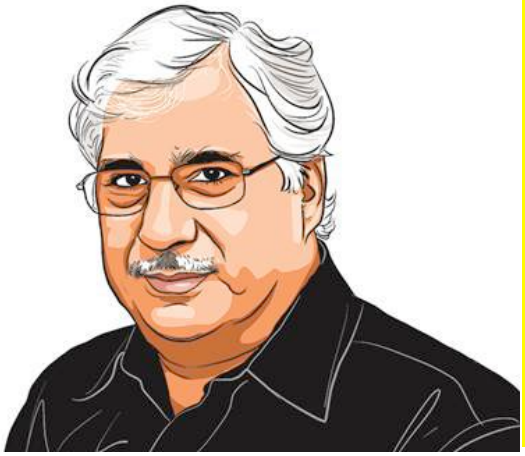


STRING THEORY: PAST AND PRESENT

SPENTAFEST, BANGALORE

11-13 JANUARY 2017

HADRONIC STRINGS: OLD AND NEW

Michael B. Green

DAMTP Cambridge and Queen Mary, U. of London

MANY THANKS FOR YOUR NUMEROUS INSIGHTS
DELIVERED WITH SUCH GRACIOUSNESS

AND FOR YOUR PASSION IN CREATING THIS MAGNIFICENT INSTITUTE.

HAPPY BIRTHDAY, SPENTA

INTUITIVE MOTIVATION FOR THIS TALK

SU(N) QCD in the large-N limit is expected to be some kind of string theory in which the coupling constant is 1/N.

"Fundamental" string theory clearly fails to capture many aspects of large-N QCD.

NOTABLY: ABSENCE OF POINT-LIKE SUBSTRUCTURE – NO HARD PARTONS

FIXED ANGLE SCATTERING AMPLITUDES DECREASE EXPONENTIALLY WITH ENERGY $s \to \infty,$ fixed $\frac{t}{s} \leq 0$

Some successes within AdS/CFT: Curved extra dimensions captures certain point-like features of QCD (Polchinski and Strassler followed by many others). But weakly coupled QCD involves understanding fundamental string theory in a highly curved holographic dual.

Polyakov's discussion of QCD in terms of Liouville theory captures other qualitative features – but also involves highly curved space-time.

- THE FUNDAMENTAL STRING IS A DIFFUSE OBJECT WITH NO POINT-LIKE SUB-STRUCTURE.
  Point-like configurations contribute with zero measure to the fundamental string functional integral.
- BUT, IN QCD, OFF-SHELL CURRENTS MUST COUPLE LOCALLY - TO POINT-LIKE ENERGY DENSITIES.
  Reconsider a "prehistoric" procedure for reweighting the measure so that point-like configurations contribute significantly in the relevant kinematic regimes.

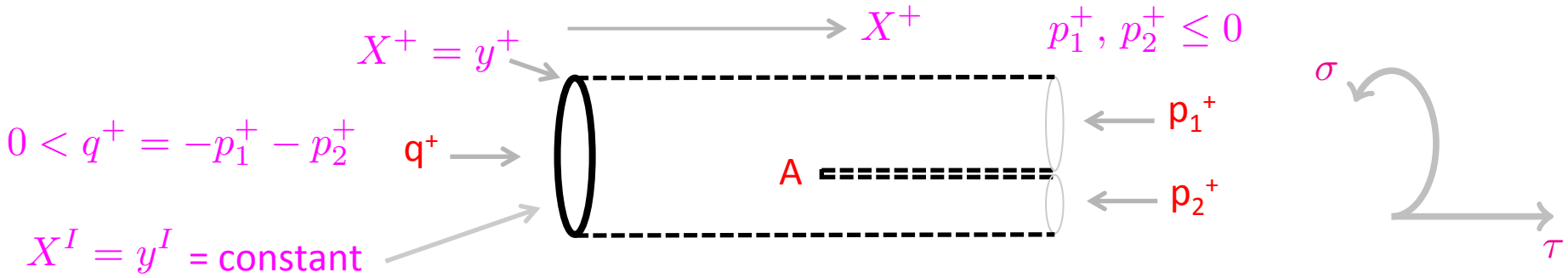
I WILL HERE CONCENTRATE ON CLOSED STRINGS (GLUEBALL STRINGS).

