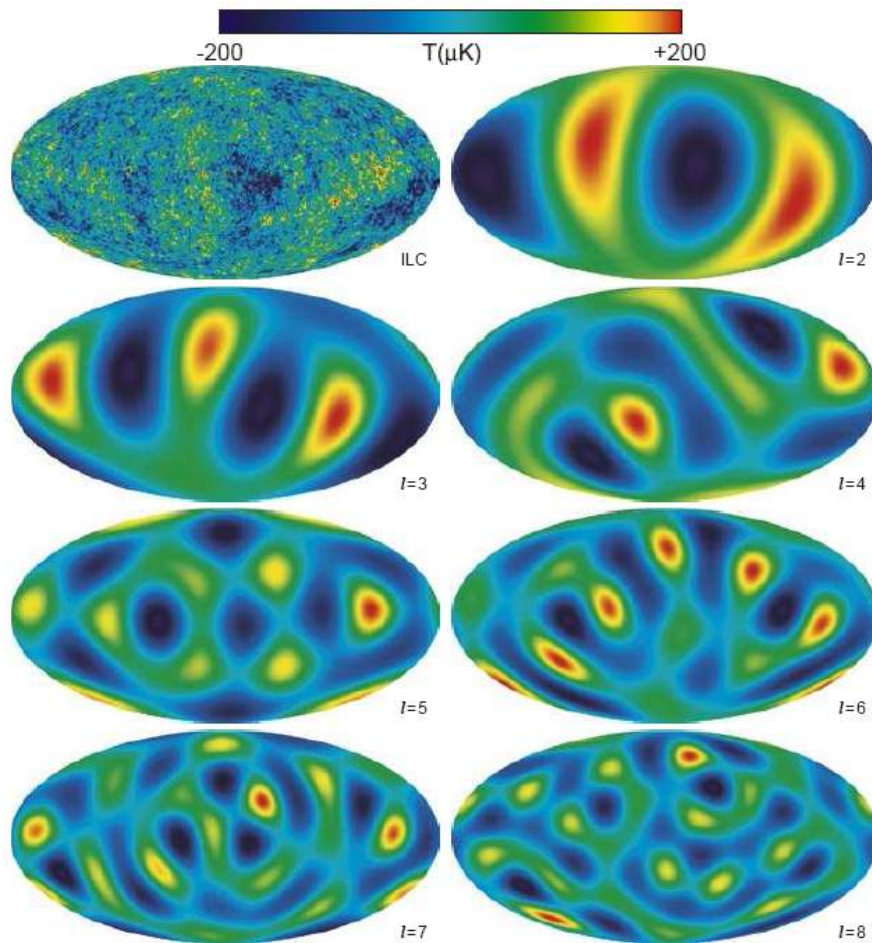
Coherent oscillations in photon-baryon plasma, excited by density perturbations on *super-horizon* scales ...



