Quantum Entropy of Black Holes & Localization in Supergravity

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Outline

- Part 1 : Quantum Entropy of Black Holes
- Part 2 : Localization in Quantum Field Theory
- Part 3 : Localization in Supergravity
- Part 4 : Non-perturbative Effects
- Part 5 : Quantum Holography
- Part 6 : Assessment and Open Problems

Today and tomorrow, I'll give an overview in the first half of the lecture.

Quantum Entropy of Black Holes

- Classical black holes
- Semi-classical black holes
- Quantum black holes

Quantum black holes provide us with an invaluable tool to learn about the *short distance (UV) structure* of quantum gravity by studying *long distance (IR) properties.*

- A black hole is at once the ***most simple*** and yet the ***most complex*** object.
- Understanding the ***simplicity*** is in the realm of ***classical gravity*** and understanding the ***complexity*** is in the realm of ***quantum gravity.***

A great deal of **quantitative** information about semiclassical and quantum properties of a black hole has been obtained entirely on the strength of theoretical considerations. Makes for an interesting study in history of science.

Classical Black Holes

- ***No hair theorem***:

  A black hole is completely specified by mass, spin, charge much like an elementary particle.

  Kerr-Newman Metric.

  A black hole (unlike a star) is simple!

- ***Event Horizon:***

  A one way surface that ***causally*** separates the outside from the inside of the black hole.

Paradox I

- What happens if you throw a bucket of hot water into a black hole? The entropy of the world outside the black hole would decrease, violating the second law of thermodynamics.

Bekenstein

- ***Resolution****: Second law can be saved if the black hole also has entropy. Then the total entropy of black hole + bucket can increase in accordance with the second law.*

Paradox II

- If a black hole has entropy and mass then by ***first*** law of thermodynamics, it must also have temperature. But then it must radiate which is impossible for a classical black hole because the event horizon is a one way surface.
- **Resolution**: Because of quantum pair creation near the event horizon, a black hole can radiate. Metric is still treated classically.

Hawking

Semi-classical Black Holes

- A black hole has temperature and entropy

$$S = \frac{Ac^3}{4\hbar G} = \frac{A}{4l^2}$$

- This `Bekenstein-Hawking' area formula is remarkably general and involves all three fundamental constants of nature. Enormous entropy signifying a huge complexity.

Paradox III

- If a black hole has entropy then in quantum theory it must be an ensemble of microstates according to Boltzmann. How do we associate so many microstates with a hole in spacetime?
- To resolve this paradox we really need a quantum theory of gravity with a well-defined quantum Hilbert space.

Partially understood in string theory.

Quantum Gravity

- One of the enduring challenges of theoretical physics is to find a consistent framework for Quantum Gravity that unifies General Relativity with Quantum Mechanics.
- String theory offers a promising route towards such a Quantum Theory of Gravity:

perturbatively UV finite, strong-weak dualities, AdS/CFT holography…

However…..

Hurdles for String Theory

- We do not have a microscope like a super-LHC to probe the theory directly at Planck scale.
- We do not even know which phase or `compactification' of the theory may correspond to the real world.

How can we be sure that string theory is the right approach to quantum gravity in the absence of direct experiments?

How can we proceed?

- In a such a situation, a good strategy is to focus on **universal** features that must hold in **all phases** of the theory. Analogy with water.
- Entropy of a black hole is one such quantity which gives very precise quantitative thermodynamic information.
- In a quantum gravity, it should be possible to interpret a black hole as an ensemble of states in the Hilbert space of the theory.

Quantum Black Holes

Any black hole in ***any*** phase of the theory should be interpretable as an ensemble of quantum states ***including finite size effects.***

- ***Universal*** *and* ***extremely stringent*** *constraint*
- *An* ***IR*** *window into the* ***UV***
- Connects to a broader problem of

  Quantum Holography at finite N.