POINT-LIKE COUPLING TO OFF-SHELL CURRENTS

Physical intuition: LIGHT-CONE FRAME DIAGRAMS – $X^+ = x^+ + p^+\tau$ width in $\sigma = p^+$

MANDELSTAM

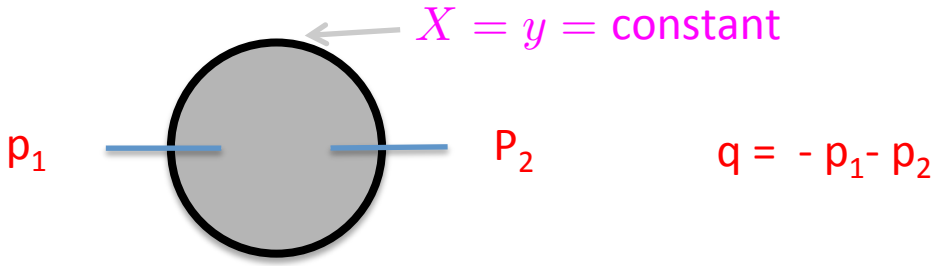
e.g. FORM FACTOR - string collapses to a point at $X^+ = y^+$, $X^I = y^I$.



POINT-LIKE BOUNDARY STATE c.f. first order contribution of a D-instanton

Integrate over the joining point A

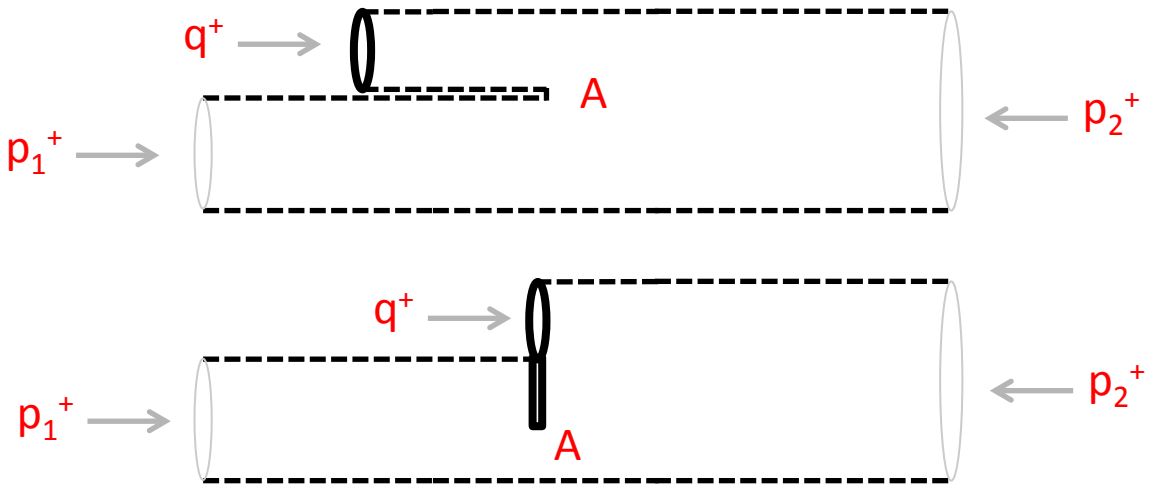
Conformal map from upper half z-plane



- Power dependence on q^2 (modulo divergences in bosonic string).

