INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

DISCUSSION MEETING
STRING THEORY: PAST AND PRESENT

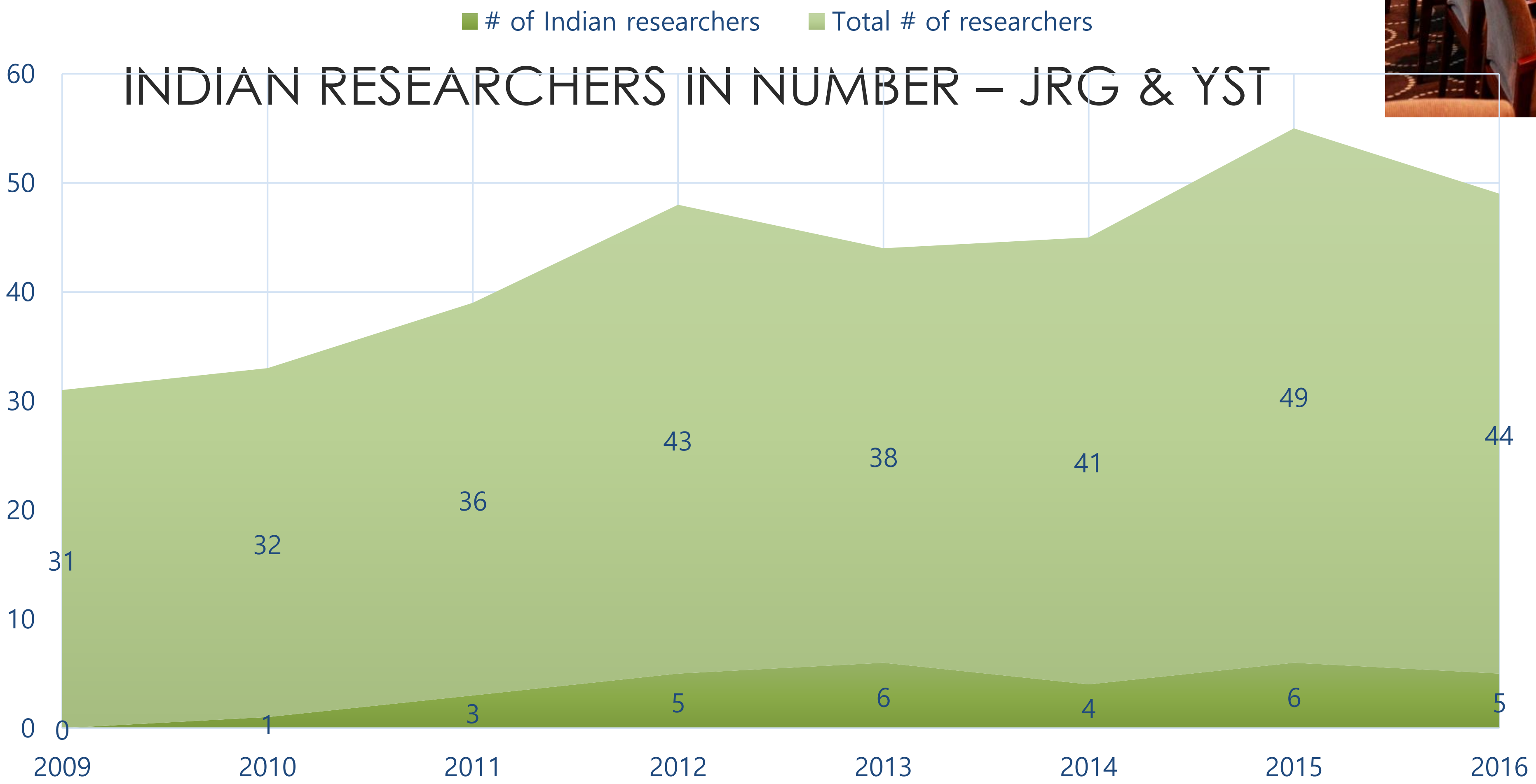
11 January 2017 to 13 January 2017
Ramanujan Lecture Hall, ICTS, Bengaluru

On the thermodynamic stability of Black Holes

BUM-HOON LEE

APCTP, POHANG (former)
CQUeST, SOGANG UNIVERSITY, SEOUL

Thanks to Spenta's leading role, young researchers in Asia region has been more interactive,
Indian researchers in Korea increasing.



Spenta has been serving as the science council member of APCTP since 2010.

Member of CQUeST			
	Name	Position	Period
1	Pichai Ramadevi	Visiting Professor	2011.03 ~ 2011.08
2	K.P Yogendran	Researcher	2005.12 ~ 2008.09
3	Rashmi Rekha Nayak	Researcher	2007.01 ~ 2007.10
4	Shesansu Pal	Researcher	2009.09 ~ 2011.12
5	Raju Roychowdhury	Researcher	2011.08 ~ 2012.11
6	Shailesh Kulkami	Researcher	2011.10 ~ 2013.09

Many more Indian visitors & participants to the activities, thanks to Spenta's initiation.

1. Motivations

Low energy effective theory from string theory

→ Einstein Gravity + higher curvature terms

Gauss-Bonnet term is the simplest leading term.

Q : What is the physical effects of Gauss-Bonnet terms?

1) Effects to the Black Holes.

No-Hair Theorem of Black Holes

Stationary black holes (in 4-dim Einstein Gravity) are completely described by 3 parameters of the Kerr-Newman metric :

mass, charge, and angular momentum (M, Q, J)

Werner Israel(1967),
Brandon Carter(1971,1977),
David Robinson (1975)

Hairy black hole solution ?

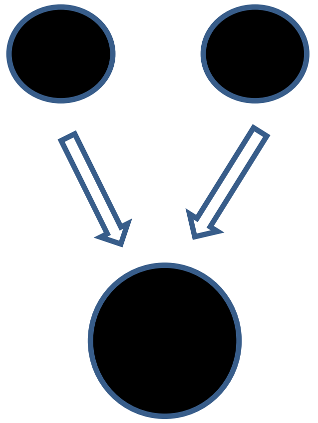
In the dilaton-Gauss-Bonnnet theory → Yes!

Exists the minimum mass of BH

Affects the stability, etc.

2) Effects in the Early Universe.

Motivations - continued

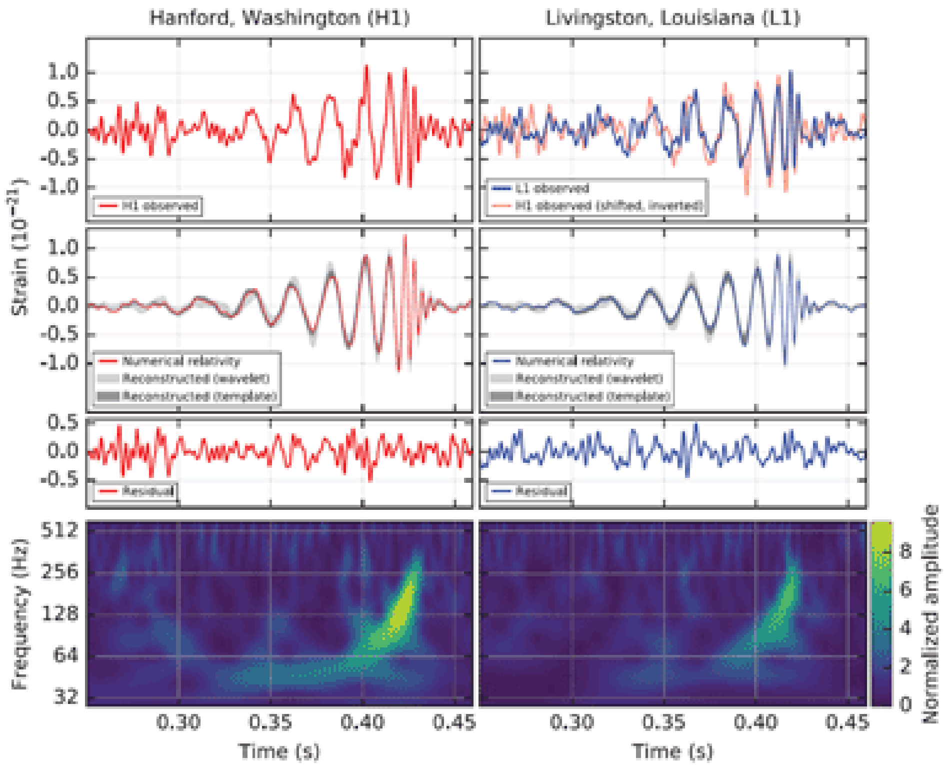


A Black Hole Merger

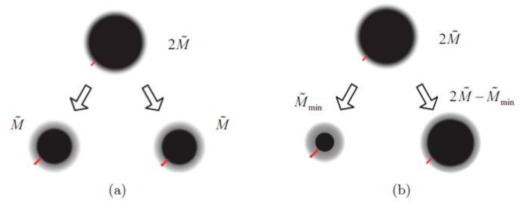
Colliding Black Holes :

A Black Hole Merger + Gravitational Wave

GW150914



Q: A Black Hole unstable ?
splitting
into two Black Holes ?



