INTRO

- AIM: To explore some UNIVERSAL aspects of dynamics of QFT out of equilibrium — using holographic techniques

- In equilibrium, an important ingredient in understanding the physics of RG
  - Explains decoupling of scales
  - Helps identify which features of the physics is insensitive to microscopic details

  This is crucial in our ability to write down simple models to describe complex situations — UNIVERSALITY

- Away from equilibrium situation is far from clear. Methods of equilibrium RG cannot be applied in a straightforward manner — Models vs Real systems?

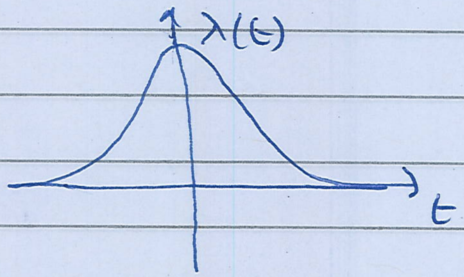
  In addition, the number of theoretical tools is rather limited — particularly for strong coupling

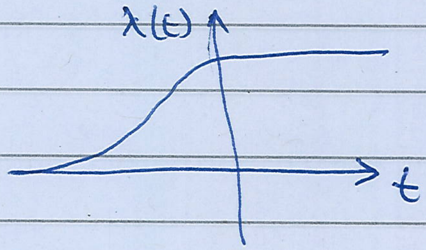
- The typical situation we will discuss

(i) Start the system in equilibrium
e.g $T = 0$ ground state
$T \neq 0$ thermal state

(ii) Take the system out of equilibrium.
Many ways of doing this.
One way is to turn on a time dependent source

$$H = H_0 + \int d^{d-1}x \; \lambda(t) \, \mathcal{O}_\Delta(t, x)$$





The time dependent term then excites the initial equilibrium state
- want to find out the nature of the state subsequently

- At late times, one expects that most systems settle down to a steady state
  — this resembles a THERMAL STATE.
  We might want to study
  (i) How generic is this?
  (ii) Approach to this
  (iii) In what sense is this state THERMAL

This is the issue of THERMALIZATION and RELAXATION

- In equilibrium UNIVERSALITY is ubiquitous in critical phenomena.

  In dynamical situations, one also expects interesting universal behavior which appear when the quench involves a critical point — we will concentrate on this aspect in these talks.

- When the disturbance which takes the system out of equilibrium is of small amplitude — one can use perturbation theory to analyze the system

A classic result of time dependent perturbation theory is the result for the RESPONSE

$$\langle \psi(t) | A | \psi(t) \rangle = \langle \psi(0) | A | \psi(0) \rangle - i \int_0^t dt' \langle \psi(0) | [A(t), \overline{H}_1(t)] | \psi(0) \rangle + \cdots$$

where

$$A(t) = e^{iH_0 t} A_s e^{-iH_0 t}$$

$$\overline{H}_1(t) = e^{iH_0 t} H_1(t) e^{-iH_0 t}$$

If $H_1(t) = \int d^{d-1}x \, dt \, \lambda(x,t) \, \mathcal{O}(x,t)$

Then the LINEAR RESPONSE term

$$-i \int_0^t dt' dx' \, \lambda(x',t') \langle \psi(0) | [\mathcal{O}(x,t), \mathcal{O}(x',t')] | \psi(0) \rangle$$

$$G_R(x,t;x',t') = i\Theta(t-t')\langle \psi(0) | [\mathcal{O}(x,t), \mathcal{O}(x',t')] | \psi(0) \rangle$$

— theory of Retarded Green's functions.

- In fact the most important universal aspect of universality in non-equilibrium physics is HYDRODYNAMICS.

This is the regime of long wavelength variations — but with arbitrarily large amplitudes.