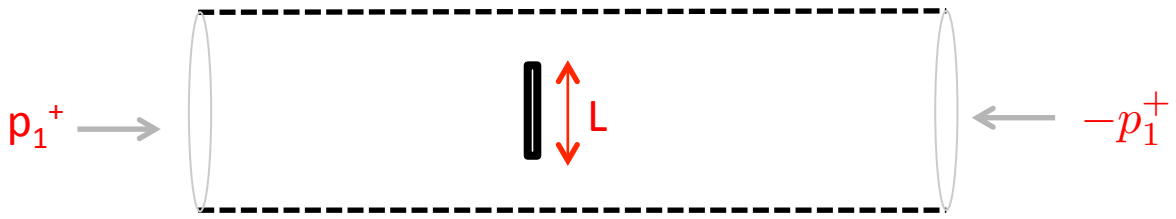
$$F(q^2) = \int d^D y \tilde{F}(y)\, e^{iq\cdot y} \sim \int_0^\infty dt\, e^{-(q^2/2+2)t} \prod_{n=1}^{\infty} (1 - e^{-2nt})^{-24}$$

SPACE-LIKE FORM FACTOR $0 < q^+ \leq -p_1^+ \, , \, -p_2^+$



Two distinct components of space-like form factor

$q^+ = 0$

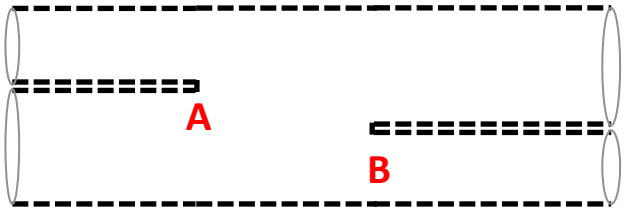


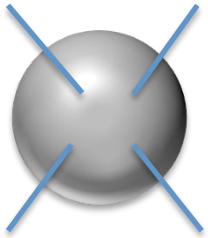
Zero-momentum boundary insertion – **POINT-LIKE CORRECTION TO PROPAGATOR**

Required by local coupling to off-shell currents

Must interpret problematic singularities: Dilaton singularity at L=0 ; open string divergence at L= p_1^+

TREE-LEVEL SCATTERING AMPLITUDE WITH A DIRICHLET CORRECTION

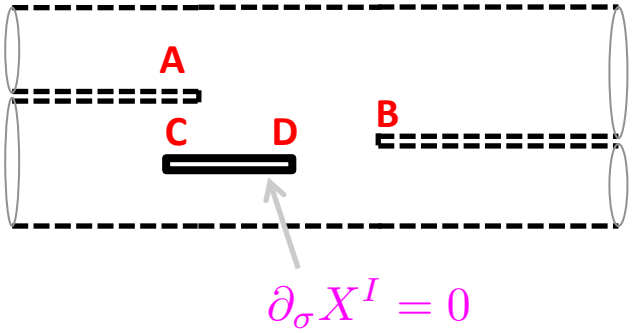


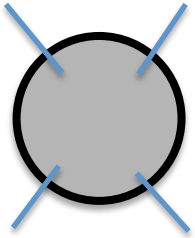


Sphere

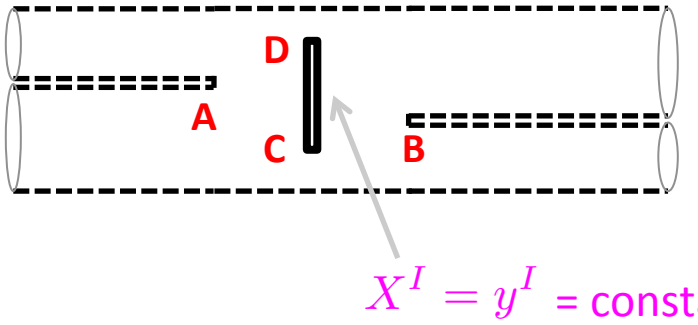
Exponential fixed-angle scattering

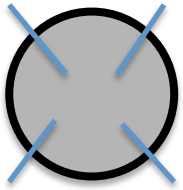
$$A \sim e^{-s f(\cos\theta)}$$





Neumann Disk





Dirichlet Disk

Power behaved Fixed-angle scattering

$$A \sim \frac{1}{s^4} f(\cos\theta)$$

(bosonic string)

