Introduction to Particle Physics

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TIFR

Lecture 3

The Structure of Hadrons

- In 1897, Thomson discovered the *electron*.
- In 1911, Rutherford (with Geiger & Marsden) discovered the nucleus

The hydrogen nucleus must be a single positively-charged *proton*.

These were the only elementary particles known at the time

(and the *photon* – postulated by Einstein)

Problem of atomic number: $A > Z$

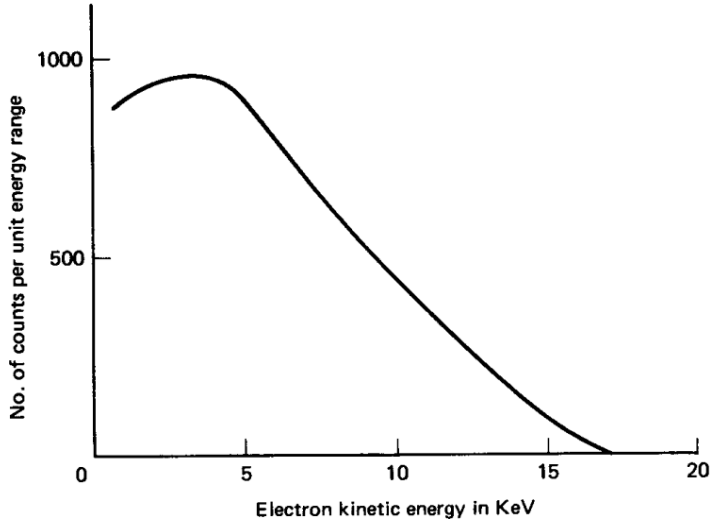
If a nucleus has A protons, it should have charge A ....

...what is reducing the charge?

Curie (1911): there are $A - Z = N$ electrons in the nucleus

- explains the charge puzzle; negligible change in mass due to Ne^{-}
- oppositely charged particles could help in binding the nucleus

- explains beta decay ${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^{-}$



The beta decay spectrum of tritium (${}^{3}_{1}H \rightarrow {}^{3}_{2}He$). (*Source:* G. M. Lewis, London: Wykeham, 1970), p. 30.)

atomic electrons will have energies

~ 10 eV

beta electrons have energies

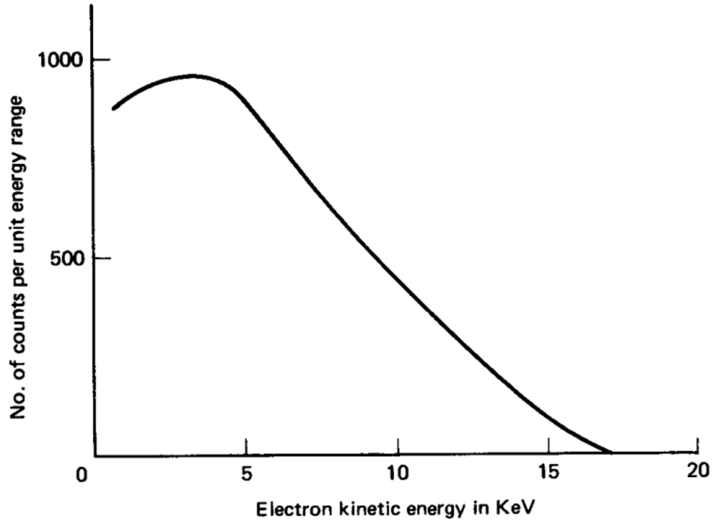
~ 10 keV

$\Rightarrow$ must be of nuclear origin

sort of ‘evaporation’ process

Curie’s theory was generally accepted...

- explains beta decay ${}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;A}Y + e^{-}$



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Curie's theory was generally accepted...

... till the discovery of the Uncertainty Principle (Heisenberg 1925)

$$\Delta x\, \Delta p > 1 \qquad \Rightarrow \quad p > \Delta p > \Delta x^{-1}$$

For a nucleus $\Delta x \sim 1$ fm $\Rightarrow p >$ MeV i.e. $E >$ MeV

Beta electrons (positrons) cannot be in equilibrium inside the nucleus; or, outside the nucleus...

