Black Hole Dynamics at large D

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References

- Talk based on

  ArXiv 1504.06613 S. Bhattacharyya, A. De, S.M, R. Mohan, A. Saha

  ArXiv 1511.03432 S. Bhattacharyya, M. Mandlik, S.M and S. Thakur

  ArXiv 1607.06475 Y. Dandekar, A. De, S. Mazumdar, S.M.

  ArXiv 1609.02912 Y. Dandekar, S. Mazumdar, S.M. and A. Saha

  ArXiv 1611.09310 S. Bhattacharyya, A. Mandal, M. Mandlik, U. Mehta, S.M., U. Sharma , S Thakur
- And ongoing work

  Membrane in more general backgrounds

  S. Bhattacharyya, P. Biswas, B. Chakrabarty, Y. Dandekar, S. Mazumdar, A. Saha

  Love Numbers

  Y. Dandekar, S. Mazumdar S.M. and A. Saha
- Builds on observations and earlier work

  Early work (qnms + . . .) Emparan, Suzuki, Tanabe (EST) and collaborators
- Other related recent work:

  About 9 papers ArXiv 1504.06489...1605.08854 T, ST, EST + collaborators.

Introduction

- In this talk I study the classical dynamics of objects that have always fascinated Spenta: black holes. Classical black hole dynamics is well understood in principle. Governed by the classical vacuum Einstein or Einstein Maxwell equations with no other matter.
- While the equations of classical black hole dynamics are easy to state they are difficult to solve in sufficiently dynamical situations, e.g. those that are of interest to LIGO.
- In order to make progress I change the problem. I the dynamics of black holes in D dimensions (rather than 4 dimensions) and then take D to be large. While very different in detail, the $1/D$ expansion I will use is similar in spirit to the $1/N$ expansion that Spenta has used so effectively for over 35 years in his studies of gauge thoeries, matrix models (and black holes).

Chief Result

- In this talk I will not have the time to even outline derivations. I will simply state our main results and try to explain their consequences
- In the limit of a large number of dimensions, - and under certain conditions - the classical dynamics of black holes is equivalent or 'dual' to the following non gravitational problem.
- Consider flat non gravitational spacetime with one or more holes eaten out of it. The boundary of these holes is a codimension one surface we call the membrane. The shape of the membrane is dynamical; in addition the membrane carries a charge and world volume velocity field.
- The membrane carries a charge current and stress tensor that are functions of the charge shape, charge density and velocity field on the membrane. These currents are given, to leading and first subleading order in $1/D$ by

Stress Tensor and Charge Current

$$8\pi T_{AB} = \left(\frac{\mathcal{K}}{2}\right) u_A u_B - \left(\frac{\nabla_A u_B + \nabla_B u_A}{2}\right) + \left(\frac{1}{2}\right) K_{AB}$$
$$+ \frac{1}{2}\left(u_A\left(\frac{\nabla_B \mathcal{K}}{\mathcal{K}} + \nabla^2 u_B\right) - u_B\left(\frac{\nabla_A \mathcal{K}}{\mathcal{K}} + \nabla^2 u_A\right)\right)$$
$$- \mathcal{P}_{AB}\left(\frac{1}{2} u \cdot K \cdot u + \frac{1}{2}\frac{\mathcal{K}}{D} - \frac{K^{MN}\left(\nabla_M u_N + \nabla_N u_M\right)}{2\mathcal{K}}\right)$$

$$J^A = \left[Q\mathcal{K}u^A - \left(\frac{\mathcal{K}}{D}\right)\left(Q(u\cdot\partial)u_C + \partial_C Q\right)\mathcal{P}^{CA}\right]$$

Equations of Motion

The stress tensor and charge current of this ‘membrane at the boundary of the universe; are both conserved. Conservation implies that the dynamical membrane fields obey the following equations of motion

$$
\begin{aligned}
&\nabla . u = 0 \\
&\left(\frac{\nabla^2 u}{\mathcal{K}} - (1 - Q^2) \frac{\nabla \mathcal{K}}{\mathcal{K}} + u \cdot K - (1 + Q^2)(u \cdot \nabla) u \right) \cdot \mathcal{P} = 0, \\
&\frac{\nabla^2 Q}{\mathcal{K}} - u \cdot \nabla Q - Q \left(\frac{u \cdot \nabla \mathcal{K}}{\mathcal{K}} - u \cdot K \cdot u \right) = 0,
\end{aligned}
$$

Totally D equations. Same as number of variables. The equations of motion appear be a well posed dynamical system, i.e. appear to define a good initial value problem.

