Growth dynamics and size fluctuations of bacterial cells

Sanjay Jain
Department of Physics and Astrophysics, University of Delhi;
Santa Fe Institute

Work done in collaboration with

Parth Pratim Pandey

Harshant Singh

Exponential growth of single cells

presented by other authors indicate that this dimension remains approximately constant during the development of individual cells (Adolph & Bayne-Jones, 1932; Deering, 1958; Maclean & Munson, 1961). The apparent refractive index of the cells also remained constant during their development. Measurement of the refractive index with the interference microscope has shown little variation among individuals

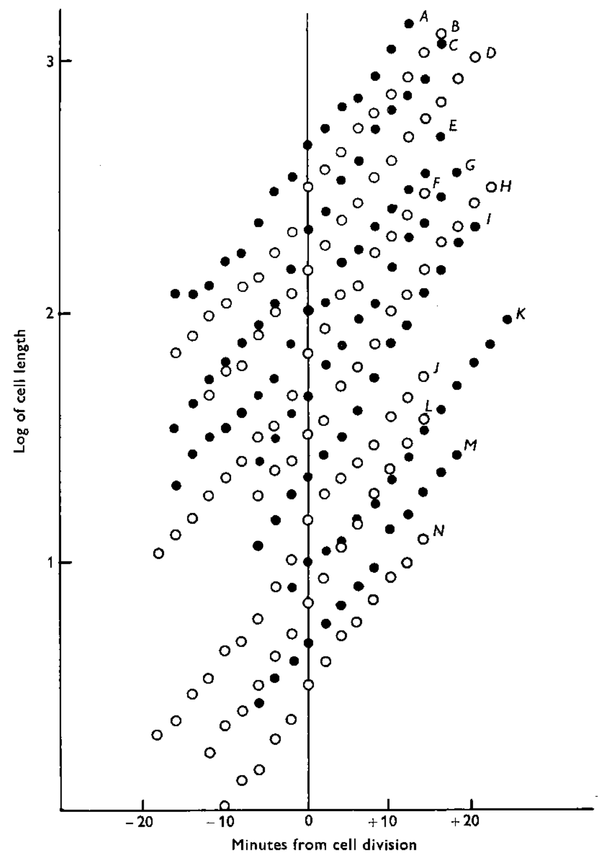


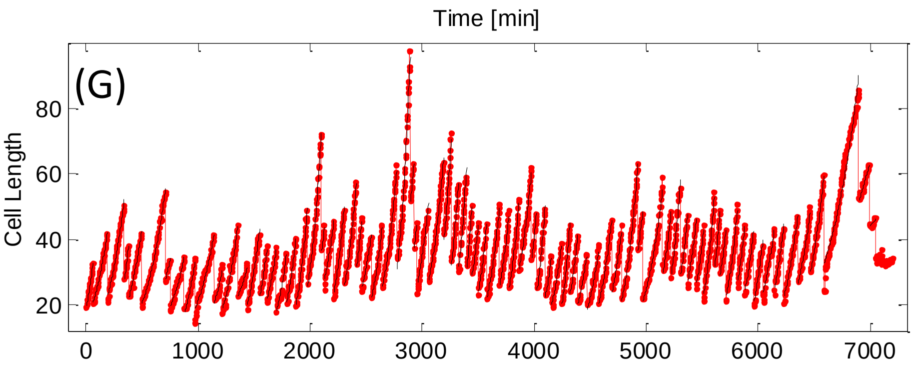
Fig. 1. Elongation of individual cells. The length of *E. coli* B/r cells (Expt. E-1) was measured every 2 min. The length of the resulting daughter cells was added. The measurements are presented in the following manner: the logarithm of cell length in arbitrary units is plotted along the ordinate. Zero on the abscissa represents the time of division. The size at division of individual cells is spaced at equal intervals along the ordinate. The length of the cells at the time of division was (in alphabetical order): 4·1, 4·9, 5·0, 5·1, 5·1, 5·2, 5·4, 5·5, 5·7, 5·7, 5·8, 5·9, and 6·0 μ.

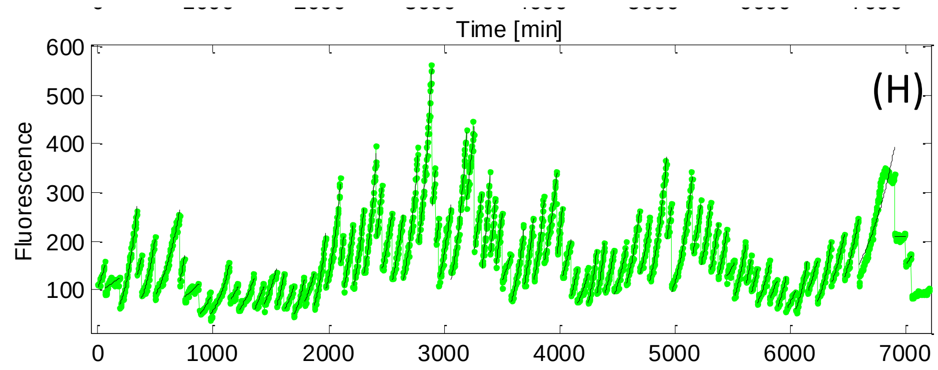
Schaechter et al (1962) J. Gen. Microbiol.