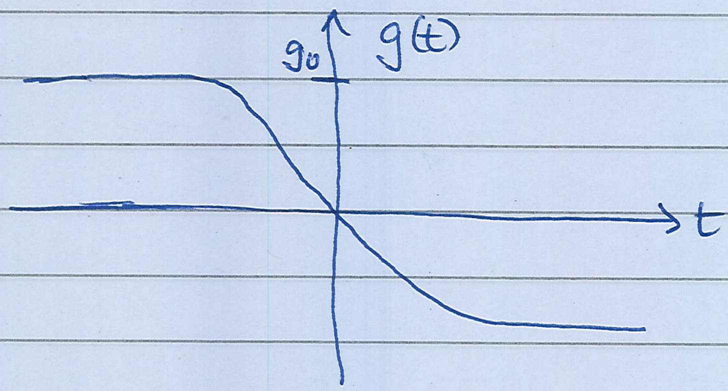
This regime is described by a few variables — for a typical fluid $T, \vec{v}$:

- The most difficult and the most interesting regime is when nothing is small ~~of~~ · — this is the regime of tough dynamics

- Even in this regime we will see that UNIVERSAL FEATURES do arise — but the theory is not very well understood.

An important example is Kibble Zurek scaling. Consider

$$H = H_0 + \int d^d x \, g(t) \, \mathcal{O}(x,t)$$



$g = 0$ is an equilibrium critical point with a correlation length exponent ν:

$$\xi(g) \sim |g|^{-\nu}$$

Energy gap is

$$E(g) \sim |g|^{z\nu}$$

z = dynamical critical exponent.

· At very early times the system can be treated in an adiabatic approximation.

Consider for a moment t as a parameter rather than time
Instantaneous eigenstates

$$H(t)|n\rangle_t = E_n(t)|n\rangle_t$$

and write

$$|\psi(t)\rangle = \sum_n a_n(t)\, e^{-i\int_0^t E_n(t')dt'} |n\rangle_t$$

$\Rightarrow$

$$\frac{da_k}{dt} = \sum_{n\neq k} \frac{a_n}{\omega_{kn}} \exp\left[i\int_0^t \omega_{kn}(t')dt'\right] \langle k|\frac{\partial H}{\partial t}|n\rangle$$

where

$$\omega_{kn}(t) \equiv E_k(t) - E_n(t)$$

Lowest order of adiabatic approx: $\partial H/\partial t = 0$. Thus state remains.

Suppose at $t = 0$ state is $|m\rangle$ of H_0

The first correction is obtained by considering RHS to be a const. Then

$$a_k(t) \sim \frac{1}{\omega_{km}^2} \langle k | \frac{\partial H}{\partial t} | m \rangle \left(e^{i\omega_{km} t} - 1 \right)$$

Thus the approximation is good so long as a_k stays small

$$\langle k | \frac{\partial H}{\partial t} | m \rangle \ll \omega_{km}^2 .$$

- If initially the system is in the ground state, the approx breaks down whenever this is violated for the first excited state. This gives

$$\frac{1}{E_g^2} \frac{dE_g}{dt} \sim 1$$

- Since $g = 0$ is a critical point $E(0) = 0$, this is going to happen at a time earlier than $t = 0$.

Suppose $g(t) \sim g_0 (vt)^r$ near the critical point.

Assume self consistently that this adiabaticity breakdown happens when $g(t)$ can be approximated as such.

$$E_g(t) = [g_0 (vt)^r]^{z\nu}$$

$$\frac{dE_g}{dt} = g_0^{z\nu} v^{z\nu r} t^{rz\nu - 1}$$

$$\frac{1}{E_g^2}\frac{dE_g}{dt} = g_0^{-\nu z} v^{-z\nu r} t^{-(1+rz\nu)}$$

Thus adiabaticity breaks when

$$t = t_Q = (g_0 v^r)^{-\frac{z\nu}{rz\nu+1}}$$

The gap at this time is

$$E(-t_Q) = (g_0 v^r)^{\frac{z\nu}{rz\nu+1}}$$

When is our assumption self consistent? Consider e.g

$$g(t) = g_0 (\tanh vt) \quad (r=1$$

Since $\nu t = \left(g_0^{-z\nu} \nu\right)^{\frac{1}{z\nu+1}}$ $(r=1)$
and the approx $g \sim g_0 \nu t$ is valid when $\nu t \ll 1$, we must have

$$\nu \ll g_0^{z\nu}$$

But g_0 is the initial coupling and the initial gap is $g_0^{z\nu}$.
Thus

$$\nu \ll E_g(t \Rightarrow -\infty).$$

- For $t > -t_Q$ we cannot use any useful standard technique.

Kibble : Assume that at $t = t_Q$ the system makes a transition to DIABATIC regime

$\Rightarrow$ For $t > t_Q$ treat the system in SUDDEN APPROXIMATION.