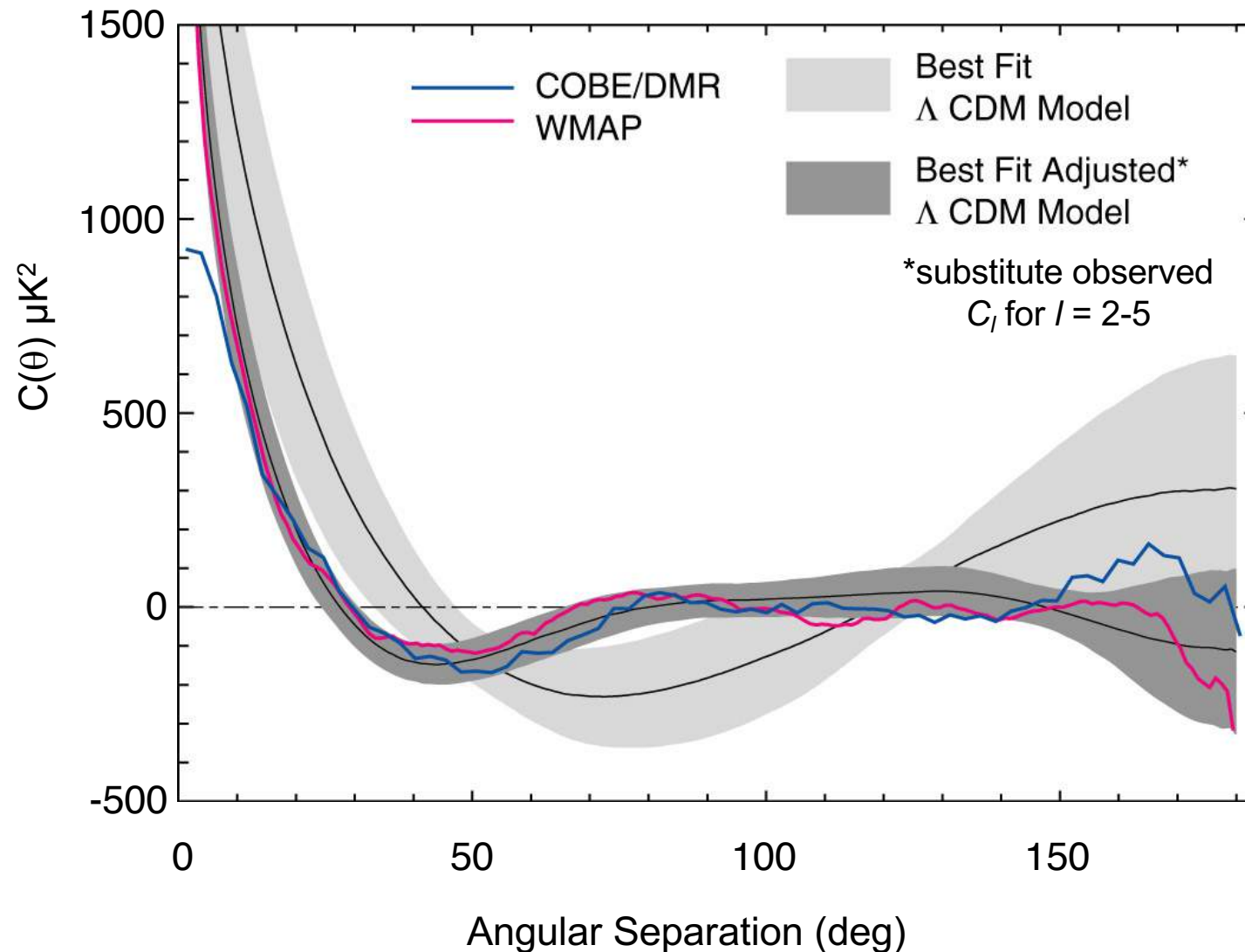
(Hubble radius at  $t_{\text{rec}}$ )

$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$



The lack of power on large angular scales is most striking, although it is claimed to be *not* unlikely taking cosmic variance and foreground subtraction uncertainties into account  
 → chance probability of  $O(1\%)$ ?