IP: 192.16.204.215
On: Thu, 19 May 2016 05:27:36

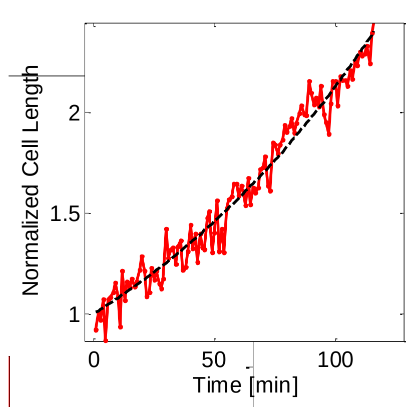
Cell size

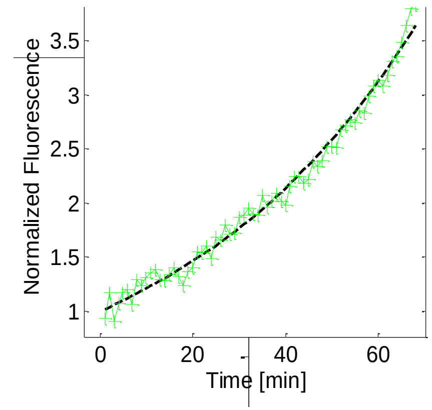
Intracellular molecular population

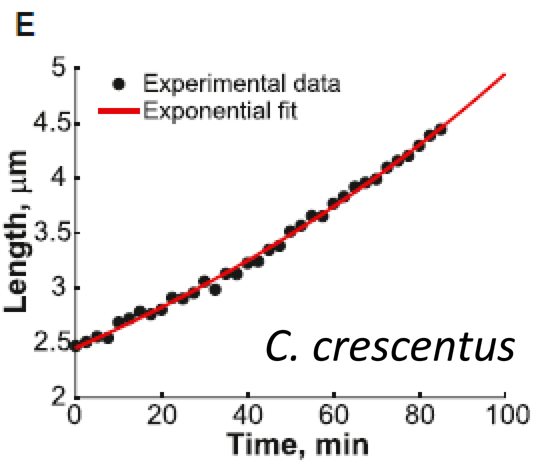


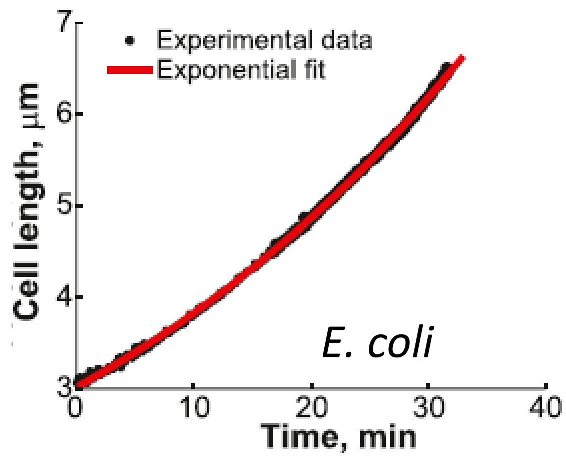


Brenner et al (2015)
Eur. Phys. J.