This means that the system is frozen at the state $t = t_Q$.

After some time, roughly $t = +t_Q$ the system again reverts to a close to adiabatic time evolution.

At the end of the quench — can read off various quantities by evaluating their values at $t = -t_Q$

- Second assumption: Even if the coupling is changing very fast in the impulse regime, assume that the only scale in the problem is
$$\xi(t_Q) \quad \text{or} \quad E_g(t_Q).$$
This means
$$\langle \mathcal{O} \rangle = [\xi(t_Q)]^{-\Delta}$$
$$\sim \left(g_0 v^r\right)^{\frac{\Delta\nu}{r z\nu + 1}}$$

- Modification: assume that ξ is not really frozen $-t_Q < t < t_Q$ but
$$\xi(t) = \xi(t_Q)\, f\left(\left|\frac{t}{t_Q}\right|\right)$$
where f is a slowly varying fn.

- Apply this to relativistic theory

$$S = S_{CFT} + \int d^{d-1}x \, dt \; \lambda(t)\, \mathcal{O}_\Delta(x,t)$$

$$\lambda(t) \sim \lambda_0 (vt)^{*}$$

Then the gap should behave as

$$E \sim \lambda^{\frac{1}{d-\Delta}} \qquad \Rightarrow \nu = \frac{1}{d-\Delta}$$

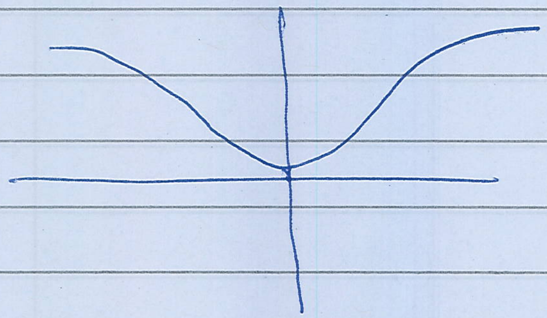
This gives

$$t_Q = (\lambda_0 v)^{-\frac{1}{d-\Delta+1}}$$

$$\langle \mathcal{O}_\Delta \rangle \sim (\lambda_0 v)^{\frac{\Delta}{d-\Delta+1}}$$

- Equilibrium critical phenomena
  - decoupling of all scales is justified by RG.
  - One of the things we want to understand is : how and why other scales decouple.

Other protocols in KZ



General scaling theory
Chandran, Erez, Gubser & Sondhi

- In the other extreme is an INSTANTANEOUS QUENCH, e.g.
  Another set of universal results expected

$$H = H_0 + \Theta(-t)\,\lambda\,\mathcal{O}$$

Initially — a gapped theory with Hamiltonian $H_0 + \lambda\mathcal{O}$ $\quad t<0$ and the system is in its ground state.

This is an excited state in the critical hamiltonian — want to study evolution of this state

- Typically such a state is rather complicated

Consider e.g. a one point function

$$\langle\Psi_0|\,\mathcal{O}(t,\vec{x})\,|\Psi_0\rangle \equiv \langle\Psi_0|\,e^{iHt}\,\mathcal{O}(\vec{x})\,e^{-iHt}\,|\Psi_0\rangle$$

- Cardy–Calabrese: for the purpose of long distance physics one may write

$$|\Psi_0\rangle = e^{-\tau_0 H}\,|B\rangle$$

where $|B\rangle$ is a conformally invariant state : BOUNDARY STATE

- Consider e.g. mass quench in a scalar field theory

$$S = -\int d^{d-1}x \left[(\partial\phi)^2 + m^2(t)\,\phi^2 \right]$$

$$m(t) = \theta(-t)\, m_0$$

Momentum space equation

$$\frac{d^2\phi_k}{dt^2} + \left[k^2 + m^2(t) \right]\phi_k = 0$$

has following independent solns.

