Higher Dimensional Field Theories

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String Theory: Past and Present
SpentaFest
ICTS, India
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Park, Masato Taki, Cumrun Vafa, Futoshi Yagi, Ho-Ung Yee

Canonical Quantization Of Nonabelian Gauge Theory In The Schrodinger Picture: Applications To Monopoles And Instantons

Spenta Wadia

(City Coll., N.Y.). Jan 1979. 137 pp.
UMI Thesis Server

4 dimensions

- 4d Yang-Mills Theories: Asymptotically Free
  - UV free, IR strong
- 4d QED: Landau pole
  - UV strong/incomplete, IR free

5 dim & 6 dim

- Yang-Mills theory: $[1/g^2] = M$ (5d), M2 (6d)
    - coupling expansion = $1/M$ or $1/M^2$ expansion
    - IR free, UV: strong, Landau pole?
- No well defined QFTs in higher dimensions?

5 dim

- Seiberg (9608111): 5d N=1 Super Yang-Mills theories with SU(2) gauge group and n fundamental hypers have a UV fixed point with enhanced global symmetry $E_{n+1} \supset SO(2N_f)xU(1)_I$

- Morrison, Seiberg (9609070): For n=0, one can have two options, $\theta=0,\pi$, with E_1, $\bar{E}_1$ enhanced symmetry. Non-Lagrangian theory of rank 1 E_0 without any global symmetry.

- E_0, $\bar{E}_1=U(1)$, $E_1=SU(2)$, $E_2=SU(2)xU(1)$, $E_3=SU(3)xSU(2)$, $E_4=SU(5)$, $E_5=SO(10)$, E_6, E_7, E_8,

- $\tilde{E}_8$: SU(2) + 8 hyper: completed in 6d (1,0) SCFT

6d (1,0) SCFTs

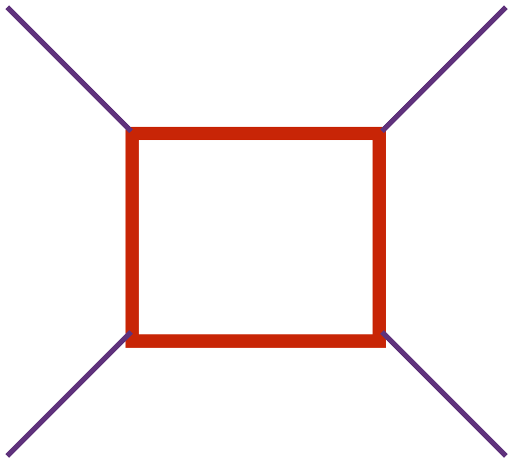
Seiberg'96,Danielsson et.al.'97

- Seiberg (9609161): Nontrivial fixed points of the renormalization group in six-dimensions
- vector multiplet ($A\mu,\lambda$)
- hyper multiplet (ϕ,ψ)
- tensor multiplet (B,Ψ,Φ)
- fermion helicity
- Anomalies

	helicity
vector	(0,1)
hyper	(1,0)
tensor	(1,0)
Q	(1,0)

Heckman,Morrison,Vafa(Heckman('13),Morrison, Rudelius, Vafa ('15)

Gauge Anomaly



$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_{\mathrm{tot}} = (\alpha_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} \alpha_R) = 0$$

$$c_{\mathrm{tot}} = (c_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} c_R) \geq 0$$

- The gauge anomaly polynomial is made of two pieces.
  - The first one should vanish by vector and hyper contributions.
  - The second one can be removed with coupling to tensor multiplet and using the Green-Schwartz mechanism

$$H^2 + \sqrt{c_R}(B \wedge \mathrm{tr} F \wedge F + \Phi F^2)$$

6d (2,0) SCFTs

- Witten[9503124], Strominger[9512059], Klebanov and Tseytlin[9604089]
- type IIB on ADE ALE space
- theory on N M5 branes, M5+OM5
- tensor multiplet (B,Ψ,Φ_I) $\gamma^6\,\Psi=\Psi$, $H=dB={}^*H$
- M2 branes between M5 branes=selfdual strings