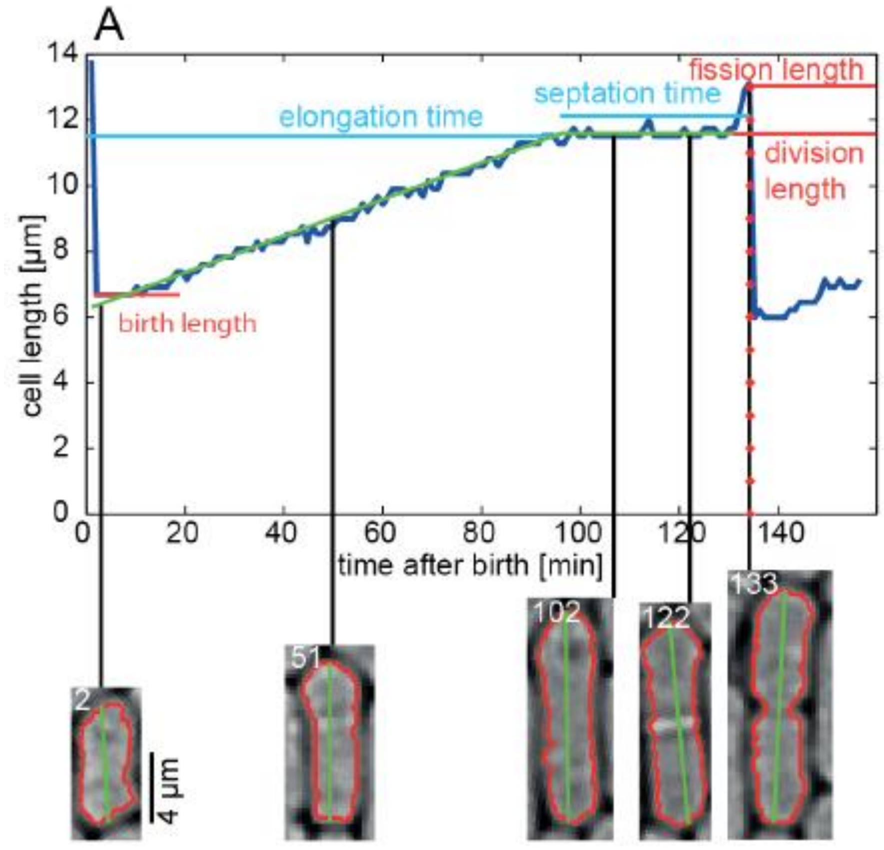




Campos et al (2014) Cell

Some organisms do not show exponential growth

S. Pombe



Nobs and Maerkl (2014) PLoS One

Mathematical models of cell growth and division

Cell Growth:

$$dX_i/dt \equiv \dot{X}_i = f_i(\mathbf{X}), \qquad i = 1, 2, \ldots, N,$$
$$dZ/dt \equiv \dot{Z} = h(\mathbf{X}),$$

Cell Division:

Above equations hold as long as $Z < Z_c$

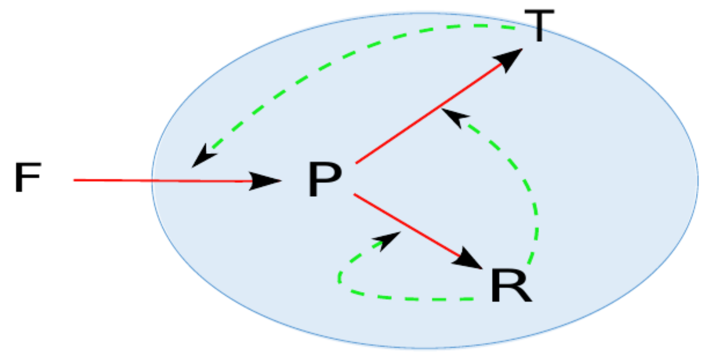
When $Z = Z_c$,

Then replace $X_i \to \frac{X_i}{2}, \quad Z \to Z_b$

Growth dynamics can be deterministic (average behaviour) or stochastic (cell-to-cell variations)

Precursor **T**ransporter **R**ibosome cell model

$$\frac{dP}{dt} = K_P T - k\frac{RP}{V},$$
$$\frac{dT}{dt} = K_T \frac{RP}{V} - d_T T,$$
$$\frac{dR}{dt} = K_R \frac{RP}{V} - d_R R,$$



The PTR cell

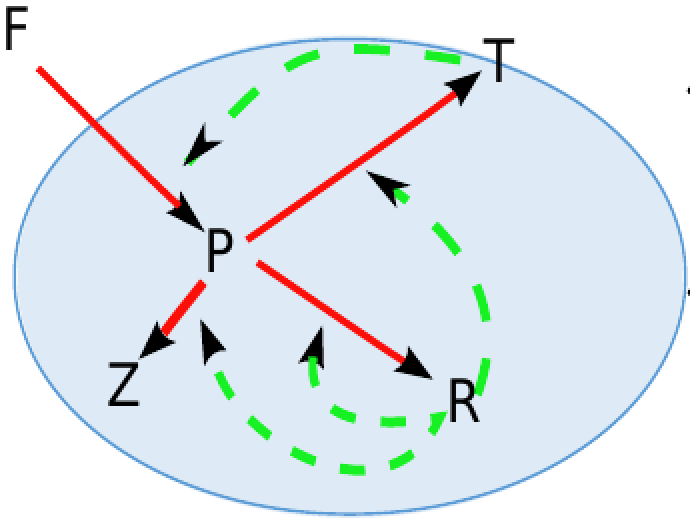
$$V = v_P P + v_T T + v_R R.$$

$$K_T = \frac{f_T k}{m_T}, \quad K_R = \frac{f_R k}{m_R}$$

$$f_T + f_R = 1$$

P.P.Pandey and SJ (2016) Th. in Biosc.