Membrane equations at subleading order

- Our membrane equations can be corrected systematically order by order in the $1/D$ expansion. At next subleading order in this expansion we find

$$\nabla \cdot u = \frac{1}{2\mathcal{K}}\left(\nabla_{(A}u_{B)}\nabla_{(C}u_{D)}\mathcal{P}^{BC}\mathcal{P}^{AD}\right)$$

and

$$\left[\frac{\nabla^2 u_A}{\mathcal{K}} - \frac{\nabla_A \mathcal{K}}{\mathcal{K}} + u^B K_{BA} - u \cdot \nabla u_A\right]\mathcal{P}^A_C$$
$$\left[\left(-\frac{u^C K_{CB} K^B_A}{\mathcal{K}}\right) + \left(\frac{\nabla^2\nabla^2 u_A}{\mathcal{K}^3} - \frac{u \cdot \nabla\mathcal{K}\nabla_A\mathcal{K}}{\mathcal{K}^3} - \frac{\nabla^B\mathcal{K}\nabla_B u_A}{\mathcal{K}^2} - 2\frac{K^{CD}\nabla_C\nabla_D u_A}{\mathcal{K}^2}\right)\right.$$
$$\left(-\frac{\nabla_A\nabla^2\mathcal{K}}{\mathcal{K}^3} + \frac{\nabla_A\left(K_{BC}K^{BC}\mathcal{K}\right)}{\mathcal{K}^3}\right) + 3\frac{(u\cdot K\cdot u)(u\cdot\nabla u_A)}{\mathcal{K}} - 3\frac{(u\cdot K\cdot u)(u^B K_{BA})}{\mathcal{K}}$$
$$\left.6\frac{(u\cdot\nabla\mathcal{K})(u\cdot\nabla u_A)}{\mathcal{K}^2} + 6\frac{(u\cdot\nabla\mathcal{K})(u^B K_{BA})}{\mathcal{K}^2} + \frac{3}{(D-3)}u\cdot\nabla u_A - \frac{3}{(D-3)}u^B K_{BA}\right]\mathcal{P}^A_C = 0$$

Nature of Duality

- Each of the solutions of the equations of motion described above describes the dynamics of a bunch of holes eaten out of spacetime. Each such motion is dual to the dynamics of a collection of black holes at large D provided that the membranes configurations vary on length scales unity rather than, e.g. $1/D$ and provided also that the membrane does not crumple in too many directions at any one point. The last condition is met, for instance, in a membrane configuration that preserves $SO(D-p-3)$ invariance with p held fixed as $D \to \infty$.
- The holes in spacetime in our membrane solutions map to the interior regions of the black holes. The boundary of these holes - the membrane itself - maps to the event horizon of actual black hole solutions.

Nature of Duality

- The black hole solution dual to any particular membrane motion is obtained as follows. At fractional distance $\rho/r_0 \gg \frac{1}{D}$ away from the membrane surface the deviation of the metric from flat space turns out to be small, and is well described by linearized Einstein gravity. The corresponding linearized solution is obtained in the usual manner, by convoluting the membrane stress tensor against the appropriate retarded Greens function. r_0 is the local Schwarzshild radius.
- When $\rho/r_0 \ll 1$, on the other hand, the metric is not a small deviation from flat space but there is a new simplification. In this region the spacetime metric is locally determined by the corresponding membrane degrees of freedom in a very explicit manner presented in our papers.

Nature of Duality

- In the region
$$\frac{1}{D} \ll \frac{\rho}{r_0} \ll 1$$
both the approximations described in the previous slide apply and they agree there.
- It is very important that the metric deviates significantly away from that of flat space is a shell of thickness $\frac{1}{D}$ around the black hole. In particular the membrane world volume - which is time like when viewed as a submanifold of flat space - is null when viewed as a submanifold of the true metric - as had to be the case given that it is the event horizon.

Greens functions at large D

- Metric in linearized region given by convolution of the retarded Green's function against the stress tensor. We now study the Greens function for ∇^2 in D dimensions. Work in Fourier space in time but coordinate space in the spatial coordinates.
- $G_\omega(r)e^{-i\omega t}$. Let $G_\omega(r) = \psi_\omega(r)/r^{-(D-3)/2}$. Away from $r = 0$ ψ obeys the equation

$$-\partial_r^2\psi_\omega + \frac{(D-4)(D-2)}{4r^2}\psi_\omega + \omega^2\psi_\omega = 0$$

- Effective Schrodinger problem with $\bar{h}^2/2m = 1$, $E = \omega$ and

$$V(r) = \frac{(D-4)(D-2)}{4r^2}$$

Decay and radiation

- Potential is positive and of order $\mathcal{O}(D^2)$. At energies ω of order unity the Greens function has to tunnel through a very high potential barrier and so decays rapidly, explaining why the linearized approximation works well for $\rho/r_0 \gg \frac{1}{D}$.
- The decay continues untill $r = \frac{D}{2\omega}$. At this value of r the mode finally emerges and begins to propagate as radiation. At ω of order unity the amplitude of this mode is severely suppressed by the tunneling, and turns out to be of order $\frac{1}{D^D}$.
- Note that there is no suppression for $\omega \sim D/r_0$. So processes that occur over time scales $1/D$ can lead to a substantial emission of radiation.