Micro from Macro

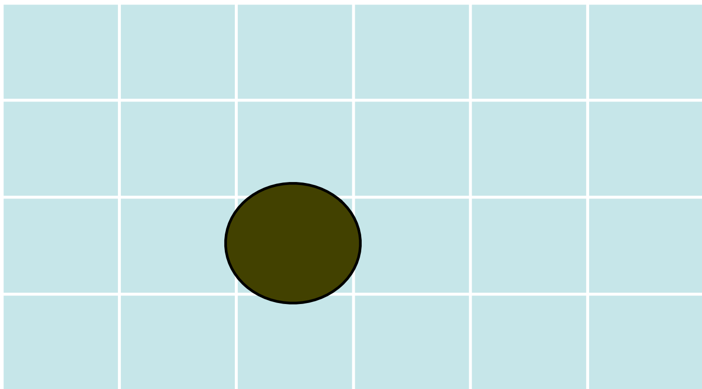
- In the absence of a microscope, one can often learn a lot about the microstructure from thermodynamic properties.
- For example, temperature dependence of specific heat of a metal tells you whether phonons or electrons are the relevant degrees of freedom.

Quantum properties of black holes can be put to good use in an analogous fashion.

Historical Analogy

- Kinetic theory of gases was a triumph of 19th century physics that formed the basis for the atomic hypothesis & later for quantum theory.
- It started with the attempts to explain **macroscopic** properties of ideal gases in terms of **microscopic `atoms'** even though there were no microscopes at the time that could establish the reality of atoms directly.

Entropy of dilute Nitrogen gas



$$\Omega = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$$

d(N) is the number of ways N molecules of size λ (de Broglie wavelength) can be distributed in volume V. **Microscopic counting** explains **Macroscopic Entropy!**

This line of reasoning has already led to some important advances in the 1990s.

- A large class of supersymmetric (charged) black holes can indeed be interpreted as ensembles of microstates as required by Boltzmann.
- Study of black branes (extended versions of black holes) led to holographic equivalence between a theory with gravity (AdS) and a theory without gravity (CFT).

Mostly for large area or large charge.

Black Hole as an Ensemble

- Does this entropy satisfy Boltzmann relation?

$$S = \log(d)$$

- Yes! For example, for a susy black hole with three charges Q_1, Q_2, Q_3 (all large)

$$\frac{A(Q_1, Q_2, Q_3)}{4} = 2\pi\sqrt{Q_1 Q_2 Q_3}$$

Macroscopic *Microscopic*

Bekenstein-Hawking *Strominger-Vafa*

A black hole is simple not because it is like an elementary particle but rather because it is like a thermodynamic ensemble.

This explains why it is both simple & complex!

Can we mine further this very important clue about quantum gravity?

Quantum Entropy

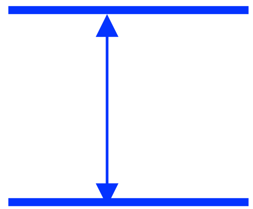
Given such a beautiful approximate agreement, it is natural ask:

- *What **exact** formula does it approximate? What is a **quantum generalization** of the Bekenstein-Hawking area formula for entropy?*
- *How to **systematically compute** the corrections to this quantum entropy and compare with microscopic counting?*

More subtle statistical considerations can yield more interesting information. Let us return to our historical analogy.

For example, we need the N! in our counting because all molecules of nitrogen molecules are identical. *Gibbs* deduced this important fact about the **microstructure** from extensivity of **macroscopic** entropy decades before the spin-statistics theorem in QFT.

- Classical equipartition theorem gives wrong specific heat for nitrogen. Because vibrational degrees are frozen at low temperature



- **Maxwell** regarded this as the *`greatest difficulty of classical molecular theory'* as early as **1859**. **Jeans** made a prescient remark in **1890** that somehow *`the degrees of freedom seem to be frozen.'*
- Serious crisis of classical physics.

What is new?

- Much of earlier work on black holes and holography is for black holes with large area.
- Our focus here will be on **finite size (finite charge) corrections to the black hole entropy**.
- Unlike the leading Bekenstein-Hawking formula, these **depend** sensitively on the phase under consideration & provide a useful window into the **UV structure** of the theory.

Why obsess with black holes?

- *Universal and extremely stringent constraint*

  In **any** phase of the theory that admits **any** black hole as a solution, it should be possible to view it as an ensemble of quantum states.

- *Quantum Entropy as* ***IR*** *window into the* ***UV***

  Finite size corrections to quantum entropy of a black hole give very precise quantitative information about the UV of the theory.

What is the **exact quantum generalization** of the celebrated Bekenstein-Hawking formula?

$$S = \frac{\mathbf{A}}{\mathbf{4}} + c_1 \log(A) + c_2 \frac{1}{A} \ldots + e^{-A} + \ldots$$

Generalization due to Wald is applicable only for local actions. We need a definition that includes *nonlocal* quantum effects from massless loops.