PTR(Z) model to understand cell size fluctuations and the 'adder mechanism'



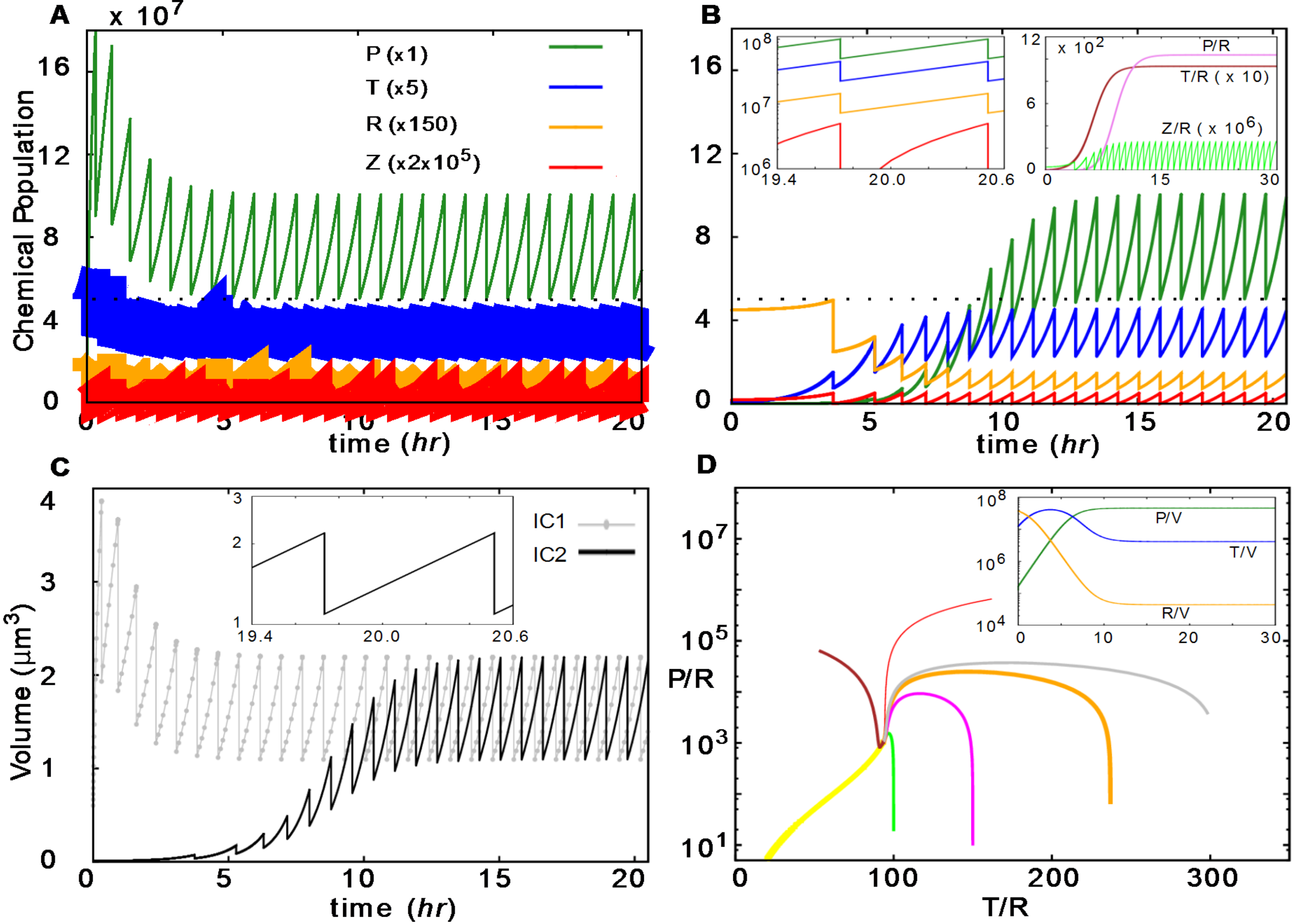
$$\frac{dP}{dt} = K_P T - k\frac{RP}{V},$$
$$\frac{dT}{dt} = K_T\frac{RP}{V} - d_T T,$$
$$\frac{dR}{dt} = K_R\frac{RP}{V} - d_R R,$$