Membrane Paradigm

- The fact that the interior of the black hole is simply a hole in space time with no degrees of freedom living on it is simply a reflection of the fact that the black hole interior is causally disconnected from the exterior and so has no impact on anything an external observer can see at any future time.
- The novelty at large D is that the balck hole metric to the exterior of the event horizon is truely nontrivial only over a small thickness of order $1/D$. The membrane equations are the almost decoupled (fluid gravity like) description of this thin sliver of spacetime. Our large D membrane paradigm is a little like a classical version of the AdS/CFT correspondence. The parameter that ensures decoupling is $\frac{1}{D}$ rather than low energies. Radiation represents the failure of decoupling and occurs at order $1/D^D$.

Quasinormal Modes about RN black holes

- Simplest solution: static spherical membrane. Dual to RN black hole. Linearizing the membrane equations yields

$$
\begin{aligned}
\omega^r_{l=0} &= 0 \\
r_0\omega^r_l &= \frac{-i(l-1) \pm \sqrt{(l-1)(1-lQ_0^4)}}{1+Q_0^2} \quad (l \geq 1) \\
r_0\omega^Q_l &= -il \quad (l \geq 0) \\
r_0\omega^v_l &= \frac{-i(l-1)}{1+Q_0^2} \quad (l \geq 1)
\end{aligned}
\tag{1}
$$

- Note highly dissipative. Can compare with direct gravity analysis of QNMs at large D. Turns out two kinds of modes. Light, $\omega \sim \frac{1}{r_0}$. Heavy, $\omega \sim \frac{D}{r_0}$. Spectrum above in perfect agreement with light modes. Our membrane equations: nonlinear effective theory of light modes obtained after 'integrating out' the heavy stuff.

Zero modes

- $$\begin{aligned} \omega^r_{l=0} &= 0 \\ r_0\omega^r_l &= \frac{-i(l-1) \pm \sqrt{(l-1)(1-lQ_0^4)}}{1+Q_0^2} \quad (l \geq 1) \\ r_0\omega^Q_l &= -il \quad (l \geq 0) \\ r_0\omega^v_l &= \frac{-i(l-1)}{1+Q_0^2} \quad (l \geq 1) \end{aligned} \tag{2}$$

- $\omega^r_{l=0}$, $\omega^r_{l=1}$ $\omega^Q_{l=0}$ and $\omega^v_{l=1}$ are all zero modes. Each has a simple physical interpretation - exact zero mode. All other modes dissipate away. Dynamics very stable about static spheres.

Dissipation and the Entropy Current

- The highly dissipative nature of our membrane equations is related to the fact that membrane equations are thermodynamical in nature and obey a local version of the second law. Using our explicit construction of the metric dual to any membrane motion we find
- $$J^S_M = \left(I + \mathcal{O}\left(\frac{1}{D^2}\right)\right) \frac{u^M}{4}$$
- At leading nontrivial order

$$\nabla . J^S = \frac{\nabla . u}{4} == \frac{1}{8\mathcal{K}} \left(\nabla_{(A} u_{B)} \nabla_{(C} u_{D)} \mathcal{P}^{BC} \mathcal{P}^{AD} \right) + \ldots$$

Gregorry Laflamme Instability

- The spherical membrane solution - about which we linearized above - has an obvious generalization, namely the solution with $u = -dt$ and shape $S^{D-p-2} \times R^p \times$ time. This configuration is dual to the 'black p brane'. The special case $p = 1$ id dual to a black string.
- Black branes are known to suffer from 'Gregorry Laflamme' instabilities at every D. In order to see this instability from the membrane equations we focus on configurations that preserve the $SO(D-p-1)$ isometry and linearize the membrane equations about the exact black brane solutions.
- Turns out that the 'radius' zero mode of the previous slide develops the following dispersion relation

$$w = i\left(-\frac{\tilde{k}}{\sqrt{n}} - \frac{\tilde{k}^2}{n}\right)$$

where $\tilde{k}$ is the momentum along R^p and $n = D - 3$.