$$C(\theta) = \langle T(n_i)T(n_j) \rangle$$

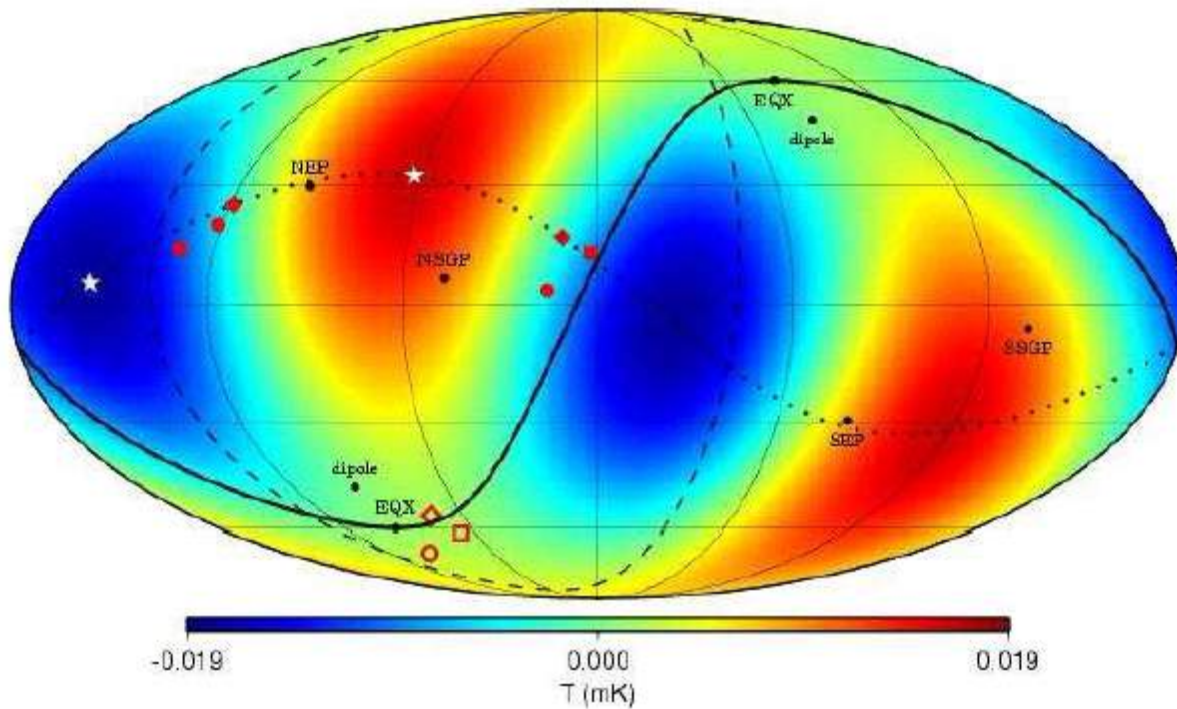
$$n_i \cdot n_j = \cos \theta$$

$$S = \int_{60^\circ}^{180^\circ} C(\theta)^2 d\theta$$

*A posteriori*  
 likelihood of  
 observed  $S$  is only  
 (0.15 - 0.3) %

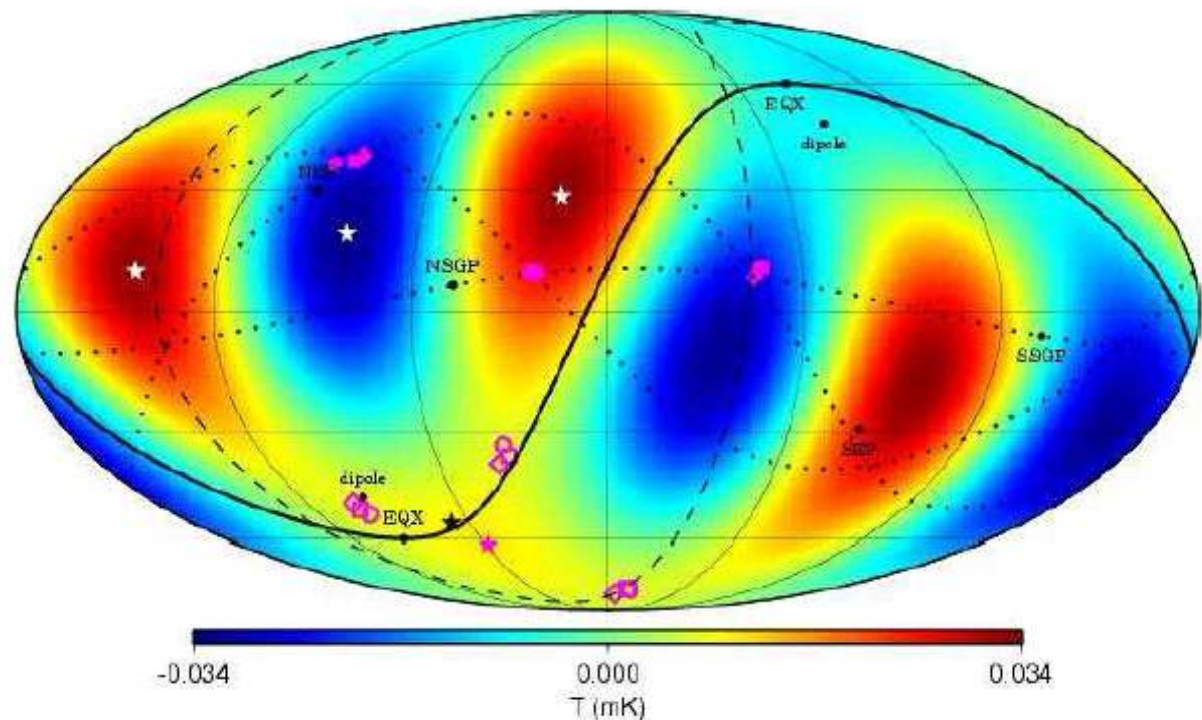
Moreover there is an unexpected alignment of low multipoles, a 'cold spot', and an asymmetry between the North and South ecliptic hemispheres