Comes from region in which points A and B pinch the line CD

OR

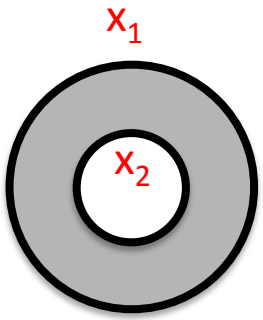
Region in which 3 vertices approach boundary of the disk

World-sheets loops with Dirichlet boundaries

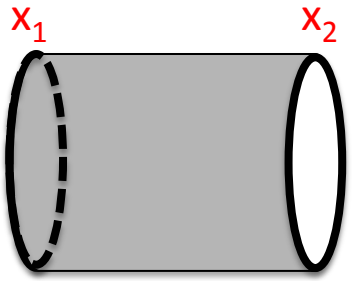
Hamiltonian for open string with fixed end-points winding m times around a circle radius R

$$L_0^D = \frac{1}{4\alpha'}\left(\frac{x_2 - x_1}{\pi} + 2mR\right)^2 + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

Two-current amplitude (viz. e^+- e^- annihilation)



$$\int_0^\infty \frac{dt}{t}\,\mathrm{Tr}\, e^{-(L_0^D - 1)t}$$

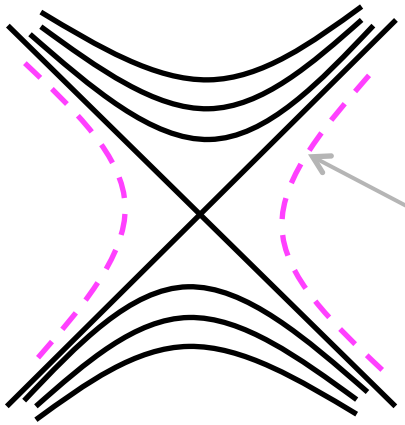


$$\langle B_1 | \frac{1}{L_0 + \tilde{L}_0 - 2} | B_2 \rangle$$

$X(\sigma)\,|B_i\rangle = x_i\,|B_i\rangle$
Dirichlet boundary state

Singularities at:

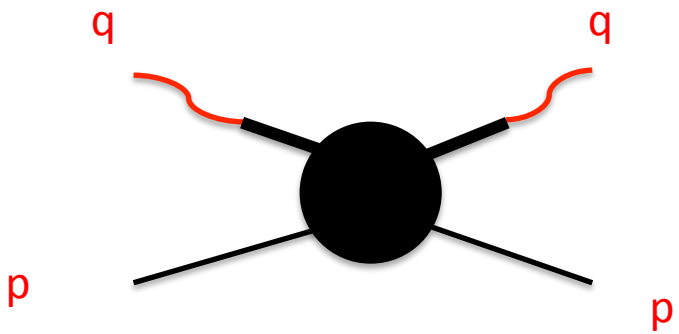
$$(x_2 - x_1)^2 = 4\pi\alpha'(1 - n)\,, \quad n \geq 0$$



Space-like singularity due to open-string tachyon.
Absent in certain superstring models (with BPS boundary Dirichlet states)

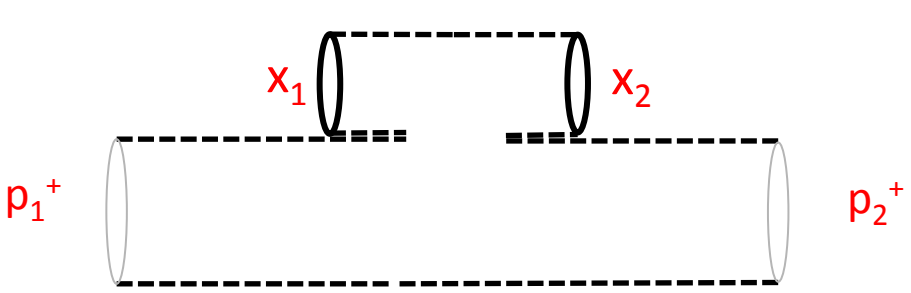
"Deep inelastic scattering"