Scaling and endpoint

- Note the instability for $\tilde{k} < \sqrt{n}$. Rayleigh instability of the membrane.
- Factor of $\sqrt{n}$ in the frequency suggests that the interesting physics happens at length scale $\frac{1}{\sqrt{n}}$. Also the form of eigenfunction suggests $u_i \sim \frac{1}{\sqrt{n}}$ and $\delta r \sim \frac{1}{n}$. Substituting these scalings into the membrane equations yields a collection of effective nonlinear equations that capture not just the GL instability but also its end point.
- These effective 'black brane' equations - first obtained by EST using different techniques - are now seen to emerge simply as a scaling limit of our more general membrane equations.

Stationary solutions

- Entropy production must vanish on stationary solutions. It follows that σ_{MN} vanishes on such solutions. Recall that $\nabla . u$ also vanishes. It can be shown that a veolcity field has these properties if and only if it is proportional to a killing vector on the manifold on which it lives.
- In the simplest solution the membrane has a unique killing vector ∂_t. Easy to demonstrate that the lowest order membrane equation reduces to $\mathcal{K} = const$ in agreement with a direct analysis by EST of static solutions.
- Another simple situation: the manifold preserves some axial symmetries. In this case the velocity field has to be that of rigid rotations. Plugging this into the membrane equations we once again recover the equation $\mathcal{K} = \propto \gamma$ of EST ($\gamma = \frac{1}{\sqrt{1-v^2}}$). Easy to explicitly solve.
- Generalizes to charge. $Q \propto \gamma$. Can construct charged rotating solutions.

Polarization by gravity waves

- We have so far studied spacetimes that reduce to exactly flat space far away from the black hole (apart from radiation). Another interesting phenomenon one might want to study, however, is what happens when you shine a gravity wave (or EM wave) onto a black hole.
- In order to answer this question we need to generalize our considerations as follows. Consider any reference horizon free solution of Einstein's equation (e.g. a long wavelength gravity wave). Problem: construct black hole solutions of GR that rapidly asymptote to the given reference solution away from the black hole.
- Given by eating holes out of the new reference space time. At first order the membrane equations are given by the'Equivalence principle'. We simply promote every flat space derivative to a covariant derivative in the ref spacetime.

Polarization

- Can now compute the 'polarizability' of black holes. I.e. compute the response of the membrane to an incident gravity wave of any angular momentum. Related to the so called 'Love numbers' defined by GR practitioners.
- Find simple analytic expressions for angular momentum dependent polarizabilities as a function of frequency. The ω dependence of these polarizabilies is particularly simple: e.g. in the case of scalar modes for uncharged black holes we find that the polarizability is proportional to

$$\frac{\omega^r_{l+}\omega^r_{l-}}{(\omega - \omega^r_{l+})(\omega - \omega^r_{l-})}$$

where $\omega^r_{l\pm}$ are the two quasinormal mode frequencies listed above.

Collision of two black holes

- Consider two holes eaten out of spacetime that collide with each other. Over time scales of order $\frac{1}{D}$ before and after this collision our membrane equations do not apply. Heavy modes with frequencies of order D are excited in this process and presumably propagate to infinity as an intense burst of radiation. At larger times the two holes presumably merge into one. The boundary of this hole gets smoothened out and our membrane equations take over.
- It is of great interest to be able to reliably control the solution over this rapid collision time scale, both to be able to determine the nature of the emitted ratiation as well as to be able to determine the effective initial condiions for the membrane equations once they take over. We are trying to make progress here.

Conclusions

- We have demonstrated that the near horizon geometry of charged and uncharged black holes decouples from asymptotic infinity at large D. At the classical level the decoupled theory is governed by a set of equations that describe the propagation of a membrane in flat space.
- The degrees of freeedom of this membrane are its shape and a velocity and a charge density. The membrane carries a conserved stress tensor and charge current whose explicit form we have detrmined at low orders. Membrane equatons of motion are simply the staement of conservation of these currents and appear to define a well posed non gravitational initial value problem.
- Radiation reflects the failiure of decoupling and occurs at order $1/D^D$. The explicit form of radiation fields is obtained by coupling the membrane stress tensor and charge current to the linearized exterior metric and gauge fields in the usual manner.

Future Directions?

- Have studied black holes in spaces that are flat away from the membrbane region. Would be interesting to generalize to solutions that reduce to other solutions (e.g. waves) of Einstein equations. Also useful to generalize to gravity with a cosmological constant. In progress (see refs)
- It would be interesting to perform a structural analysis of the constraints on membrane equations that follow from the requirement that they carry a conserved entropy current. Perhaps this analysis could shed light on the still mysterious second law of thermodynamics in higher derivative gravity.
- Could be interesting to use the membrane to study the large D versions of complicated gravitational phenomena. E.g. black hole collsisons. $D = 4$?
- Could the membrane equations derived presented above turn out to be the hydrodynamical equations for a consistent quantum theory?