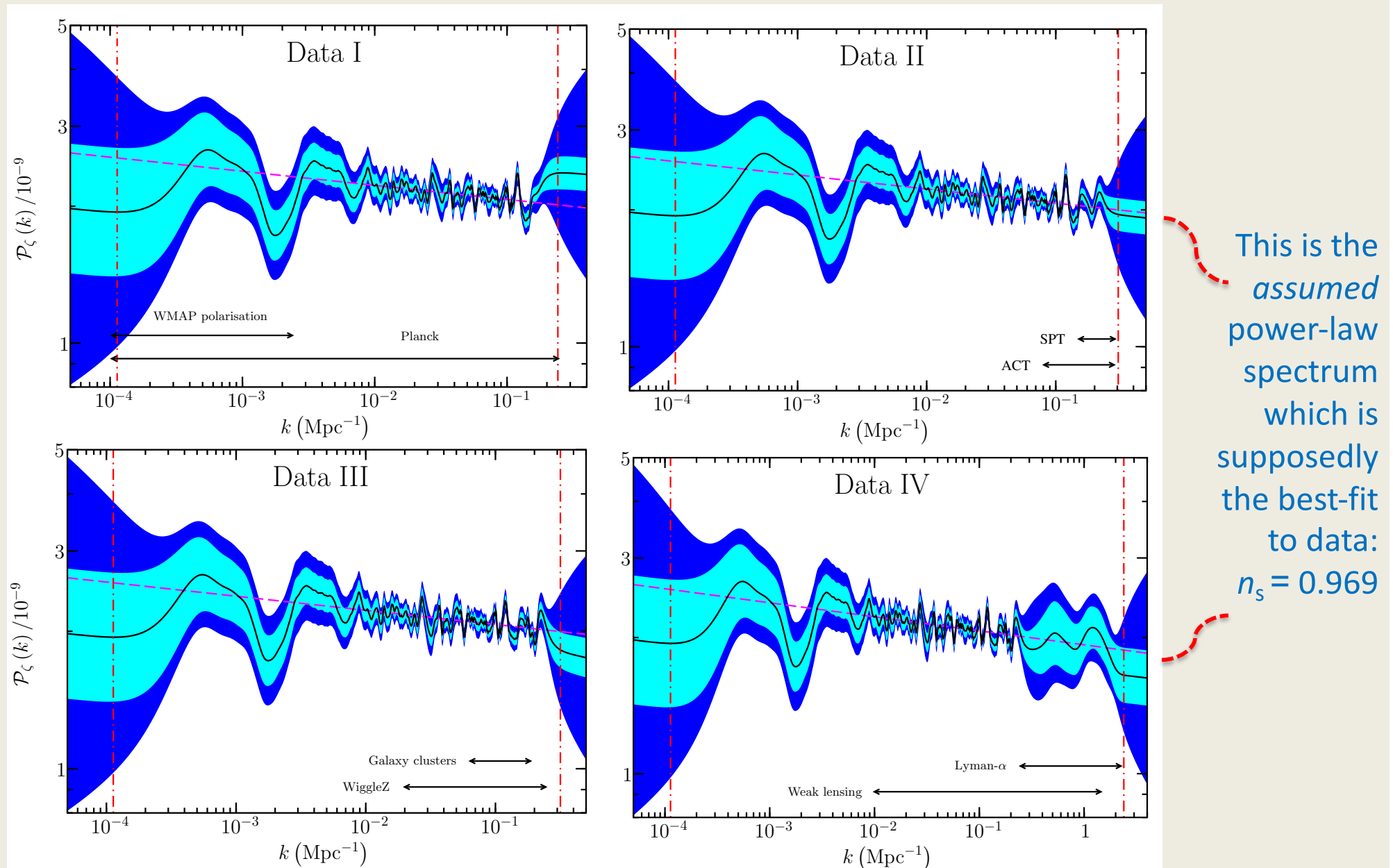


Curious alignment of quadrupole and octupole (along the ecliptic)  
 Power concentrated in plane tilted by  $\sim 30^\circ$  from the Galactic plane  
 ( $m = \pm l$  in suitable coord. system)

Probability of low quadrupole  
 + alignment + “planarity”:  
 $\sim 4 \times 10^{-5}$  (Tegmark *et al* 2004)



The primordial spectrum of perturbations can be deconvoluted from CMB & LSS data *non-parametrically*, using 'Tikhonov regularisation' (Hunt & S.S., JCAP **12:052**,2015)