6d (1,0), (1,1), (2,0) LSTs

- Seiberg[9705221], Losev, Moore Shatashvili[9707250]

- (1,1) & (2,0) LSTs on NS5 branes with $g_s \to 0$, l_s: fixed
- (1,1) LST as the UV completion of 6d (1,1) SYM

- (1,0) LSTs: Bhardwaj, Del Zotto, J. Heckman, Morrison, Rudelius, Vafa [1511.05565]

Today

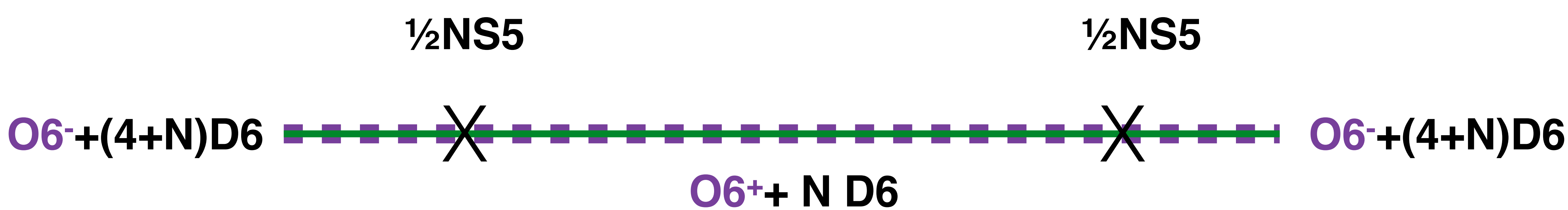
- Focus on a very small part of this theory space
- generalization of 5d Seiberg's result of SU(2) theory to larger gauge group
- relate that to 6d (1,0) SCFTs
- Enhanced Global Symmetry and Gopakumar-Vafa invariants

Start with 6d (1,0) SCFTs

- Seiberg: 6d (1,0) SCFT with E_8 global symmetry
- $[E_8]$-T : M5 brane near M9 Horava-Witten Wall
- $[E_7]$-T-$[SU(2)]$: part of E_7 conformal matter
- $[E_6]$-T-$[SU(3)]$: part of E_6 conformal matter
- $[G_2]$-T-$[F_4]$: part of E_8 conformal matter
- $[SO(8)]$-T-$[SO(8)]$: D_4 conformal matter

D_{4+k} conformal matter

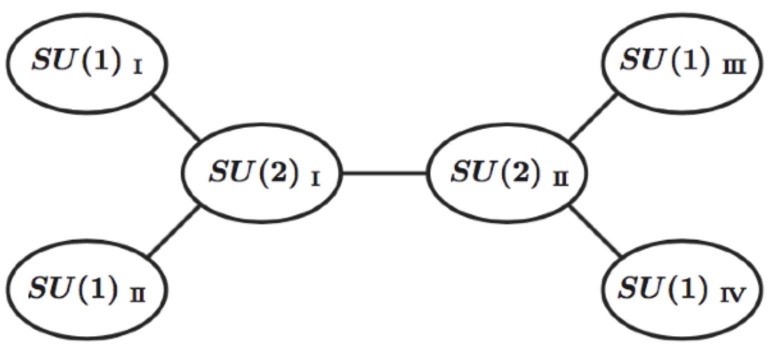
- $[SO(2k+8)]$-$Sp(k)_T$-$[SO(2k+8)]$
- $Sp(k)$+ tensor + $(2k+8)$ fundamental hyper
- global symmetry = $SO(4k+16)$
- for $k=0$, $SO(16)$ get enhanced to E_8