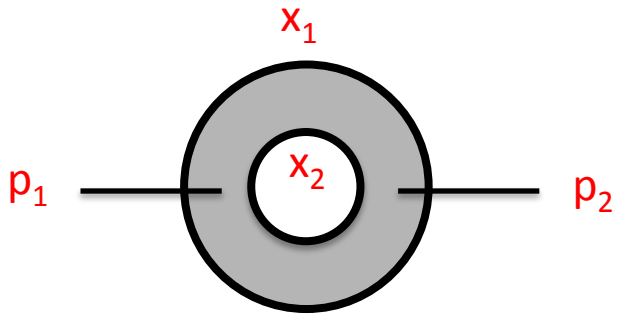
Electron-proton total cross section $q^2 >> 0$ (space-like)



Total cross-section for current carrying large space-like momentum.
Light-cone dominance $q^2\ ,\ q:p >> 1/\alpha'$

$$\mathrm{Im}\, A(q,p) = F(q^2, q\cdot p)$$

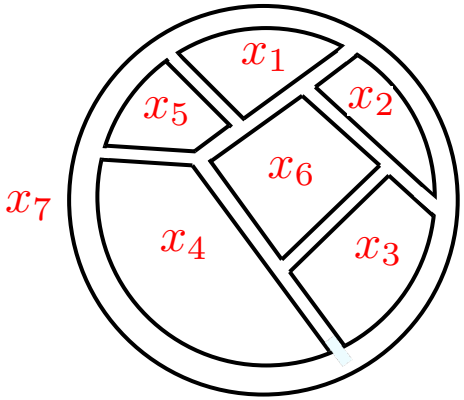




$$F(q^2, q\cdot p) = \int dx_1\, dx_2\, e^{iq\cdot(x_1-x_2)}\, \tilde{F}(x_1, x_2, p)$$

$$\tilde{F}(x_1, x_2, p) \sim \sum_{n=0}^{\infty} \frac{c_n(p\cdot(x_2-x_1))}{[(x_1-x_2)^2 + 4\pi\alpha'(n-1)]^2}$$

A CONDENSATE OF DIRICHLET BOUNDARIES

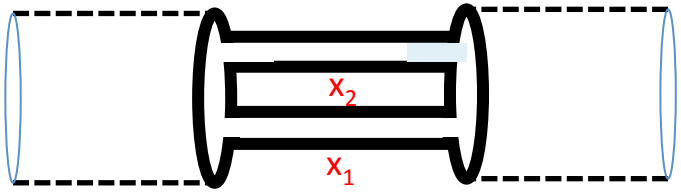


Express as integration over position-space variables,

Large-α' behaviour determined by singularities at $(x_i - x_j)^2 = 0$.

- E.g.Two boundary case. Integration over x_i generates log correction in 4-dimensional compactified theory (modulo problematic divergences of bosonic string theory) – could these indicate asymptotic freedom?

This diagram is NOT in light-cone gauge - stretch the two slits.



HIGH TEMPERATURE LIMIT OF THE CONFINING PHASE (Polchinski 1991)

- QCD has a deconfinent phase transition. In the large-N limit this should correspond roughly to the Hagedorn transition of the equivalent string theory.

- Finite temperature analysis of large-N QCD. Consider the correlation function of two Wilson-Polyakov loops winding k times around the euclidean time dimension of radius $\beta = 1/kT$.

$$W_k(\mathbf{x}) = \frac{1}{N}\mathrm{Tr}\, P \exp\left(i \int_0^{\beta k} d\tau A^\tau(\tau, \mathbf{x}) \right)$$

- Confinement phase signalled by unbroken Z_N in the centre of $SU(N)$.

$$\langle W_k(0)\, W_{-k}(L) \rangle \sim e^{-M_k(\beta)\, L}\,, \qquad L \to \infty$$

- Lowest mass glueball exchange becomes tachyonic at $\beta = \beta_c$.

