

Spenta Fest



ICTS 13th January 2017

TOY MODEL $D=1$?

March, 1980

EFI 80/15

80-6-4



$N=\infty$ PHASE TRANSITION IN A CLASS OF EXACTLY SOLUBLE
MODEL LATTICE GAUGE THEORIES*

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$D=4$ HAWKING PAGE TRANSITION?

On Singularities and Holography For Spenta: A Toy Model in the big picture

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Works with Jose F Barbon - Madrid



J'ai souhaité être parmi
et avec ceux d'aujourd'hui
à offrir à Carrier un Hommage
qui resterait chez lui
Marc Chagall

- **What do singularities reflect?**
- **Usually that something was missed out(QM, massless degrees of freedom).**
- **GR has horizons. What are they hiding? Are singularities different.**
- **Time like singularities- String theory.**
- **Space like singularities**

Singularities:
Can one live with them?
Can they heal?
What can one learn about
them?

Are they simple or complicated?
Some are “simple”

What are you trying to say:

Two very different looking boundary theories

Same BULK SINGULARITY

Complementary- Dual

d=1 Toy Model

Introducing the bare “TOY” model

$$H = \frac{P^2}{2m} + \frac{\lambda}{2} x^n$$

$$H = p^2 / 2m + \lambda / 2 x^n$$

$$H = 1/2 \, \varepsilon(\lambda, m) (p^2 + x^n)$$

$$H = -\frac{1}{2m} \frac{d^2}{dr^2} + \frac{N^2}{8mr^2} - \frac{e^2}{r} \quad (6)$$

If now we rescale the radial coordinate, defining $r = N^2 R$, then in terms of R , H becomes

$$H = \frac{1}{N^2} \left(-\frac{1}{2mN^2} \frac{d^2}{dR^2} + \frac{1}{8R^2 m} - \frac{e^2}{R} \right) \quad (7)$$

Apart from the overall factor of $1/N^2$, which only determines the overall scale of energy or time, the only N in this Hamiltonian is the N^2 that appears with the mass in the kinetic energy term. The Hamiltonian (7) describes a particle with an effective mass $M_{\text{eff}} = mN^2$, moving in an effective potential

$$V_{\text{eff}} = \frac{1}{8R^2 m} - \frac{e^2}{R} \quad (8)$$

$$H = P^2 / 2m + \lambda / 2 x^n$$

$$H = 1/2 \, \varepsilon(\lambda, m) (p^2 + x^n)$$

n is NOT -2

$$H = \frac{P^2}{2m} + \frac{\lambda}{2} x^n$$

$$H = \frac{1}{2} (p^2 + \lambda x^{-2})$$

Conformal Quantum Mechanics

CONTEXT OF SPACE LIKE SINGULARITIES

Magic of String Theory

No concept* in Math remains
unambiguous

Magic of String Theory

Metric

With extended objects

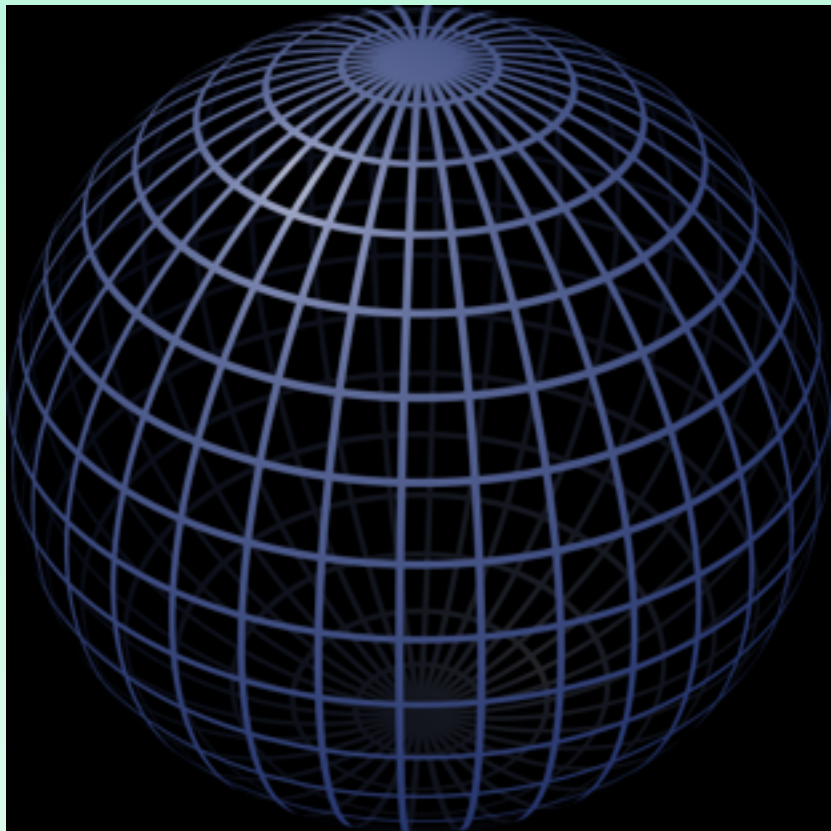
Large=Small

$R' = 1/R$ T Duality

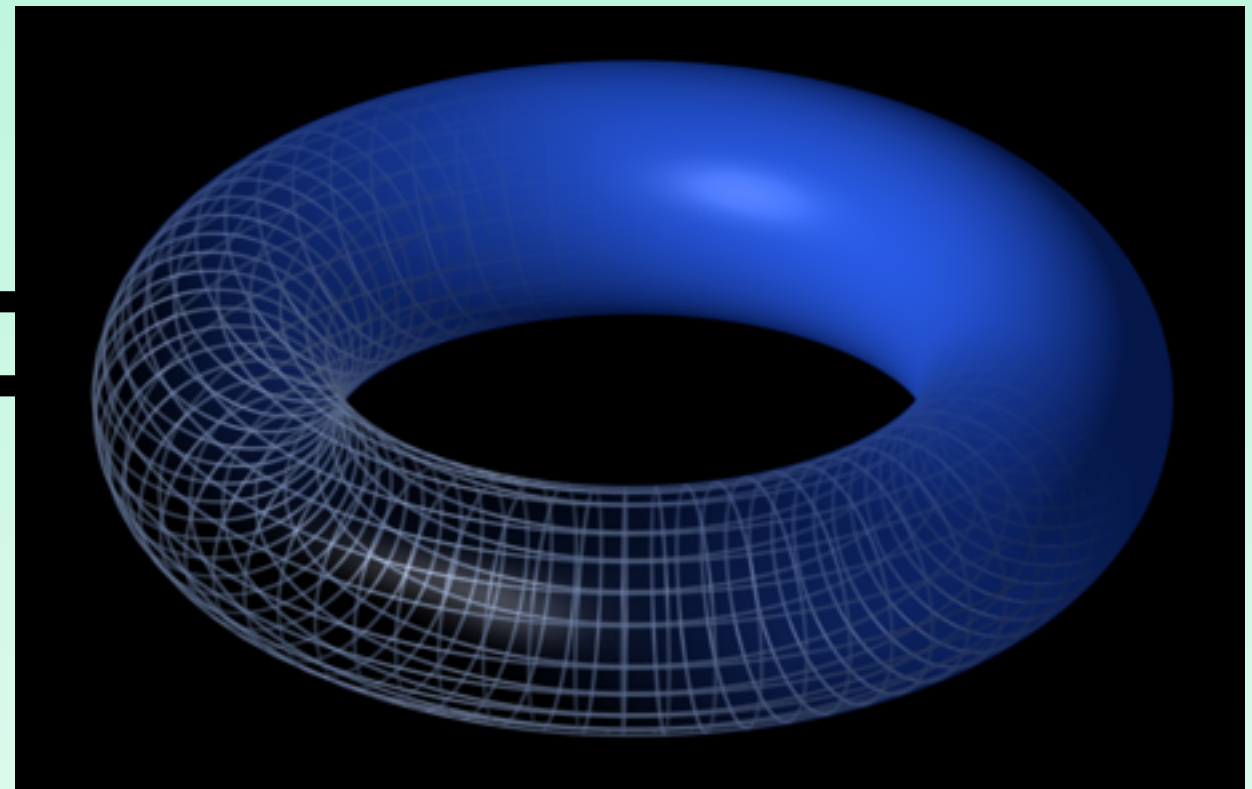
Magic of String Theory

Topology

With extended objects
Surface of a Sphere=Torus



=



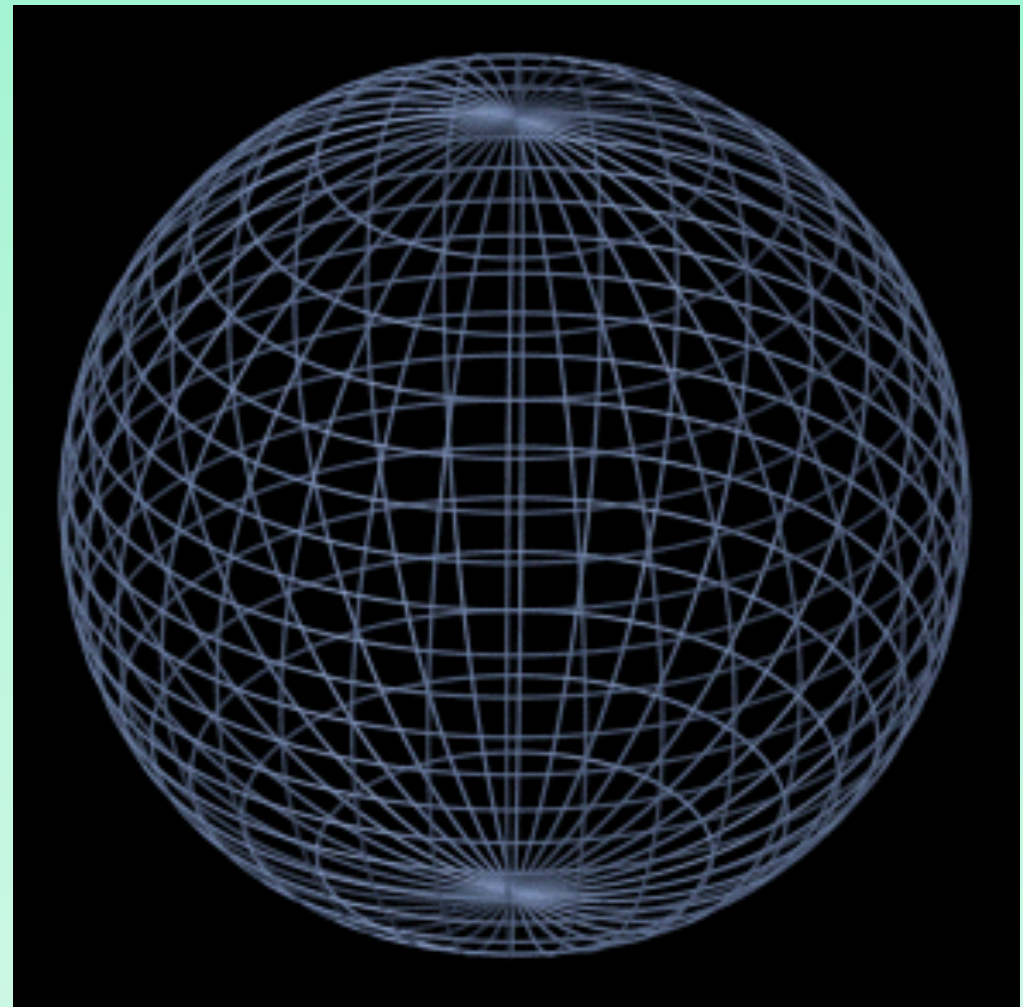
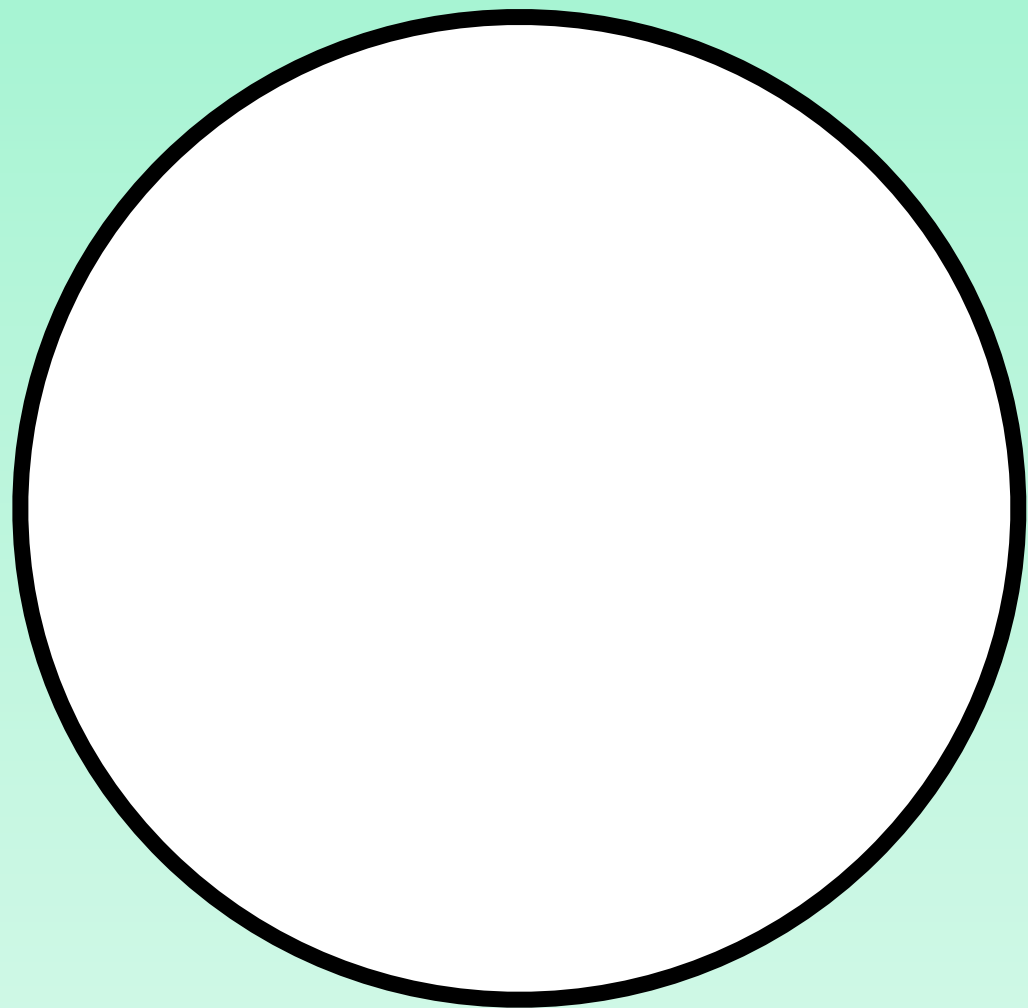
Magic of String Theory

