Aspects of superstring perturbation theory:

Lecture 1: Motivation & Strategy.

Lectures 2 & 3: Details.

UV & IR divergences in perturbative QFT

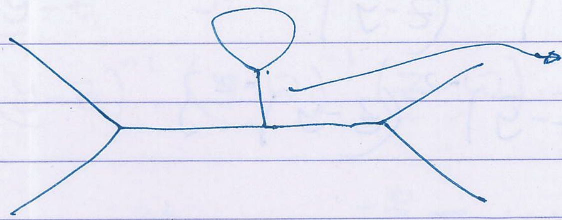
UV: loop momentum $\to \infty$.

Renormalizable QFT ✓

Non-renormalizable QFT ✗ (includes gravity)

IR: One or more internal propagator (tree or loop) goes on-shell.

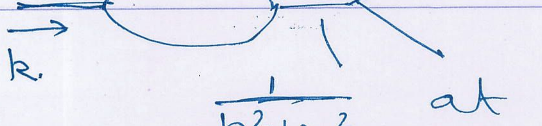
Examples. ① Massless tadpoles (in massless ϕ^3).



$$\frac{1}{k^2}\Big|_{k=0} \to \infty.$$

→ signals wrong choice of vacuum.

② Mass renormalization (in massive ϕ^3)



$\frac{1}{k^2+m^2}$ at $k^2+m^2=0 \Rightarrow \infty$.

→ Signals mass renormalization.

m^2 is not the correct mass of external states.

(3) We can also have IR divergence from loops.

→ Wrong choice of external states.

Schwinger parametrization

(1) $$\frac{1}{k_i^2+m_i^2} = \int_0^\infty ds_i\, e^{-s_i(k_i^2+m_i^2)}$$

for every propagator:

(2) Do loop momentum integrals.

→ gaussian.

(a) UV divergence from $\sum_{i \in \text{loop}} s_i \to 0$.

(b) IR divergence from $s_i \to \infty$ for one or more propagator.

UV: removed by renormalization.

What about IR divergences?

A useful approach ~~to~~ for dealing with IR divergences:

1PI effective action

One particle irreducible

→ generating function of off-shell 1PI amplitudes.

$\Gamma(\psi)$ → fields.

→ No IR divergence of tadpole or mass renormalization type.

IR divergences in loops are removed when we put external states off-shell

Given $\Gamma(\psi)$.

① Find vacuum by solving $\frac{\delta\Gamma}{\delta\psi} = 0$.
→ ψ_{vac}.

② Expand $\Gamma(\psi_{vac} + \phi) = \Gamma(\psi_{vac}) + \phi K \phi + \cdots$

Zeroes of K in $-k^2$-plane ⇒ physical mass^2

(3) Calculate off-shell amplitudes from tree diagrams of $\Gamma(\Phi_{vac}+\phi)$.

(4) Calculate S-matrix elements using LSZ prescription.

Steps 1 & 2 remove tadpole & mass renormalization divergences.

Need some more work to remove IR divergence from loops before step 4. → Well known.

String theory: Different starting point.

2-d world-sheet (S)CFT of matter + ghost

Q_B: BRST operator $Q_B^2=0$.

Physical states: $Q_B|A\rangle=0$.

"Gauge equivalence": $|A\rangle \equiv |A\rangle + Q_B|B\rangle$ $\forall\ |B\rangle$.

Imposes $L_0=0$, $\bar{L}_0=0$.

A can be taken to be dimension 0 primaries $c\bar{c}\,V_{matter}$. (Bosonic s.t.

$L_0 + \bar{L}_0 = \frac{k^2}{2} + N$ → takes discrete values.

$\Rightarrow \frac{k^2}{2} + N = 0$

$\Rightarrow$ Allowed N-values $\Rightarrow$ allowed $\frac{m^2}{2}$.

$Q_B |A\rangle = 0 \Rightarrow$ mass-shell condition.