+

$$\dot{z} = K_Z\frac{RP}{V}$$

PTR Dynamics

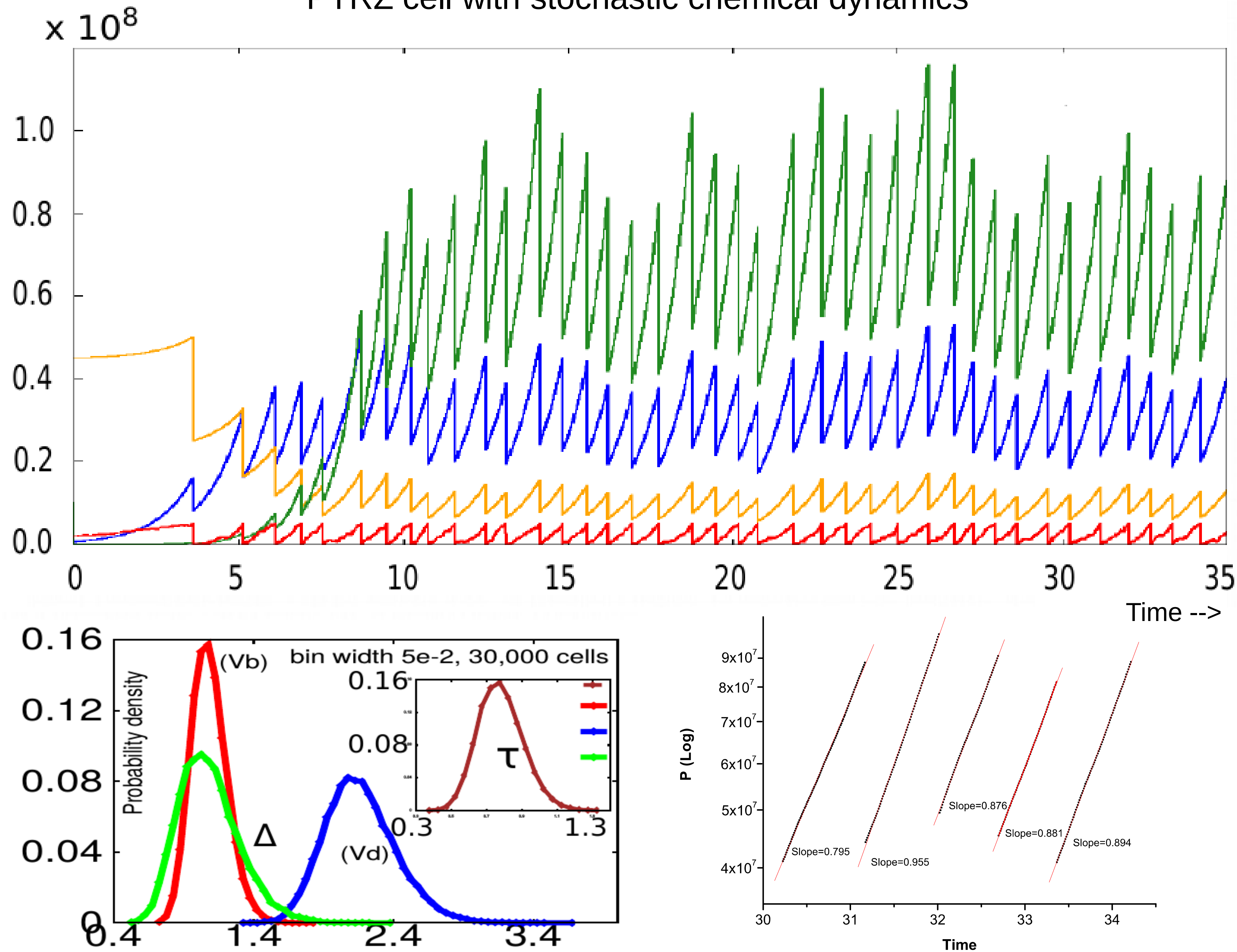
Z Dynamics



$K_P = 500hr^{-1}$, $k = 10^{-3}hr^{-1}(\mu m)^3$, $d_T = 0.1hr^{-1}$, $d_R = 0$ $m_T = 400$,$m_R = 10000$,

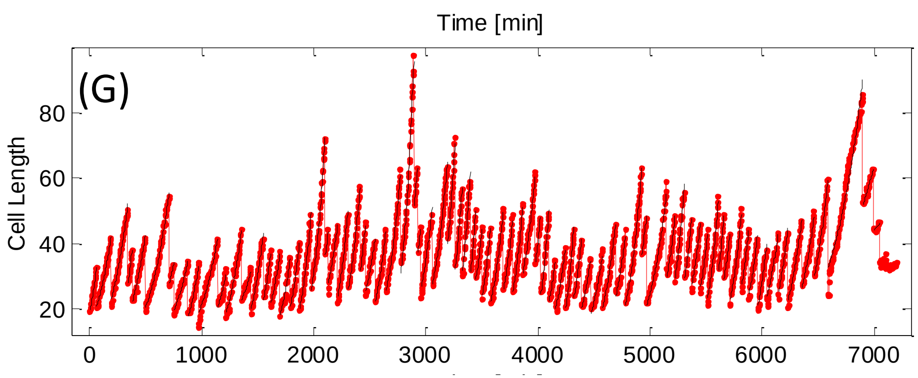
$v_P = v_T = v_R = 2 * 10^{-8}(\mu m)^3$ $f_R = 0.19377, K_Z = 10^{-11}hr^{-1}(\mu m)^3, z_c = 25, \tau_1 = 0.$