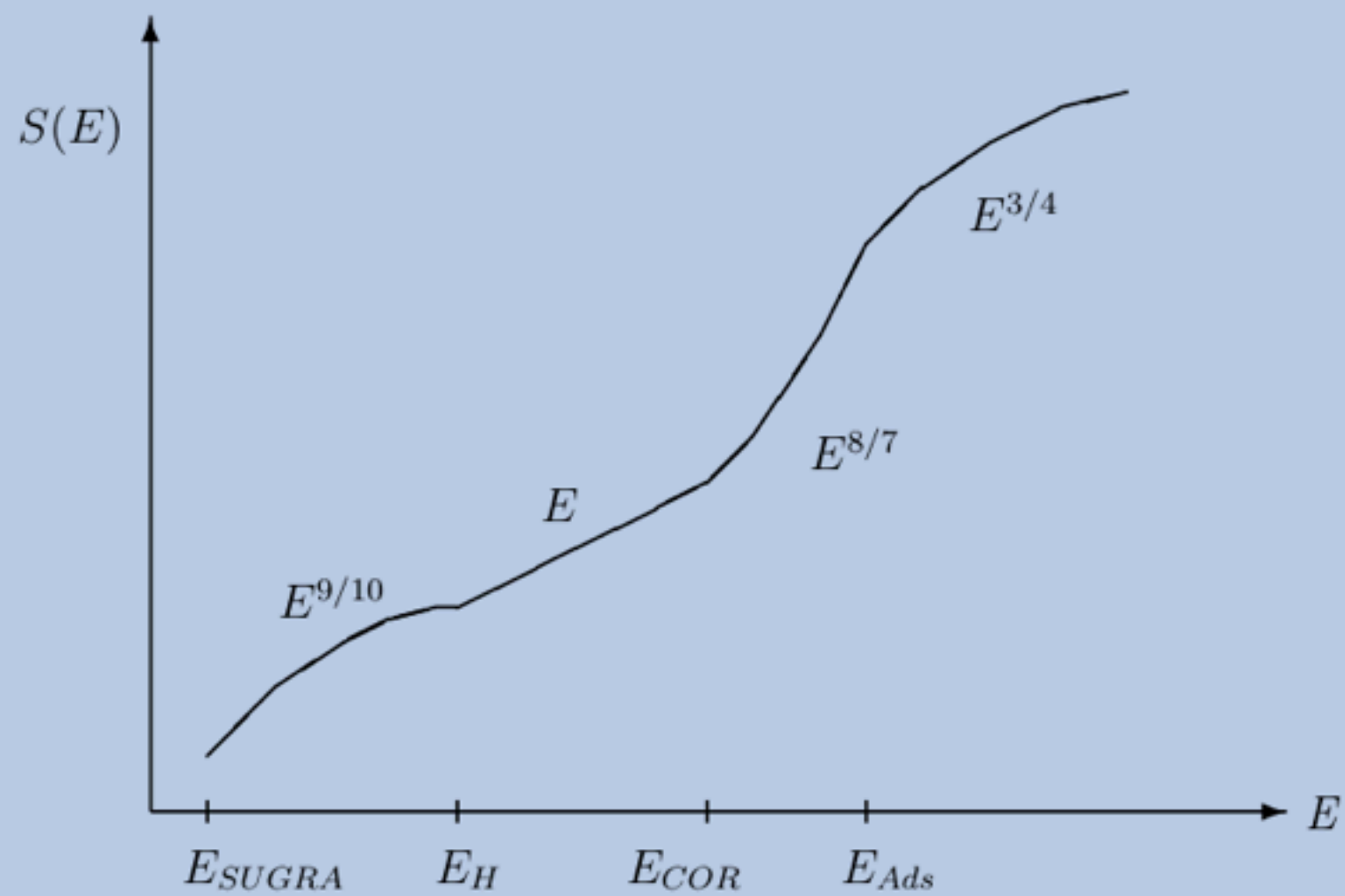
Dimension

With extended objects

$$1=3, 4=10$$

$SU(2)$ $\dim=3$, $\text{rank}=1$



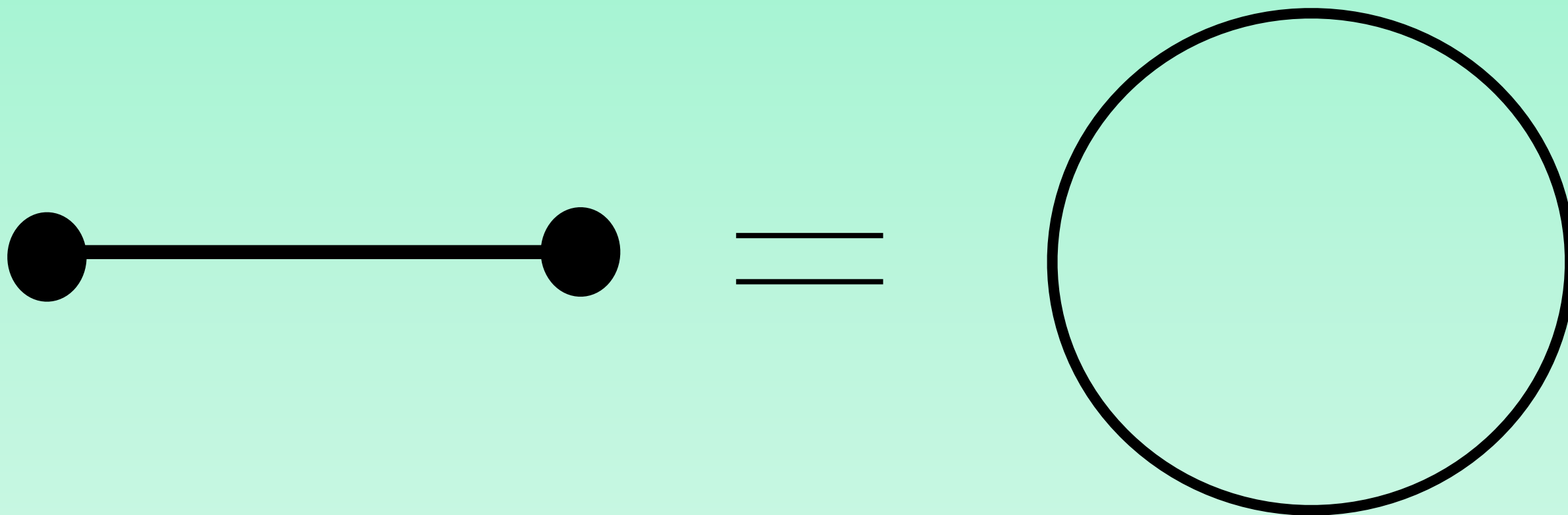


Magic of String Theory

Singularities

With extended objects

Time Like Singularity



Magic of String Theory

COMMUTATIVITY ?

With extended objects

$$[x,y]=0 \quad =$$



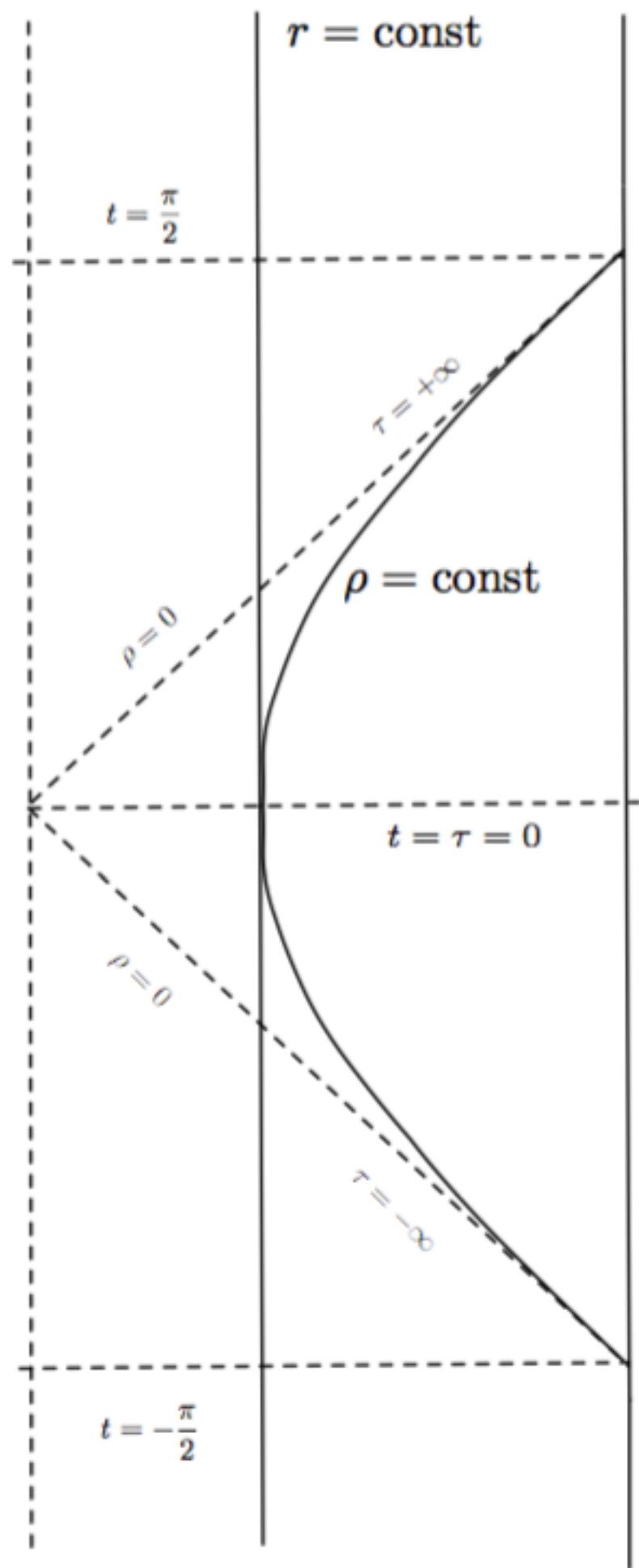
Dualities

- Geometry
- Topology
- Number of dimensions, small and large
- (non-)Commutativity
- Singularity structure
- Associativity
- NO UNIFIED FRAMEWORK

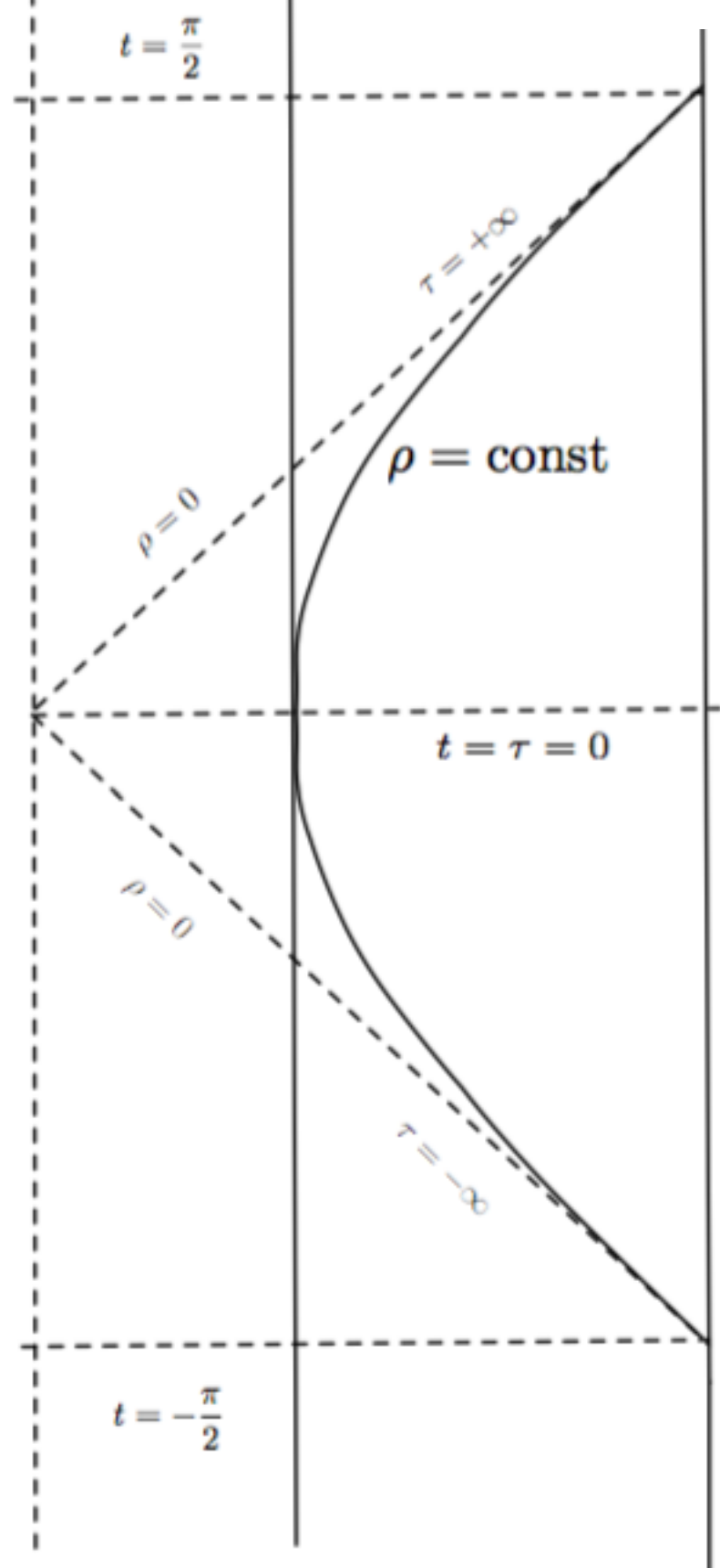


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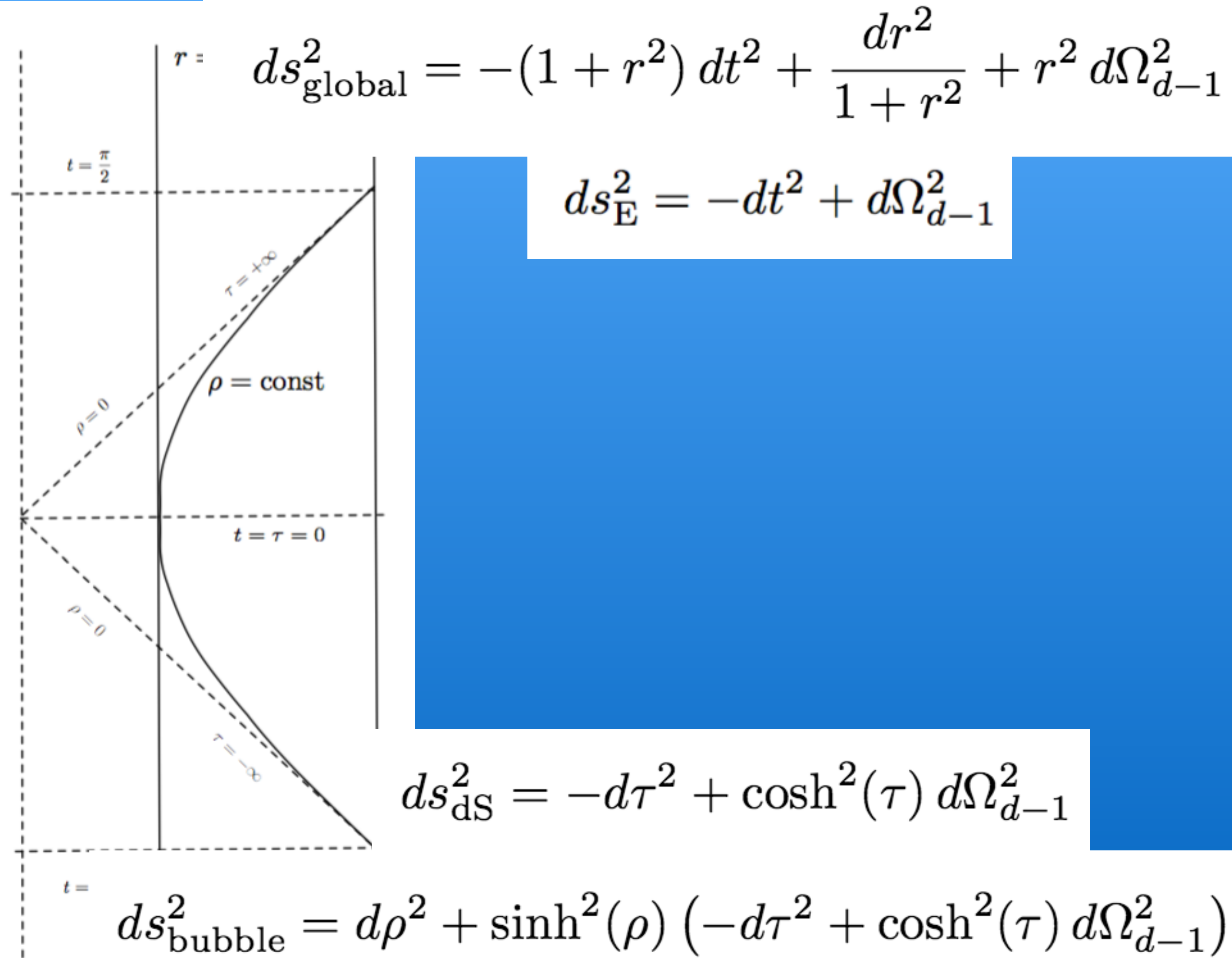