Continue to $\beta << 1$ $\qquad M_k^2(\beta) = -\dfrac{2 g_{YM}^2(\beta)\, N}{\pi^2 \beta^2 k^2}$ $\qquad g_{YM}(\beta)$ Running coupling

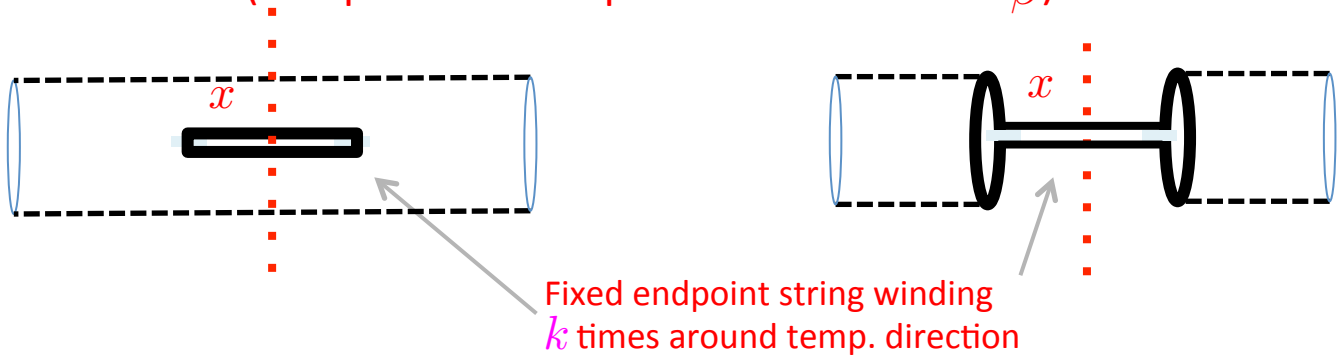
(Polchinski 1991)

- Compare with typical closed string theory result.

$$\alpha' M_k^2(\beta) = -c - \frac{1}{16\pi^2\alpha'}\beta^2\, k^2\,, \qquad c > 0$$

NOW CONSIDER CLOSED STRING PROPAGATOR WITH A DIRICHLET BOUNDARY INSERTION AT FINITE TEMP. (compactified on spatial circle of radius β)

Winding number k



Open string propagator $\Sigma = \dfrac{1}{\frac{4\,k^2\beta^2}{\alpha'} + N_o - 1}$ ($N_o = 0$ tachyon absent in type 0)

Closed string mass renormalisation. Renormalised closed string propagator to lowest order in $w_d\, g_s$ with winding number: k

$$\Delta(\beta) = \frac{1}{L_0 + \tilde{L}_0 - 2 - g_s w_D\, \Sigma} \qquad L_0 + \tilde{L}_0 = \alpha' p^2 + N_c + \tilde{N}_c + \frac{\beta^2 k^2}{16\pi^2\alpha'}$$

$N_o = 1$ term (massless vector) dominates as $\beta \to 0$: $\Sigma \sim \dfrac{\alpha'}{4k^2\beta^2}$

$$\Delta(\beta) \sim \frac{1}{p^2 - \frac{g_s w_D}{4k^2\beta^2}}$$

If we suppose that $w_D = N$ and $g_s = g_{YM}^2/4\pi$, there is a pole at

$$p^2 = -M_k^2(\beta) = \frac{g_{YM}^2 N}{16\pi^2\beta^2 k^2}$$

SOME COMMENTS/QUESTIONS

- Qualitative connection with fixed angle n-particle scattering of (scalar) glueball states in SU(N) QCD.
- Generalisation to superstring theory (far from QCD).
- Type IIB D-instanton boundary states are BPS − holographic duals of YM instantons.

  Precise connection in supersymmetric − low energy - limit
- Interpretation of logarithmic corrections in D=4 compactifications?
- Compactfication to a variety of D-instanton boundary states.
- May be a rôle for the Type 0 theory (no closed-string fermions).
- Entertaining extension to open-string (meson) strings - dynamical massive end-points.
- **HOW DOES THE DIRICHLET CONDENSATE AFFECT THE SPECTRUM?**

  (is there a massless spin-2 state ??)