$$\phi_k^{(in)}(t) = \frac{1}{\sqrt{2\omega_{in}}}\, e^{-i\omega_{in}t}\,\theta(-t) + \frac{1}{\sqrt{2\omega_{out}}}\left(a\, e^{-i\omega_{out}t} + b\, e^{i\omega_{out}t} \right)\theta(t)$$

$$\phi_k^{(out)}(t) = \frac{1}{\sqrt{2\omega_{in}}}\left(a\, e^{-i\omega_{in}t} - b\, e^{i\omega_{in}t} \right)\theta(-t) + \frac{1}{\sqrt{2\omega_{out}}}\left(\cancel{a}\, e^{-i\omega_{out}t} \right)\theta(t).$$

$$\omega_{in} \equiv \sqrt{k^2 + m_0^2} \qquad \omega_{out} = |k|$$

$$a = \frac{\omega_{in} + \omega_{out}}{2\sqrt{\omega_{in}\omega_{out}}} \qquad b = \frac{\omega_{out} - \omega_{in}}{2\sqrt{\omega_{in}\omega_{out}}}$$

These sets are related by a Bogoliubov transformation

$$\phi_k^{in} = \alpha_k \phi_k^{out} + \beta_k \phi_{-k}^{out\,*} \qquad \begin{matrix} \alpha = a \\ \beta = b. \end{matrix}$$

· One can now expand the field in either set of modes

$$\Phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[A_{in}\phi_{in} + A_{in}^{\dagger}\phi_{in}^{*} \right]$$

$$= \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[A_{out}\phi_{out} + A_{out}^{\dagger}\phi_{out}^{*} \right]$$

$$\begin{pmatrix} A_{out} \\ A_{out}^{\dagger} \end{pmatrix} = \begin{pmatrix} \alpha & \beta^* \\ \alpha^* & \beta \end{pmatrix} \begin{pmatrix} A_{in} \\ A_{in}^{\dagger} \end{pmatrix}$$

The Heisenberg picture state is then $|0\rangle_{in}$

$$A_{in}|0\rangle_{in} = 0 \qquad \forall\, k$$

May now use Bogoliubov to show

$$|0\rangle_{in} = \prod_k \exp\left[\gamma(k)\, A_{out}^{\dagger}(k)\, A_{out}^{\dagger}(-k) \right] |0\rangle_{out}$$

$$\gamma(k) = \frac{\beta^*(k)}{\alpha^*(k)}.$$

• $$\gamma = -\frac{1}{2}\frac{\sqrt{k^2+m^2}-k}{\sqrt{k^2+m^2}+k}$$
$$\simeq -\frac{1}{2}\left[1 - \frac{2|k|}{m} + O\left(\frac{k^2}{m^2}\right)\right]$$

Thus for $|k| \ll m$ we get a state of the final theory

$$|0\rangle_{in} = \exp\left[-\frac{1}{2}\sum A^{+}_{out}(k)\, A^{+}_{out}(-k)\right]|0\rangle_{out}$$
$$\equiv |B\rangle$$

$|B\rangle$ is a boundary state

$$\left[A_{out}(k) + A^{+}_{out}(-k)\right]|B\rangle = 0$$

• It can be easily seen that one may always write

$$|0\rangle_{in} = e^{-\sum_k \tau_0(k) \cdot |k|\, A^{+}_{k} A_{k}}\, |B\rangle$$

Calculate $\tau_0(k) = \frac{2}{m} - \frac{1}{3}\frac{k}{m^2} + \cdots$

Thus for quantities which receive contributions from low $k \ll m$

$$e^{-\tau_0 H}\,|B\rangle$$

• May now write

$$\langle \Psi_0 | \mathcal{O}(t,x) | \Psi_0 \rangle$$

$$= \langle B | e^{-\tau_0 H} e^{itH} \mathcal{O}(r) e^{-itH} e^{-\tau_0 H} | B \rangle$$

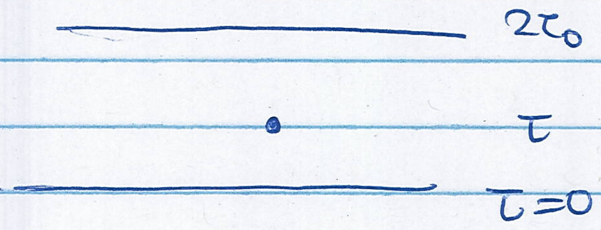
Normalize by $\langle B | e^{-2\tau_0 H} | B \rangle$

Analytic continuation

$$it = \tau - \tau_0$$

$$\langle \mathcal{O} \rangle = \langle B | e^{-(2\tau_0 - \tau)H} \mathcal{O} e^{-\tau H} | B \rangle$$

→ Slab geometry



May represent by Euclidean path integral with boundary conditions corr. to $|B\rangle$.