Holography

(asymptotic) AdS Black Hole in d+1 dim

↔

Quantum System in d dim.

Instability of Black Holes

↔ instability of Quantum System

Hence,

instability of AdS BH

↔ phase transitions in Quantum System

* Black holes in higher dimensions are quite diverse !

Contents

√ 1. Motivation

2. Black Holes in the Dilaton Gauss-Bonnet theory

Black Hole Stability

Perturbative stability in D=4 and higher dim.

Fragmentation Instability

for Rotating AdS BH

Charged AdS

4. Fragmentation Instability with Gauss-Bonnet Term

3. G-B gravity in Inflationary Universe

Model

Preheating, etc.

2. Black Holes in the Dilaton Gauss-Bonnet theory

W.Ahn, B. Gwak, BHL,
W.Lee,
Eur.Phys.J.C (2015)

Hairy black holes in Dilaton-Einstein-Gauss-Bonnet (DEGB) theory

Action

$$I=\int_{\mathcal{M}}\sqrt{-g}d^4x\left[\frac{R}{2\kappa}-\frac{1}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi+\alpha e^{-\gamma\Phi}R^2_{\rm GB}\right]+\oint_{\partial\mathcal{M}}\sqrt{-h}d^3x\frac{K-K_o}{\kappa}, \quad (1)$$

where $g=\det g_{\mu\nu}$ and $\kappa\equiv 8\pi G$

Guo,N.Ohta & T.Torii, Prog.Theor.Phys. 120,581(2008);121 ,253 (2009);
N.Ohta &Torii,Prog.Theor.Phys.121,959; 122,1477(2009);124,207 (2010);
K.i.Maeda,N.Ohta Y.Sasagawa, PRD80, 104032(2009); 83,044051 (2011)
N. Ohta and T. Torii, Phys.Rev. D 88 ,064002 (2013).

The Gauss-Bonnet term :

$$R^2_{\rm GB}=R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Note :

1) The symmetry under $\gamma\to-\gamma,\Phi\to-\Phi.$

allows choosing γ positive values without loss of generality.

2) The coupling α dependency could be absorbed by the r → r/√α transformation.
with non-zero α coupling cases being generated by α scaling.
However, the behaviors for the α = 0 case cannot be generated in this way.
Hence, we keep the parameter α, to show a continuous change to α = 0.

The Einstein equations and the scalar field equation are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \Phi \partial^\rho \Phi + T^{GB}_{\mu\nu} \right) , \qquad (2)$$

$$\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi] - \alpha \gamma e^{-\gamma \Phi} R^2_{\rm GB} = 0,$$

where

$$\begin{aligned} T^{GB}_{\mu\nu} &= -8\alpha (R_{\mu\rho\nu\sigma} \nabla^\rho \nabla^\sigma e^{-\gamma\Phi} - R_{\mu\nu} \Box e^{-\gamma\Phi} + 2\nabla_\rho \nabla_{(\mu} e^{-\gamma\Phi} R^\rho{}_{\nu)} - \frac{1}{2} R \nabla_\mu \nabla_\nu e^{-\gamma\Phi}) \\ &\quad +4\alpha (2R^{\rho\sigma} \nabla_\rho \nabla_\sigma e^{-\gamma\Phi} - R \Box e^{-\gamma\Phi}) g_{\mu\nu} , \end{aligned} \qquad (4)$$

and $\Box \equiv \nabla_\mu \nabla^\mu$ is the d'Alembertian.

Note :

1. All the black holes in the DEGB theory with given non-zero couplings α and γ have hairs.
   I.e., there does not exist black hole solutions without a hair in DEGB theory.
   (If we have Φ = 0, dilaton e.o.m. reduces to R^2_{GB} = 0. so it cannot satisfy the dilaton e.o.m..)

2. For the coupling α = 0, the solutions become a Schwarzschild black hole in Einstein gravity.

3. For $\gamma = 0$, DEGB theory becomes the Einstein-Gauss-Bonnet (EGB) theory.
   The EGB black hole solution is the same as that of the Schwarzschild one.
   However, the GB term contributes to the black hole entropy and influence stability.

We consider a spherically symmetric static spacetime with the metric

P. Kanti et al., PRD54, 5049 (1996).

$$ds^2 = -e^{X(r)}dt^2 + e^{Y(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \qquad (5)$$

The asymptotic form of the solutions takes **(M = ADM mass, Q = scalar charge)**

$$e^X \simeq 1 - \frac{2M}{r} + \mathcal{O}(1/r^3)\,, \qquad (18)$$

$$\Phi \simeq \Phi_\infty + \frac{Q}{r} + \mathcal{O}(1/r^2)\,, \qquad (19)$$

The mass of a hairy black hole is represented as follows

$$M(r) = M(r_h) + M_{\rm hair}\,.$$

where $M(r_h) = \frac{1}{2}r_h$ **is the BH mass subtracting the scalar hair contribution.**
M_{hair} **represents the contribution from the scalar hair.**

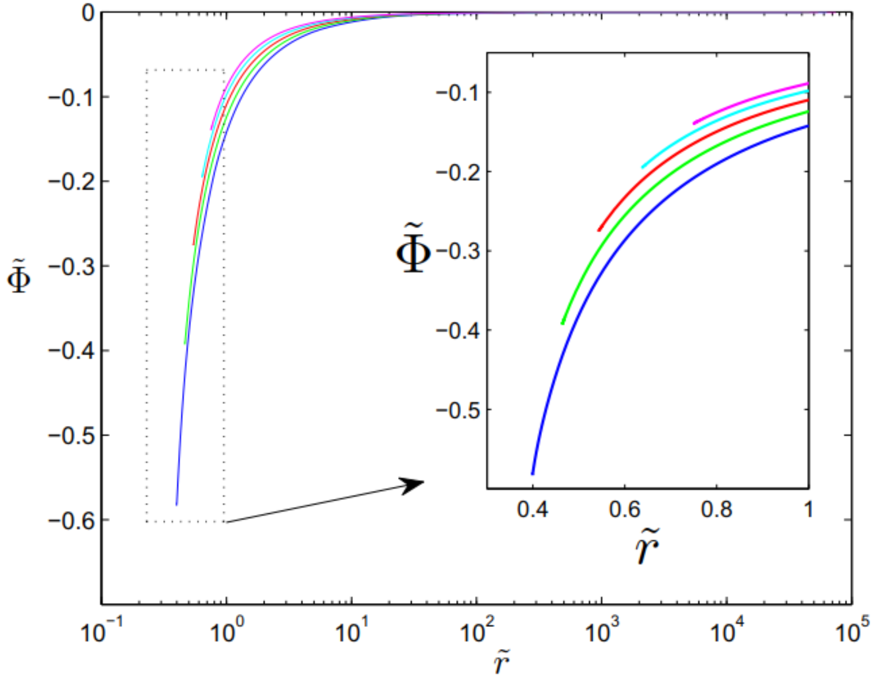
Note :

Hair Charge Q is not zero, and is not independent charge either.

