# Physics at different length scales

The Institute of Mathematical Sciences, Chennai

Spentafest, ICTS,
$11^{\text{th}}$ January, 2017.

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# Re-interpreting the Nambu Jona-Lasinio Model

VOLUME 52, NUMBER 12 PHYSICAL REVIEW LETTERS 19 MARCH 1984

**Nambu–Jona-Lasinio Model: An Effective Lagrangian for Quantum Chromodynamics at Intermediate Length Scales**

A. Dhar and Spenta R. Wadia
*Tata Institute of Fundamental Research, Bombay 400 005, India*
(Received 19 September 1983)

A Nambu–Jona-Lasinio type model is proposed as an effective Lagrangian for quantum chromodynamics at intermediate length scales. It is shown that the $\mathrm{SU}(n) \otimes \mathrm{SU}(n)$ nonlinear $\sigma$ model, including the Wess-Zumino term, can be derived from it in the low-energy

PHYSICAL REVIEW D VOLUME 31, NUMBER 12 15 JUNE 1985

**Nambu—Jona-Lasinio—type effective Lagrangian: Anomalies and nonlinear Lagrangian of low-energy, large-$N$ QCD**

Avinash Dhar,* R. Shankar, and Spenta R. Wadia
*Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India*
(Received 18 December 1984)

We present a qualitative derivation of the chiral model from QCD. This is based on using a Nambu—Jona-Lasinio—type effective Lagrangian as an intermediate step. A detailed derivation of

VOLUME 58, NUMBER 21 PHYSICAL REVIEW LETTERS 25 MAY 1987

**Large-$N$ Baryons: From Quarks to Solitons**

Sourendu Gupta and R. Shankar
*Tata Institute of Fundamental Research, Bombay 400005, India*
(Received 7 October 1986)

We consider baryons in a two-flavor Nambu–Jona-Lasinio–type model for self-interacting quarks in

# A peek at Planck scales

PHYSICAL REVIEW D VOLUME 32, NUMBER 10 15 NOVEMBER 1985

## Conformal invariance and string theory in compact space: Bosons

Sanjay Jain, R. Shankar, and Spenta R. Wadia
*Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay-400005, India*
(Received 19 February 1985)

The area law of the Nambu-Goto string is generalized to include a solid-angle-type term, which is purely topological in nature. Such a term exists and is unique provided the manifold $M$ in which the string lives satisfies certain topological conditions. This generalization may be useful to maintain conformal invariance in case $M$ is compact. Using methods of Polyakov and Friedan we identify the conformal anomaly coefficient with the central charge of the Virasoro algebra of this string theory. As an illustration we choose $M$ to be a compact Lie group and compute the anomaly coefficient following the work of Knizhnik and Zamolodchikov.

### I. INTRODUCTION

Superstring theory in 10 dimensions offers an attractive possibility of unifying all known interactions including gravity.[1] For special gauge groups like SO(32) and $E_8 \times E_8$ the theory is free of gauge and gravitational anomalies.[2] This theory is based on a supersymmetric generalization of the Nambu-Goto string where the action

If we write the monopole term in polar coordinates $r,\theta,\phi$, it becomes $i(k/2)\int_\Sigma \sin\theta\, d\theta\, d\phi$. The integral over $\Sigma$ is the solid angle subtended by $\Sigma$ at the origin. This immediately explains why $S(C)$ is independent of local deformations of $\Sigma$. Also, since the total solid angle of a closed surface enclosing the origin is $4\pi$, it is clear that if $e^{-S(C)}$ is to be totally independent of $\Sigma$, $k$ must be an integer.

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# Bose $\rightarrow$ Fermi in 2d

Modern Physics Letters A, Vol. 5, No. 8 (1990) 593–603


## ON BOSE-FERMI EQUIVALENCE IN A U(1) GAUGE THEORY WITH CHERN-SIMONS ACTION

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Received 27 December 1989

We show, using path integral methods, that a complex scalar field in 2 + 1 dimensions coupled to an abelian gauge field with Chern-Simons action is equivalent to a free Dirac fermion. We show the equivalence of the vacuum functional and construct the fermion fields explicitly. Our proof is independent of the long wavelength approximation.

# Bose $\rightarrow$ Fermi in 2d

Path integral for a 2d relativistic fermion (Polyakov)

$$K(x'|x) = \sum_{C_{xx'}} e^{-mL_C + i\pi W_C}$$

$W_C$: “Writhe" of the curve.