Comparison with Monte Carlo simulations shows  $\sim 2\sigma$  deviations from a power-law spectrum

## RECONSTRUCTION OF A DIRECTION-DEPENDENT PRIMORDIAL POWER SPECTRUM FROM PLANCK CMB DATA

We consider a **direction-dependent** component of the power spectrum of the CMB fluctuations, which is moreover allowed to vary with the scale (wave number):

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{LM} g_{LM}(k) Y_{LM}(\hat{\mathbf{k}})$$

... and focus on the **quadrupole** modulation (NB: density field is *real*, hence symmetry requires  $L$  to be *even* – see Hajian & Souradeep 2005, Pullen & Kamionkowski 2007)

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{M=-2}^2 g_{2M}(k) Y_{2M}(\hat{\mathbf{k}})$$

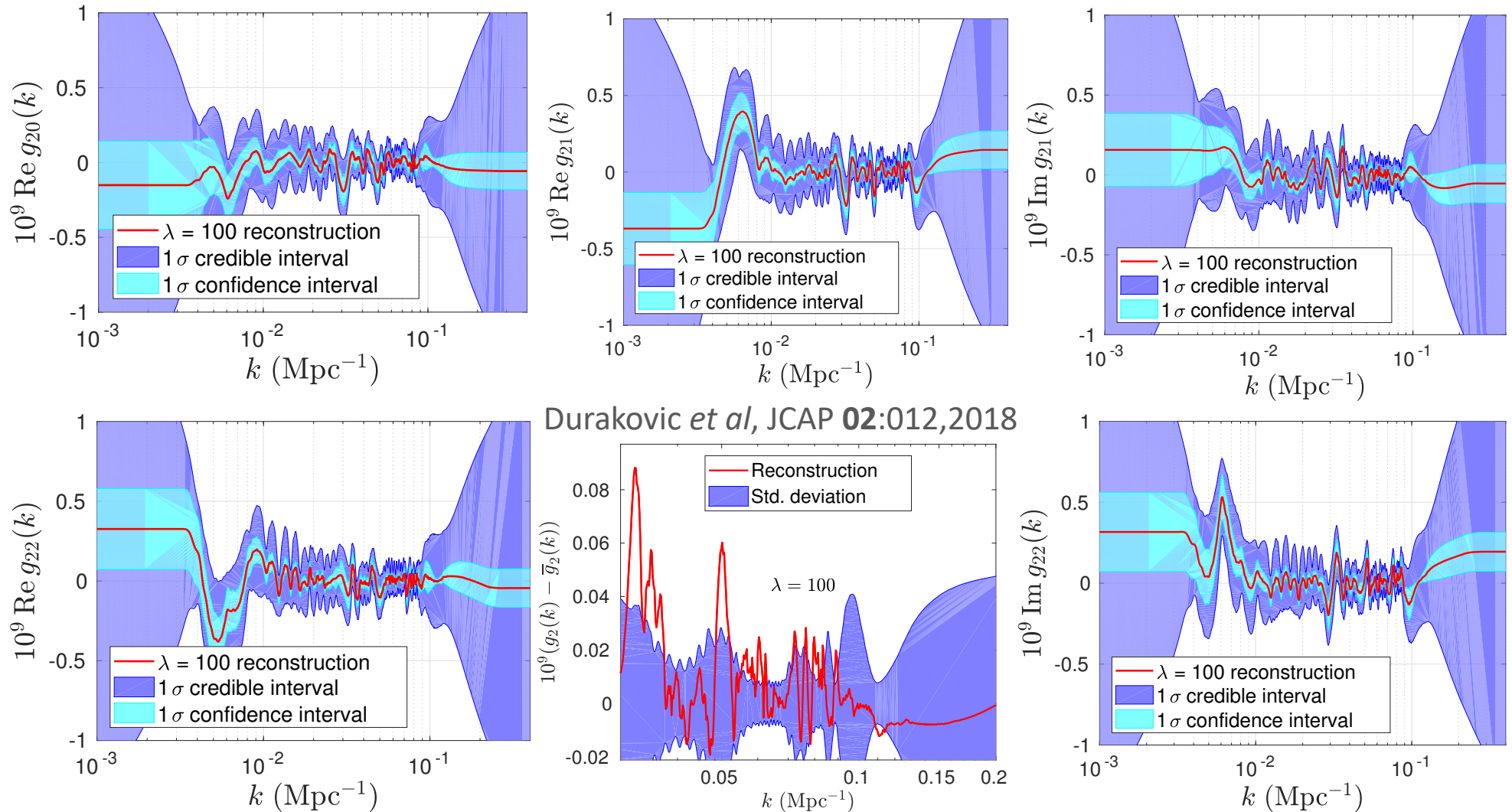
We compute these ‘**bipolar spherical harmonics**’ for the Planck DR2-2015 SMICA map, and estimate the noise covariance from *Planck Full Focal Plane 9* simulations