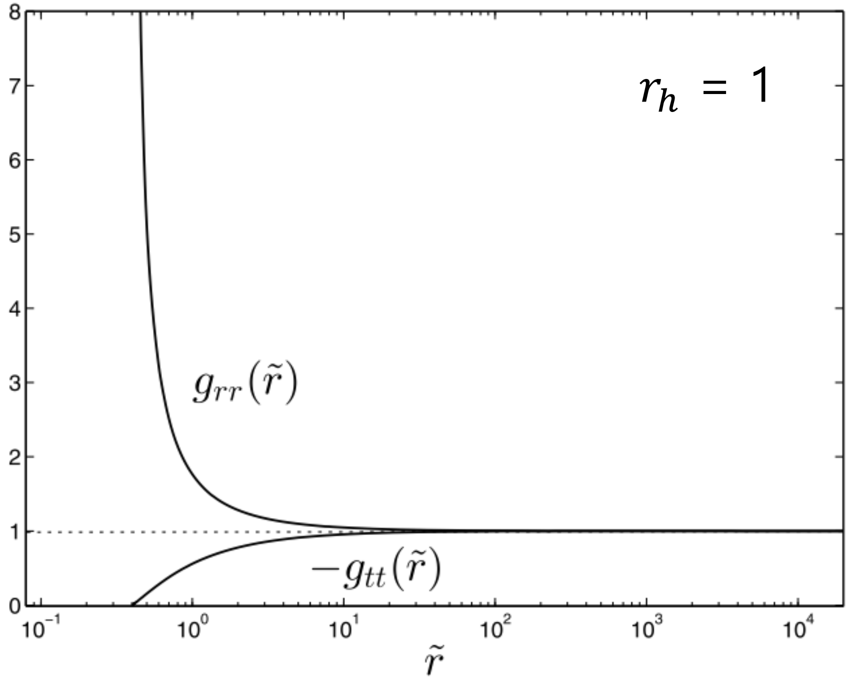
Numerical Construction of DEGB Black Hole solutions

an event horizon at $g^{rr}(r_h) = 0 \text{ or } g_{rr}(r_h) = \infty.$

the rescaling $\tilde{\Phi} = \Phi - \Phi_\infty \quad r \to \tilde{r} = r e^{\gamma \Phi_\infty /2} \quad M \to \tilde{M} = M e^{\gamma \Phi_\infty /2} \quad Q \to \tilde{Q} = Q e^{\gamma \Phi_\infty /2}.$



γ=1/6,
α=1/16

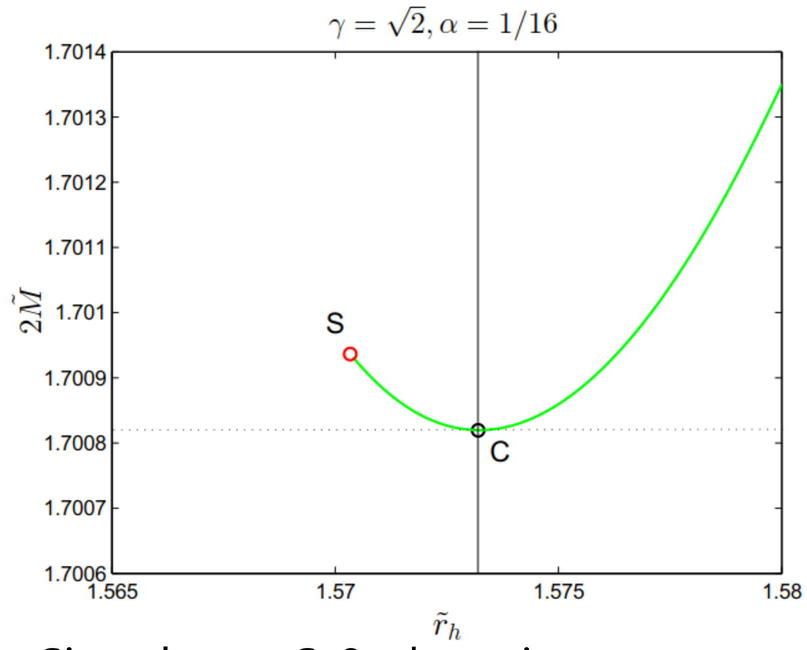


Note :

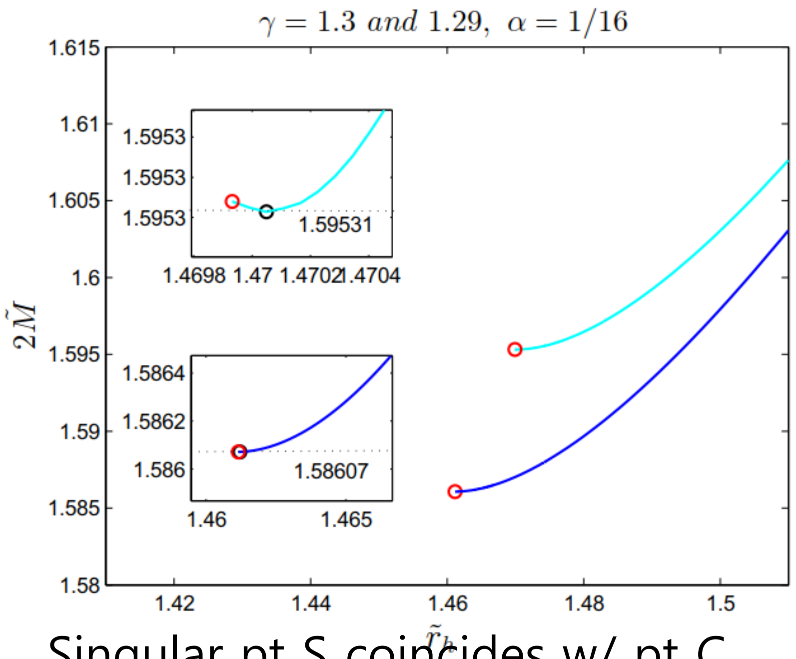
1. If DEGB black hole horizon becomes larger, the magnitude of the scalar field becomes smaller.
2. In the large horizon radius limit, the scalar field approaches zero, and then the black hole becomes a Schwarzschild black hole.

Coupling γ dependency of the minimum mass for fixed α 1/16.

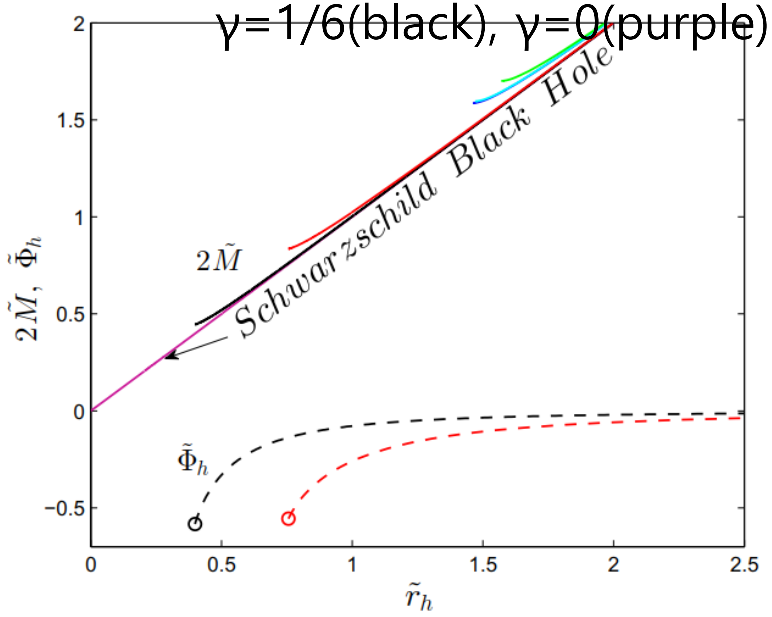
γ=√2(green), γ=1.3(cyan), γ=1. 29(blue), γ=1/2(red), γ=1/6(black), γ=0(purple)



Singular pt S & the min. mass C exist for γ = √2.



Singular pt S coincides w/ pt C btwn γ=1.29(blue) & 1.30(cyan). No lower branch below γ=1.29



As γ→0, the solution →Schw BH.

Note :

1. For large γ, sing. pt S & extremal pt C (with minimum mass $\widetilde{M}$) exist.

2. The solutions between point S and C are unstable for perturbations and end at the singular point S , (which saturates to equality in Eq. (18).) In other words, there are two black holes for a given mass in which the smaller one is unstable under perturbations.