Question: Where is the spinor structure ?
Answer: The directions of the initial and final velocities of the particle are the intitial and final spinor polarizations

The ansatz for the fermion field is then,

$$\Psi_{\hat{\eta}}(x) = Z(c)\delta^2(\hat{\eta} - \hat{B}(x)) \exp\left(i \int d^3x A_\mu(x) j_c^\mu(x)\right) \phi(x) \,, \tag{24a}$$

$$\bar{\Psi}_{\hat{\eta}}(x) = \bar{Z}(c)\phi^*(x) \exp\left(-i \int d^3x A_\mu(x) j_c^\mu(x)\right) \delta^2(\hat{\eta} - \hat{B}(x)) \,, \tag{24b}$$

# Exclusion statistics

PHYSICAL REVIEW
LETTERS

VOLUME 72 | 6 JUNE 1994 | NUMBER 23

**Haldane Exclusion Statistics and Second Virial Coefficient**

M. V. N. Murthy and R. Shankar
*Institute of Mathematical Sciences, Madras 600 113, India*
(Received 25 January 1994)

We show that Haldane's new definition of statistics, when generalized to infinite dimensional Hilbert spaces, is determined by the high temperature limit of the second virial coefficient. We thus show that this exclusion statistics parameter $g$ of anyons is nontrivial and is completely determined by its exchange statistics parameter $\alpha$. We also compute $g$ for quasiparticles in the Luttinger model and show that it is equal to $\alpha$.

VOLUME 73 | 19 DECEMBER 1994 | NUMBER 25

**Thermodynamics of a One-Dimensional Ideal Gas with Fractional Exclusion Statistics**

M. V. N. Murthy and R. Shankar
*The Institute of Mathematical Sciences, Madras 600 113, India*
(Received 28 April 1994)

We show that the particles in the Calogero-Sutherland model obey fractional exclusion statistics as defined by Haldane. We construct anyon number densities and derive the energy distribution function. We show that the partition function factorizes in the form characteristic of an ideal gas. The virial expansion is exactly computable and interestingly it is only the second virial coefficient that encodes the statistics information.

# Exclusion statistics

Exchange Phase:

$$\psi(\vec{x},\vec{y}) = e^{i\alpha\pi}\psi(\vec{y},\vec{x})$$

$\alpha = 0$ : Bosons, $\alpha = 1$ : Fermions else Anyons

Generalised exclusion principle (Haldane, 1991):

$$d_{N+1} = d_0 - gN$$

$g = 0$ : Bosons, $g = 1$ : Fermions else Anyons

| | |
|---|---|
| Question: | Are $\alpha$ and $g$ related ? |
| Answer: | $g = \alpha(2-\alpha)$ for anyon gas |
| | $g = \alpha = \left(NR + \frac{M}{R}\right)^2$ for Luttinger liquid |

# Exclusion statistics

The above example gives a clear insight into the mechanism of the phenomenon. What is happening is that the addition of a quasiparticle causes a phase shift of every other quasiparticle, resulting in an energy shift of $\frac{\alpha \pi v_F}{L}$ per particle . When we count the dimension of the single particle space with a fixed (smooth) cutoff, there are $\alpha$ states missing. The important thing here is that all the single particle levels shift up by the same amount, however high the energy. This is why we get $g$ to be well defined and nontrivial in the cutoff going to infinity limit.

# The Hofstader butterfly

PHYSICAL REVIEW B **93**, 125134 (2016)

## Effects of interaction in the Hofstadter regime of the honeycomb lattice

Archana Mishra, S. R. Hassan, and R. Shankar
*The Institute of Mathematical Sciences, C.I.T. Campus, Chennai 600 113, India*
(Received 20 November 2015; revised manuscript received 15 February 2016; published 21 March 2016)

We investigate phases of spinless fermions on the honeycomb lattice with nearest-neighbor interaction in the Hofstadter regime. The interaction induces incompressible nematic and ferrielectric phases with broken translation symmetry. Some of the transitions are accompanied by changes in the Hall conductivity. We study pair correlations and show that the quantum metric, averaged over the Brillouin zone, characterizes the shape of the pair correlation function.