- The exponential of the quantum entropy must yield an **integer**. This is extremely stringent.
- Subleading corrections **depend** sensitively on the phase & provide a window into the **UV structure.**

Holography

- **Paradox IV**: If the entropy in a volume of space scales with volume then the spontaneous process of collapse into a black hole would decrease entropy violating second law. Unless the initial entropy scales with area and not volume.
- A (d+1)-dimensional theory must have the degrees of freedom of a d-dimensional theory like the holographic imprint of a 3d object onto a 2d hologram.

Holography in String Theory

- This heuristic idea is realized concretely in string theory in the AdS/CFT correspondence. Maldacena
- A remarkable quantum equivalence between

a theory ***with*** gravity **AdS**

(strings moving in Anti de Sitter spacetime)

&

and a theory ***without*** gravity **CFT**

(Conformal Field Theory in one less dimension).

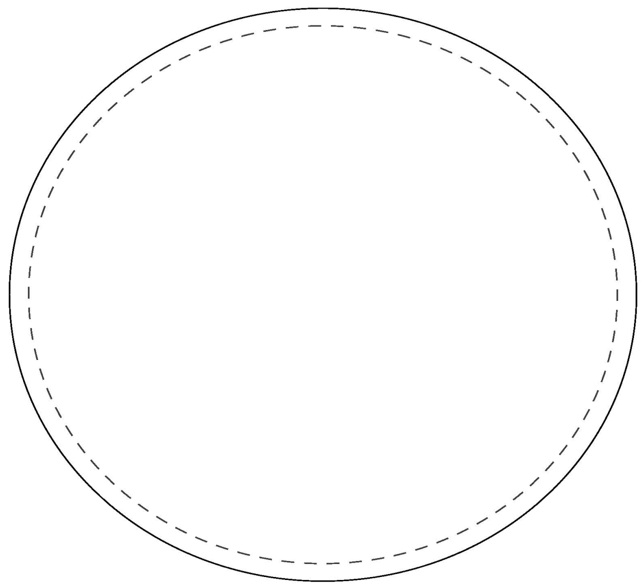
$$AdS_{p+2}/CFT_{p+1}$$

- Near horizon limit of a ***p-brane*** geometry is ***Anti de Sitter spacetime*** AdS_{p+2} .
- Low energy modes of the p-brane are desribed by ***Conformal Field Theory*** CFT_{p+1}

RG scale of the CFT gives an additional dimension. Much evidence for this equivalence, its full implications for quantum gravity are far from being understood.

AdS_2/CFT_1 and Quantum Entropy

- The near horizon geometry of a extremal charged (non-spinning) black hole is *AdS_2* .
- Quantum entropy can then be defined as a functional integral *$W(Q)$* in *AdS_2 over all string fields with appropriate boundary conditions, operator insertion, and a renormalization procedure.* ***Sen***
- For large charges, logarithm of *$W(Q)$* reduces to Bekenstein-Hawking-Wald entropy.



- Euclidean AdS_2 space is a disk with a metric

$$ds^2 = (r^2 - 1)d\theta^2 + \frac{dr^2}{r^2 - 1}$$

- Put a cutoff at $r = r_0$

W(Q)	**d(Q)**
Black Hole (charge Q)	Brane (charge Q)
Quantum Entropy	Counting of States
AdS_2	CFT_1
Spacetime Geometry	Hilbert Space

This gives the proper quantum generalization of

Bekenstein-Hawking ---- *Boltzmann*

Can we compute both sides?

How to evaluate it?

- *d(Q)* is the number of bound states of a brane system which is an extremely difficult dynamical problem in general.
- *W(Q)* is given as a formal functional integral and it is far from clear what to do with it.

A lot of progress has been made in several models in these computations. I will describe a couple of simpler examples for illustration.

Computing d(Q)

- For a large class of models this problem has been solved through the work of many people over several years.
- For a black hole with electric charge vector *q* and magnetic charge vector *p*, the degeneracy often depends only on a few duality invariants.
- Degeneracy given in terms of Fourier coefficients of ***modular forms***.