Durakovic, Hunt, Mukherjee, S.S. & Souradeep, JCAP **02**:012,2018

Previous work by: Groeneboom & Eriksen (2009), Kim & Komatsu (2013)

Theoretical models by: Ford (1989), Chibisov (1989), Ackerman *et al* (2007), Pitrou *et al* (2008), Himmetoglu *et al* (2009), Watanabe *et al* (2009), Bartolo *et al* (2013, 2018), ...

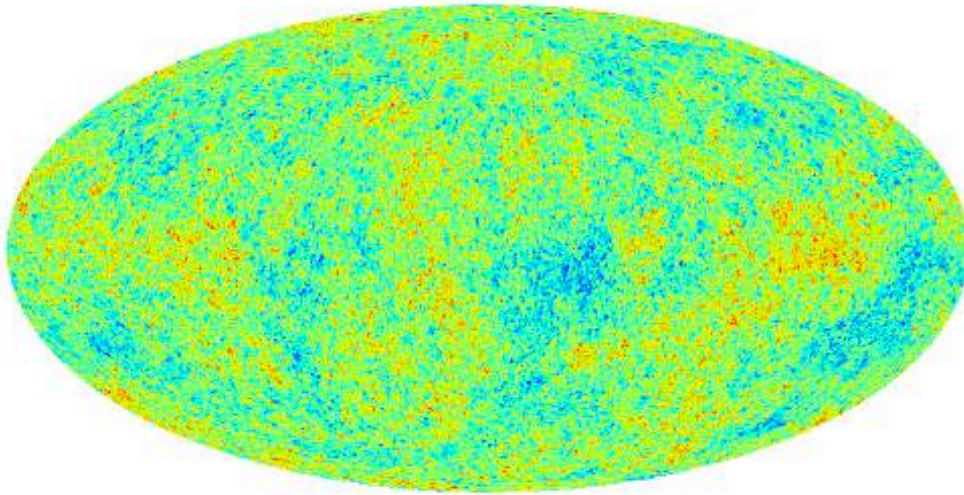
When a constant **quadrupolar modulation** is fitted to Planck data in the range  $0.005 \leq k/\text{Mpc}^{-1} \leq 0.008$ , its **preferred directions** are found to be *related* to the **cosmic hemispherical asymmetry**, and the **CMB dipole**



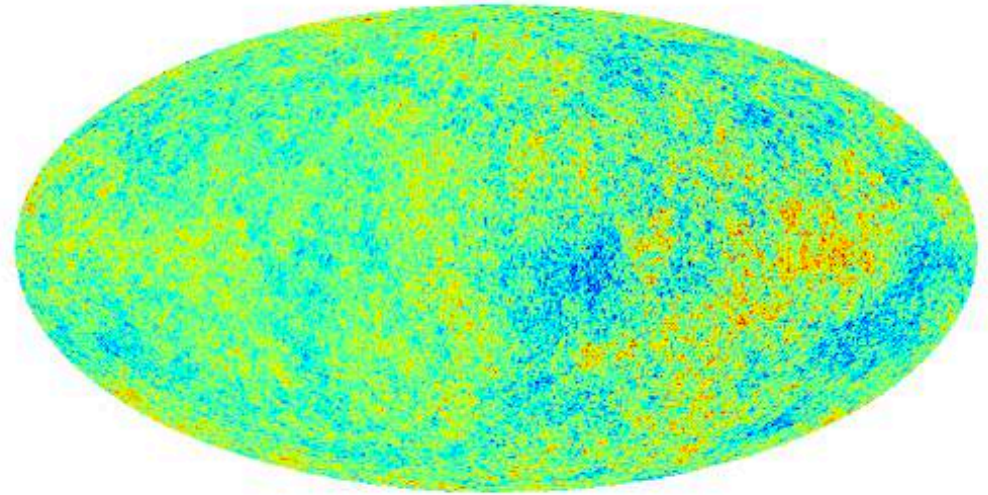
The significance is  $2.1\sigma$  with a test statistic sensitive only to the amplitude of the modulation ... but with a statistic sensitive also to the direction, it rises to  $6.9\sigma$ !