## Translational symmetry breaking and the disintegration of the Hofstadter butterfly

Archana Mishra, S. R. Hassan, and R. Shankar
*The Institute of Mathematical Sciences, C.I.T. Campus, Chennai 600 113, India*
(Dated: December 31, 2016)

We study the effect of interactions on the Hofstadter butterfly of the honeycomb lattice. We show that the interactions induce charge ordering that breaks the translational and rotational symmetries of the system. These phase transitions are prolific and occur at many values of the flux and particle density. The breaking of the translational symmetry introduces a new length scale in the problem and this affects the energy band diagram resulting in the disintegration of the fractal structure in the energy flux plot, the Hofstadter butterfly. This disintegration increases with increase in the interaction strength. Many of these phase transitions are accompanied with change in the Hall conductivity. Consequently, the disintegration of the Hofstadter butterfly is manifested in the Landau fan diagram also.

# The Hofstader Regime

The system:
Electrons in a plane in the presence of a normal magnetic field and a periodic potential.

The length scales
Magnetic length: $l_c = \sqrt{\frac{\hbar}{eB}}$
Lattice spacing: $a$

The Hofstader regime

$$a \sim l_c$$

# The Fractal

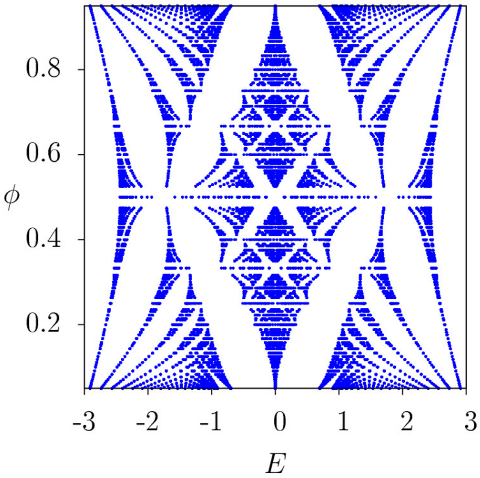



FIG. 4: (Color Online) Hofstadter butterfly for the honeycomb lattice. Here the x-axis represents the single particle energy $E$ and y-axis is the magnetic flux per plaquette $\phi$ of form $p/q$. In this plot $q \leq 20$.

# The topological invariants

Two integer invariants, $t, s$.
$t$ is the Chern-invariant, $\sigma_H = \frac{e^2}{h} t$

If,
$r/q =$number of particles in the unit cell
$p/q =$number of flux quanta passing through the unit cell
Then,

$$r = pt + qs$$

$$\frac{r}{q} = t\frac{p}{q} + s$$

if $s = 0$, $t = \nu$ (no periodic potential)

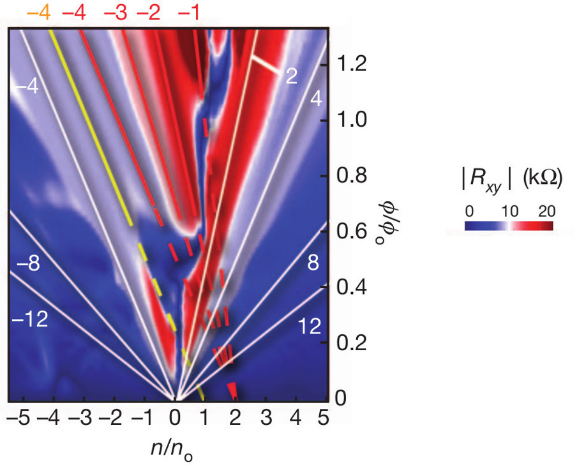
Non-zero $s$ has been experimentally measured in graphene Moire patterns recently.

# An experiment

doi:10.1038/nature12186

## Hofstadter's butterfly and the fractal quantum Hall effect in moiré superlattices

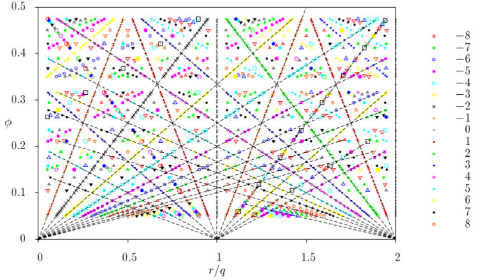
C. R. Dean[1], L. Wang[2], P. Maher[3], C. Forsythe[3], F. Ghahari[3], Y. Gao[2], J. Katoch[4], M. Ishigami[4], P. Moon[5], M. Koshino[5], T. Taniguchi[6], K. Watanabe[6], K. L. Shepard[7], J. Hone[2] & P. Kim[3]