They must be created in the nucleus and immediately emitted...

Return of the problem of atomic number: $A > Z$

If a nucleus has A protons and no electrons, it should have charge A ....

Neutron hypothesis:

The nucleus contains Z protons and $A - Z = N$ neutrons

- explains the charge puzzle if the neutron mass = proton mass
- explains beta decay: ${}_0^1n \rightarrow {}_1^1p + e^-$
- energy of the beta electron depends on the mass difference

- In 1932, Chadwick discovered the *neutron*

Solved the problem of nuclear charge but created a new problem

Problem of nuclear stability:

Inside the nucleus we have Z protons – all positive – and N neutrons – all neutral. Why doesn't the mutual repulsion of the protons cause the nucleus to break up?

Heisenberg: Must have an attractive force inside the nucleus

Estimate this force...must balance out the repulsion...

$$F \sim \frac{e^2}{r^2} = \frac{e^2}{a_0^2}\left(\frac{a_0}{r}\right)^2 = \frac{e^2}{a_0^2} \times \left(\frac{0.5\ \text{Å}}{1\ \text{fm}}\right)^2 = \frac{e^2}{a_0^2} \times 2.5 \times 10^9$$

Few *billion* times stronger than the atomic force...

Supported only on ‘weak’ van der Waals forces between atoms...

Heisenberg became the discoverer of the strong (nuclear) interaction...

Neutrons must participate in this interaction as well as protons:

- there are more and more neutrons in nuclei as Z increases
- we observe deuteron (pn) and tritium (pnn) bound states

Apart from the charge, the neutron is very similar to the proton...

1. Masses are almost equal: $M_p = 938.2 \text{ MeV}$ and $M_n = 939.5 \text{ MeV}$
2. Charge symmetry of strong interactions: $V_{pp} \approx V_{nn}$
3. Charge independence of strong interactions: $V_{pp} \approx V_{nn} \approx V_{np}$

Clearly, so far as the strong interaction is considered, there is no difference between the proton and the neutron...

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Heisenberg 1932 : can they be different states of the same particle?

Isospin

Could the $({}^{1}_{1}p, {}^{1}_{0}n)$ be different states of the same particle (*nucleon*)?

Recall that an electron comes in two states:

viz., spin + ½ and spin -½

For a free electron these are degenerate;

in a magnetic field B there is a splitting by $2\mu_B B$

$$e = \begin{pmatrix} e \uparrow \\ e \downarrow \end{pmatrix}$$

Perhaps there is an internal quantum number

(call it *isospin*) such that

proton has isospin + ½ and neutron has isospin -½ ...

$$N = \begin{pmatrix} p \uparrow \\ n \downarrow \end{pmatrix}$$

For some reason, the isospin + ½ partner is charged...

... electromagnetic self-energy is adequate to explain mass difference

Model the isospin quantum number on spin:

Spin-½ states correspond to a 2-dim representation of the *rotation group*, i.e. the states are $|s, s_3\rangle$ with $s = ½$:

$$v = \begin{pmatrix} |½, +½\rangle \\ |½, -½\rangle \end{pmatrix}$$

Under a passive rotation by Euler angles (ψ, θ, φ), we expect

$$v \to v' = D_{½}(\psi, \theta, \varphi) v$$

where

$$D_{½}(\psi, \theta, \varphi) = e^{i\varphi\frac{\sigma_3}{2}} e^{i\theta\frac{\sigma_2}{2}} e^{i\psi\frac{\sigma_3}{2}} = \begin{pmatrix} e^{\frac{i(\psi+\varphi)}{2}} \cos\frac{\theta}{2} & e^{\frac{-i(\psi-\varphi)}{2}} \sin\frac{\theta}{2} \\ -e^{\frac{i(\psi-\varphi)}{2}} \sin\frac{\theta}{2} & e^{\frac{-i(\psi+\varphi)}{2}} \cos\frac{\theta}{2} \end{pmatrix}$$

Note that $D^{\dagger}_{½}(\psi, \theta, \varphi) D_{½}(\psi, \theta, \varphi) = I$ and $\det D_{½}(\psi, \theta, \varphi) = +1$

This makes it an SU(2) matrix, i.e. rotation group ⇔ SU(2) of spin

Imagine the isospin as another SU(2) symmetry of the Hamiltonian of strong interactions...