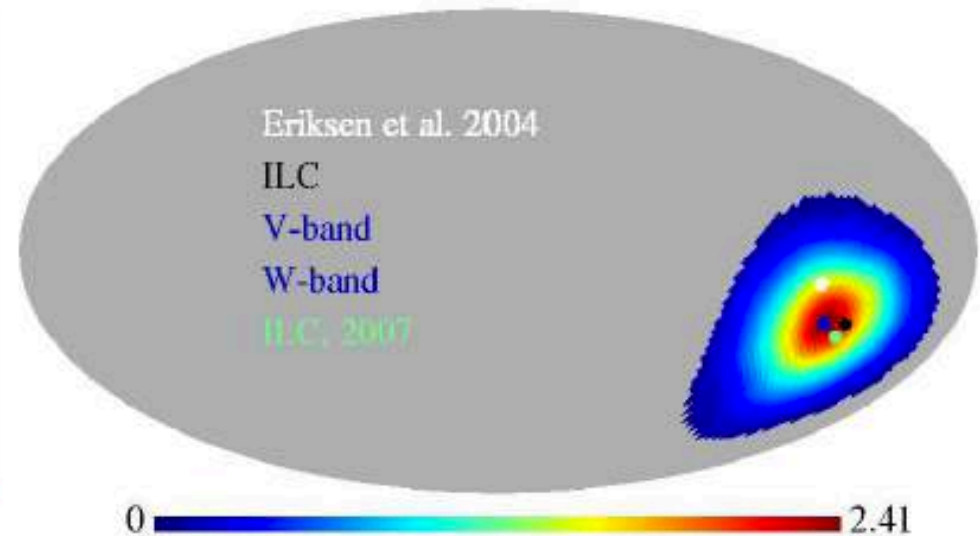
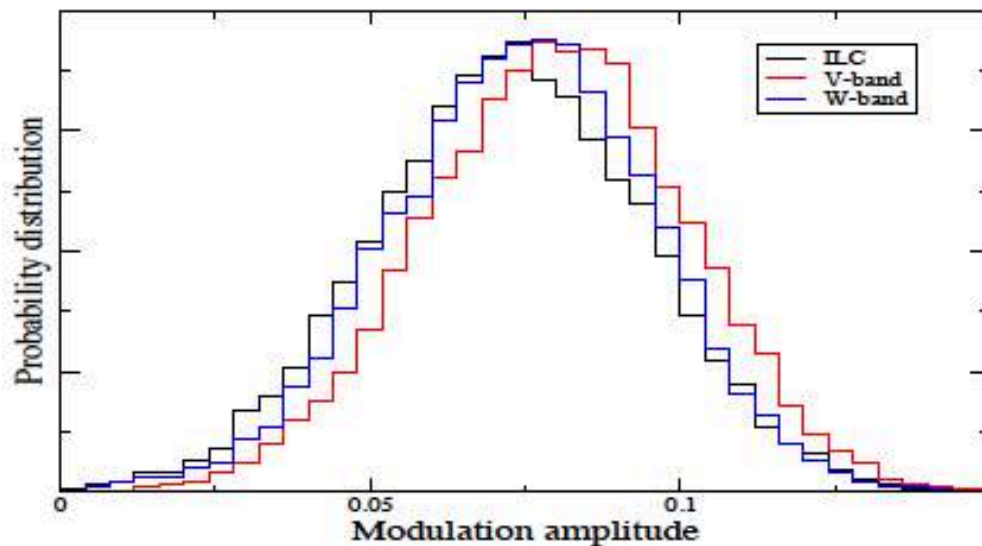
Eriksen *et al* (2004) found that the CMB fluctuations are stronger in one hemisphere of the sky than in the other ( $@3\sigma$ ) ... as if the perturbations are modulated by a *dipole*