3. As γ smaller, the singular point S gets closer to the minimum mass point C.

4. Below γ=1.29, the solutions are perturbatively stable and approach the Schwarzschild black hole in the limit of γ going to zero. These solutions depend on the coupling γ.

Q: How about the properties, such as

Stability
Implication to the cosmology
etc ?

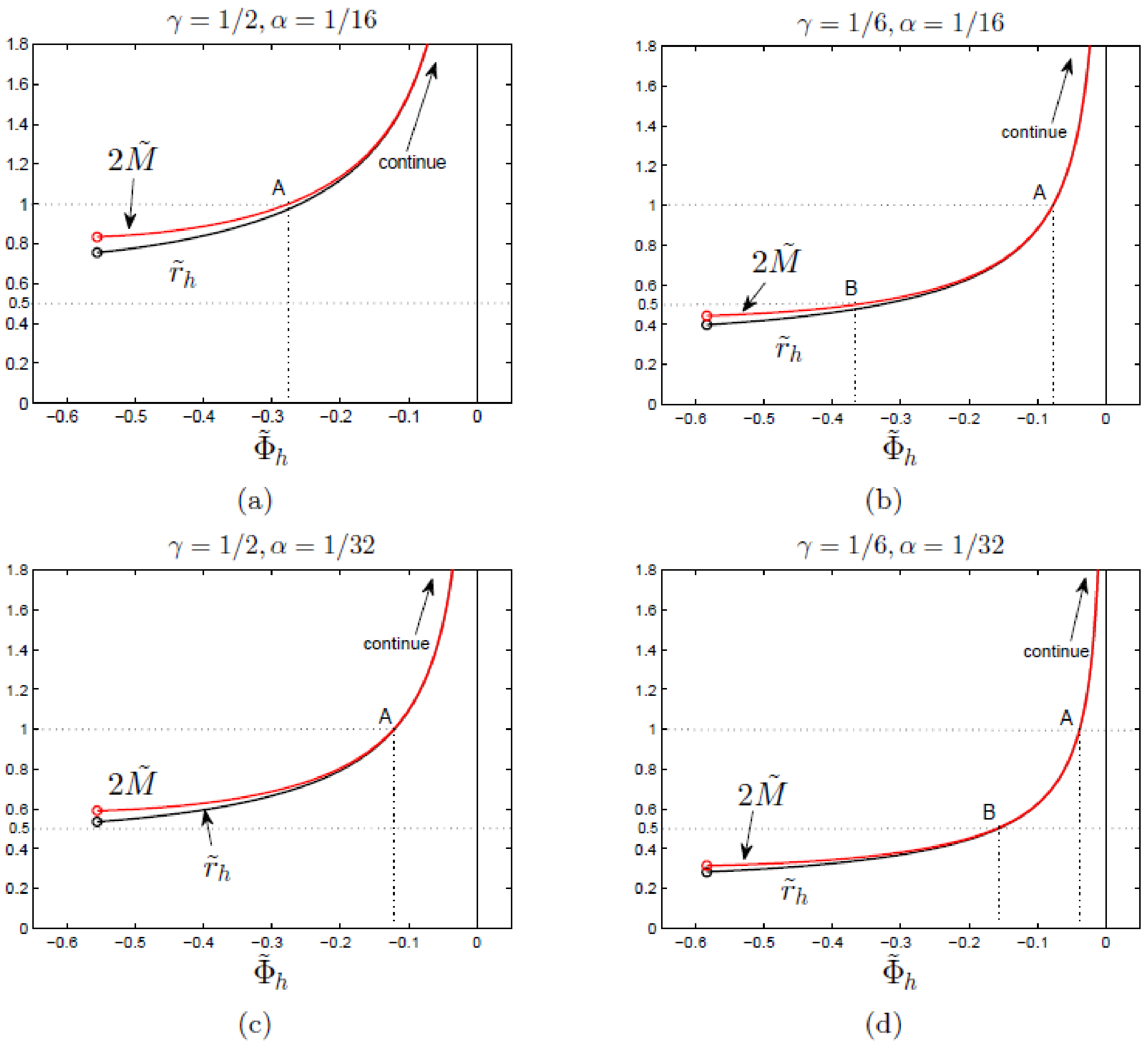


Figure 3: Rescaled black hole mass $\widetilde{M}$(black) and $\tilde{r}_h$ (red) with respect to $\widetilde{\Phi}_h$.
The hairy black hole solutions depends on coupling α and γ.

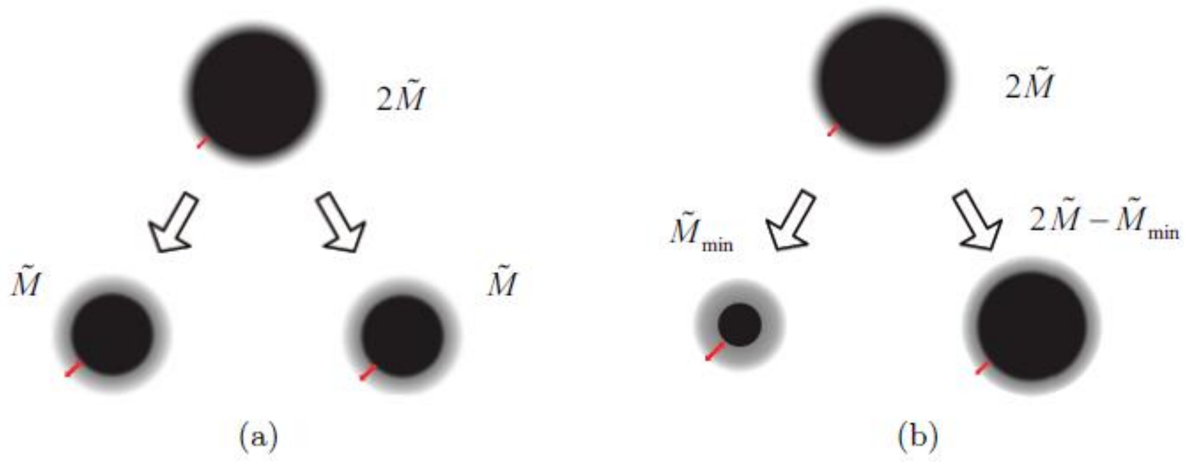
2. Black Hole Stability

perturbative

non-perturbative

Fragmentation instability is based on the entropy preference between the solutions.

Emparan and Myers, JHEP 0309, 025 (2003).



2.1 Perturbative Gravitational (in)stability

Perturbations of a black hole space-time

by adding fields or

by perturbing the metric.

The typical equations in the linear approximation :

$$-\frac{d^2R}{dr_*^2}+V(r,\omega)R=\omega^2R,$$

The quasinormal spectrum of a stable black hole is an infinite set of complex frequencies which describes damped oscillations.

If there is at least one growing mode, the space−time is unstable with the instability growth rate proportional to the imaginary part of the growing QNM.

4-dimensional BH : Perturbative Stability

Konoplya and Zhidenko, RMP (2011)
(arXiv:1102.4014)

Most of the 4-dim. black holes proved to be stable.
(Sch, SdS, SAdS, RNdS, Kerr, KdS, KAdS,

Extreme Kerr & RN BHs are unstable.