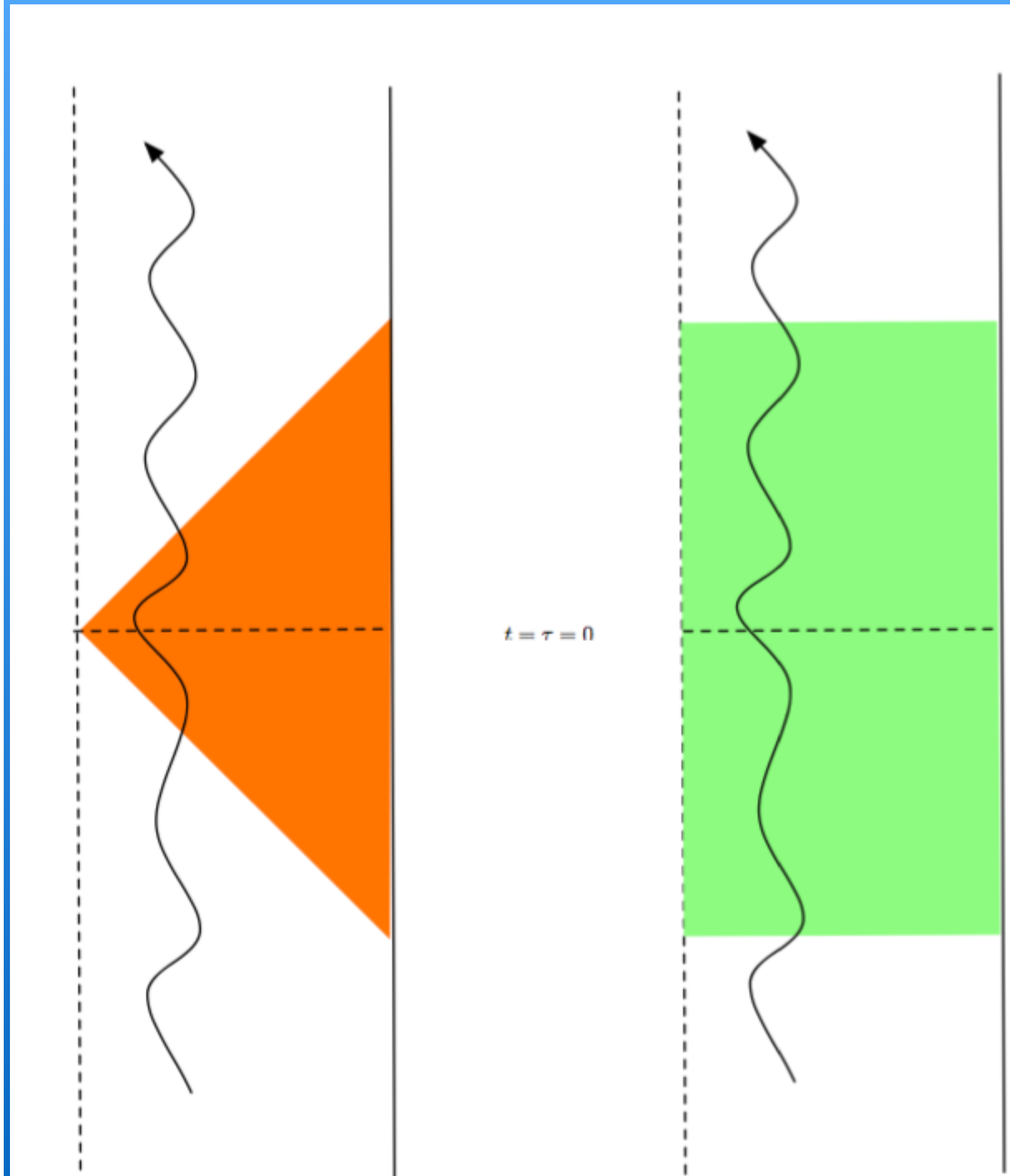
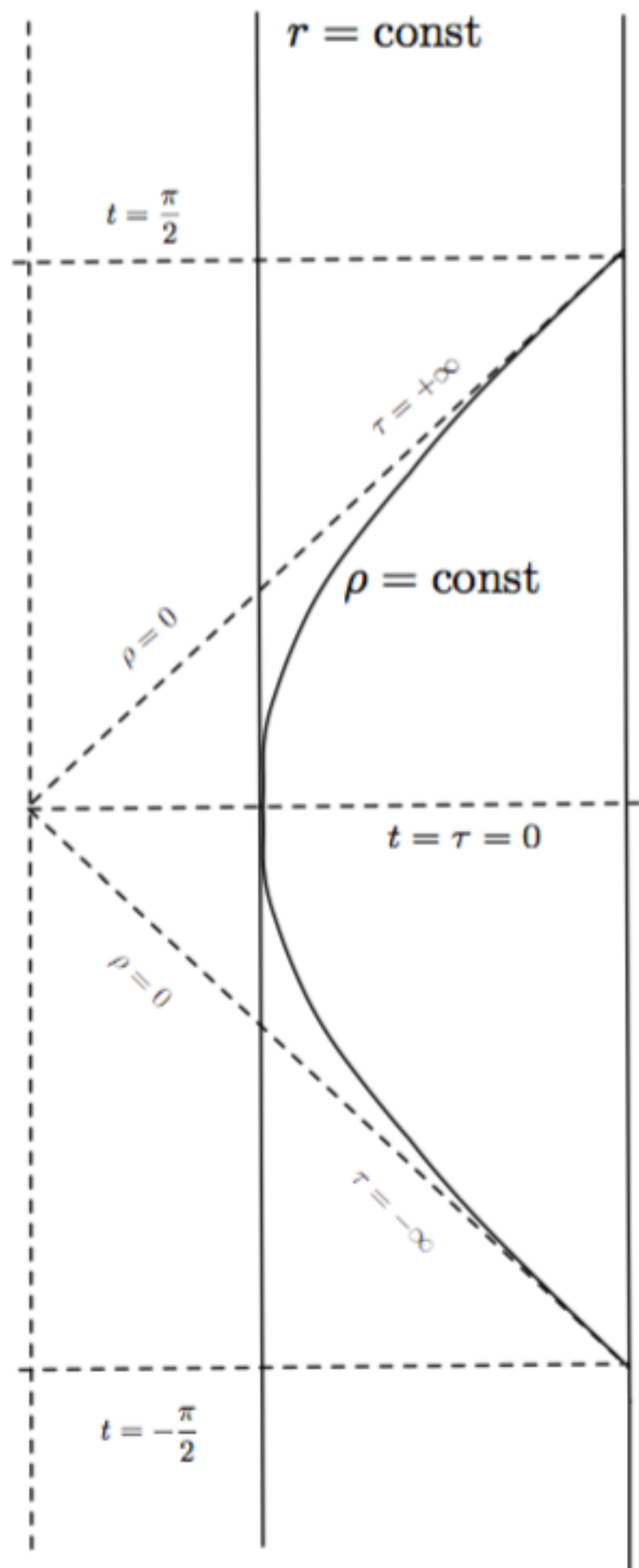


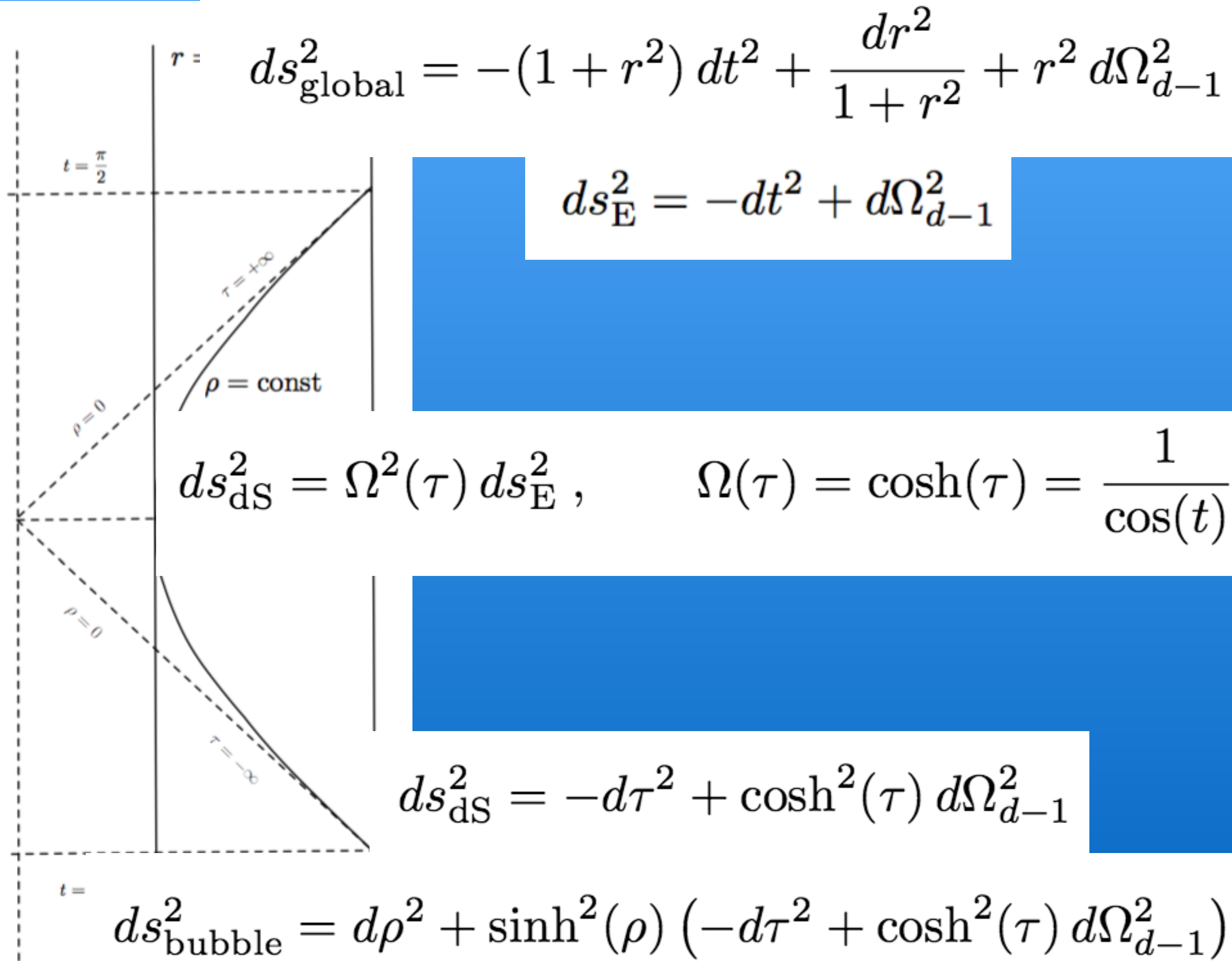
$$r = \quad ds_{\text{global}}^2 = -(1 + r^2) dt^2 + \frac{dr^2}{1 + r^2} + r^2 d\Omega_{d-1}^2$$

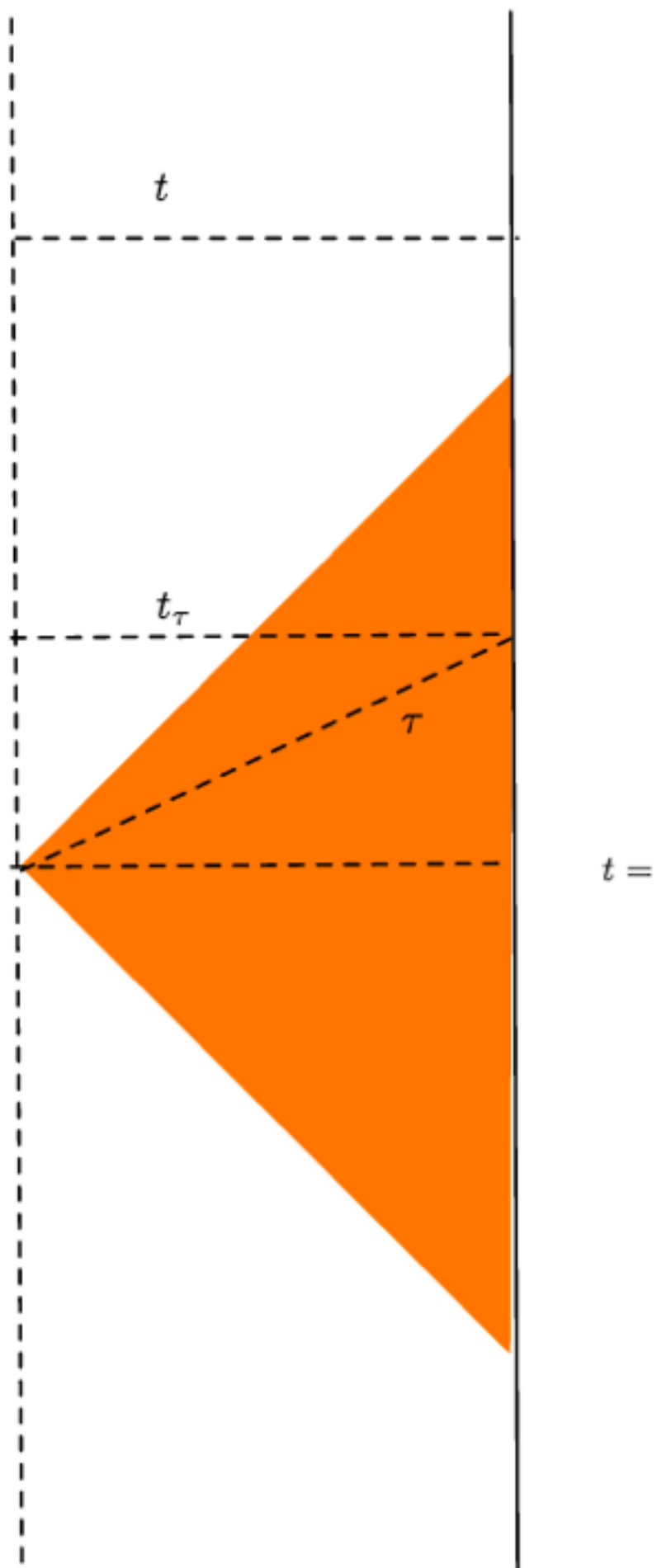


$$ds_{\text{E}}^2 = -dt^2 + d\Omega_{d-1}^2$$









**One Can in some cases
continue**

**beyond the coordinate
singularity**

**One Can in some cases
continue**

**beyond the coordinate
singularity**

How about the following case?