S-matrix (g-loop, n-point).

$$= \int_{\mathcal{M}_{g,n}} d^{6g-6+2n} m \; \langle \prod_i A_i \times \text{ghost} \rangle_{\Sigma_{g,n}}$$

described later

Moduli space of ~~genus~~ $\Sigma_{g,n}$.

Vertex ops. for external states inserted at punctures.

genus g Riemann surface with n marked points (punctures)

$\langle \quad \rangle_{\Sigma_{g,n}}$: Finite in the interior of $\mathcal{M}_{g,n}$.

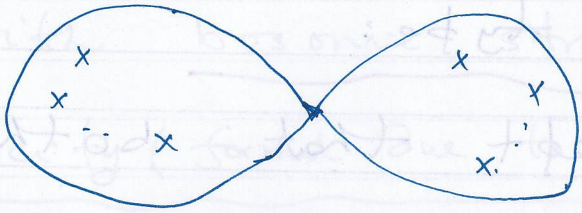
Only divergences could come from boundary of $\mathcal{M}_{g,n}$.

⇓

degenerate Riemann surfaces.

separating type — non-separating type.

Description of $\Sigma_{g,n}$ near separating type degeneration:

Σ_{g_1,n_1}  Σ_{g_2,n_2}

$g_1+g_2=g$

$n_1+n_2=n+2$

Mathematical description:

① Take Σ_{g_1,n_1} & Σ_{g_2,n_2}

② Take one puncture from each & glue Σ_{g_1,n_1} and Σ_{g_2,n_2} via:

$$w_1 w_2 = e^{-s+i\theta}$$

Complex coordinates around the puncture with $w_1=0$ & $w_2=0$ giving the locations of the punctures.

$$\mathcal{M}_{g,n} = \mathcal{M}_{g_1,n_1} \times \mathcal{M}_{g_2,n_2} \times (s,\theta)$$

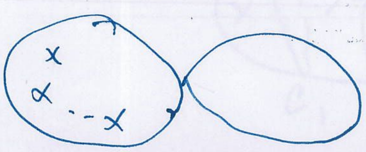
$0 \leq \theta < 2\pi$

$s \to \infty$: Degeneration limit.

$\Rightarrow$ Possible divergences in amplitude.

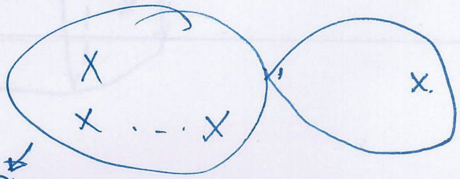
$s \Leftrightarrow$ Schwing parameter of QFT

$\Rightarrow$ Divergences in string theory are IR.



$\to$ tadpole

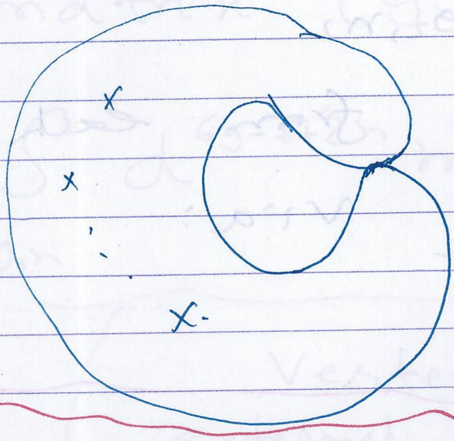
mass renormalization



Non-separating type:

→ Similar, except that both punctures are on $\Sigma_{g,n+2}$.

$w_1 w_2 = e^{-s+i\theta}$ $s \to \infty$ is degeneration



⟺ IR divergence associated with loops.

How do we deal with these IR divergences?

Strategy: Construct 1PI effective action & follow the same strategy as in a QFT.

① Need to define off-shell amplitude.

② Need to identify the analog of 1PI contribution to the amplitude.