...this must be an internal symmetry (flavour), not spacetime symmetry

We will then have a 2-dim representation $|i, i_3\rangle$ with $i = ½$:

$$N = \begin{pmatrix} |½, +½\rangle = p \\ |½, -½\rangle = n \end{pmatrix} \qquad\qquad q = i_3 + ½$$

There will be iso-rotations in the internal space, of the form

$$N \to N^{'} = UN$$

where U is an SU(2) matrix, belonging to SU(2) of isospin.

Written explicitly $N^{'} = UN$ means

$$p^{'} = U_{11}p + U_{12}n$$
$$n^{'} = U_{21}p + U_{22}n$$

i.e. the proton and neutron states can mix freely without changing the strong interaction Hamiltonian i.e. this is a symmetry of the H'tonian

Yukawa's theory (1935)

The electron corresponds to spin-½ states, and forms a doublet

The photon corresponds to spin-1 states, and forms a triplet

$$A^{\mu} = (\varphi, \vec{A}) \quad \text{where} \quad \partial_t \varphi + \vec{\nabla}.\vec{A} = 0 \quad \text{(Lorenz gauge)}$$

Now, we can write three independent polarisation vectors for $\vec{A}$,

i.e. the states are $|s, s_3\rangle$ with $s = 1$:

$$\varepsilon(p) = \begin{pmatrix} |1,+1\rangle \\ |1,\ \ 0\rangle \\ |1,-1\rangle \end{pmatrix} \quad \text{transforms as} \quad \varepsilon(p) \to \varepsilon'(p) = U\varepsilon(p)$$

Can we have the same thing happen with isospin?

i.e. can we have a 3-dim representation $|i, i_3\rangle$ with $i = 1$?

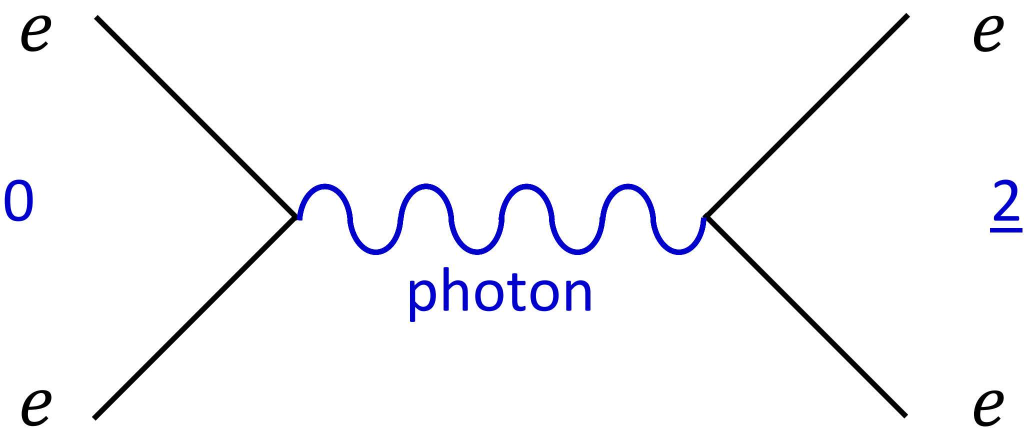
$$\pi = \begin{pmatrix} |1,+1\rangle = \pi^+ \\ |1,\ \ 0\rangle = \pi^0 \\ |1,-1\rangle = \pi^- \end{pmatrix}$$

$$q = i_3 + \frac{B}{2}$$

Another role for the pion:

The photon field is the mediator of electromagnetic interactions

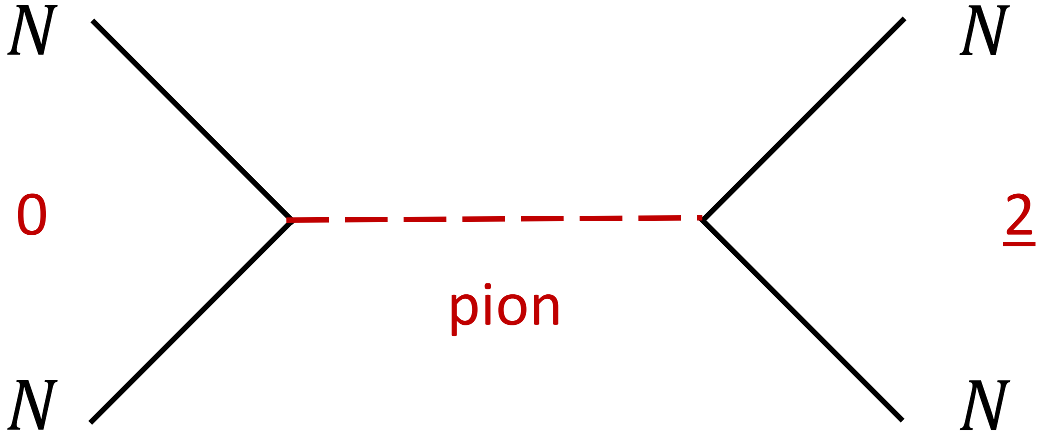
$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$



$\underline{2} \otimes \underline{2} = \underline{3} \oplus \underline{1}$

Can the pion be the mediator of the strong interaction?

$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$



$\underline{2} \otimes \underline{2} = \underline{3} \oplus \underline{1}$

Photons are vectors under SU(2)$_S$,
but scalars under SU(2)$_I$

Obey the wave equation...

$$\Box A^{\mu} = Q\delta^3(\vec{x})$$

with a point source

Go to the static limit...

$$\nabla^2\varphi = -Q\delta^3(\vec{x})$$

Solution is well-known

$$\varphi = \frac{Q}{4\pi\, r}$$

Long-range interaction
i.e. range is infinite

Photons are vectors under $SU(2)_S$, but scalars under $SU(2)_I$

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Long-range interaction

i.e. range is infinite

Pions are vectors under $SU(2)_I$, but scalars under $SU(2)_S$

Obey the Klein-Gordon equation...

$$(\Box + M^2)\pi = -g_s\delta^3(\vec{x})$$

with a point source

Go to the static limit...

$$(\nabla^2 + M^2)\pi = -g_s\delta^3(\vec{x})$$

Solution is well-known

$$\pi = \frac{g_s}{4\pi}\frac{e^{-Mr}}{r}$$

Short range interaction

i.e. range is M^{-1}

Set $M^{-1} \sim 1\ \mathrm{fm} \Rightarrow M \sim 200\ \mathrm{MeV}$

Pions were discovered in cosmic rays showers by Powell and his group at Bristol (1947)