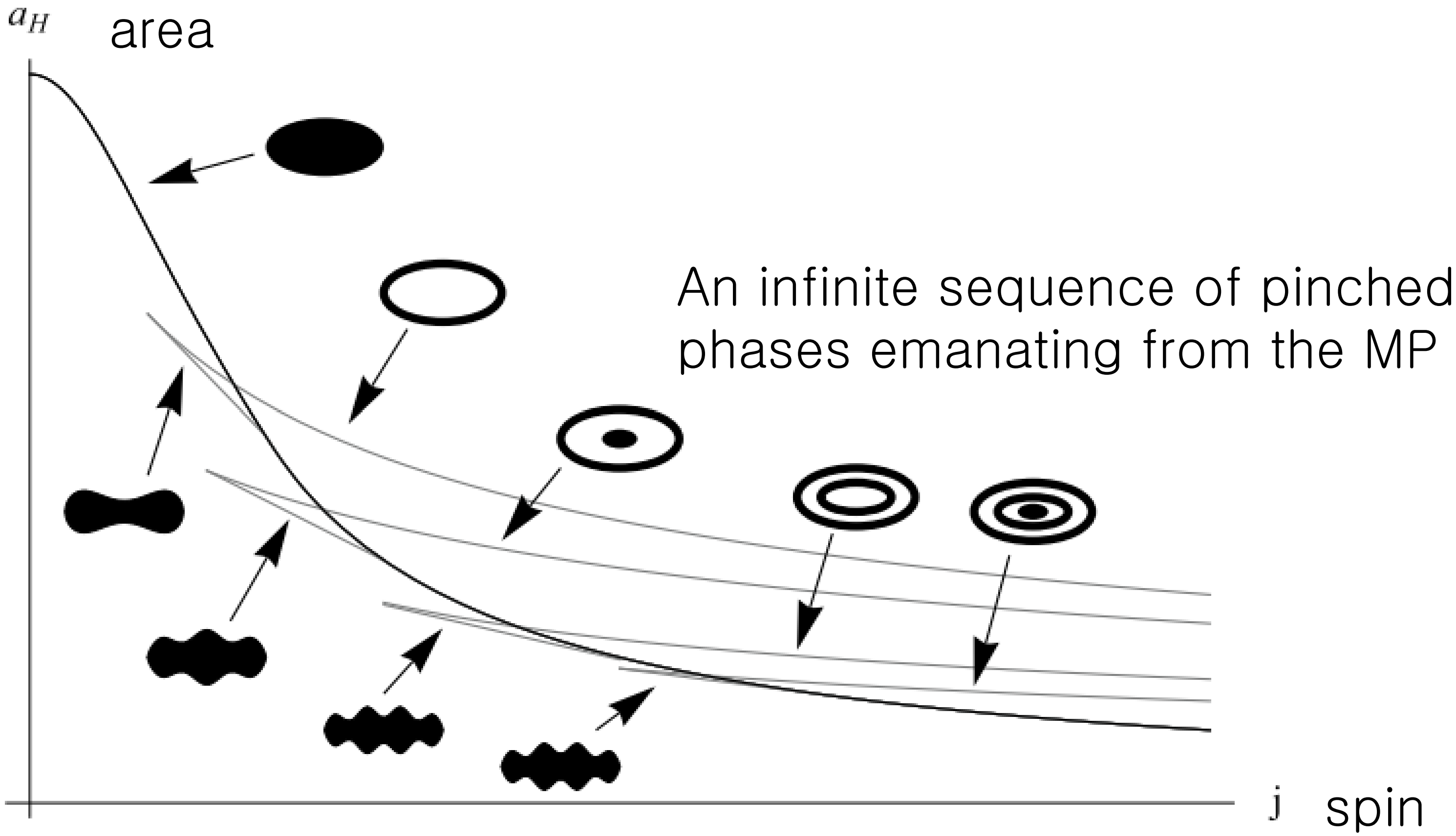
Higher (D ≥ 5) dim BH & stability

wide class of objects : black strings, black branes, black ring, saturn, etc.

There exists various instabilities : (non) Gregory-Laflamme instabilities, etc.

Q : How about nonperturbative stability?

The qualitative phase diagram for the black objects in D ≥ 6



An infinite sequence of pinched black hole phases emanating from the MP curve.

If thermal equilibrium is not imposed, multi-rings are possible in the upper region of the diagram.

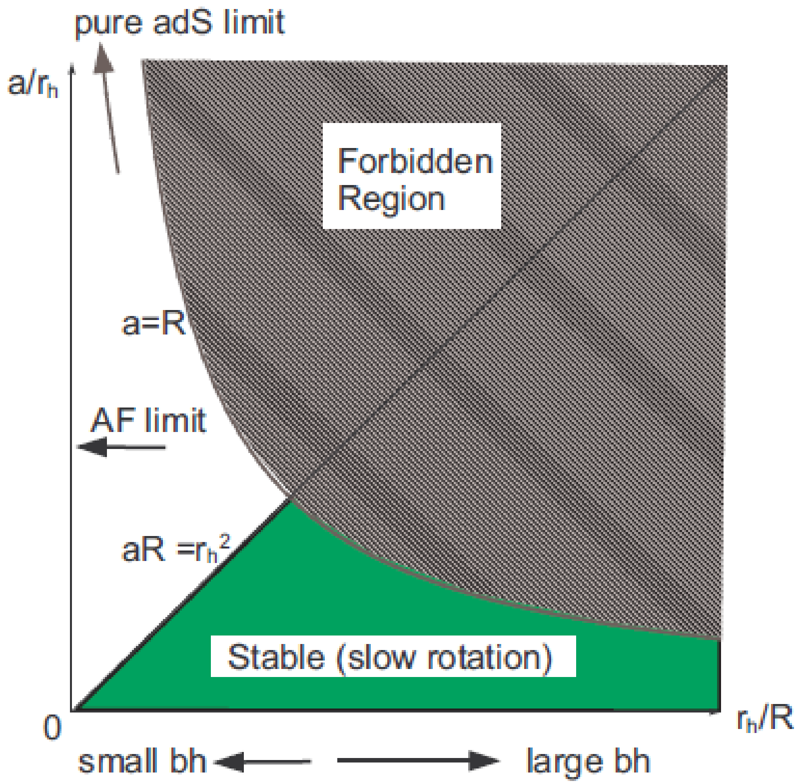
Black hole (perturbative) stability in Einstein gravity (Higher dim.)

Schwarzschild BH: stable

AdS Schwarzschild BH: stable

Kerr BH: stable for superradiance

AdS Kerr BH: stable in green region
→ superradiance instability

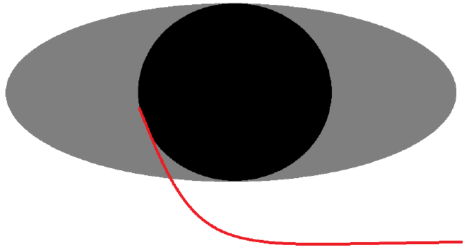


In Hideo Kodama, R.A. Konoplya, Alexander Zhidenko, Phys.Rev. D79 (2009) 044003

RN BH is stable under neutral and charged perturbations.
The stability of RN-AdS blackhole depends on the regions.

Particle absorption : Thermodynamics of Rotating AdS Black Holes:

Based on arXiv: 1106.1483(PRD), 1202.2244(CQG),



Kerr Black Hole / Myers-Perry BH with single rotation

As a result of Particle absorption, the black hole entropy changes

$$S_{\rm BH} = \frac{1}{4}\Omega_{N-2} M_{\rm ir}^2, \qquad \delta S_{\rm BH} = \frac{4\pi r_h |\Sigma p^r|}{(N-3)r_h^2 + (N-5)a^2},$$

•The equality: radial momentum=0 at the horizon.

MP BHs with multi-rotation

•Consider the two types of values of Angular momenta: a_i…; a, a, a,…, b, b, b,…

•The particle changes black holes:

•Irreducible mass:

$$S_{\mathrm{BH}} = \frac{1}{4}\Omega_{N-2}M_{\mathrm{ir}}^2, \qquad \delta S_{\mathrm{BH}} = \frac{4\pi r_h^3 |p^r| \rho^2}{r_h^2(2b^2(-1+m) + r_h^2(-2+2n-\epsilon)) - a^2(2b^2 + r_h^2(2+2m-2n+\epsilon))} \geq 0.$$

Note :

The entropy and irreducible mass have a one-to one correspondence.

The particle absorption can change the entropy and irreducible mass.

The changes in entropy and irreducible mass are only proportional to the particle radial momentum pr at the horizon.