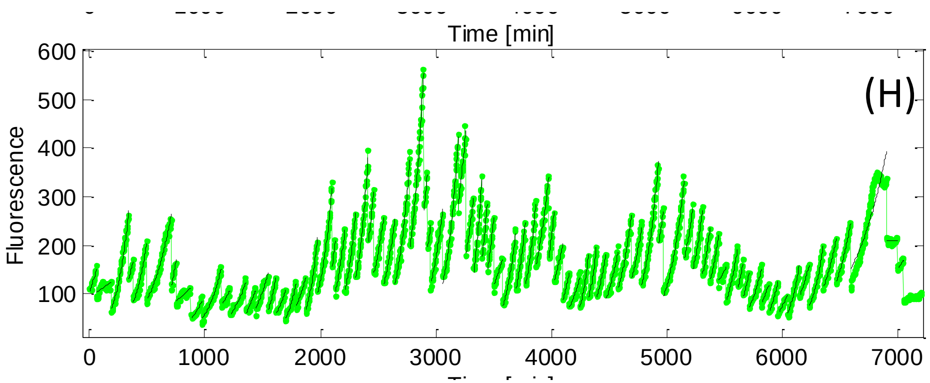
PTRZ cell with stochastic chemical dynamics

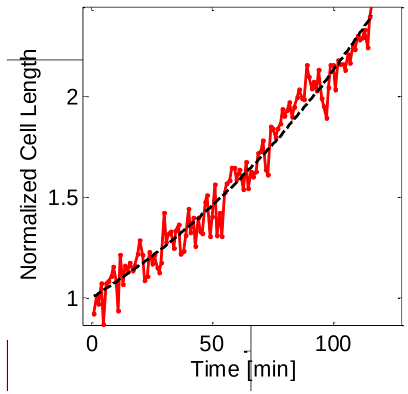


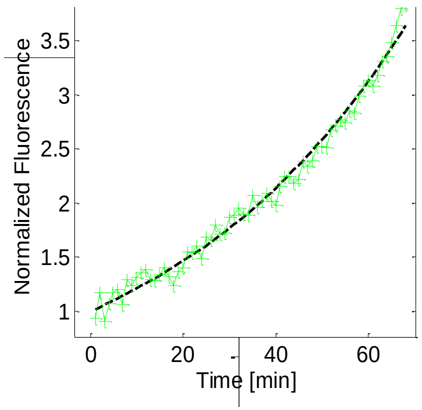
Cell size

Intracellular molecular population









Brenner et al (2015) Eur. Phys. J.

Reason for exponential growth

Class-I systems

$$\dot{X}_i = f_i(\mathbf{X}), \qquad i = 1, 2, \ldots, N$$

with the homogeneous degree-1 condition

$$f_i(\beta \mathbf{X}) = \beta f_i(\mathbf{X}), \quad i = 1, 2, ..., N$$

Class-I systems have an exponentially growing solution:

$$X_i(t) = X_i(0) e^{\mu t} \quad \text{for all} \quad i = 1, \ldots, N$$

For autocatalytic systems typically this solution is an attractor of the dynamics

Class-I property always arises when

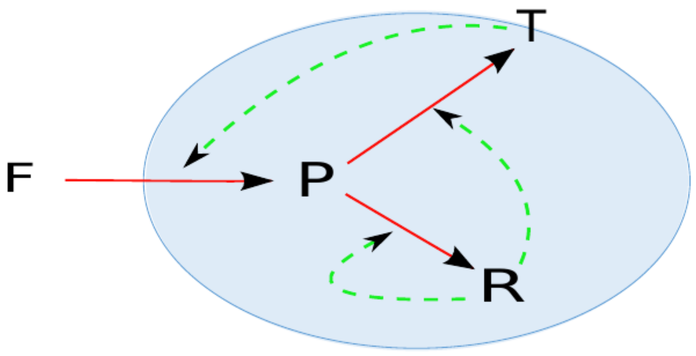
(a) the underlying chemical dynamics is given by mass action kinetics

$$\dot{x}_i = g_i(\mathbf{x}), \qquad x_i = \frac{X_i}{V}$$

(b) V is a linear function of the populations $V = \sum_{i=1}^{N} v_i X_i$

$$\frac{dP}{dt} = K_P T - k\frac{RP}{V},$$
$$\frac{dT}{dt} = K_T\frac{RP}{V} - d_T T,$$
$$\frac{dR}{dt} = K_R\frac{RP}{V} - d_R R,$$