$$ds_{\text{FRW}}^2 = -dt^2 + G(t)ds_{\mathbf{H}^d}^2$$

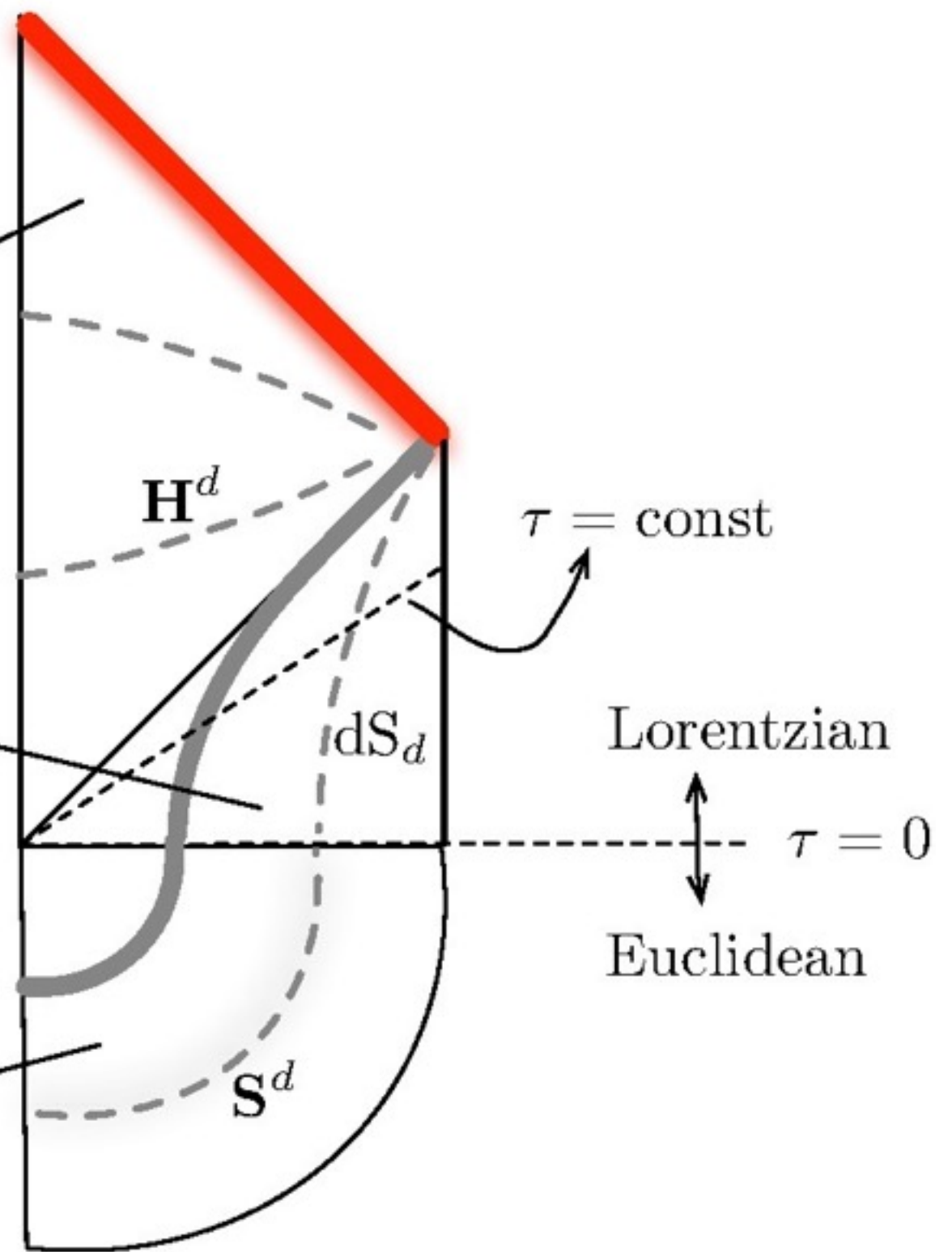
FRW
patch

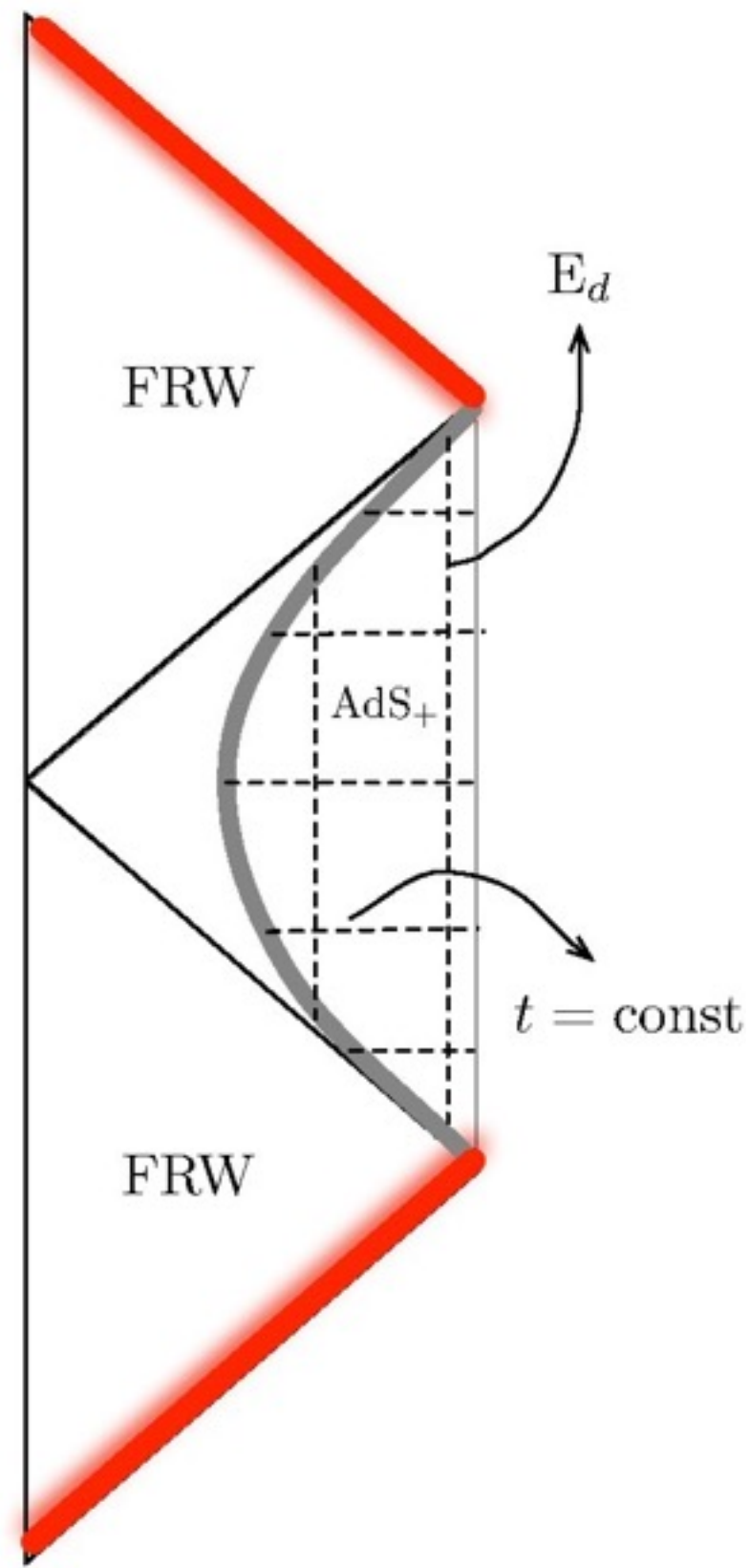
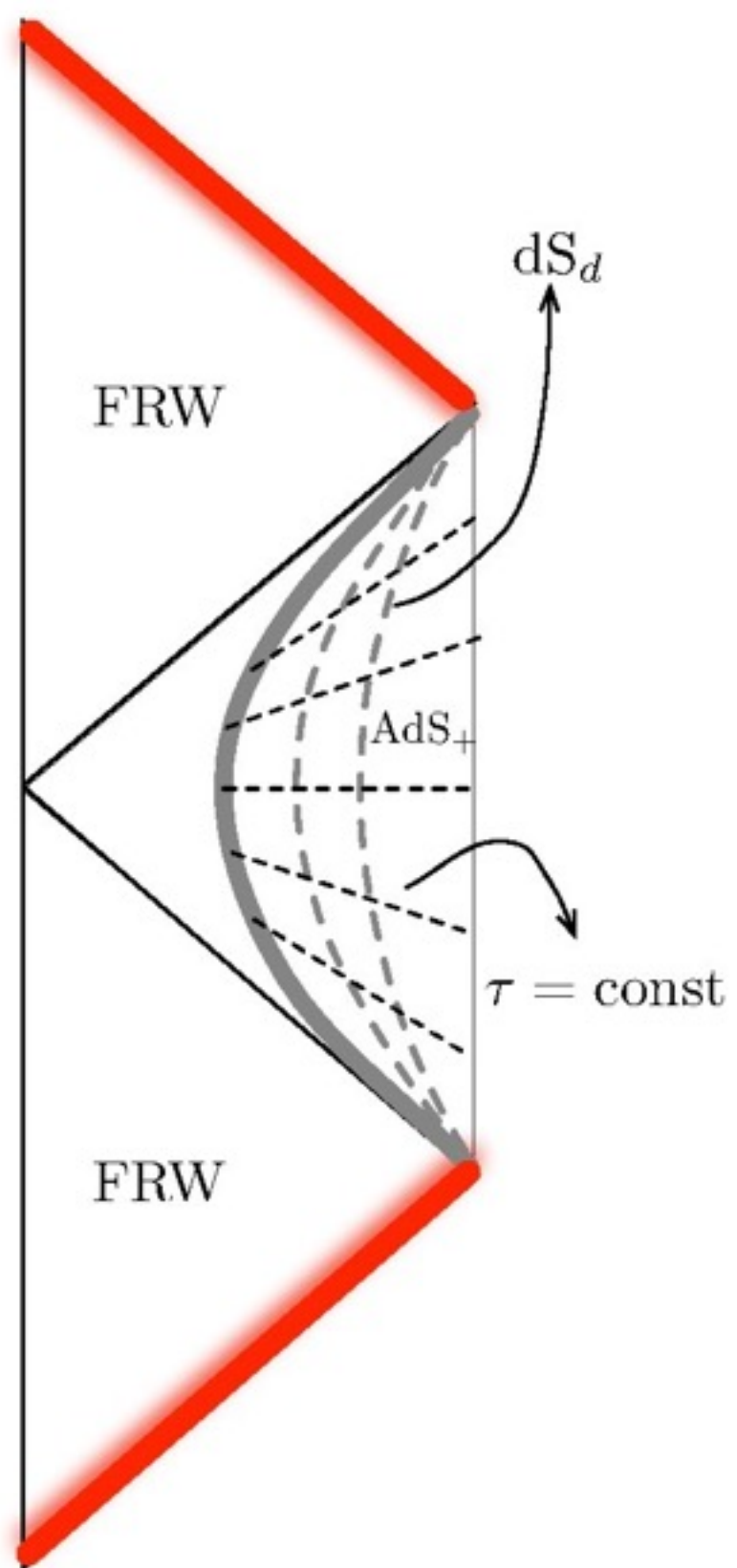
$$ds_{\text{Bubble}}^2 = d\rho^2 + F(\rho)ds_{dS_d}^2$$

bubble
patch

$$ds_{\text{Ball}}^2 = d\rho^2 + F(\rho)ds_{\mathbf{S}^d}^2$$

ball
patch





- **The Singularity Reaches the Boundary/UV in**
- **a Finite Time and thus can be**
- **Described by Local Boundary Operators.**

