Construction of 1PI effective action

$\mathcal{H}$: Hilbert space of CFT satisfying

$L_0^- |A\rangle = 0$, $b_0^- |A\rangle = 0$ if $|A\rangle \in \mathcal{H}$.

$L_0^\pm = L_0 \pm \bar{L}_0$, $b_0^\pm = b_0 \pm \bar{b}_0$, $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$

Given $|A_1\rangle, \cdots |A_n\rangle \in \mathcal{H}$, define:

$$\{A_1 \cdots A_n\} \equiv \sum_{g=0}^{\infty} (g_s)^{2g} \int_{R_{g,n}} \omega_{6g-6+2n}(A_1, \cdots A_n)$$

→ 1PI amplitude of $A_1, \cdots A_n$

Define $[A_1 \cdots A_n] \in \mathcal{H}$ via

$\langle A_0 | c_0^- | [A_1 \cdots A_n] \rangle = \{A_0 A_1 \cdots A_n\}$ $\forall A_0$.

$\langle A|B\rangle$: BPZ inner product of CFT

Define ghost no:

$b, \bar{b}$: -1, $c, \bar{c}$: 1, matter : 0.

String field $|\psi\rangle$

→ A state in $\mathcal{H}$ of ghost no. 2.

If $\{|\phi_r\rangle\}$ is a basis of $\mathcal{H}$ of ghost no. 2

$|\psi\rangle = \sum_r a_r |\phi_r\rangle$

$\hookrightarrow$ dynamical variables.

$$\sum_r \rightarrow \int d^D k \sum_s$$

$\hookrightarrow$ discrete quantum nos.

$$\sum_r a_r |\phi_r\rangle$$

$$= \int d^D k \sum_s a_s(k) \, |\phi_s(k)\rangle$$

$\downarrow$

Fourier transform of fields.

1PI action $S(\psi)$ → should give a number for any given $|\psi\rangle$.

$$S(\psi) = \frac{1}{g_s^2}\left[\frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\psi^n\}\right]$$

$$\{\underbrace{\psi\,\psi \cdots \psi}_{\hookrightarrow\, n\text{-times}}\}$$

$S(\psi)$ is invariant under

$$\delta|\psi\rangle = Q_B|\Lambda\rangle + \sum_{n=0}^{\infty}\frac{1}{n!}[\psi^n \Lambda]$$

$|\Lambda\rangle$: an infinitesimal state of $\mathcal{H}$ of ghost no. 1.

⇒ ∞ dimensional gauge invariance (includes general coordinate transformation)

Different choice of $S_{g,n}$ & $R_{g,n}$.

$\Rightarrow$ different 1PI action $\tilde{S}(\psi)$.

One can show that $S(\psi)$ and $\tilde{S}(\psi)$ are related by field redefinition.

$$S(\psi) = \tilde{S}(f(\psi)) \text{ for some } f(\psi).$$

S-matrix elements.

Naive approach:

① Gauge fix. $b_0^+|\psi\rangle = 0 \rightarrow$ Siegel gauge.

② Derive Feynmann rules.

③ Compute tree amplitudes. on-shell

→ Gives us back the usual on-shell amplitudes of string theory with its IR divergences.

Correct approach

① Find the correct vacuum by solving eqs. of motion:

$$Q_B|\psi\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}[\psi^{n-1}] = 0.$$

Note: $n=1 \Rightarrow [\ ] \neq 0$ from $g \geq 1$ terms.

$\Rightarrow |\Psi\rangle = 0$ is not a soln. to eqs. of motion.

② If $|\Psi_{vac}\rangle$ is the soln., define

$$|\Phi\rangle = |\Psi\rangle - |\Psi_{vac}\rangle.$$

③ Expand S in powers of Φ:

$$\tilde{S}(\Phi) = S(|\Psi_{vac}\rangle + |\Phi\rangle)$$

$$= S(|\Psi_{vac}\rangle) + \frac{1}{g_s^2}\Big[\langle\Phi|c_0^- \hat{Q}_B|\Phi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\Phi^n\}'\Big].$$

$\hat{Q}_B^2 = 0$

$$\hat{Q}_B|A\rangle = Q_B|A\rangle + \sum_{n=0}^{\infty}[\Psi_{vac}^n\, A]\frac{1}{n!}$$

$$\{A_1 \cdots A_n\}' = \sum_{m=0}^{\infty}\frac{1}{m!}\{A_1 \cdots A_n\, \Psi_{vac}^m\}$$

for $n \geq 3$.

