Lecture 3 (India)

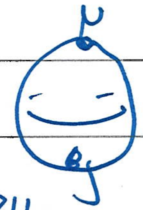
Considered 2d $\mathcal{N}=(2,2)$ theories on S^2
$$\begin{cases} SU(2|1)_A \longrightarrow Z_A \\ SU(2|1)_B \longrightarrow Z_B \end{cases}$$

$SU(2|1)_A$ $\quad \{Q_\alpha, \bar{Q}_\beta\} = \gamma^m_{\alpha\beta} J_m - \frac{1}{2}\epsilon_{\alpha\beta}$.

- $\exists$ two classes of deformations.

$$z F = \int d^2\theta\, z\, \Phi_c \qquad \tau G = \int d^2\tilde{\theta}\; \tau . Y_{tc}$$

Consider the following supercharge $\quad Q^2 = J_3 + \frac{R}{2}$



Decoupling theorems:

$\langle O_a(Y)\, \bar{O}_{\bar{b}}(\bar{Y})\rangle$

$\cancel{\langle \Phi_a(N)\, \bar{\Phi}_{\bar{b}}(S)\rangle}$ is Q-invariant

- Independence of Z_A and Z_B on g^2
- $Z_A(\tau, \bar{\tau})$; $Z_B(z, \bar{z})$? Weyl anomaly

: Showed that $\langle O_a(Y)\, \bar{O}_{\bar{b}}(\bar{Y})\rangle = r \int_{S^2} dY_0\, d\bar{Y}_0\, e^{-4\pi i r(\tilde{W}(Y_0) + \bar{\tilde{W}}(\bar{Y}_0))}\, O_a(Y_0)\, O_b(Y_0)$

$\cancel{\langle \Phi_a(N)\, \bar{\Phi}_{\bar{b}}(S)\rangle =}$

~~What does Z_{S^2} compute?~~ ~~From this we can show that~~

$\tilde{W} = \sum_a \tau_a O_a +$

$\cancel{\partial_a \partial_{\bar{b}} \log Z_{S^2} = \int d^2x\, d^2y\, \langle O_a(x)\, \bar{O}_{\bar{b}}(y)\rangle}$

$\cancel{= \langle}$

~~This allows us~~

This allows us to understand what $Z_A(S^2)$ computes

Consider deforming SCFT by exactly marginal operators

$$\widetilde{W} = \sum_a \tau_a O_a(Y)$$

Calculate $\partial_a \partial_{\bar{b}} \log Z_A = \langle \int d^2x \int d^2y \sqrt{g}$

$$\cdot \int d^2\tilde{\theta}\, O_a(Y(x)) \int d^2\theta\, \bar{O}_{\bar{b}}(\bar{Y}(y)) \rangle .$$

$- \langle O_a(N)\ \bar{O}_{\bar{b}}(S) \rangle = G_{a\bar{b}}$ = Zamolodchikov metric

$$G_{a\bar{b}} = -\partial_a \partial_{\bar{b}} \log Z_{S^2}$$

Kahler ‖

$\partial_a \partial_{\bar{b}}$ ~~log~~

$$\boxed{Z_{A^2} = e^{-k(\tau,\bar{\tau})}}$$

$$\boxed{Z_B = e^{-k(z,\bar{z})}}.$$

Ambiguities $\qquad G_{a\bar{b}}$ invariant under Kahler transf

$$\downarrow \qquad Z \to Z e^{-F-\bar{F}}$$

Lecture 3 (Zudia) : ③

~~Last lecture : Two inequivalent~~

$$c(\lambda) \log(r\Lambda)$$

$$\int d^2x \sqrt{g}$$

$$Z_{S^2} = e^{f(\lambda)(\Lambda r)^2 + \underbrace{c(\lambda) \log(r\Lambda)} + F(\lambda)}$$

$$\int d^2x \sqrt{g}\, f(\lambda) \qquad\qquad \underbrace{\int d^2x \sqrt{g}\, F(\lambda) R}_{\text{ambiguous.}}$$

$\nexists$ a local counterterm that can cancel that

$$Z_{S^2} \sim (r\Lambda)^{c/3}$$

This term is the conformal anomaly $\log(r\Lambda)$

$$\delta_\sigma \log Z = \int d^2x \sqrt{g}\, \sigma\, c\, R .$$

- Z_{S^2} ~~Kä~~ is ambiguous.
  - in a ~~S~~CFT

- What happens in the presence of SUSY ?
  - Impose that counterterms are SUSYc $\Rightarrow$ SUGRA invariant
  - $\Rightarrow$ space of counterterms is reduced $\Rightarrow$ ambiguities may be reduced

$$\mathcal{L}(\underbrace{\lambda^i, \ldots}_{\text{multiplet}} ; \underbrace{g_{\mu\nu}, \ldots}_{\text{SUGRA multiplet}})$$

can we supersymmetrize $\int d^2x \sqrt{g}\, R\, f(\lambda)$?

Depends on the theory:

$D=2$ $\mathcal{N}=(2,2)$. $\exists$ a finite SUGRA counterterm

$$\int d^2\tilde\theta\, \mathcal{E}\, \underset{\substack{\uparrow \\ \text{curvature} \\ \text{superfield}}}{R}\, F(\underset{\substack{\uparrow \\ \tau \text{ not } \bar\tau}}{\tau}) + c.c. \xrightarrow[S^2]{} \oint F(\tau) + \bar{F}(\bar\tau)$$

$$R|_{\text{top}} = R + iF$$

$\Longrightarrow$ Z_{S^2} has a Kähler ambiguity, so Z_{S^2} has a restricted ambiguity, precisely the Kähler ambiguity

$$Z \to Z e^{-F-\bar F}$$

- For 4d $\mathcal{N}=2$ ambiguity is reduced $Z_{S^4} \to Z_{S^4} e^{-F-\bar F}$

- But not in 4d $\mathcal{N}=1$, $\Longrightarrow$ Z_{S^4} is ambiguous. real!

$$\int d^2\theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\, R\bar{R}\, F(\Phi^i, \bar\Phi^i)$$