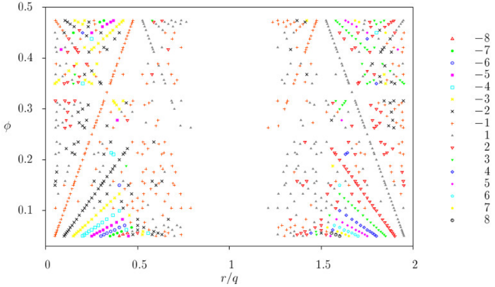
# Effect of interactions

- Repulsive nearest neighbour interactions induce charge ordering leading first order transitions into anisotropic (nematic) phases that break the lattice translation symmetry.
- These transitions are prolific. Many of them are topological, i.e. accompanied by changes in the Hall conductivity (Chern number).
- The toplological invariants of resulting states do not obey the Diophantine equation.
- They lead to a disintegration of the fractal structure.

# The disintegration




**V=0**




**V=4**

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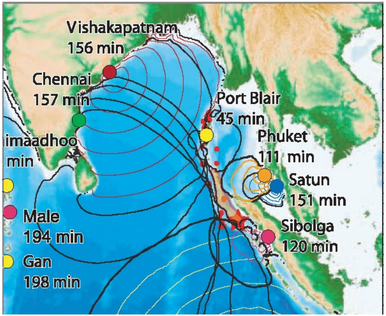
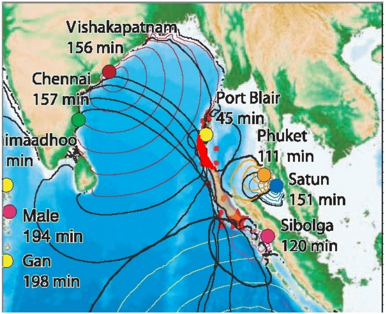
# The 2004 Bay of Bengal Tsunami

## Comment on "The Great Sumatra-Andaman Earthquake of 26 December 2004"

S. Neetu,[1*†] I. Suresh,[1*] R. Shankar,[2] D. Shankar,[1] S. S. C. Shenoi,[1] S.R. Shetye,[1] D. Sundar,[1] B. Nagarajan[3]

Lay *et al.* (Research Articles, 20 May 2005, p. 1127) estimated a 600-km length for the tsunami source region. Adding tide-gauge data from Paradip, the northernmost of the Indian east-coast stations and therefore the most critical constraint on the northern extent of the source, we estimate that its length was greater by ~30%.

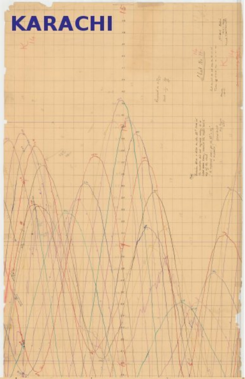


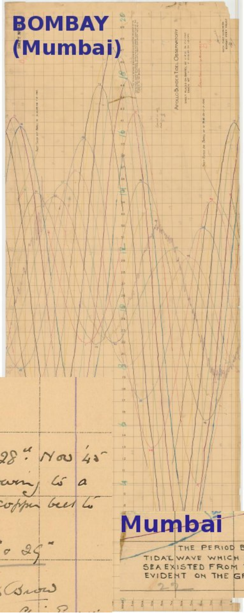



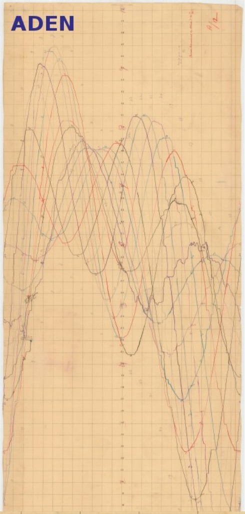
# The 1945 Arabian Sea Tsunami

**21:56 UTC, 27 November, 1945 Mw=8.0**

# The 1945 Arabian Sea Tsunami




**Karachi**

Note

Between 0810 & 0830 on the 28th Nov '45 the recorder did not register owing to a tidal wave having caused the copper belt to slip off the wheel.