④ Linearized eq. of motion for $|\Phi\rangle$:

$$\hat{Q}_B|\Phi\rangle = 0.$$

Physical mass.

soln. $e^{ik\cdot X(0)}|\tilde{A}\rangle$ for $k^2 + m_{ph}^2 = 0$.

↳ discrete q. nos.

⑤ Derive Feynman rules & Calculate S-matrix elements using tree amplitude of $\tilde{S}(\phi\phi)$.

→ Free from tadpole & mass renormalization divergences.

For $D \geq 5$ there are also no IR divergences from loops.

In $D=4$, on-shell there are IR divergences from loops which can be removed by standard QFT method.

Generalization to superstrings (type II & heteroti

Focus on heterotic for definiteness.

Matter system: $(c_L, c_R) = (26, 15)$.

Ghost $b, c, \bar{b}, \bar{c}$ $(c_L, c_R) = (-26, -26)$

~~Ghos~~ β, γ $(c_L, c_R) = (0, 11)$

Commuting.

"Bosonization:"

$\gamma = \eta e^{\phi}$, $\beta = \partial \xi e^{-\phi}$

(ξ, η) : fermions. ϕ : scalar.

Ghost no: $c, \bar{c}, \gamma$: 1

$b, \bar{b}, \beta$: -1

η : 1, ξ : -1, ϕ ~~ϕ~~ : 0.

Picture no: $e^{2\phi}$: 2, ξ : 1, η : -1.

γ : 0, β : 0.

~~On~~ NS sector states: picture no. -1

R sector states: picture no. $-1/2$

On genus g need total picture no. $(2g-2)$.

A genus g amplitude with m NS-sector states and n R-sector states need insertion of an operator of picture no. $2g-2+m+\frac{n}{2}$.

Achieved with the help of picture changin operator $X(z) = \{Q_B, \xi(z)\}$.

Need $m + \frac{n}{2} + 2g - 2$ insertions of X at $y_1, y_2, \ldots y_{m+\frac{n}{2}+2g-2}$.

Off-shell amplitude depends on y_i's.

$P_{g,n}$ now contain data on y_i's.

Fiber direction: ~~Loc~~ Coordinate system w_a around punctures and y_i's.

Plumbing fixture induces PCO locations on $\Sigma_{g_1+g_2, n_1+n_2-2}$ from Σ_{g_1,n_1} and Σ_{g_2,n_2}.

Exception: If the puncture where surfaces ar glued are Ramond punctures then we need ~~extra~~ one extra insertion on $\Sigma_{g,n}$. $\to$ excercis

Taken to be $X_0 \equiv \oint \frac{dw}{w} X(w)$.

$\hookrightarrow$ A contour around the glued puncture.

Choice of $S_{g,n}$ should respect this.

Definition of ω_p is similar.

Additional information needed: Contraction of ω_p with the tangent vectors $\frac{\partial}{\partial y_i}$.

Result: Replace $X(y_i)$ by $-\partial\xi(y_i)$.

IPI amplitude:

$$\{A_1 \cdots A_n\} = \sum_{g=0}^{\infty} (g_s)^{2g} \int_{R_{g,n}} \omega_{6g-6+2n}(A_1, \cdots A_n)$$

$[A_1 \cdots A_n] \to$ defined similarly.

String field $|\Psi\rangle = |\Psi_{NS}\rangle + |\Psi_R\rangle$

$$\underset{\substack{\text{ghost no. } 2 \\ \text{picture no. } -1}}{|\Psi_{NS}\rangle} \qquad \underset{\substack{\text{ghost no. } 2 \\ \text{picture no. } -\frac{1}{2}}}{|\Psi_R\rangle}$$

IPI equation of motion:

$$Q_B|\Psi\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \mathcal{G}[\Psi^{n-1}] = 0$$

$\mathcal{G} = 1$ on NS-sector, X_0 on R-sector.

Gauge invariance

$$\delta|\Psi\rangle = Q_B|\Lambda\rangle + \sum_{n=0}^{\infty} \frac{1}{n!} \mathcal{G}[\Psi^n \Lambda]$$

$\Rightarrow$ includes local SUSY and general coordinate transformation.

References.

A.S. arXiv: 1408.0571, 1411.7478, 1501.00988

Related references:

Zwiebach, hep-th/9206084

Pius, Rudra, A.S., arXiv: 1311.1257, 1401.7014,
1404.6254

A.S. and Witten, arXiv: 1504.00609