Computing W(Q)

- Integrate out massive string modes to get a Wilsonian effective action for massless fields.
- Still need to make sense of the formal path integral of supergravity fields. Using it do explicit computations is fraught with danger.
- It helps to have microscopic degeneracies $d(Q)$ from brane counting to compare with:

$$W(Q) = d(Q)$$

Localization in Supergravity

- Localization has enabled many powerful computations in QFT over the past two decades. One could access aspects of strongly coupled QFT which were otherwise inaccessible.
- Our results can be viewed as a beginning of a similar program for quantum gravity.
- Rules are less clear. Comparison with boundary results is a useful guide. One hopes these methods will develop further in the coming years.

Modular forms

- A holomorphic function $F(\tau)$ on the upper half complex plane is a modular form of weight k, if it transforms as

$$F(\frac{a\tau+b}{c\tau+d}) = (c\tau+d)^k F(\tau)$$

for *a, b, c, d, k* integers and *ad-bc =1*

The matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *form the group SL(2, Z) under matrix multiplication.*

Electric States in Heterotic String

- Degeneracy *d(q)* depends not only on the duality invariant $q^2/2 = N$. It is given by

$$
\begin{aligned}
F(\tau) &= \frac{1}{q\prod_n (1-q^n)^{24}}\,, \qquad q = e^{2\pi i\tau} \\
&= \sum_n c(n) q^n \\
d(N) &= c(N).
\end{aligned}
$$

Here $F(\tau)$ is a modular form of weight -12.

One-eighth BPS states in N=8

- Type-II compactified on T^6
- Dyonic states with charge vector ***(Q, P)***
- U-duality invariant $\Delta = Q^2P^2 - (Q \cdot P)^2$
- Degeneracy given by Fourier coefficients $C(\Delta)$ of

$$\frac{\vartheta(\tau, z)^2}{\eta(\tau)^6}$$

$$d(\Delta) = (-1)^{\Delta+1} C(\Delta)$$

Computing $W(\Delta)$

- The structure of the microscopic answer suggests that $W(\Delta)$ should have an expansion

$$W(\Delta) = \sum_{c=1}^{\infty} W_c(\Delta)$$

- Each $W_c(\Delta)$ corresponds to a $\mathbb{Z}_c$ orbifold of the Euclidean near horizon black hole geometry.
- The higher *c* are exponentially subleading. Unless one can evaluate each of them *exactly* it is not particularly meaningful to add them.

Localization enables us to do this.

Path Integral on AdS_2

- Including the M-theory circle, there is a family of geometries that are asymptotically $AdS_2 \times S^1$:

$$ds^2 = (r^2 - \frac{1}{c^2})d\theta^2 + \frac{dr^2}{r^2 - \frac{1}{c^2}} + R^2 \left(dy - \frac{i}{R}(r - \frac{1}{c})d\theta + \frac{d}{c}d\theta \right)^2$$

- These are $\mathbb{Z}_c$ orbifolds of BTZ black hole. These geometries $\mathcal{M}_{c,d}$ all have the same asymptotics and contribute to the path integral.
- Related to the $SL(2,\mathbb{Z})$ family in AdS_3

  Maldacena Strominger (98), Sen (09), Pioline Murthy (09)

Localization

- If a supersymmetric integral is invariant under a localizing supersymmetry Q which squares to a compact generator H, then the path integral localizes onto fixed manifold of the symmetry Q.
- We consider localization in N=2 supergravity coupled to n_v vector multiplets whose chiral action is governed by a prepotential *F*.
- *We find the localizing submanifold left invariant by Q and evaluate the renormalized action.*

Off-shell Localizing Solutions

- We found ***off-shell*** localizing instantons in *AdS*$_2$ for supergravity coupled to n_v vector multiplets with scalars X^I and auxiliary fields Y^I

$$X^I = X_*^I + \frac{C^I}{r}\,, \qquad Y^I = \frac{C^I}{r^2}\,, \qquad \begin{array}{c} C^I \in \mathbb{R} \\ I = 0, \ldots, n_v \end{array}$$

- These solutions are *universal* in that they are *independent of the physical action* and follow entirely from the off-shell susy transformations.

Renormalized Action

- The renormalized action for prepotential ***F*** is

$$\mathcal{S}_{ren}(\phi, q, p) = -\pi q_I \phi^I + \mathcal{F}(\phi, p)$$

$$\mathcal{F}(\phi, p) = -2\pi i \left[F\left(\frac{\phi^I + ip^I}{2}\right) - \bar{F}\left(\frac{\phi^I - ip^I}{2}\right) \right]$$

$\frac{1}{2}(\phi^I + ip^I)$ is the off-shell value of X^I at the origin.