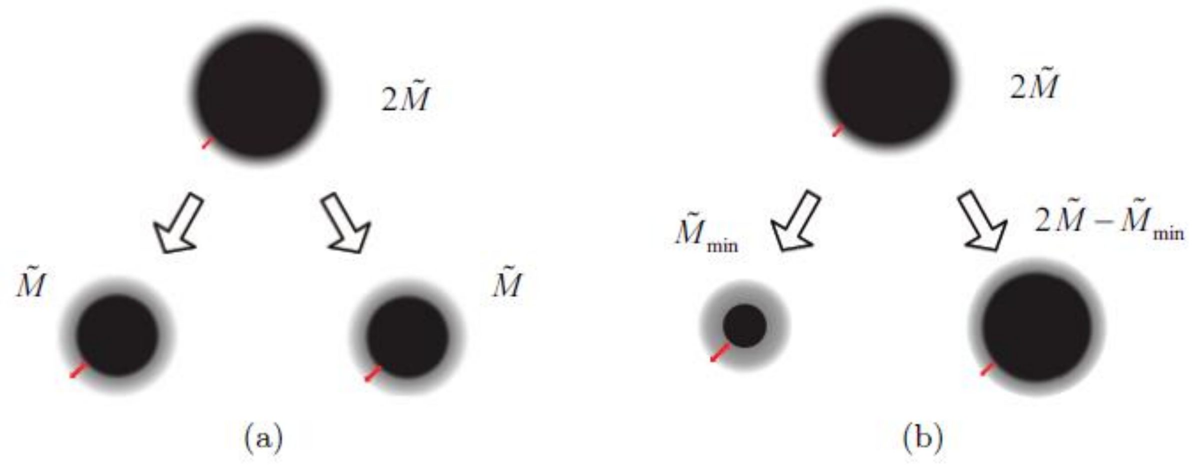
The particle radial momentum increases the black hole energy degeneracy, the entropy.

If the particle rotates in the same direction as the black hole, the black hole mass is increased.
For a particle with a nonzero radial momentum, the entropy and irreducible mass are increased.
interpreted as an irreversible process.

If the particle rotates in the opposite direction to the black hole, the black hole mass can decrease.
The black hole mass increases when the radial momentum is bigger than $\frac{a|L|}{\Sigma}$
and decreases when smaller.
For a particle with a nonzero radial momentum, pr, the entropy and irreducible mass still increase as an irreversible process.

3. Fragmentation Instability

Black holes may break apart into smaller black holes .
The initial phase is a single black hole.
The final phase is two black holes far from each other.



Apply thermodynamic 2nd law to initial (one black hole) and final(fragmented two black holes) phase.

entropy of 1 BH < entropy of 2 fragmented BHs

$\longrightarrow$ (transition to) instability

Investigation to unstable parameter region which is stable under linear perturbation.

Myers–Perry blackhole : Rotating Black hole in higher dimensions
There doesn’t exist any upper limit on the angular momentum

Myers–Perry blackhole becomes unstable for large angular momentum into fragmentation.

RN blackhole is also thermodynamically unstable in specific parameter region.

Instability of a charged AdS black hole
→ Instability of Schwarzschild-AdS black hole
Also the flat limit

The fragmentation instability of hairy black holes in the theory with a Gauss-Bonnet term in asymptotically flat space time

Fragmentation allows the upper or lower bound of black hole charges.
B. Gwak and B.-H. Lee, arXiv:1405.2803 PRD91 (2015) 6, 064020.

Mass ratio δ is the mass ratio 0≤δ ≤ ½ .
The minimum mass ratio has a finite value in DGB Black Hole, because the black hole has minimum mass $\tilde{M}_{min}$.
The black holes can be fragmented only when it exceeds twice of minimum mass. Black holes with mass below twice of minimum mass are absolutely stable.
Effects of Gauss-Bonnet terms to the Black Holes.

The initial phase decays to the final phase if the entropy is larger than that of the initial phase.

For Schwarzschild black hole

$$\frac{S_f}{S_i} = \frac{(\delta\,\tilde{r}_h)^2 + ((1-\delta)\,\tilde{r}_h)^2}{\tilde{r}_h^2} = \delta^2 + (1-\delta)^2\,, \tag{22}$$

The entropy ratio is always smaller than 1.
Therefore, a Schwarzschild black hole is always stable under fragmentation.
The entropy ratio marginally approaches 1 in
$\delta \to 0,$

These phenomena become different in the theory with the higher order of curvature term.

For a black hole in EGB theory

The EGB black hole solution is the same as the Schwarzschild one. However, the GB term contributes to the black hole entropy and influence stability.

The initial black hole entropy is

$$S_i = \frac{A_H}{4G}\left(1 + \frac{8\alpha\kappa}{\tilde{r}_h^2}\right) = \frac{\pi}{G}\left(\tilde{r}_h^2 + 8\alpha\kappa\right) . \tag{24}$$

Unlike Schwarzschild black holes, the fragmentation instability occurs depending on the fragmentation ratio δ. For the case of $(\delta, 1-\delta)$ fragmentation, the final phase entropy is given

$$S_f = \frac{\pi}{G}\left((\delta\tilde{r}_h)^2 + 8\alpha\kappa\right) + \frac{\pi}{G}\left(((1-\delta)\tilde{r}_h)^2 + 8\alpha\kappa\right) . \tag{25}$$

The EGB black hole is unstable if,

$$\frac{S_f}{S_i} = \frac{\left((\delta\tilde{r}_h)^2 + 8\alpha\kappa\right) + \left(((1-\delta)\tilde{r}_h)^2 + 8\alpha\kappa\right)}{\left(\tilde{r}_h^2 + 8\alpha\kappa\right)} > 1 . \tag{26}$$

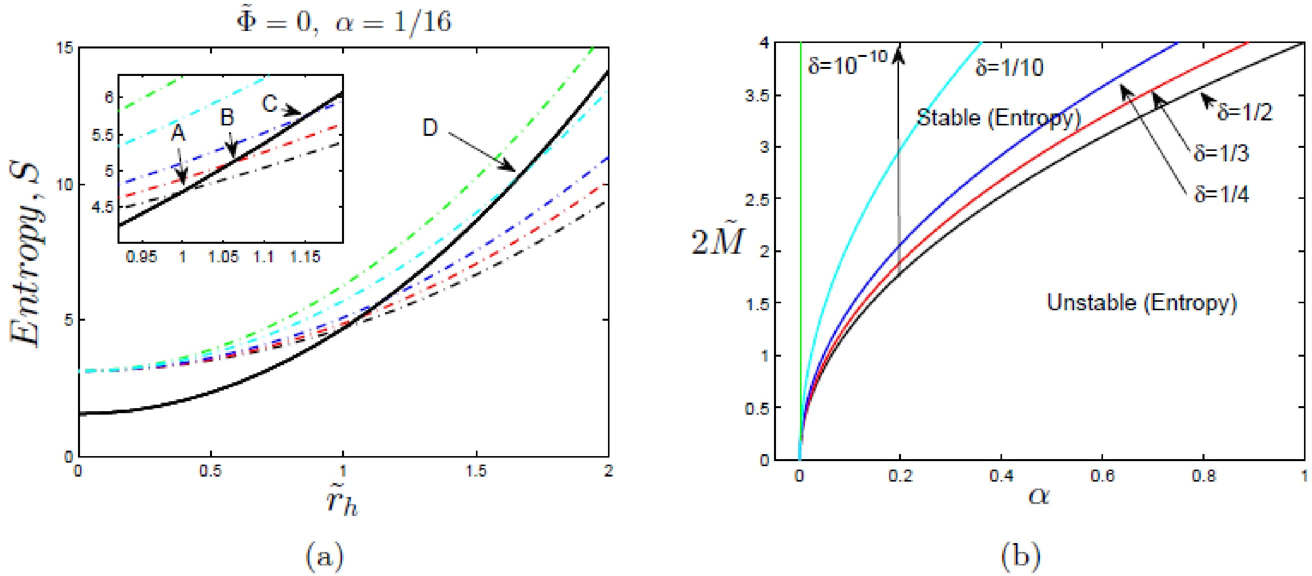


Figure 4: (a) Fragmentation ratio and EGB black hole entropy. The black solid line is initial phase entropy. The black, red, blue, cyan, and green dashed-dot lines are cases of $(\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{3}, \frac{2}{3})$, $(\frac{1}{4}, \frac{3}{4})$, $(\frac{1}{10}, \frac{9}{10})$, and $(10^{-10}, 1-10^{-10})$. The crossing points go up from point A to D with changing δ. We fix κ=1. (b) Phase diagram of EGB black hole for δ=1/2 (black solid line), δ=1/3 (red solid line), δ=1/4 (blue solid line), δ=1/10 (cyan solid line) and δ=10^{-10} (green solid line) fragments corresponding to crossing points between initial and final phase of the black hole entropy in each color of line same to those in figure (a).