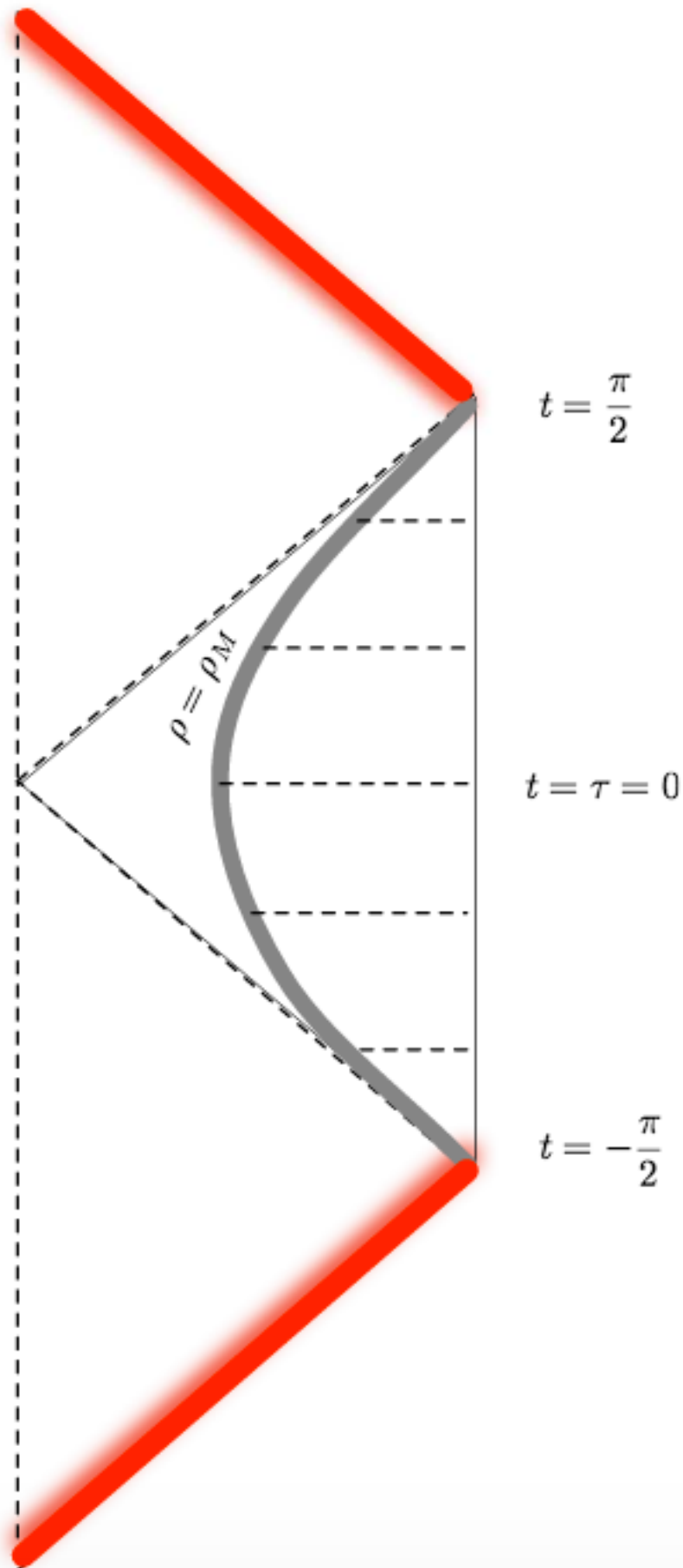
Engineering Big Crunch Singularities

Add

Marginal or Relevant

operators on the boundary

AN EFFECTIVE ACTION OF A BRANE INDUCES A LG BOUNDARY ACTION



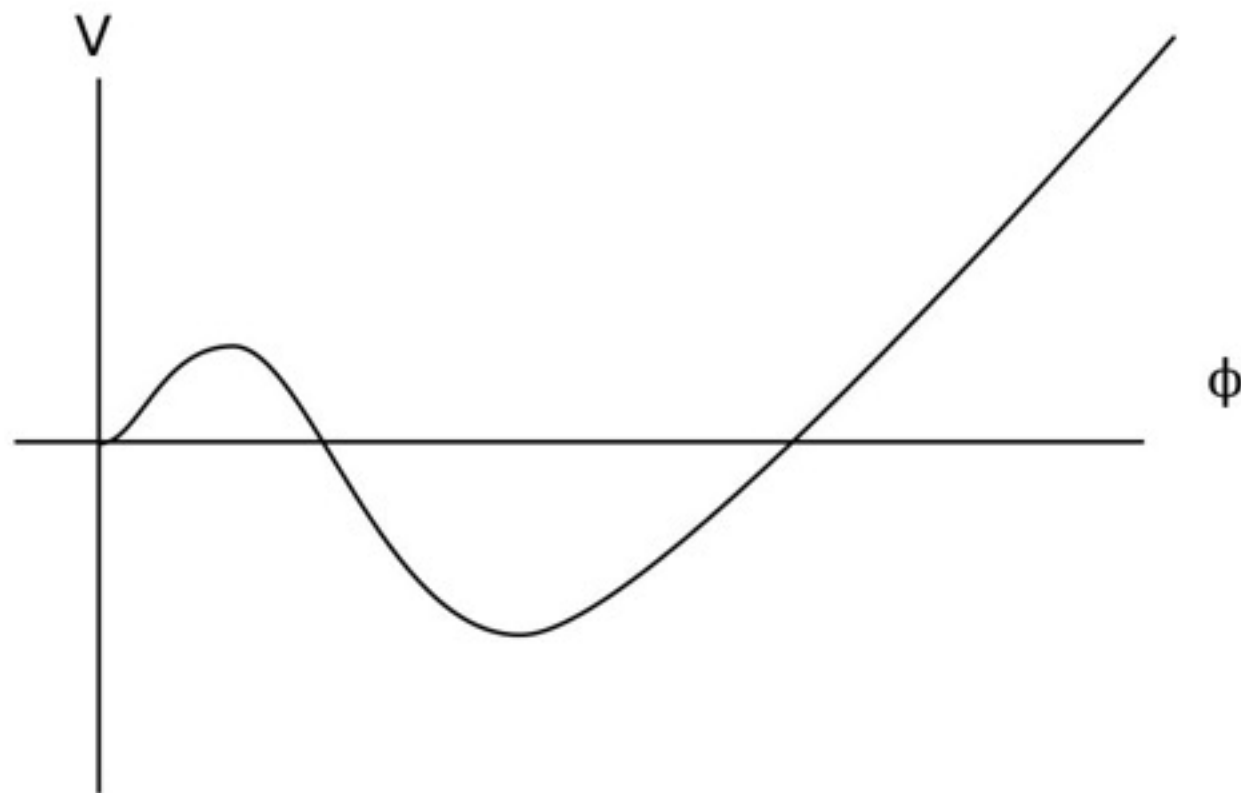
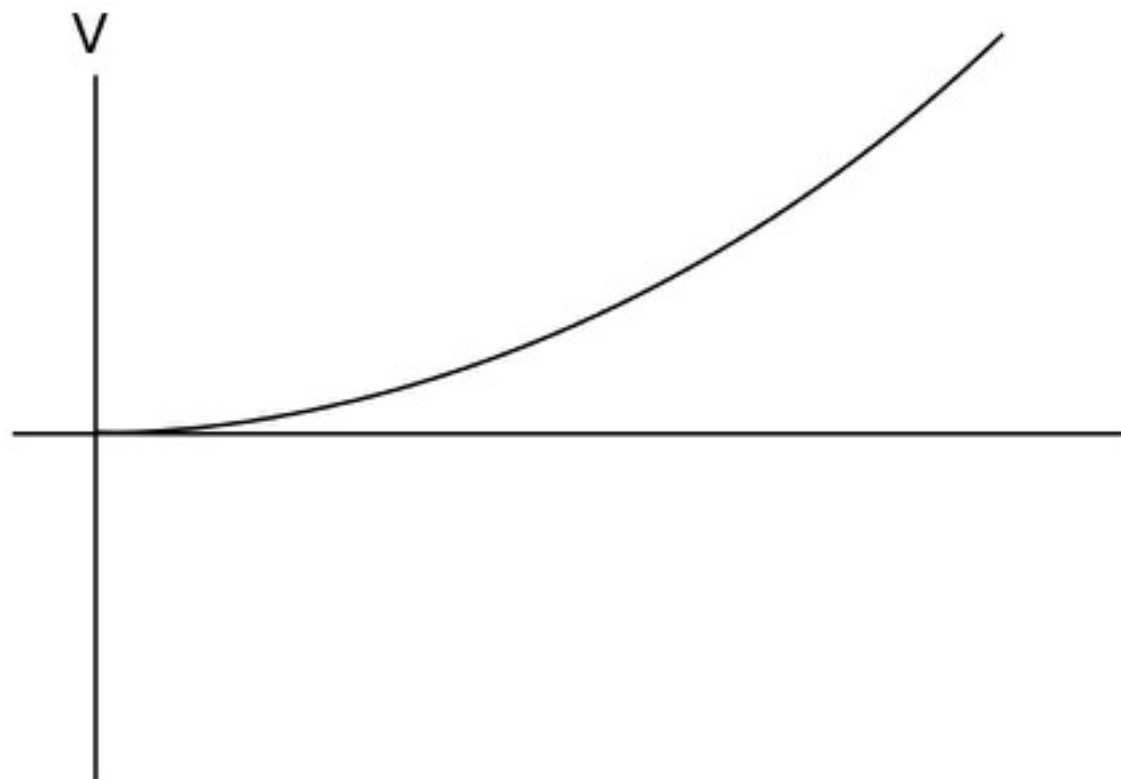
$$\rho_M \sim \log \langle \phi \rangle_M \sim \log(M)$$

$$\langle \phi \rangle_M \sim M^{\frac{d-2}{2}}$$

Non perturbative definition of the theory.

There are several possible QFT duals on the boundary

BOUNDARY



World Volume dS!



- If the boundary theory is well defined so is the crunch in the bulk.
- For the bulk crunch example above the boundary theory is well defined. Possible to describe a crunch.
- It is well defined on a world volume which is dS but there is no gravitational coupling.
- To see the crunch change coordinates on the boundary.

A simple classical model: $O(N)$ on de Sitter

$$S_{\text{dS}}[\vec{\phi}] = - \int_{\text{dS}_4} \left(\frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{\vec{\phi}^2}{R^2} + \lambda \left(\vec{\phi}^2 \right)^2 + \varepsilon M^2 \vec{\phi}^2 \right)$$

$MR \gg 1 \longrightarrow$ Phases are clear-cut

$$\varepsilon > 0 \quad \text{UV}_{O(N)} \xrightarrow{\text{RG flow}} \text{IR}_{\text{gap}}$$

$$\varepsilon < 0 \quad \text{UV}_{O(N)} \xrightarrow{\text{RG flow}} \text{IR}_{O(N-1)}$$

$$ds_{\text{E}}^2 = -dt^2 + d\Omega_{d-1}^2,$$

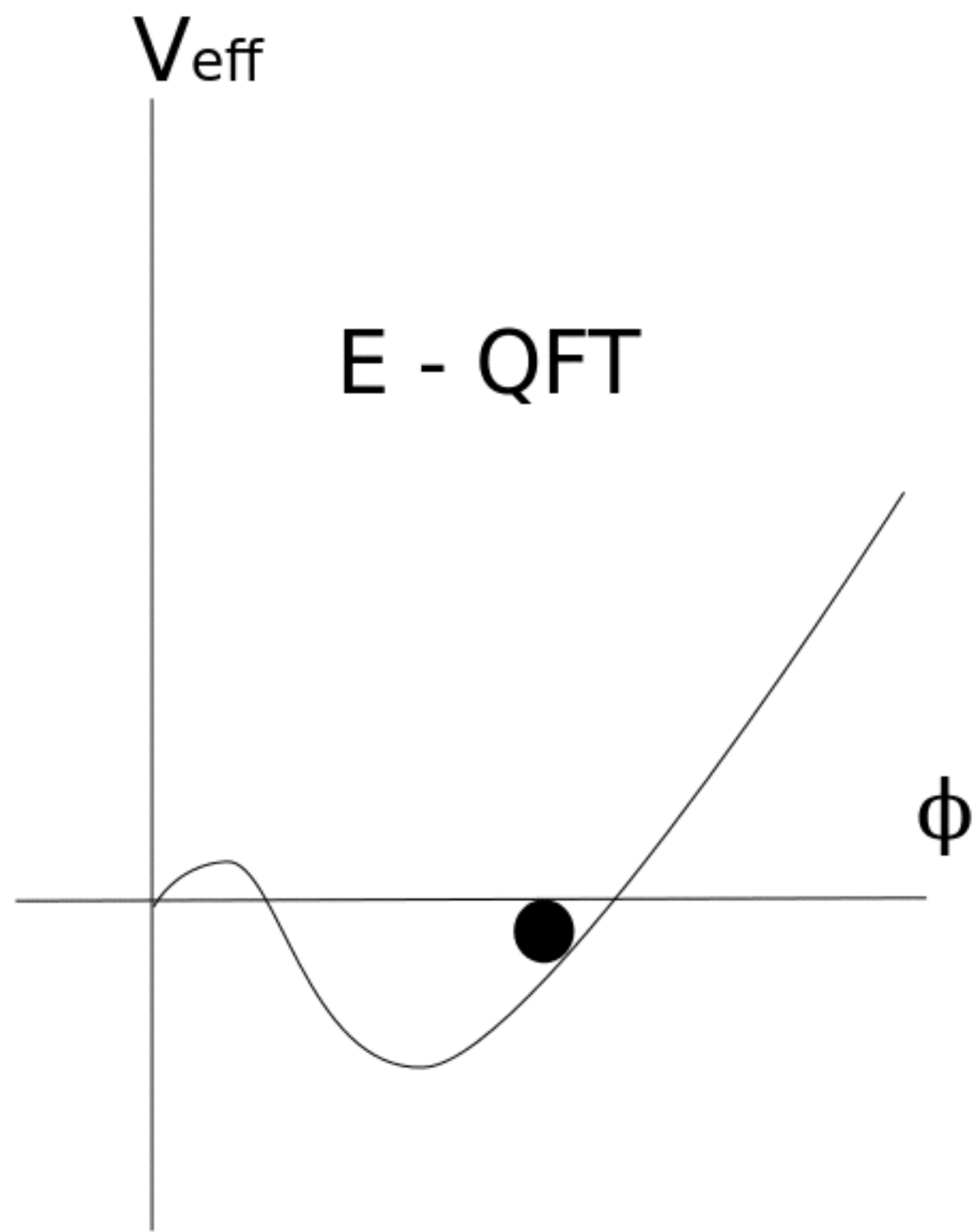
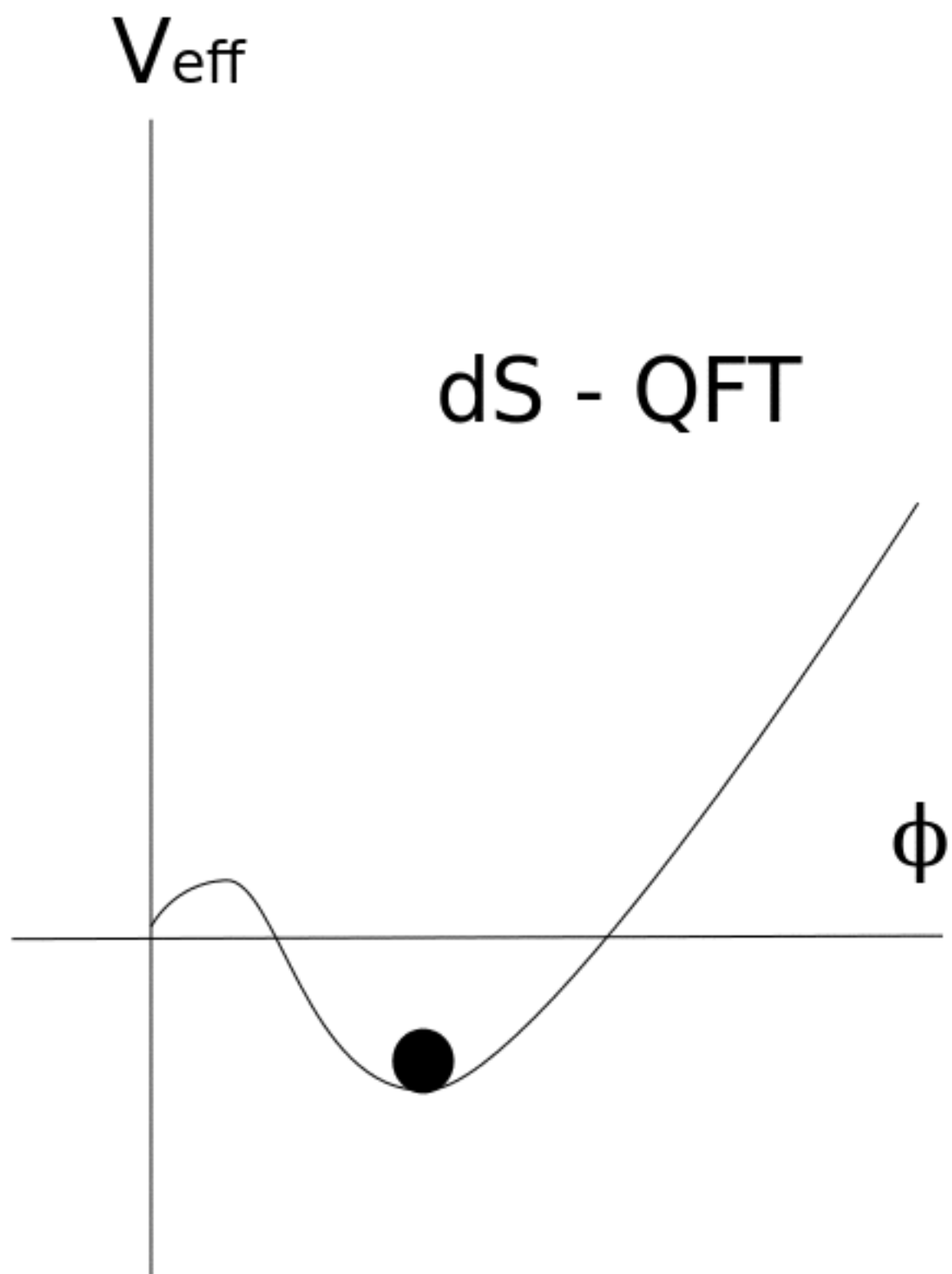
$$ds_{\text{dS}}^2 = \Omega^2(t) \, ds_{\text{E}}^2 \, , \qquad \Omega(t) = \cosh(\tau) = \frac{1}{\cos(t)} \, ,$$

$$t = \int \Omega^{-1}(\tau) d\tau = 2 \tan^{-1} [\tanh(\tau/2)].$$

Over at the E-frame...