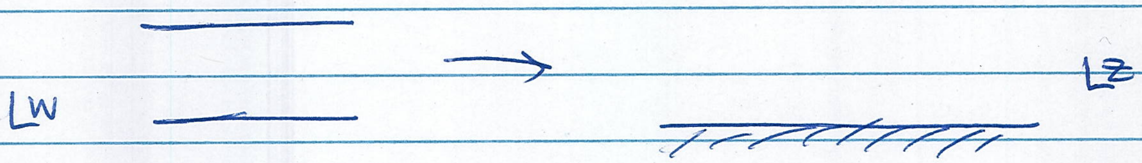
- In 1+1 dimensions this comes in handy.

$$z = e^{\frac{\pi w}{2\tau_0}} \quad \text{maps strip to UHP}$$

$$w = \sigma + i\tau$$

$$\tau = 0 \qquad z = e^{\pi\sigma/2\tau_0} > 0 \text{ real}$$

$$\tau = 2\tau_0 \qquad z = -e^{\pi\sigma/2\tau_0} < 0 \text{ real}$$



On the plane

$$\langle O(z)\rangle \approx \frac{1}{|2\,\mathrm{Im}\, z|^{x}}$$

x = Conformal dimension

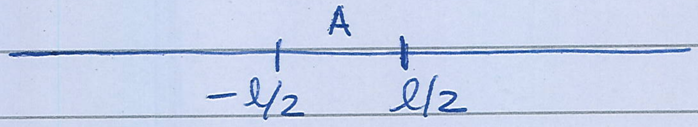
$\Rightarrow$

$$\langle O(w)\rangle = \left(\frac{\pi}{4\tau_0 \sin\frac{\pi\tau}{2\tau_0}}\right)^{x}$$

$$= \left(\frac{\pi}{4\tau_0 \cosh\frac{\pi t}{2\tau_0}}\right)^{x}$$

$$\sim e^{-\pi t x/2\tau_0}$$

Entanglement Entropy



Consider the state $|\Psi_0\rangle = e^{-\tau_0 H}|B\rangle$

Reduced density matrix ρ_A

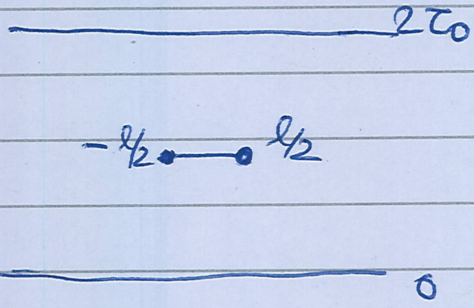
Field representation

$$\rho(\phi_A, \phi_A') = \int \mathcal{D}\phi_B \, \Psi(\phi_B, \phi_A) \, \Psi^*(\phi_B, \phi_A')$$

Renyi Entropy $\quad S = \mathrm{Tr}\, \rho_A^n$

$$S_A(t) = -\frac{\partial}{\partial n} \mathrm{Tr}\, \rho_A^n \Big|_{n=1}$$

Path integral representation of $\rho(t)$.



$\tau_1 = \tau_0 + it$

n sheeted Reimann joined along $(-l/2, l/2)$

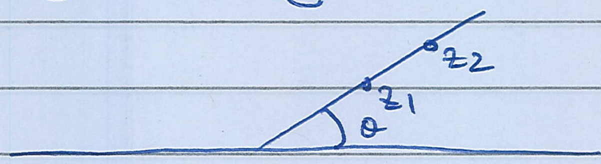
Mapping to plane $\quad z = e^{\frac{\pi}{2\tau_0}(\sigma + i\tau)}$

If $\sigma_1 = -\ell/2$ $\qquad\qquad$ $\sigma_2 = +\ell/2$

$\quad\ \tau_1 \qquad\qquad\qquad\qquad\ \tau_1$

$$z_1 = e^{-\frac{\pi \ell}{4\tau_0}} \, e^{\frac{i\pi\tau_1}{2\tau_0}}$$

$$z_2 = e^{+\frac{\pi \ell}{4\tau_0}} \, e^{\frac{i\pi\tau_1}{2\tau_0}}$$



$$\theta = \frac{\pi\tau_1}{2\tau_0}$$

- Result : $\mathrm{Tr}\,\rho_A^n$ is the 2 point fn. of a twist operator Φ_n

$$\langle \Phi_n(z_1)\ \Phi_{-n}(z_2) \rangle \qquad \Delta_n = \frac{c}{24}\left(1 - \frac{1}{n^2}\right)$$

$$= c_n \left(\frac{|z_1 - \bar{z}_2|\,|z_2 - \bar{z}_1|}{|z_1 - z_2|\,|\bar{z}_1 - \bar{z}_2|\,|z_1 - \bar{z}_1|\,|z_2 - \bar{z}_2|} \right)^{2n\Delta_n}$$