- For each orbifold one obtains a Laplace integral of $|Z_{top}|^2$ reminiscent of OSV conjecture.

Ooguri Strominger Vafa (04) *Cardoso de Wit Mohaupt (00)*

Final integral

The prepotential for the truncated theory is

$$F(X) = -\frac{1}{2}\frac{X^1}{X^0}\sum_{a,b=2}^{7} C_{ab}X^a X^b \qquad (n_v = 7)$$

(dropping the extra gravitini multiplets)

The *path* integral reduces to the Bessel integral

$$W_1(\Delta) = N\int \frac{ds}{s^{9/2}}\exp\left[s + \frac{\pi^2\Delta}{4s}\right]$$

$$W_1(\Delta) = \tilde{I}_{7/2}(\pi\sqrt{\Delta})$$

Hardy-Ramanjuan-Rademacher Expansion

An exact convergent expansion (using modularity)

$$C(\Delta) = N \sum_{c=1}^{\infty} c^{-9/2} \tilde{I}_{7/2}\left(\frac{\pi\sqrt{\Delta}}{c}\right) K_c(\Delta)$$

$$\tilde{I}_{7/2}(z) = \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{ds}{s^{9/2}} \exp[s + \frac{z^2}{4s}]$$

$$\sim \exp\left[z - 4\log z + \frac{c}{z} + \ldots\right] \qquad z = A/4$$

The c=1 Bessel function sums *all perturbative* (in 1/z) corrections to entropy. The c>1 are non-perturbative

Generalized Kloosterman Sum $K_c(\Delta)$

$$\sum_{\substack{-c\leq d<0;\\(d,c)=1}} e^{2\pi i \frac{d}{c}(\Delta/4)}\ M^{-1}(\gamma_{c,d})_{\nu 1}\ e^{2\pi i \frac{a}{c}(-1/4)}$$

$$\nu = \Delta \mod 2$$

Relevant only in exponentially subleading nonperturbative corrections . *Even though highly subleading, conceptually very important for* ***integrality.***

New results concerning these nonperturbative phases

Multiplier System

$M^{-1}(\gamma)$ is a particular two-dimensional representation of the $SL(2,\mathbb{Z})$ matrix

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$M^{-1}(T) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad M^{-1}(S) = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Use the continued fraction expansion:

$$\gamma = T^{m_t} S \ldots T^{m_1} S$$

Degeneracy, Quantum Entropy, Wald Entropy

Δ	$C(\Delta)$	$W_1(\Delta)$	$\exp(\pi\sqrt{\Delta})$
3	8	7.972	230.765
4	-12	12.201	535.492
7	39	38.986	4071.93
8	-56	55.721	7228.35
11	152	152.041	22506.
12	-208	208.455	53252.
15	513	512.958	192401

- Note that $C(\Delta)$ are alternating in sign so that $d(\Delta) = (-1)^{\Delta+1} C(\Delta)$ is strictly positive.

This is a ***prediction from IR*** *quantum gravity for black holes which is* ***borne out by the UV****.*

- This explains the Bessel functions for all *c* with correct argument because for each orbifold the action is reduced by a factor of *c*
- *What about the Kloosterman sums?*

It was a long standing puzzle how this intricate number theoretic structure could possibly arise from a supergravity path integral.

Kloosterman from Supergravity

- Our analysis thus far is local and insensitive to global topology. The Chern-Simons terms in the bulk and the boundary terms are sensitive to the global properties of $\mathcal{M}_{c,d}$
- Additional contributions to renormalized action and additional saddle points specified by holonomies of flat connections. Various phases from CS terms for different groups assemble neatly into precisely the Kloosterman sum!

Kloosterman and Chern-Simons

$$I(A) = \int_{\mathcal{M}_{c,d}} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A^3\right)$$

In our problem we have three relevant groups

$U(1)^{n_v+1}$ $\quad SU(2)_L \quad SU(2)_R$

$$\sum_{\substack{-c\leq d<0;\\ (d,c)=1}} e^{2\pi i \frac{d}{c}(\Delta/4)} \; M^{-1}(\gamma_{c,d})_{\nu 1} \; e^{2\pi i \frac{a}{c}(-1/4)}$$

Dehn Twisting

The geometries $\mathcal{M}_{c,d}$ are topologically a solid 2-torus and are related to $\mathcal{M}_{1,0}$ by Dehn-filling.