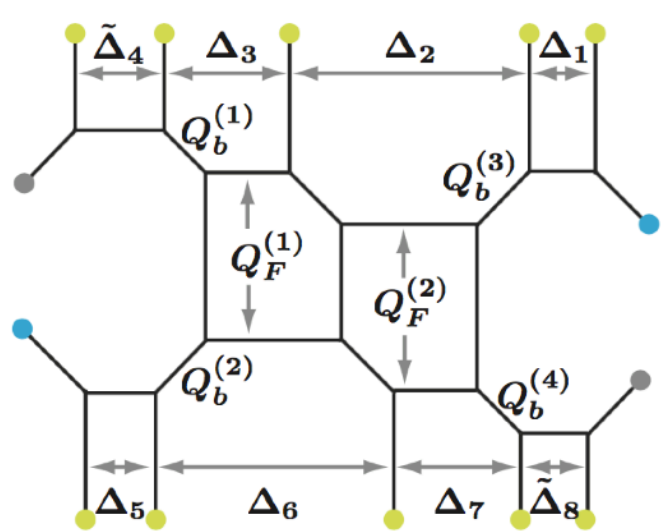
5d reduction

Hyashi,S.S.Kim,KL,Taki,F.Yagi 2015,Yonekura2015

- 6d: M5 brane exploring D_{k+4} singularity
- circle compactification: a single D4 brane exploring D_{k+4} singularity = D_{k+4} type quiver theory

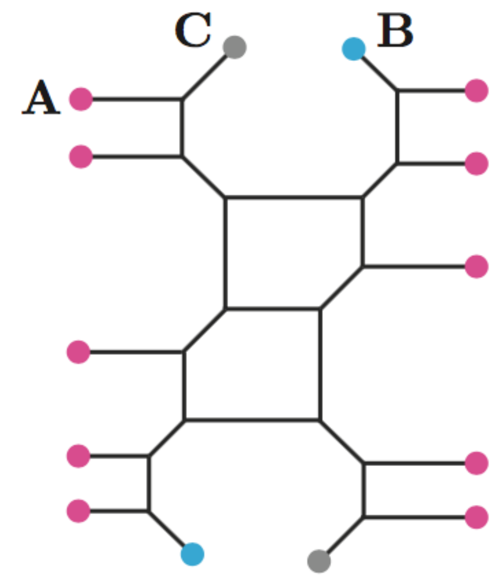


- [p,q] 5 brane web



5d reduction

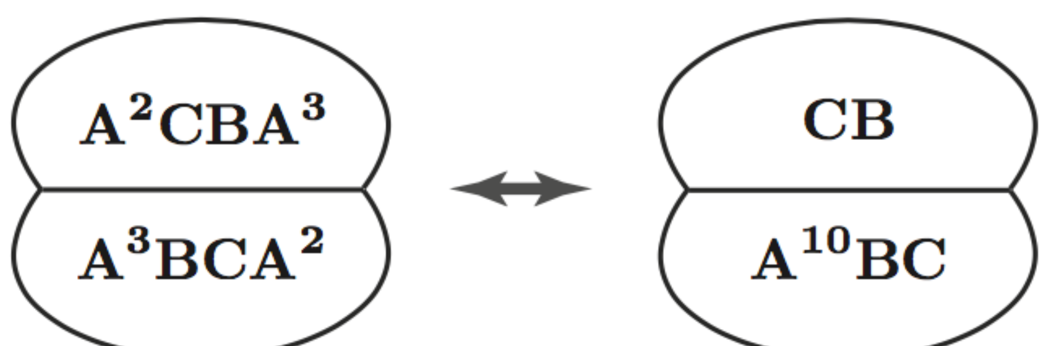
- S-dual of the [p,q] brane web



- 5d SU(3) gauge theory with 10 fundamental hypermultiplets and zero CS level: global symmetry $U(10)\mathrm{x}U(1)_I$

- Enhanced global symmetry: $SO(20)\mathrm{x}U(1)_{KK}$

- 7-brane argument



$D^{10}=A^{10}BC$

5d SU(3)$_\kappa$ + N$_f$ fund hyper

- removing hyper induce the Chern-Simons level
- rank of global symmetry= N$_f$ + 1