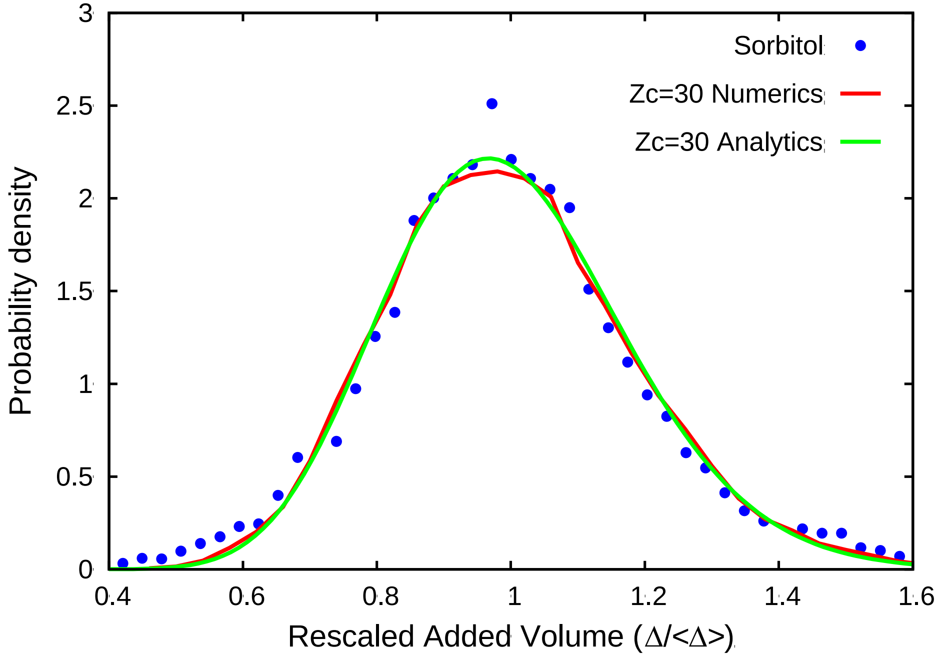
$$V = v_P P + v_T T + v_R R.$$



The PTR cell

PTR is a class-1 system -----> allows exponential solution

$$P(t) = P_b e^{\mu t}, \quad T(t) = T_b e^{\mu t}, \quad R(t) = R_b e^{\mu t}$$

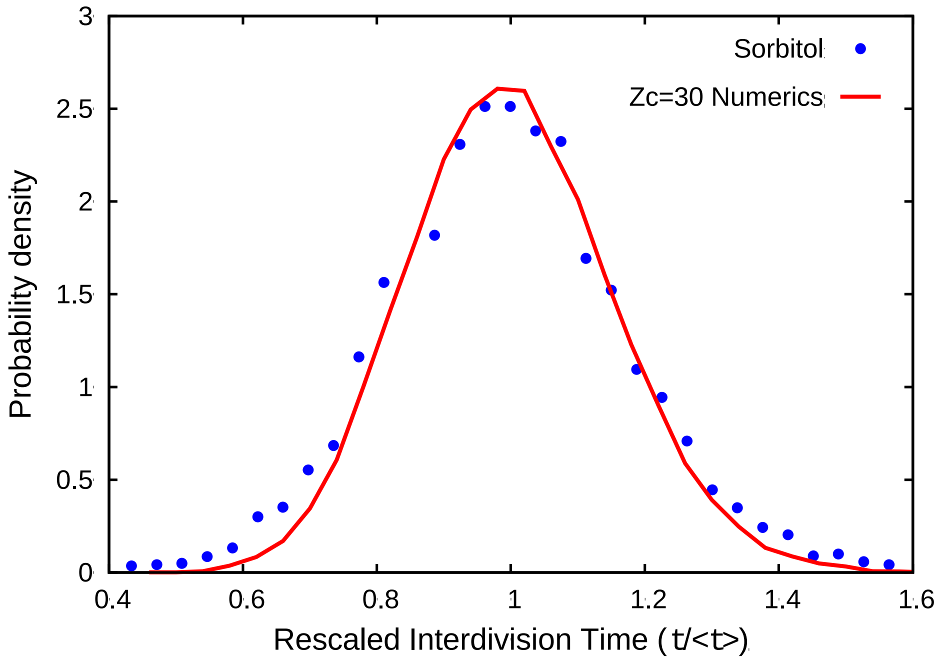


Analytical formula for Δ distribution:

$$P(u) = N^N \frac{u^{N-1}}{(N-1)!} e^{-Nu}$$

where $u \equiv \Delta / \langle \Delta \rangle$

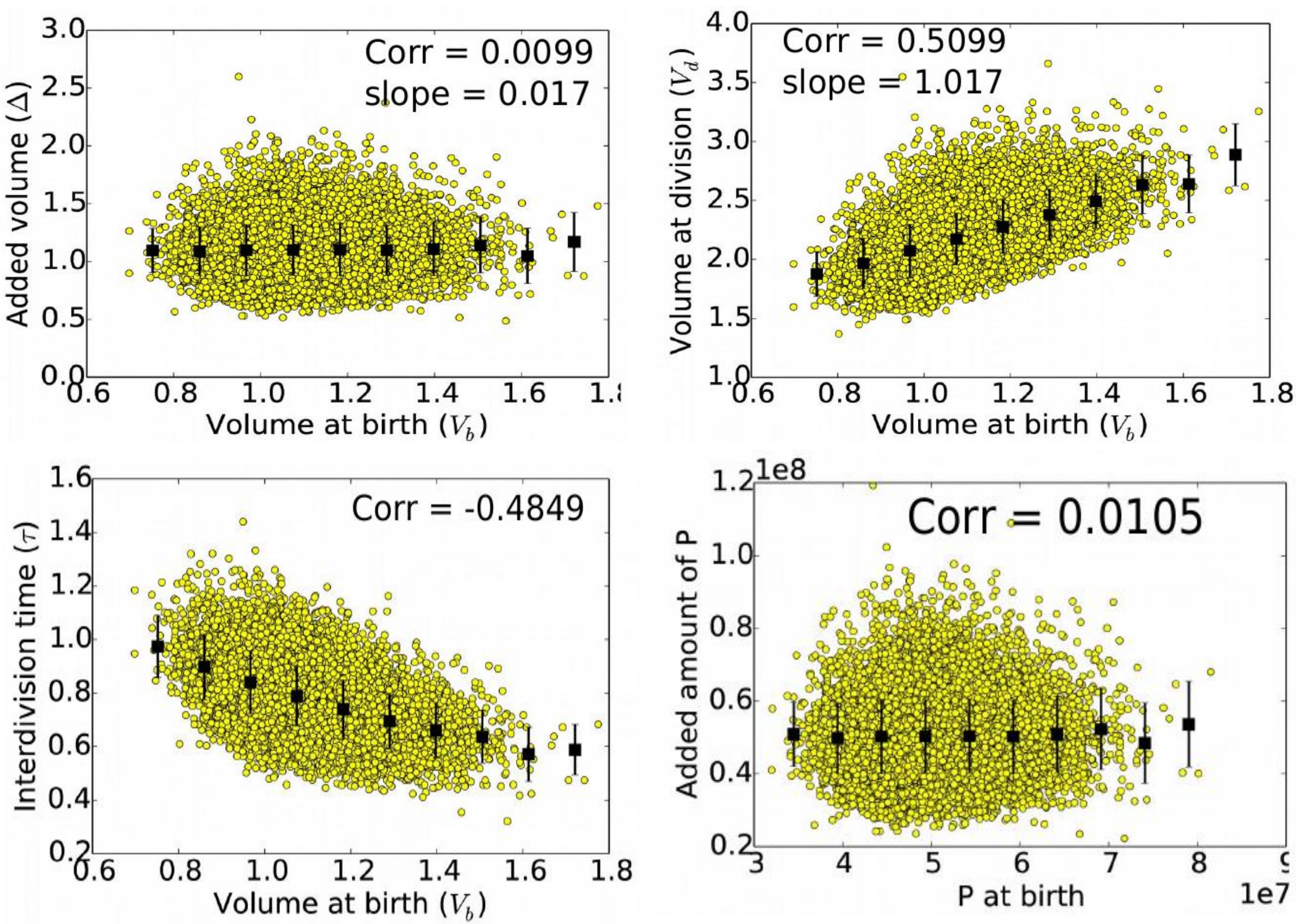
$N = Z_c - Z_b$



Universal result independent of the form of the functions $f_i(X)$, as long as they are homogeneous degree one functions of the *X* variables

Experimental distributions for E. coli in sorbitol medium from Taheri-Araghi et al (2015) Curr. Biol.

PTRZ reproduces adder property



Adder property of cell division

Proposed: Voorn et al (1993) Curr. Top. Mol. Gen.

Model: Amir (2014) Phys. Rev. Lett.

Observations and models:
Campos et al (2014) Cell
Taheri-Araghi et al (2015) Curr. Biol.