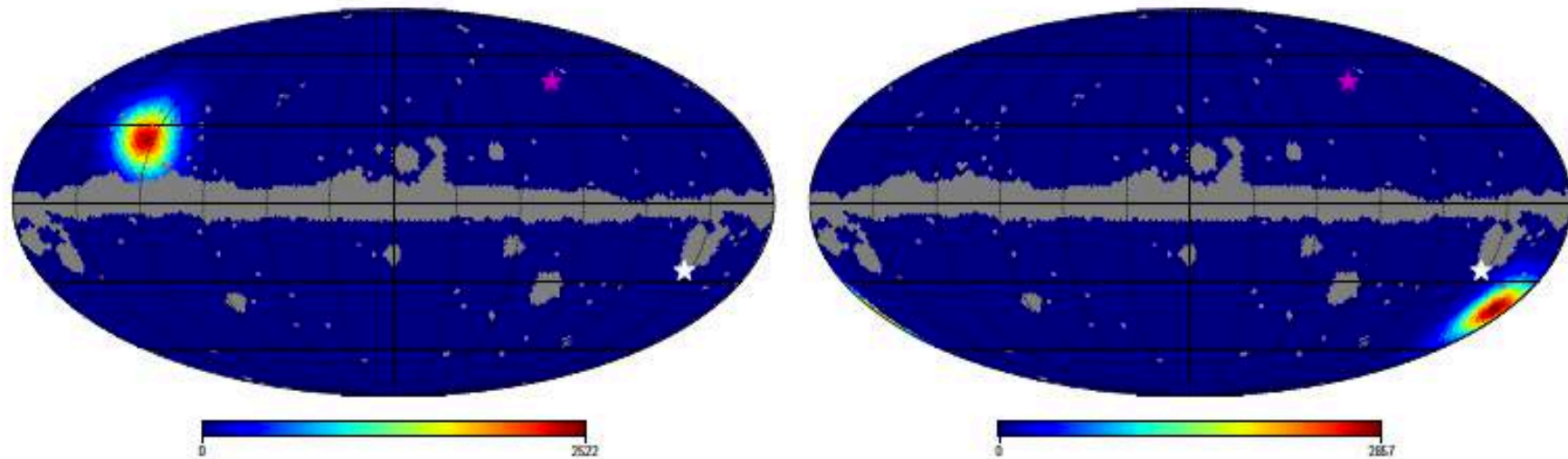
$$T_{\text{iso}}(\hat{\mathbf{n}})$$



$$T_{\text{iso}}(\hat{\mathbf{n}})(1 + A\hat{\mathbf{p}} \cdot \hat{\mathbf{n}})$$



# Alignments on the sky : What does this imply for initial conditions?



Hot ( $10^9 g_A = 0.76 \pm 0.22$ ) quadrupole modulation (left panel),  
 and cold ( $10^9 g_A = -0.82 \pm 0.21$ ) modulation (right panel).  
 The magenta and white stars indicate the direction of the CMB  
 dipole and of the hemispherical asymmetry respectively.

For $k = 0.005\text{-}0.008 \text{ Mpc}^{-1}$ :		Angular distances to:	
Amp. $10^9 g_A$	Direction ( $l, b$ )	CMB dipole ( $264^\circ, 48^\circ$ )	Hemisph. asym. ( $213^\circ, -26^\circ$ )
$0.76 \pm 0.22$	$(128^{+14}_{-14}, 25^{+11}_{-9})$	$97^\circ$	$97^\circ$
$-0.82 \pm 0.21$	$(191^{+15}_{-14}, -41^{+10}_{-11})$	$110^\circ$	$24^\circ$



# A 'TILTED' UNIVERSE?

- There is a dipole in the recession velocities of host galaxies of supernovae  
⇒ we are in a 'bulk flow' stretching out well beyond the scale ( $\sim 100$  Mpc) at which the universe supposedly becomes statistically homogeneous.
- The inference that the Hubble expansion rate is accelerating may be an artefact of the local bulk flow (there is a dipole in  $q_0$  aligned with the bulk flow, and the monopole drops in significance to be consistent with zero at  $2\sigma$ )
- The distribution of radio galaxies at  $z \gtrsim 1$  has a dipole in the same direction – but 4 times *bigger* than that in the CMB – so is at  $2.8\sigma$  tension with it
- There is a scale-dependent quadrupolar modulation of CMB anisotropy ...  
the direction is  $\sim$ orthogonal to the CMB dipole

Could all this be an indication of new horizon-scale physics?

The 'standard' assumptions of isotropy and homogeneity are questionable – data from forthcoming surveys (Euclid, LSST, SKA) will provide large enough datasets to enable definitive tests ... meanwhile: **'Caveat Emptor'!**