N_f	$G_{\lvert\kappa\rvert}$ (κ is the Chern-Simons level)				
10	$SO(20)_0$				
9	$SO(20)_{\frac{1}{2}}$				
8	$SU(10)_0$,	$\left[SO(16)\times SU(2)\right]_1$			
7	$\left[SU(8)\times SU(2)\right]_{\frac{1}{2}}$,	$SO(14)_{\frac{3}{2}}$			
6	$\left[SU(6)\times SU(2)\times SU(2)\right]_0$,	$SU(7)_1$,	$SO(12)_2$		
5	$\left[SU(5)\times SU(2)\right]_{\frac{1}{2}}$,	$SU(6)_{\frac{3}{2}}$,	$SO(10)_{\frac{5}{2}}$		
4	$SU(4)_0$,	$\left[SU(4)\times SU(2)\right]_1$,	$SU(5)_2$,	$SO(8)_3$	
3	$SU(3)_{\frac{1}{2}}$,	$\left[SU(3)\times SU(2)\right]_{\frac{3}{2}}$,	$SU(4)_{\frac{5}{2}}$,	$SO(6)_{\frac{7}{2}}$	
2	$SU(2)_0$,	$SU(2)_1$,	$\left[SU(2)\times SU(2)\right]_2$,	$SU(3)_3$,	$SO(4)_4$
1	$SU(2)_{\frac{5}{2}}$,	$SU(2)_{\frac{7}{2}}$			
0	$SU(2)_3$				

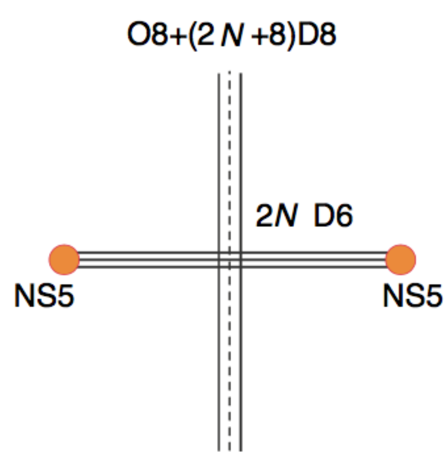
5d SU(N+1)$_\kappa$ + N$_f$ fund hyper

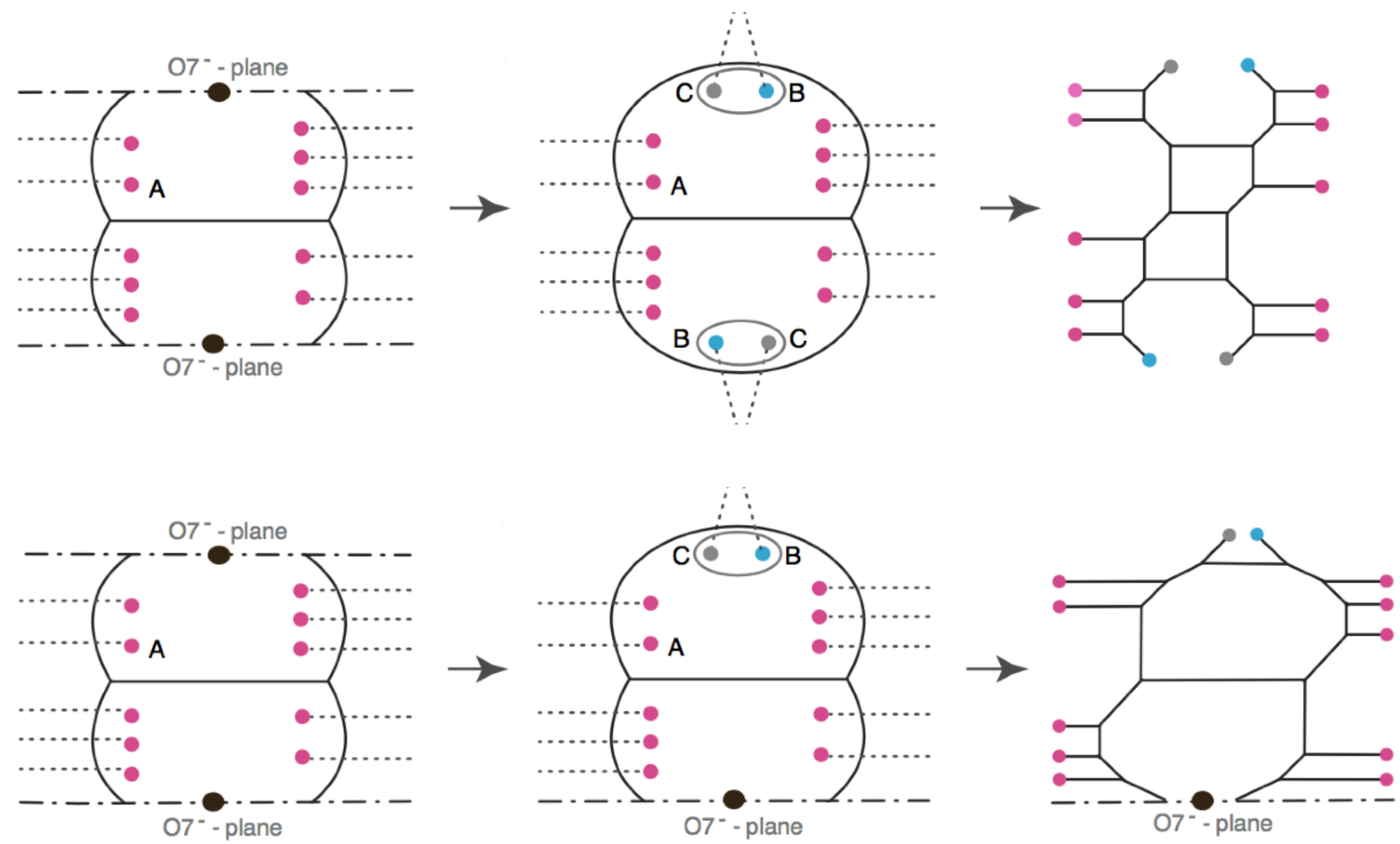
- removing hyper induce the Chern-Simons level
- rank of global symmetry= $N_f + 1$