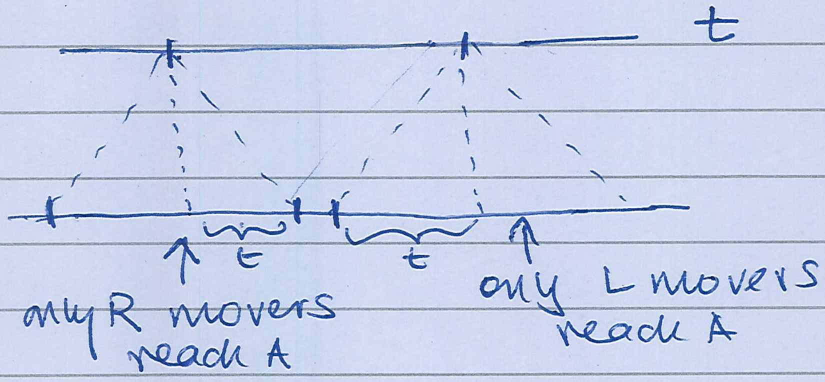
- Result

$$S_A(t) = \begin{cases} \dfrac{\pi c t}{6\tau_0} & t < \ell/2 \\[2ex] \dfrac{\pi c \ell}{12\tau_0} & t > \ell/2 \end{cases}$$

Interpretation:

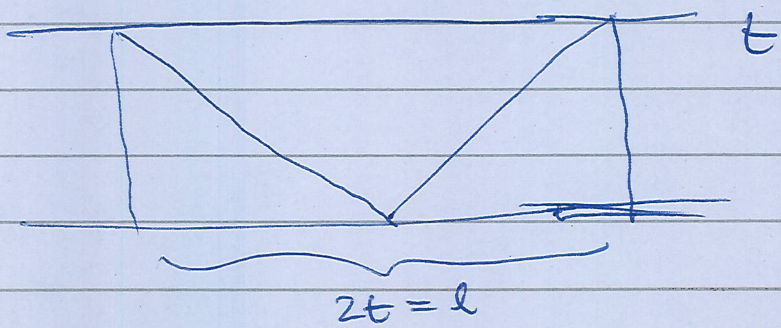
Quench at $t=0$ produces excitations
Since initial state has a gap
— only excitations produced
approx. at same point entangled.

To contribute to S_A: only the
L OR R movers must lie within A

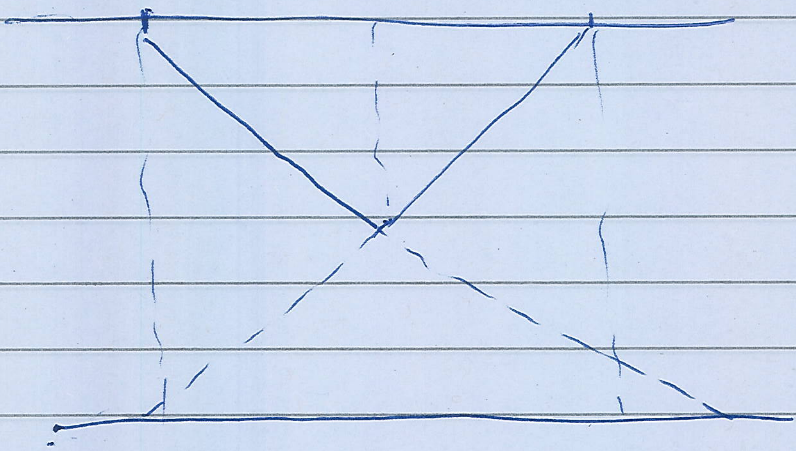


Thus $S_A = 4t$

When $2t = l$



For $2t > l$



No more excitations come in

$\Rightarrow$ entropy saturates.