The mass ratio can have continuous values, and the black hole has stable and unstable phases. The minimum unstable region is at $\delta = \frac{1}{2}$. For the limit of $\delta \to 0$, all of EGB black holes become unstable for fragmentation as shown in 4(b).

For a black hole in DGB theory

The DGB black hole has a GB term coupled with a scalar field, so additional entropy correction comes from the higher curvature term. The DGB black hole entropy is

$$S = \frac{\pi \tilde{r}_h^2}{G}\left(1 + \frac{8\alpha\kappa}{\tilde{r}_h^2} e^{-\gamma\tilde{\Phi}_h}\right), \tag{28}$$

where a EGB black hole case corresponds to $\gamma = 0$. The DGB black hole entropy ratio between the initial and the final entropy including the higher-curvature corrections

$$\frac{S_f}{S_i} = \frac{\left((\delta\tilde{r}_h)^2 + 8\alpha\kappa e^{-\gamma\tilde{\Phi}_\delta}\right) + \left(((1-\delta)\tilde{r}_h)^2 + 8\alpha\kappa e^{-\gamma\tilde{\Phi}_{1-\delta}}\right)}{\left(\tilde{r}_h^2 + 8\alpha\kappa e^{-\gamma\tilde{\Phi}_h}\right)}, \tag{29}$$

In the large mass limit $\tilde{r}_h \gg 1$, the entropy ratio becomes that of Schwarschild case,

$$\frac{S_f}{S_i} = \delta^2 + (1-\delta)^2 < 1. \tag{30}$$

Thus, massive DGB black holes are stable under fragmentation. The small mass limits are bounded to $\tilde{M}_{min}$. DGB black holes of mass $\tilde{M}_{min}$ are absolutely stable, because there are no fragmented black hole solutions. Larger than $\tilde{M}_{min}$, the black hole stability is dependent on an entropy correction term. The entropy ratio is given

$$\frac{S_f}{S_i} = \frac{\delta^2 + (\delta-1)^2 + \frac{8\alpha\kappa e^{-\gamma\bar{\Phi}_\delta} + 8\alpha\kappa e^{-\gamma\bar{\Phi}_{1-\delta}}}{\tilde{r}_h^2}}{1 + \frac{8\alpha\kappa e^{-\gamma\bar{\Phi}_h}}{\tilde{r}_h^2}}, \tag{31}$$

where the horizon radius square term is important in the small black hole. The entropy ratio may increase in smaller mass like EGB black holes, but there is ambiguity since DGB black holes have a minimum mass. In this part, there is no proper approximation to describe the instabilities of small mass DGB black holes. It should be pointed out through numerical calculation. Also, the minimum mass bounds the fragmentation mass ratio. It is not seen in the Schwarzschild black hole or EGB black hole. The DGB black holes have more variety properties and behaviors. We will obtain detailed behaviors through the numerical calculation.

Fragmentation Instability for DGB Black Holes

We investigate the fragmentation instability using a numerical analysis.

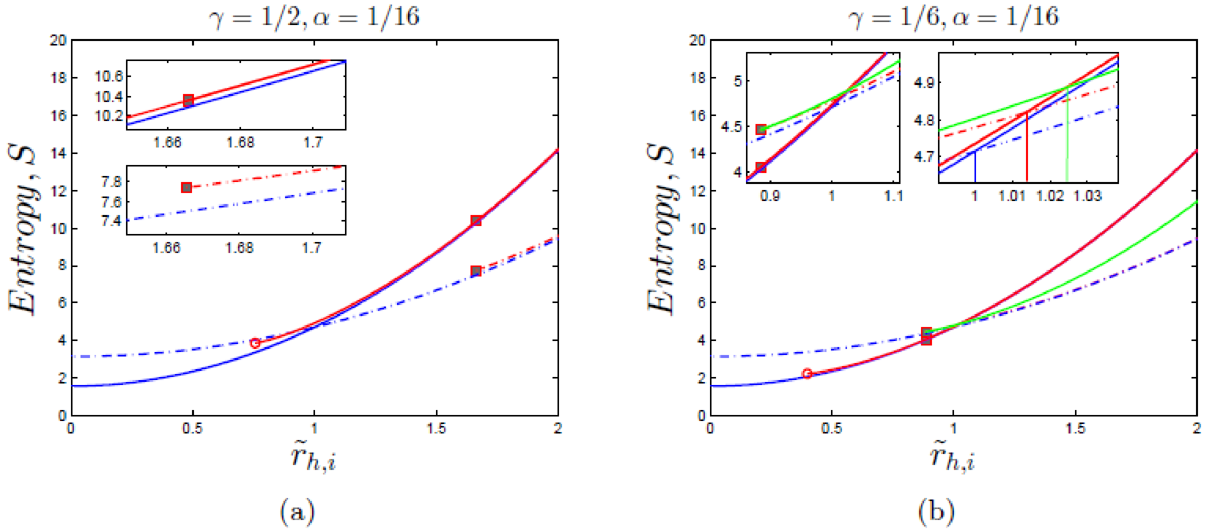
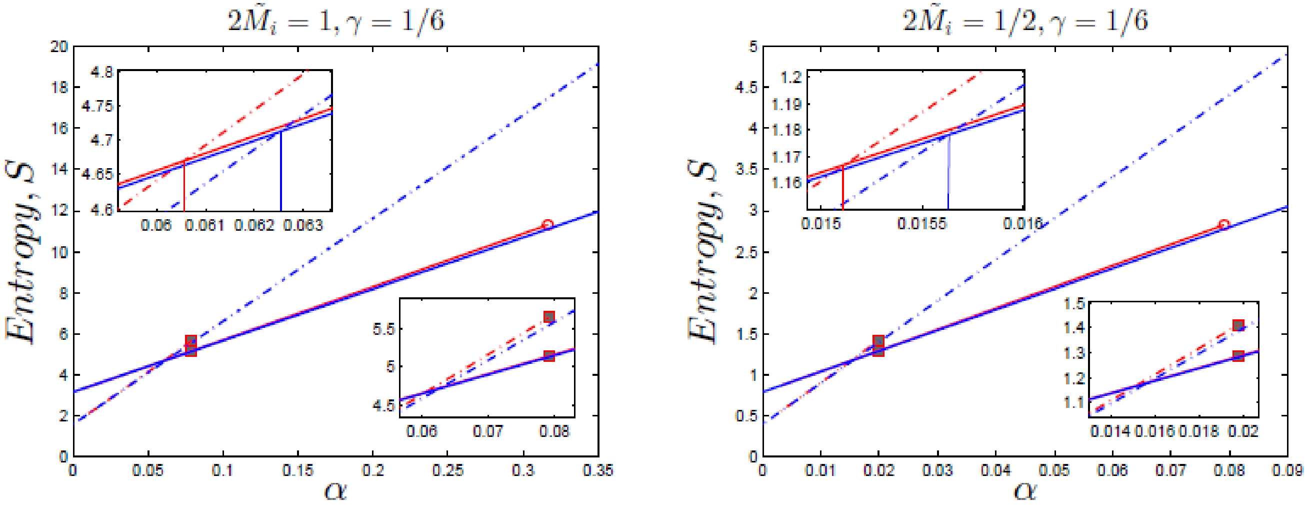


Figure 6: The initial and final phase entropies with respect to $r_{h,i}$ for given couplings γ and α. The blue solid line and blue dashed-dot line are initial and final phase entropies in EGB theory as a reference for $(\frac{1}{2}, \frac{1}{2})$. The red solid line and red dashed-dot line are initial and final phase entropies in DGB theory for $(\frac{1}{2}, \frac{1}{2})$. Initial phase exists above red circle for minimum mass. Final phase exists above red box for $(\frac{1}{2}, \frac{1}{2})$. The green solid line represents fragmentation for marginal mass ratio $\bar{\delta}$.