Typically pions decay through

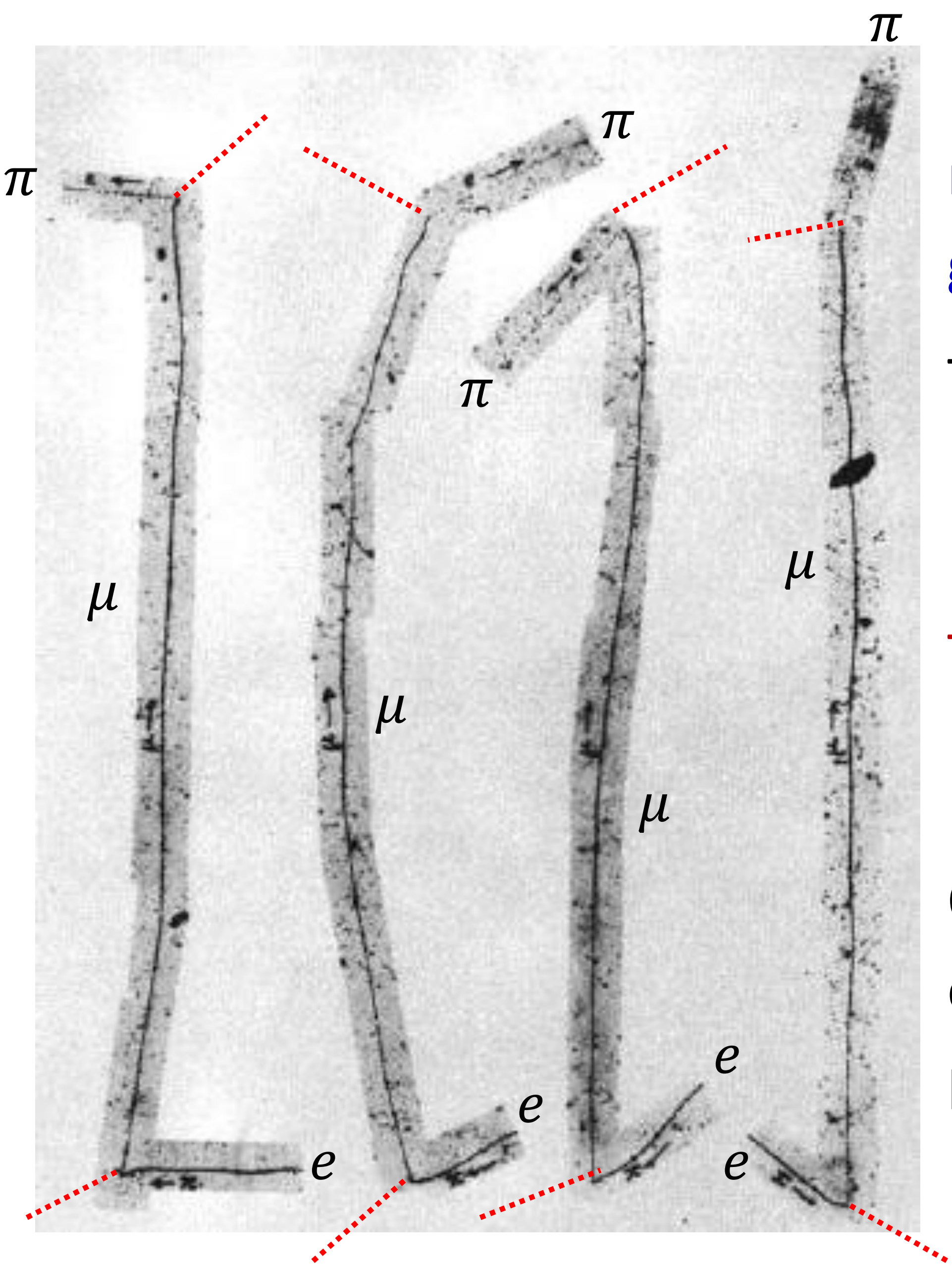
$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

These are both weak decays...

$$\tau_\pi \approx 26\,\text{ns} \qquad \tau_\mu \approx 2.1\,\mu\text{s}$$

One expects to see tracks corresponding to the charged particles, i.e. π^-, μ^- and e^-



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One can guess the existence of more particles...

$SU(2)_S$ ↓	$SU(2)_I$ →	scalar	vector
scalar		η^0 (1961)	(π^+, π^0, π^-)
vector		γ	(ρ^+, ρ^0, ρ^-) (1961)