N_f	$G_{\lvert\kappa\rvert}$
$2n+4$	$SO(4n+8)_0$
$2n+3$	$SO(4n+8)_{\frac{1}{2}}$
$2n+2$	$SU(2n+4)_0$, $\left[SO(4n+4)\times SU(2)\right]_1$
$2n+1$	$\left[SU(2n+4)\times SU(2)\right]_{\frac{1}{2}}$, $SO(4n+2)_{\frac{3}{2}}$
$2n$	$\left[SU(2n)\times SU(2)\times SU(2)\right]_0$, $SU(2n+1)_1$, $SO(4n)_2$

6d Sp(N)+ tensor + (2N+ 8) fund hyper

- 5d SU(N+2) + (2N+8) fundamental hyper
- 5d Sp(N+1) + (2N+8) fundamental hyper





5d duality between $SU(N)_\kappa$ & Sp(N-1)

Gaiotto,H.C.Kim 15

- Sp(N-1) theory: $SO(2N_f)\times U(1)$

- Enhanced global symmetry

N_f	$SU(N)_{\pm(N+1-N_f/2)}$	N_f	$SU(N)_{\pm(N+2-N_f/2)}$
$\leq 2N$	$SU(N_f+1)\times U(1)$	$\leq 2N+1$	$SO(2N_f)\times U(1)$
$2N+1$	$SU(N_f+1)\times SU(2)$	$2N+2$	$SO(2N_f)\times SU(2)$
$2N+2$	$SU(N_f+2)$	$2N+3$	$SO(2N_f+2)$

- SU(N) with $|\kappa|= N+2-N_f/2$ and N_f fund hyper = Sp(N-1) with k=0, N_f fund hyper

index function of dyonic instantons

- N=1 SUSY with Q invariant under j_R+R & j_L
  - 1/2 BPS dyonic instanton E= |k+q|, kq>0
- the Witten index

$$Z = \mathrm{Tr}(-1)^F e^{-\beta\{Q,Q\}} x^{2(j_R+R)} y^{2j_L} e^{-i\sum_i H_i m_i}$$

$$= Z_{\mathrm{pert}}(1+\sum_{k=1}^{\infty} q^k Z_k)$$

- instanton dynamics of Sp(N) theory = O(k) gauge group
  - $O(k)=O(k)_+ + O(k)_-$: determinant +1 or -1.