(A separate chart for the 28th & 29th is enclosed)

**Mumbai**

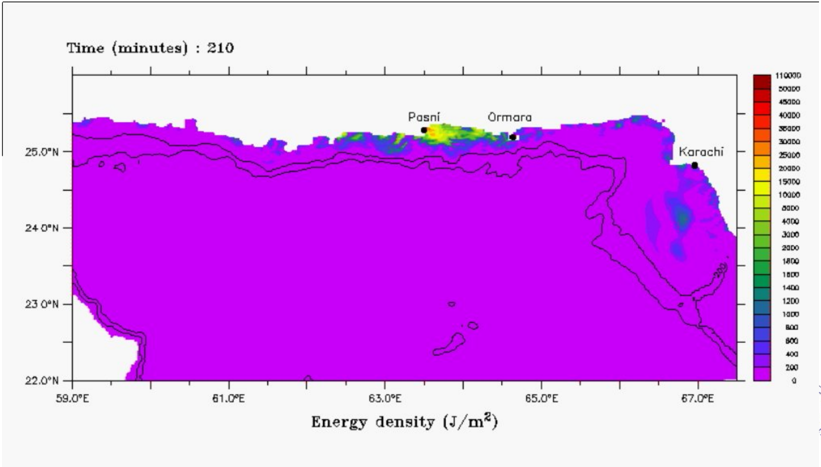
THE PERIOD BETWEEN 8-08 A.M. TO 8-22 A.M. SHOWS THE INFLUENCE OF THE TIDAL WAVE WHICH OCCURRED ON 28-11-45 A DISTURBED CONDITION OF THE SEA EXISTED FROM THAT PERIOD UNTIL ABOUT 9 A.M. ON 29TH INST. AS IS EVIDENT ON THE GRAPH.

# The 1945 Arabian Sea Tsunami

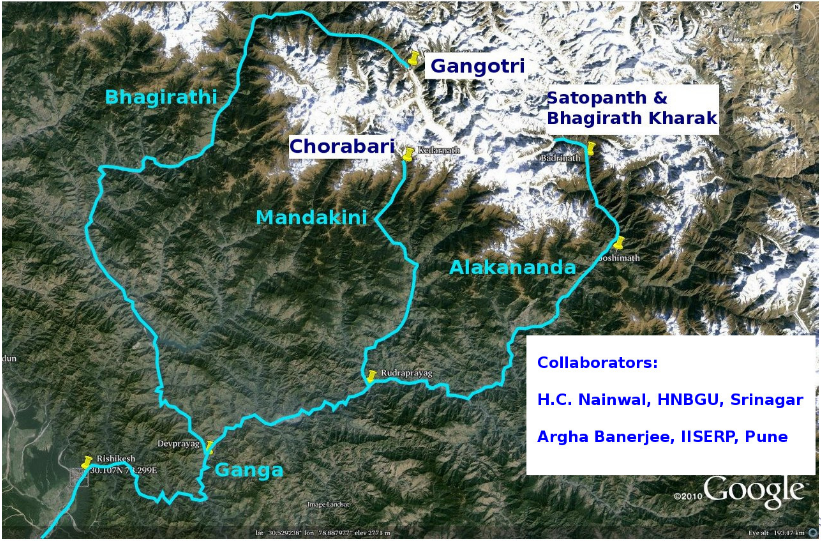
Persistant high waves in Makran coast, Karachi and Seychelles but not at Mumbai.

Previous speculations: Secondary tsunamis due to landslides.

Our work: Trapping of energy due to local bathymetric features.

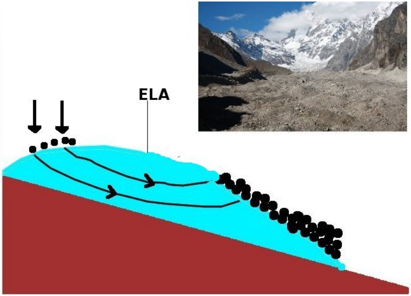


# My end game: Glaciers and Climate



# Debris covered glaciers

# Debris-ice dynamics

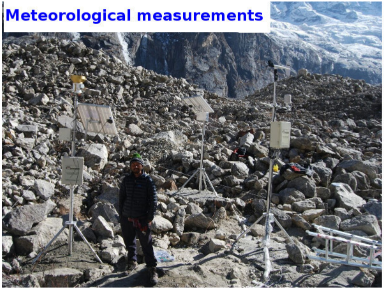



Subsurface flow

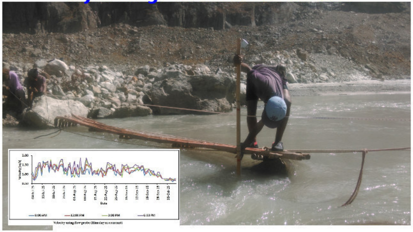
Specific mass balance ← Debris distribution

# Observations

**Meteorological measurements**



**Hydrological measurements**



**Science at Satopanth**

**Glaciological measurements**

**Ground penetrating radar (GPR)**

# THANK YOU !