$$S_E[\vec{\phi}] = - \int_{E_4} \left(\frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{\vec{\phi}^2}{2R^2} + \lambda \left(\vec{\phi}^2 \right)^2 + \varepsilon \Omega(t)^2 M^2 \vec{\phi}^2 \right)$$

Mass term blows to $\varepsilon \infty$
in finite time













$$\mathcal{O}_1 \leftrightarrow \tilde{\mathcal{O}}_1$$



$$\mathcal{O}_2 \leftrightarrow \tilde{\mathcal{O}}_2$$

$$V_{\text{dS}}[\phi] = \frac{1}{2} \xi_d \mathcal{R}_{\text{dS}_d} \phi^2 + \lambda \phi^{\frac{2d}{d-2}} - \left(\frac{\widetilde{M}}{\cosh(\tau)} \right)^{d-\Delta} \phi^{\frac{2\Delta}{d-2}}$$

d=1 is not trivial. 2d/(d-2)=-2 and!

$$\omega_k^2 = \lim_{d \rightarrow 1} \xi_d \mathcal{R}_{\mathbf{X}_k} = \lim_{d \rightarrow 1} \frac{d-2}{4(d-1)} k(d-1)(d-2) = \frac{k}{4}$$

and for the LG model on dS_d :

$$\lim_{d \rightarrow 1} \xi_d \mathcal{R}_{\text{dS}_d} = \lim_{d \rightarrow 1} \frac{d-2}{4(d-1)} d(d-1) = -\frac{1}{4}.$$

**A near boundary slow moving
particle probe in AdS2**

gives:

$$ds^2_{(k)} = -(r_k^2 + k) dt_k^2 + \frac{dr_k^2}{r_k^2 + k}$$

$$S_{(k)} = -m \int dt_k \sqrt{r_k^2 + k - \frac{1}{r_k^2 + k} \left(\frac{dr_k}{dt_k} \right)^2}$$

$$r_k \gg 1 \text{ and } |dr_k/dt_k| \ll 1 \qquad \phi(t_k) = \left(\frac{4m}{r_k(t_k)} \right)^{1/2}$$

$$S[\phi_k] = \frac{1}{2} \int dt_k \left[\left(\frac{d\phi_k}{dt_k} \right)^2 - \omega_k^2 \phi_k^2 - \frac{\lambda}{\phi_k^2} \right]$$

$$V_{\text{dS}}[\phi] = \frac{1}{2} \xi_d \mathcal{R}_{\text{dS}_d} \phi^2 + \lambda \phi^{\frac{2d}{d-2}} - \left(\frac{\widetilde{M}}{\cosh(\tau)} \right)^{d-\Delta} \phi^{\frac{2\Delta}{d-2}}$$

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Classical Maps:

$t \in [-t_\star, t_\star]$ **E= Einstien ; M= Minkowski; dS** $\tau \in \mathbf{R}$

$$dt = \frac{d\tau}{\Omega(\tau)}$$

$$\text{EM : } \omega^2 = 0, \quad \tilde{\omega}^2 = \frac{1}{4}, \quad \Omega_{\text{EM}} = \frac{1}{2} (1 + \tau^2) = \frac{1}{2 \cos^2(t/2)}$$

$$\text{EdS : } \omega^2 = -\frac{1}{4}, \quad \tilde{\omega}^2 = \frac{1}{4}, \quad \Omega_{\text{EdS}} = \cosh(\tau) = \frac{1}{\cos(t)}$$

$$\tilde{\omega}^2 = \Omega^2 \omega^2 + \frac{1}{2} \Omega \partial_\tau^2 \Omega - \frac{1}{4} (\partial_\tau \Omega)^2 \quad \text{“Anomalous”}$$

$$H(\pi, \phi)_{\text{AFF}} = \frac{1}{2} \left(\pi^2 + \frac{\lambda}{\phi^2} \right)$$

$$[D, H] = 2iH \ , \quad [D, C] = -2iC \ , \quad [H, C] = -iD$$

$$D = \frac{1}{2} \{ \phi, \pi \} \qquad C = \frac{1}{2} \phi^2 \ ,$$

SL(2,R) Invariant !

$$H(\pi, \phi)_{\text{AFF}} = \frac{1}{2} \left(\pi^2 + \frac{\lambda}{\phi^2} \right)$$

$$[D, H] = 2iH, \quad [D, C] = -2iC, \quad [H, C] = -iD$$

**SL(2,R) Yes , GCR no, dismissed for
SKY at al.**

$$H(\pi, \phi)_{\text{AFF}} = \frac{1}{2} \left(\pi^2 + \frac{\lambda}{\phi^2} \right)$$

$$[D, H] = 2iH, \quad [D, C] = -2iC, \quad [H, C] = -iD$$

For a positive coupling (and $> -1/4$)

The Spectrum is continuous

The System has no Ground State!

Note analogy to B.F Bound!

Energy is $(0, \infty)$

$$H(\pi, \phi)_{\text{AFF}} = \frac{1}{2} \left(\pi^2 + \frac{\lambda}{\phi^2} \right)$$

$$[D, H] = 2iH, \quad [D, C] = -2iC, \quad [H, C] = -iD$$

$$H_\omega = H_{\text{AFF}} + \omega^2 C \qquad C = \frac{1}{2} \phi^2$$

**Discrete Spectrum ALSO $\text{SL}(2, \mathbb{R})$
Invariant.**

Deform CQM by a potential

$$V_{\Delta}(\phi) = \varepsilon \frac{M^{1-\Delta}}{\phi^{2\Delta}}$$

$$V_{\Delta}(\phi) = \varepsilon \frac{M^{1-\Delta}}{\phi^{2\Delta}}$$

< 1 RELEVANT

Δ

=1 Marginal

Between 1 and 0 “softly “ Relevant

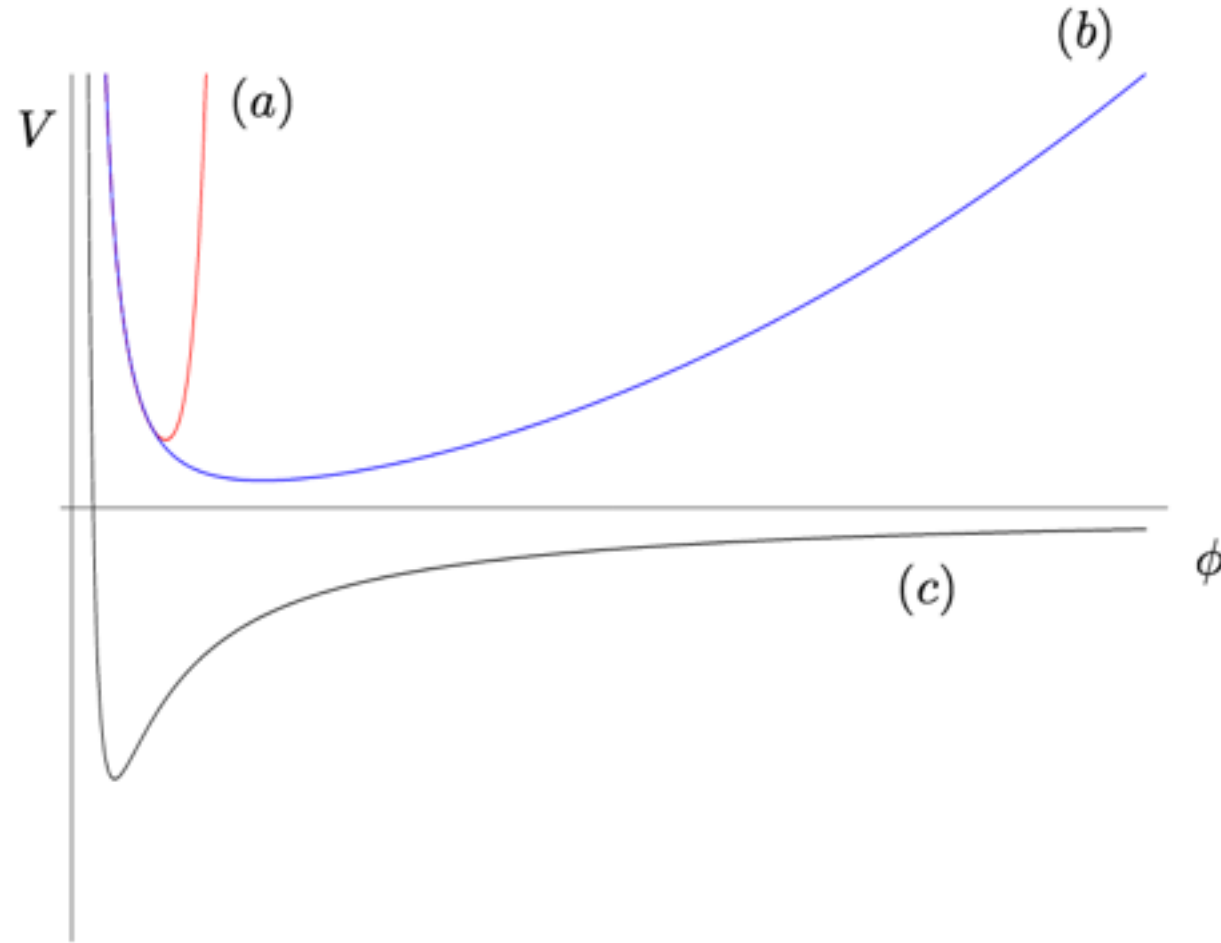


Figure 7: The AFF potential deformed by (a) a positive strongly relevant operator with $\Delta < -1$ (confinement), (b) a harmonic potential, $\Delta = -1$ (trapping), and (c) a negative, mildly relevant deformation, $0 < \Delta < 1$ (condensate).

For Large N even more stable near the origin.

Special “soft” Relevant operator =1/2

$$\left(-\frac{1}{2} \frac{d^2}{d\phi^2} + \frac{\lambda}{2\phi^2} - \frac{\sqrt{M}}{\phi} \right) U_n(\phi) = E_n U_n(\phi)$$

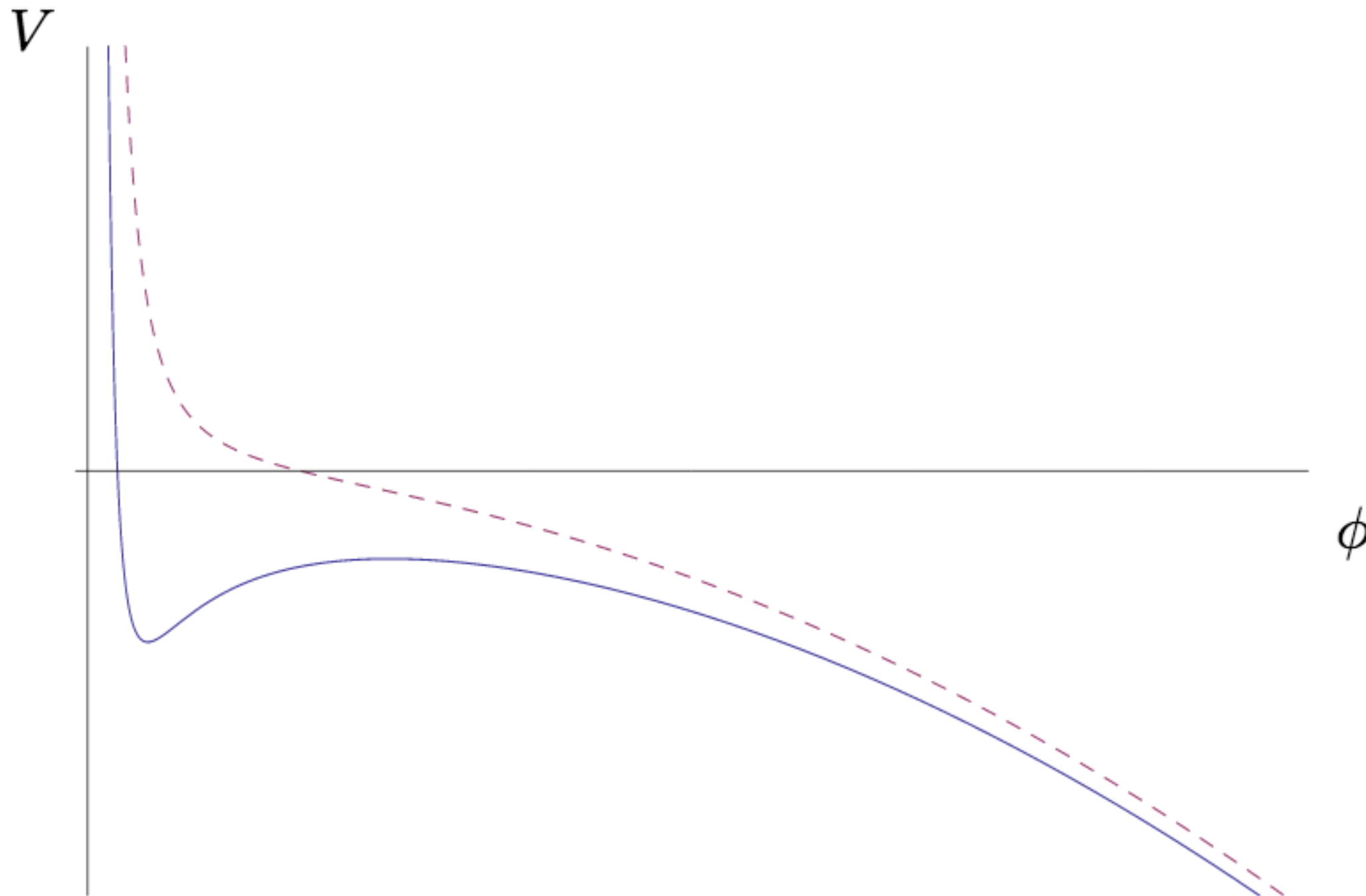
with discrete spectrum of energies

$$E_n = -\frac{2M}{(2n + 1 + \sqrt{1 + 4\lambda})^2}, \quad n \in \mathbf{Z}_+.$$

Many scattering States!