(a) (b)

Figure 7: Both the initial and the final phase entropy with respect to α for given initial mass $\widetilde{M}$ and coupling γ. The blue solid line and blue dashed-dot line are initial and final phase entropies in EGB theory as a reference. The red solid line and red dashed-dot line are initial and final phase entropies for $(\frac{1}{2}, \frac{1}{2})$ fragmentation in DGB theory. Initial phases exist below the red circle. Final phases exist below the red box.

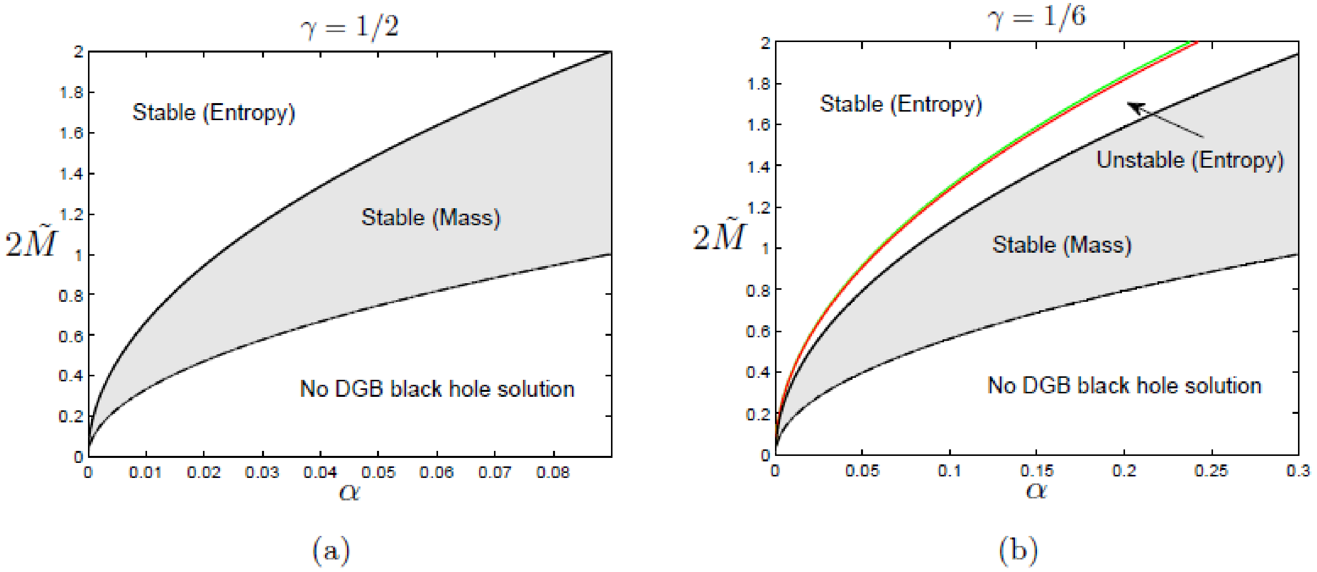


Figure 8: The phase diagrams with respect to α and $\widetilde{M}$ in fixed γ. The red solid line represents $(\frac{1}{2}, \frac{1}{2})$ fragmentation. The green solid line represents $(\bar{\delta}, 1-\bar{\delta})$ fragmentation.

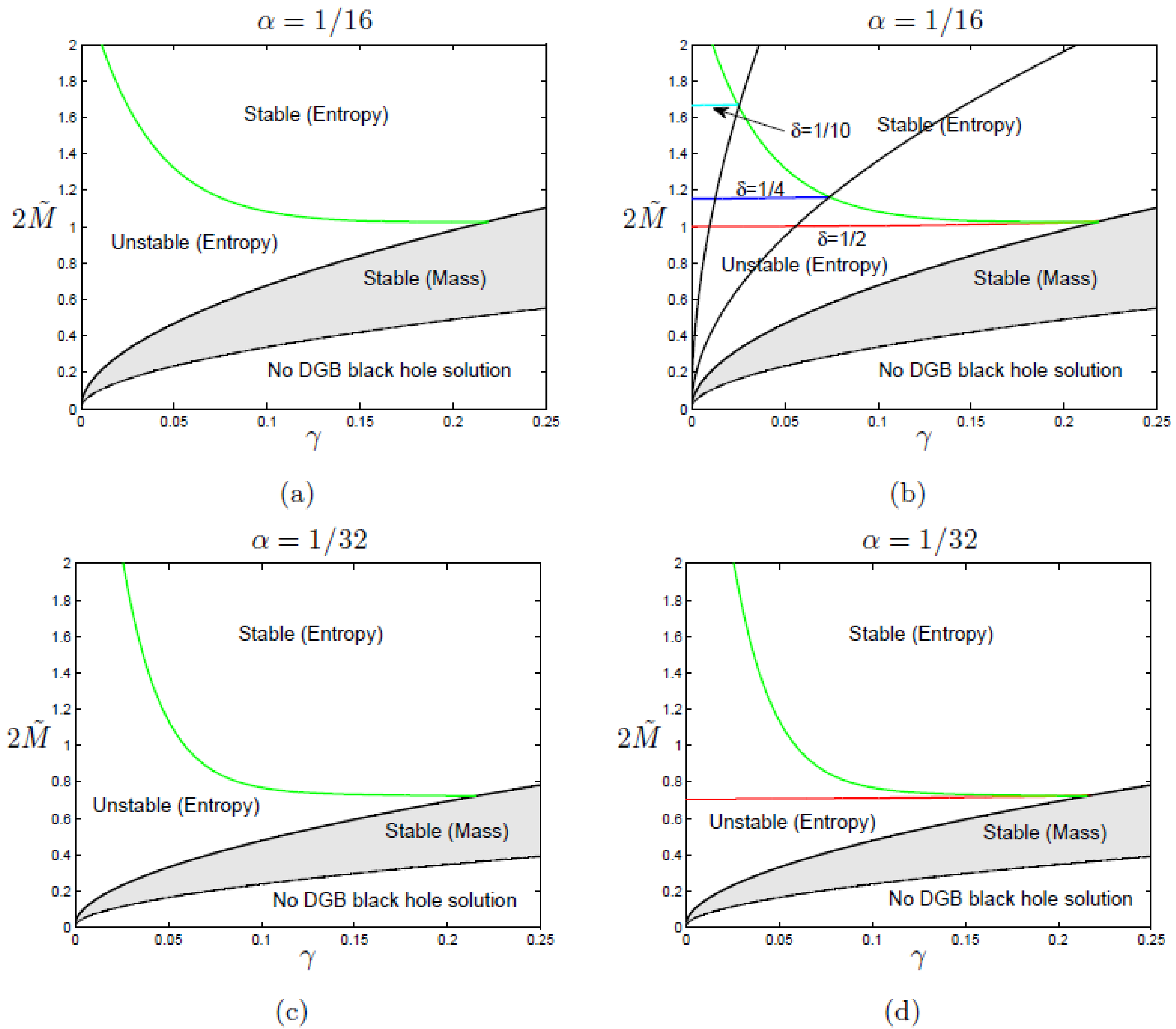


Figure 9: The phase diagrams with respect to γ and $\tilde{M}$ in fixed α for $(\frac{1}{2}, \frac{1}{2})$ (red solid line), $(\frac{1}{4}, \frac{3}{4})$(blue Solid line), $(\frac{1}{10}, \frac{9}{10})$(cyan solid line) and $(\bar{\delta}, 1-\bar{\delta})$(green solid line) fragmentation.

Summary

We have studied some of the Black Hole properties.

Theory with Gauss-Bonnet term in asymptotically flat spacetime

- Only hairy black holes exist.
- There exists the **minimum mass**.

When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

The amount of black hole hair decreases as the DEGB black hole mass increases. DEGB black hole configurations has the smooth limits to the EGB and Schwartz black hole cases

The DEGB black hole phase is unstable under fragmentation, even if these phases are stable under perturbation.

We have found the phase diagram of the fragmentation instability for a black hole mass and two couplings.

Cf. Cosmological implication in Inflationary Universe also studied (though not shown).

Congratulations
and
all the best to Spenta !