Relabeling of cycles of the boundary 2-torus:

$$\begin{pmatrix} C_n \\ C_c \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \quad \text{for} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

C_1 is contractible and C_2 is noncontractible in $\mathcal{M}_{1,0}$

C_c is contractible and C_n is noncontractible in $\mathcal{M}_{c,d}$

Boundary Conditions and Holonomies

- The cycle C_2 is the M-circle and C_1 is the boundary of AdS_2 for the reference geometry $\mathcal{M}_{1,0}$
- This implies the boundary condition

$$\oint_{C_2} A^I = \text{fixed}\,, \qquad \oint_{C_1} A^I = \text{not fixed}$$

and a boundary term

$$I_b(A) = \int_{T^2} \mathrm{Tr} A_1 A_2 d^2 x$$

Elitzur Moore Schwimmer Seiberg (89)

Contribution from $SU(2)_R$

$$\oint_{C_1} A = 2\pi i\gamma\frac{\sigma^3}{2} \qquad \oint_{C_2} A = 2\pi i\delta\frac{\sigma^3}{2}$$

$$\oint_{C_c} A = 2\pi i\alpha\frac{\sigma^3}{2} \qquad \oint_{C_n} A = 2\pi i\beta\frac{\sigma^3}{2}$$

Chern-Simons contribution to the renormalized action is completely determined knowing the holonomies. For abelian the bulk contribution is zero for flat connections and only boundary contributes.

$$I_b[A_R] = 2\pi^2\gamma\delta \qquad I[A_R] = 2\pi^2\alpha\beta$$

Kirk Klassen (90)

$$\oint_{C_1} A_R = -\frac{2\pi i}{c}\frac{\sigma^3}{2}\,, \qquad \oint_{C_2} A_R = 0$$

Supersymmetric $\mathbb{Z}_c$ orbifold $\qquad J_R = 0$

$$\gamma = -1/c\,, \quad \delta = 0\,, \quad \alpha = -1\,, \quad \beta = -a/c$$

(using $\alpha = c\gamma + d\delta\,, \qquad \beta = a\gamma + b\delta$)

The total contribution to renormalized action is

$$S_{ren} = -\frac{2\pi i k_R}{4}\frac{a}{c} \qquad k_R = 1$$

in perfect agreement with a term in Kloosterman.

Multiplier System from $SU(2)_L$

There is an explicit representation of the Multiplier matrices that is suitable for our purposes.

$$M^{-1}(\gamma)_{\nu\mu} = C \sum_{\epsilon=\pm} \sum_{n=0}^{c-1} \epsilon \, e^{\frac{i\pi}{2rc}\left[d(\nu+1)^2 - 2(\nu+1)(2rn+\epsilon(\mu+1)) + a(2rn+\epsilon(\mu+1))^2\right]}$$

Unlike $SU(2)_R$ the holonomies of $SU(2)_L$ are not constrained by supersymmetry and have to be summed over which gives precisely this matrix.

(Assuming usual shift of k going to k +2)

Knot Theory and Kloosterman

- This computation is closely related to knot invariants of Lens space $\mathcal{L}_{c,d}$ using the surgery formula of Witten. *Witten (89) Jeffrey (92)*
- This is not an accident. Lens space is obtained by taking two solid tori and gluing them by Dehn-twisting the boundary of one of them. But Dehn-twisted solid torus is our $\mathcal{M}_{c,d}$
- Intriguing relation between topology and number theory for an appropriate CS theory.

An *IR* Window into the *UV*

- The degeneracies *d(Q)* count brane bound states. These are *nonpertubative* states whose masses are much higher than the string scale.
- Our supergravity computation of *W(Q)* can apparently access this information with precision.
- If we did not know the spectrum of branes a priori in the *N=8* theory then we could in principle deduce it. For example, in *N=6* models *d(Q)* is not known but the sugra computation of *W(Q)* seems doable.