Like SSB with IR theory non trivial

“Big Crunch”

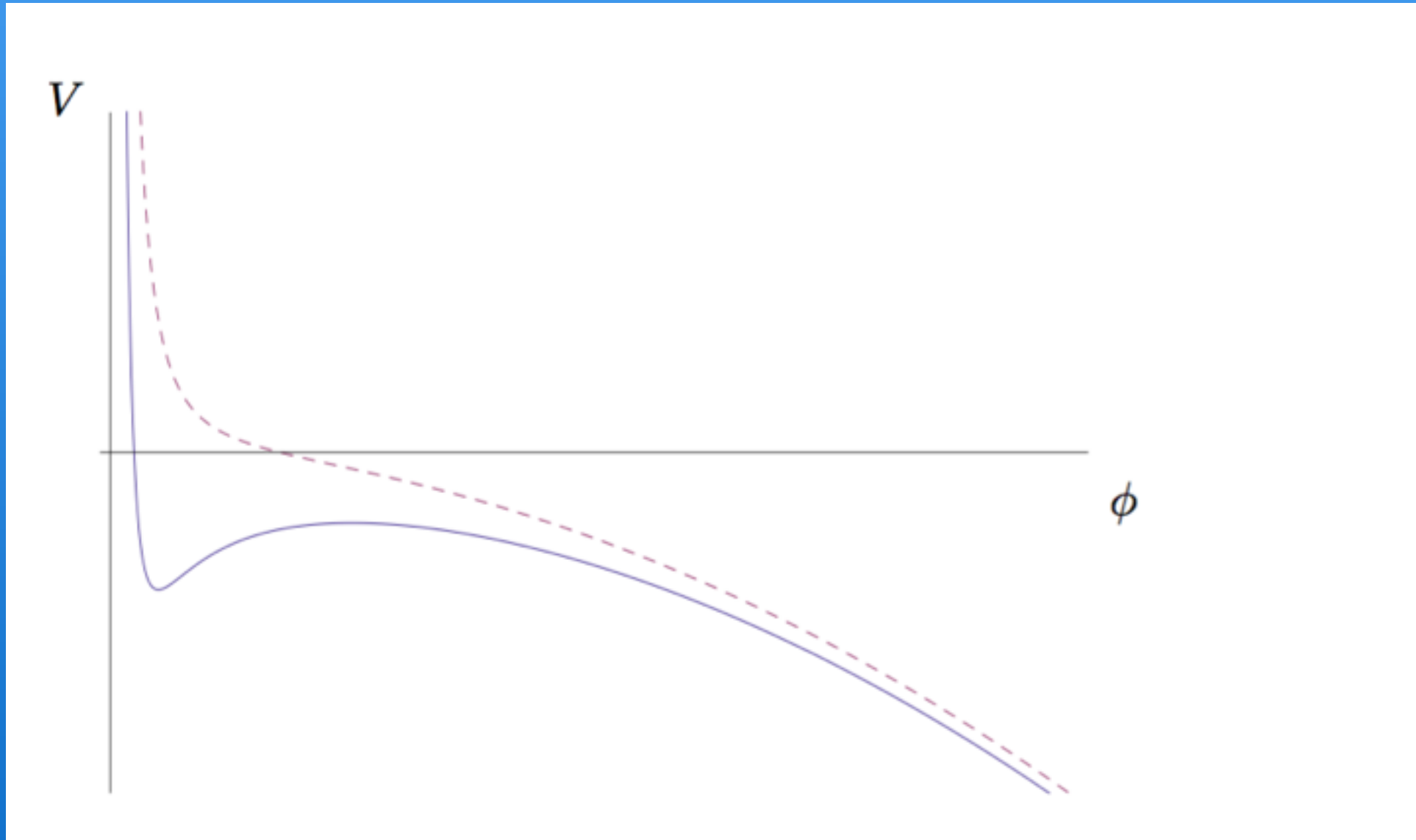


Dashed Line- Ds

Full line- Adding a negative relevant operator!

Tunneling reflects return to the bulk NOT boundary!!

Possible healing happens at $d=1$ CQM where all can be made Quantum?



$$\Psi_{\text{meta}} \approx e^{-\Gamma\tau/2} \Psi_{\text{cond}} + \sqrt{1 - e^{-\Gamma\tau}} \Psi_{\text{run}}$$

Classical Maps:

$t \in [-t_\star, t_\star]$ **E= Einstien ; M= Minkowski; dS** $\tau \in \mathbf{R}$

$$dt = \frac{d\tau}{\Omega(\tau)}$$

$$\text{EM : } \omega^2 = 0, \quad \tilde{\omega}^2 = \frac{1}{4}, \quad \Omega_{\text{EM}} = \frac{1}{2} (1 + \tau^2) = \frac{1}{2 \cos^2(t/2)}$$

$$\text{EdS : } \omega^2 = -\frac{1}{4}, \quad \tilde{\omega}^2 = \frac{1}{4}, \quad \Omega_{\text{EdS}} = \cosh(\tau) = \frac{1}{\cos(t)}$$

$$\tilde{\omega}^2 = \Omega^2 \omega^2 + \frac{1}{2} \Omega \partial_\tau^2 \Omega - \frac{1}{4} (\partial_\tau \Omega)^2 \quad \text{“Anomalous”}$$

Quantum Maps

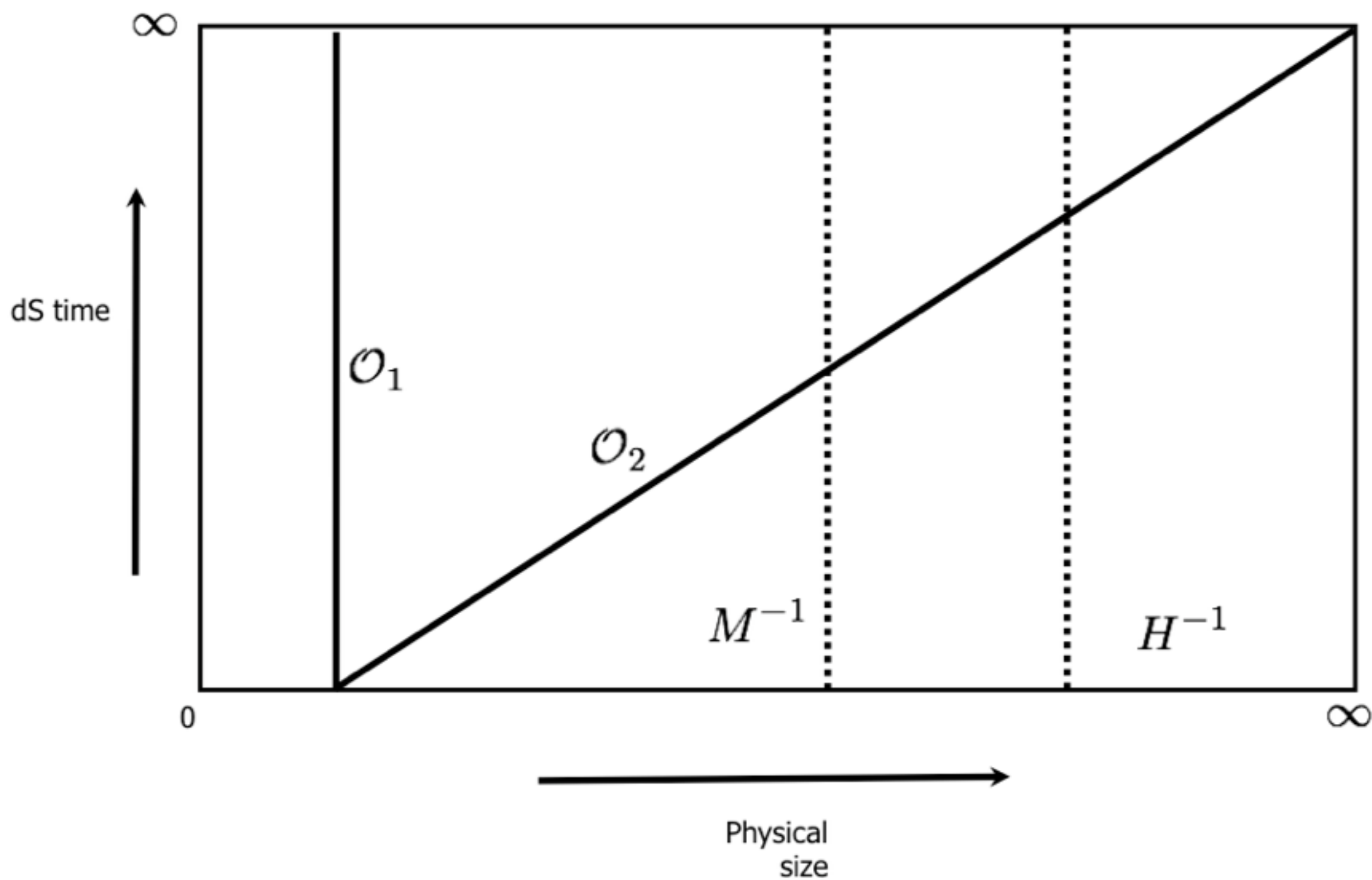
$$\Psi[\phi, \tau] \longrightarrow \tilde{\Psi}[\tilde{\phi}, t]$$

$$\tilde{\Psi}[\tilde{\phi}, t] = \Omega(t)^{\frac{1}{4}} e^{i\tilde{K}(t)} \Psi\left[\tilde{\phi}\sqrt{\Omega(t)}, \tau(t)\right]$$

$$\Psi[\phi, \tau] = \Omega(\tau)^{-\frac{1}{4}} e^{-iK(\tau)} \tilde{\Psi}\left[\phi/\sqrt{\Omega(\tau)}, t(\tau)\right]$$

$$K(\tau) = -\frac{1}{4} \phi^2 \partial_{\tau} \log \Omega(\tau) \ , \qquad \tilde{K}(t) = -\frac{1}{4} \tilde{\phi}^2 \partial_t \log \Omega(t)$$

Length scales of complete quantum state in dS variables, as a function of dS time



$$\Omega(t) \sim (t - t_\star)^{-\alpha}$$

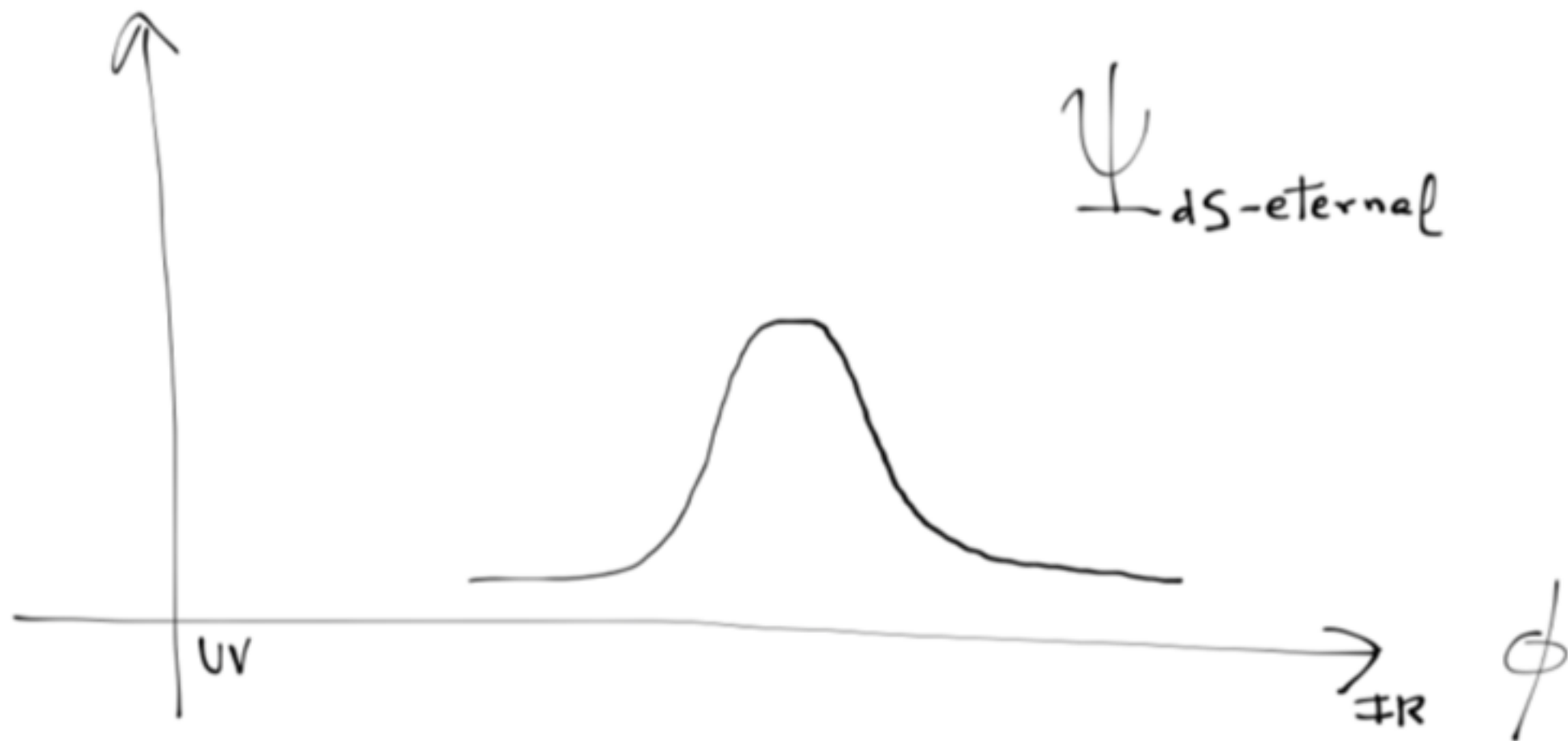
$$t = \pm t_\star = \pm \alpha\pi/2.$$

**For Candidate Crunches such as
Condensate states stationary in dS**

Crunching in E frame

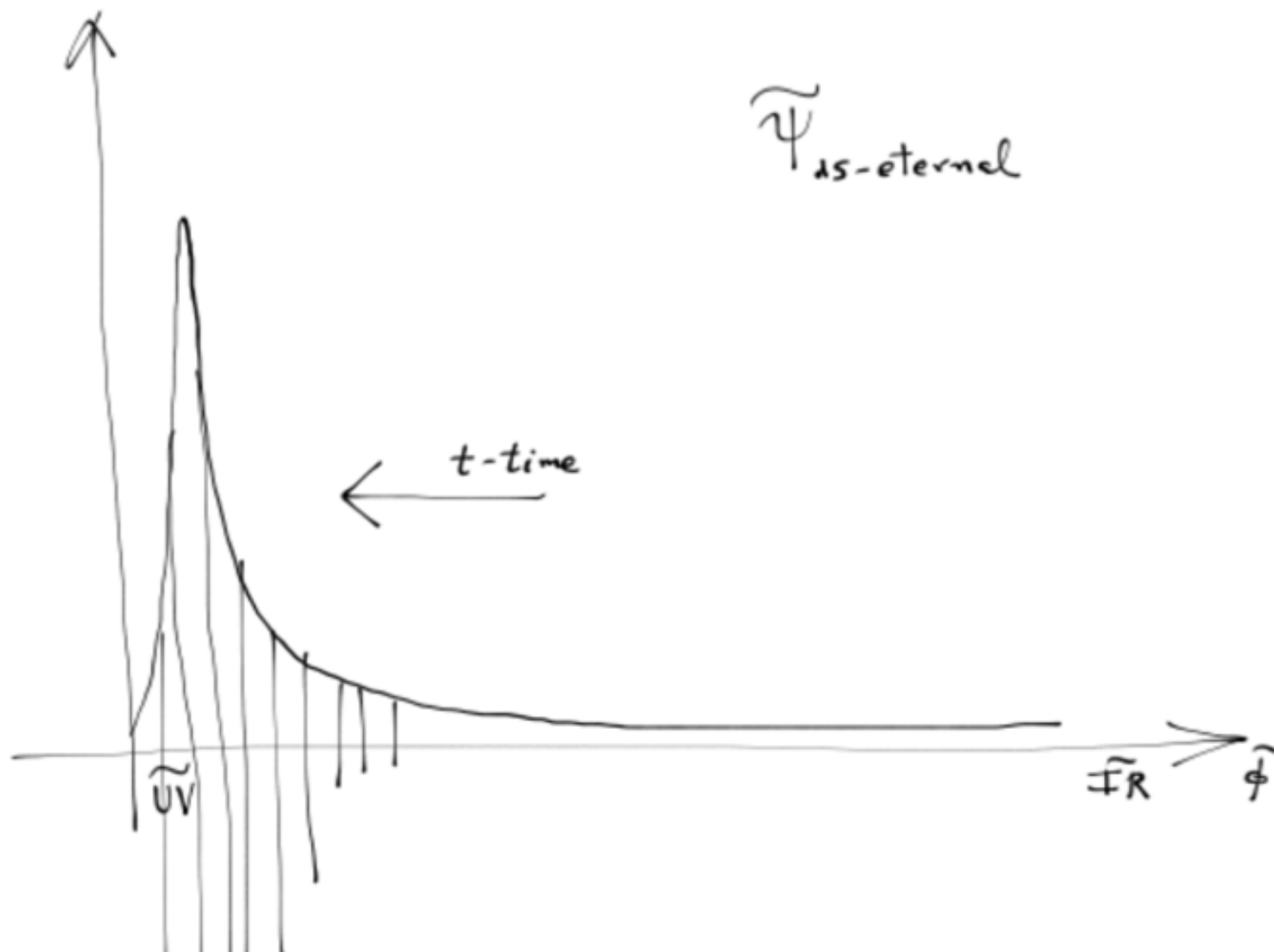
One obtains:

Stationary State in dS



**Nothing much happens
Expectation values finite.**

**In the apocalyptic frame-crunchy
contracting in the UV!
Expectation values diverge!**



In order to further fix the intuition about the meanings of the quantum map, we can consider a smooth τ -static wave function in the eternal quantum mechanics, with width Γ and centered around ϕ_0 . Its dual to the apocalyptic frame has a narrowing width $\tilde{\Gamma}(t) = \Gamma/\sqrt{\Omega(t)}$ as $t \rightarrow \pm\alpha\pi/2$, with its center migrating to the origin as $\tilde{\phi}_0(t) = \phi_0/\sqrt{\Omega(t)}$, while at the same time the phase oscillates wildly. Therefore, the $\tilde{\Psi}$ wave function is infinitely squeezed into the UV region (small $\tilde{\phi}$) as we approach the ‘apocalypse’.