gauge holonomy of $O(k)_\pm$

- O(k)+ : $$e^{i\phi_+} = \begin{cases} \mathrm{diag}(e^{i\sigma_2\phi_1}, \cdots, e^{i\sigma_2\phi_n}) & \text{for even } k\ , \\ \mathrm{diag}(e^{i\sigma_2\phi_1}, \cdots, e^{i\sigma_2\phi_n}, 1) & \text{for odd } k, \end{cases}$$
- O(k)- : $$e^{i\phi_-} = \begin{cases} \mathrm{diag}(e^{i\sigma_2\phi_1}, \cdots, e^{i\sigma_2\phi_{n-1}}, \sigma_3) & \text{for even } k\ , \\ \mathrm{diag}(e^{i\sigma_2\phi_1}, \cdots, e^{i\sigma_2\phi_n}, -1) & \text{for odd } k, \end{cases}$$
    - $k=2n+\chi$, $\chi=0,1$
- instanton dynamics of Sp(1) theory = O(k) gauge group (ADHM data)
    - N=4 susy quantum gauge dynamics with O(k) adjoint
    - O(k)xSp(N) bifundamental,
    - U(N_f)x O(k) vector fermi multiplets
    - $O(k)=O(k)_+ + O(k)_-$: determinant +1 or -1.

gauge holonomy of O(k)$_\pm$

Kim,Kim,Lee1206.6781

- Part of k instanton partition function

$$x = e^{-\gamma_1}, y = e^{-\gamma_2}$$

$$I_+^k = (2i)^{k(N_f-2N-2)-n} i^{n+2\chi} \oint [d\phi] \left[\frac{\prod_{l=1}^{N_f} \sin\frac{m_l}{2}}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \prod_{i=1}^N \sin\frac{i\gamma_1\pm\alpha_i}{2}} \prod_{I=1}^n \frac{\sin(\frac{\phi_I\pm 2i\gamma_1}{2})}{\sin\frac{\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \right]^\chi$$
$$\times \prod_{I=1}^n \left[\frac{\sinh\gamma_1}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^N \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^n \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right],$$

$$I_-^{k:odd} = \frac{(2i)^{k(N_f-2N-2)-n}}{i^{N_f-2N-n-2}} \oint [d\phi] \left[\frac{\prod_{l=1}^{N_f} \cos\frac{m_l}{2}}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \prod_{i=1}^N \cos\frac{i\gamma_1\pm\alpha_i}{2}} \prod_{I=1}^n \frac{\cos(\frac{\phi_I\pm 2i\gamma_1}{2})}{\cos\frac{\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \right]$$
$$\times \prod_{I=1}^n \left[\frac{\sinh\gamma_1}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^N \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^n \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right]$$

$$I_-^{k:even} = (2i)^{(k-1)(N_f-2N)-\frac{5}{2}k} i^{n+4} \oint [d\phi] \left[\frac{\cosh\gamma_1}{\cosh\frac{\gamma_1\pm\gamma_2}{2} \sinh^2\frac{\gamma_1\pm\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin m_l}{\prod_{i=1}^N \sin(i\gamma_1\pm\alpha_i)} \right] \qquad (3.62)$$
$$\times \prod_{I=1}^{n-1} \left[\frac{\sinh\gamma_1 \sin(\phi_I\pm 2i\gamma_1)}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2} \sin(\phi_I\pm i\gamma_1\pm i\gamma_2)} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^N \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^{n-1} \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right].$$