Platonic Elephant of M-theory

- Quantum gravity seems more like an equivalent dual description rather than a coarse-graining.
- *It is not only UV-complete (like QCD) but UV-rigid*. E. g. Small change in the effective action of an irrelevant operator will destroy integrality.
- *AdS/CFT* is just one solitonic sector of the theory. It seems unlikely that we can bootstrap to construct the whole theory from a single *CFT* which for a black hole is just a finite dimensional vector space.

Positivity

- Note that $C(\Delta)$ are alternating in sign so that $d(\Delta) = (-1)^{\Delta+1} C(\Delta)$ is strictly positive.
- Surprising for a quantum field theorist or for a number theorist, because Fourier coefficients of a modular form do not *a priori* have any positivity property.
- *This is a* ***prediction from IR*** *quantum gravity for black holes which is* ***borne out by the UV****.*

Integrality

- The area of the black hole horizon is $4\pi\sqrt{\Delta}$. *The Bessel function sums up an infinite series of perturbative corrections in inverse powers of area.* Remarkably, the functional integral quantum entropy gives an answer that is very close to the integral degeneracy.
- *By contrast, the exponential of Wald entropy is very far from the integer.* Even for $\Delta = 15$

when area is large, the nonlocal quantum corrections make a substantial contribution.

Weyl Multiplet

- In conformal supergravity in our gauge, conformal factor of the metric has been traded for a scalar field. The fiducial metric is held fixed but the physical metric has a nontrivial profile for the off-shell solution. Rather like Liouville in 2d.
- There are no additional solutions from the Weyl multiplet that contains the metric Gupta Murthy (12)
- Subtleties with localization (because metric is fluctuating) that need to be understood in better.

Quantum Entropy: An Assessment

- ✓ Choice of Ensemble: AdS_2 boundary conditions imply a *microcanonical* ensemble. *Sen (09)*
- ✓ Supersymmetry and AdS_2 boundary conditions imply that index = degeneracy and $J_R = 0$

  Sen (10) Dabholkar Gomes Murthy Sen (12)
- ✓ Path integral localizes and the localizing solutions and the renormalized action have simple analytic expressions making it possible to even evaluate the remaining finite ordinary integrals. *DGM (10, 11)*

✓ Contributions from orbifolded localizing instantons can completely account for all nonperturbative corrections to the quantum entropy.

✓ All intricate details of Kloosterman sum arise from topological terms in the path integral. *DGM (14)*

✓ (Most) D-terms evaluate to zero on the localizing solutions *de Wit Katamadas Zalk (10)* *Murthy Reys (13)*

Path integral of quantum gravity (a complex analytic continuous object) can yield a precise integer (a number theoretic discrete object).

$$W(Q) = \int d\Phi e^{-S[\Phi]} = integer$$

Open Problems

- ? Computation of the measure including one-loop determinants. Subtleties with gauge fixing.
- ? We used an N=2 truncation of N=8 supergravity. This should be OK for finding the localizing instantons because the near horizon has N=2 susy. But it's a *truncation*.
- ? We dropped the hypermultiplets. They are known not to contribute to Wald entropy but could contribute to off-shell one-loop determinants.

One-loop determinants

- Computation of one-loop determinants is subtle because of gauge fixing. The localization charge is a combination of susy and BRST. Similar to computation by Pestun.
- For vector multiplets and hypermultiplets coupled to gravity this computation has recently been done in agreement with on-shell results. Additional subtleties in supergravity in gauge-fixing conformal supergravity to Poincaré supergravity. *Murthy Reys 2015; Gupta Ito Jeon 2015*
- In the N=8 case, the net contribution of the fields that we have dropped is zero. Justifies the truncation.

Off-shell supergravity

- ? It would be useful to have off-shell realization of the two localizing supercharges on all fields of N=8 supermultiplet. Hard but doable technical problem.
- ? We have treated gravity multiplet as any other multiplet in a fixed background. This is not fully justified for very off-shell configurations.

Kloosterman sum arising from topological terms is essentially independent of these subtleties.

- Black holes continue to be an important source of new theoretical ideas. Because their thermodynamic properties are deduced from robust and well-tested physical principles, it remains our most reliable guide in the search for a coherent framework for quantum gravity.
- String theory is an extremely rigid theoretical structure and seems capable of explaining a black hole as an ensemble in a quantum Hilbert space.
- It is remarkable that gravity can 'see' the integrality of a non-perturbative count of quantum states.