This Conforms to the Intuition

Potential Energy Expectation Value

For Crunches in the Apocalyptic:

$$\lim_{|t| \rightarrow t_\star} \left\langle \tilde{V}(\tilde{\phi}) \right\rangle_{\tilde{C}}(t) = -\infty$$

For Bubbles of Nothing

$$\lim_{|t| \rightarrow t_\star} \left\langle \tilde{V}(\tilde{\phi}) \right\rangle_{\tilde{B}}(t) = +\infty .$$

For the Kinetic Energy

$$\langle \frac{1}{2} \tilde{\pi}^2 \rangle_{\tilde{\Psi}} = \Omega(t) \langle \frac{1}{2} \pi^2 \rangle_{\Psi} - \frac{1}{4} \partial_t \log \Omega \langle \{\phi, \pi\} \rangle_{\Psi} + \frac{1}{8} \Omega^{-1} (\partial_t \log \Omega)^2 \langle \phi^2 \rangle_{\Psi}$$

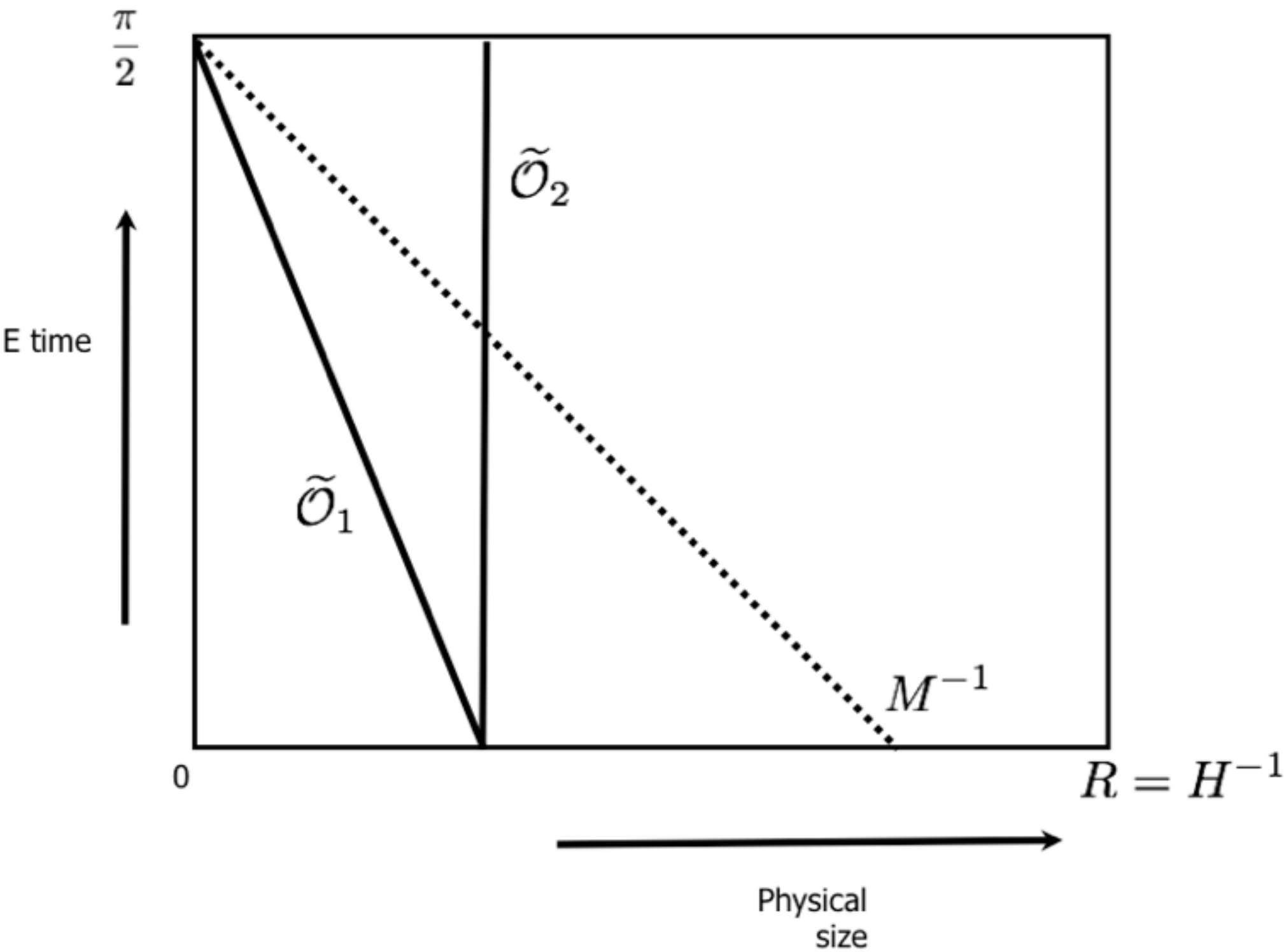
For a condensate-type state in the eternal frame, the three terms in this equation scale as $(t - t_{\star})^{-\alpha}$, $(t - t_{\star})^{-1}$ and $(t - t_{\star})^{\alpha-2}$ respectively. For either the EM or the EdS map, there is always a singular term for generic values of the eternal frame averages, confirming that the eternally stationary state is a singular state in the apocalyptic frame. The anomalous terms (second and third on the right hand side of (5.26)) are subdominant for the EM model ($\alpha = 2$), but have the same scaling as the first term in the EdS case ($\alpha = 1$).¹¹

In order to further fix the intuition about the meanings of the quantum map, we can consider a smooth τ -static wave function in the eternal quantum mechanics, with width Γ and centered around ϕ_0 . Its dual to the apocalyptic frame has a narrowing width $\tilde{\Gamma}(t) = \Gamma/\sqrt{\Omega(t)}$ as $t \rightarrow \pm\alpha\pi/2$, with its center migrating to the origin as $\tilde{\phi}_0(t) = \phi_0/\sqrt{\Omega(t)}$, while at the same time the phase oscillates wildly. Therefore, the $\tilde{\Psi}$ wave function is infinitely squeezed into the UV region (small $\tilde{\phi}$) as we approach the ‘apocalypse’.

This Conforms to the Intuition

Conversely, starting with t -static wave function with fixed width $\tilde{\Gamma}$ and centered at $\tilde{\phi}_0$ in the E-frame system, it corresponds to an eternal wave function slipping into the deep IR (large ϕ), trailing the peak at $\phi_0(\tau) = \tilde{\phi}_0\sqrt{\Omega(\tau)}$, and widening at a rate of order $\Gamma(\tau) = \tilde{\Gamma}\sqrt{\Omega(\tau)}$.

Length scales of complete quantum state in E variables, as a function
of E time



T Duality : Is R large or small ?

Locality -Winding modes

Non Local Observers



RESULT:

$$\tilde{H} = \frac{1}{2} \tilde{\pi}^2 + \frac{1}{2} \tilde{\omega}^2 \tilde{\phi}^2 + \tilde{V}(\tilde{\phi}) = \frac{1}{2} \Omega^{-1} \tilde{\omega}^2 \phi^2 + \Omega \left(\frac{1}{2} \pi^2 + V(\phi) \right)$$

$$[H, \tilde{H}] = 2i \mathcal{A} D = 2i \mathcal{A} \tilde{D}$$

$$D = \frac{1}{2} \{\phi, \pi\} = \frac{1}{2} \{\tilde{\phi}, \tilde{\pi}\} = \tilde{D} .$$

$$\mathcal{A} \equiv \frac{1}{2} (\Omega \omega^2 - \Omega^{-1} \tilde{\omega}^2)$$

AND IN GENERAL IT DOES NOT VANISH

BUT ON AdS IT DOES VANISH!

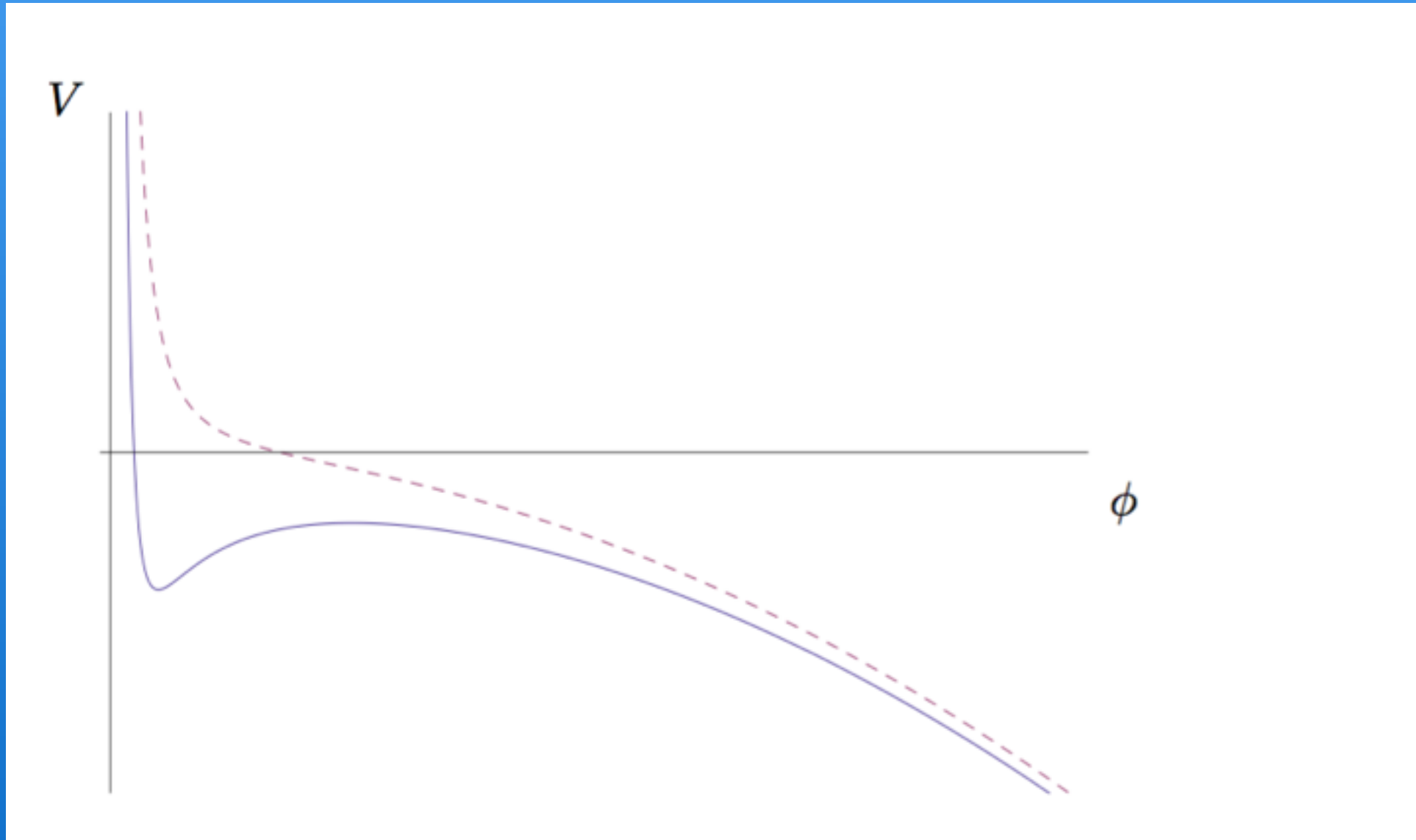
$$\tilde{H} = \frac{1}{2} \left(\tilde{\pi}^2 + \frac{\lambda}{\tilde{\phi}^2} \right) + \frac{1}{8} \tilde{\phi}^2$$

$$H = \frac{1}{2} \left(\pi^2 + \frac{\lambda}{\phi^2} \right) - \frac{1}{8} \phi^2$$

$$V_{\Delta}(\phi) = -\frac{M^{1-\Delta}}{\phi^{2\Delta}} \quad 0 < \Delta < 1$$

Only Metastable and Give:

Possible healing happens at $d=1$ CQM where all can be made Quantum?



$$\Psi_{\text{meta}} \approx e^{-\Gamma\tau/2} \Psi_{\text{cond}} + \sqrt{1 - e^{-\Gamma\tau}} \Psi_{\text{run}}$$

$$\Psi_{\text{meta}} \approx e^{-\Gamma\tau/2} \Psi_{\text{cond}} + \sqrt{1 - e^{-\Gamma\tau}} \Psi_{\text{run}}$$

$$\tilde{\Psi}_{\text{cond}} \quad e^{-\Gamma\tau/2} \sim |t_{\star} - t|^{\Gamma/2}$$

$\tilde{\Psi}_{\text{cond}}$ **Contributes to Expectation Values blowing up**

$$\Gamma \sim M \exp(-a M^{2/3}) \ll 1 \quad \textbf{Semi classically} \quad \Gamma \ll b$$

Depletion not fast enough to avoid crunch

If b of Order 1 there is QUANTUM healing !!!

CONCLUSIONS

New aspects.

There is an Apocalyptic -Eternal Duality.

The Crunch can be seen in the UV.

Must have non-local aspects.

Healing?

**Congratulations
Spenta**

**Many more years of intellectual
Stimulation and friendship!**