Nekrasov partition function =Gopakumar-Vafa Invariant

small charge expansion $e^{-\alpha}$

- **refined topological vertex: Iqbal,Kozcaz,Vafa [0701156]**
- **BPS degeneracy and Superconformal index: Iqbal,Vafa [1210.3605]**
- $\tilde{A} = q^{\frac{2}{8-N_f}} e^{-\alpha}$
- M-theory on non-compact CY3
- Z= exp(F)=PE(G)= $\exp\left[\sum_{n=1}^{\infty} \frac{1}{n} G_X(x^n, y^n, \tilde{A}^n)\right]$

$$G_X(x, y, \tilde{A}) = \sum_{C \in H_2(X,\mathbb{Z})} \sum_{j_L, j_R} N_C^{j_L, j_R} f(j_L, j_R) \tilde{A}^{n_C}$$

$$f(j_L, j_R; x, y) = \frac{(-1)^{2j_L+2j_R}(y^{-2j_L} + \cdots + y^{2j_L})(x^{-2j_R} + \cdots + x^{2j_R})}{((xy)^{1/2} - (xy)^{-1/2})((x/y)^{1/2} - (x/y)^{-1/2})}$$

more hypers and vectors

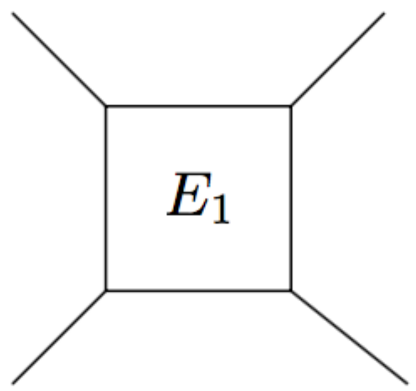
- f(0,0)Ã : perturbative hyper + instanton $f(0,0) = \frac{x}{(1-xy)(1-x/y)}$
- f(0,1/2)Ã² : perturbative vector + instantons $f(0,1/2) = \frac{1+x^2}{(1-xy)(1-x/y)}$

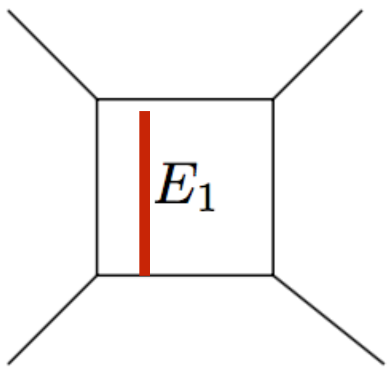
$$F_{E_6} = f(0,0)\chi_{\mu_6}\tilde{A} + f(0,\frac{1}{2})\chi_{\mu_1}\tilde{A}^2 + \Big[f(0,0) + f(0,1)\chi_{\mu_2} + f(\frac{1}{2},\frac{3}{2})\Big]\tilde{A}^3 + \Big[f(0,\frac{1}{2})\chi_{\mu_6} + f(0,\frac{3}{2})\chi_{\mu_3} + f(\frac{1}{2},2)\chi_{\mu_6}\Big]\tilde{A}^4 + \mathcal{O}(\tilde{A}^5).$$

$$\chi_{\mu_1} = 27, \quad \chi_{\mu_2} = 78, \quad \chi_{\mu_3} = \overline{351}, \quad \chi_{\mu_4} = 2925, \quad \chi_{\mu_5} = 351, \quad \chi_{\mu_6} = \overline{27},$$

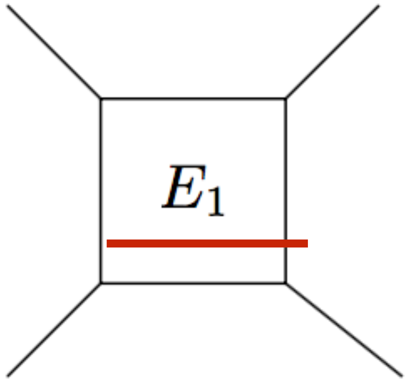
- f(0,0)Ã : 10+ 17 (8+1+ 8)
- f(0,1/2)Ã² : 1+ 26 (8+10+8)

N_f=0





W-boson=F1



instanton=D1

0 massive hypermultiplets
2 massive vector multiplets
1 massless vector

Conclusion

- There are a lot more to be learned about 5 and 6d SCFTs and LST
- Classification of 5d SCFTs are not done yet.
- There might exist some gems to be discovered.
- interesting implications in lower dimensional quantum field theories
- little string theories are less explored and need more understanding.

Spenta Wadia

Wish you many many happy returns of the day!