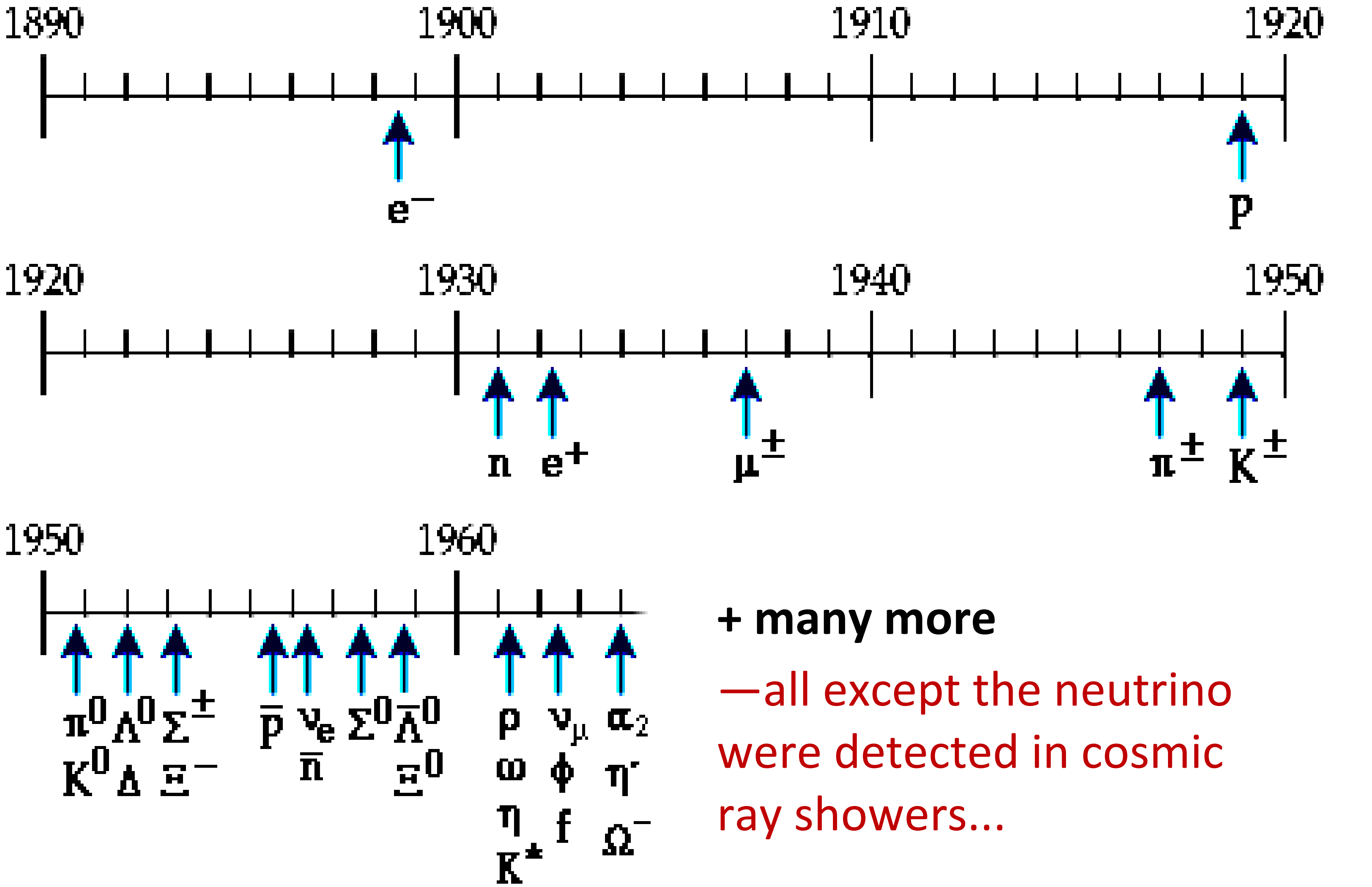
Why not higher representations? e.g. $|i, i_3\rangle$ with $i = \frac{3}{2}$ (Delta baryon)

$$\Delta = \begin{pmatrix} |3/2, +3/2\rangle \\ |3/2, +1/2\rangle \\ |3/2, -1/2\rangle \\ |3/2, -3/2\rangle \end{pmatrix} = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix} \qquad q = i_3 + \frac{B}{2}$$

(1952)

Proliferation of elementary particles during the 1950’s and 1960’s...

...mostly hadrons of different types...



+ many more

—all except the neutrino were detected in cosmic ray showers...

Cosmic rays

Primary cosmic rays arise from

(a) the Sun

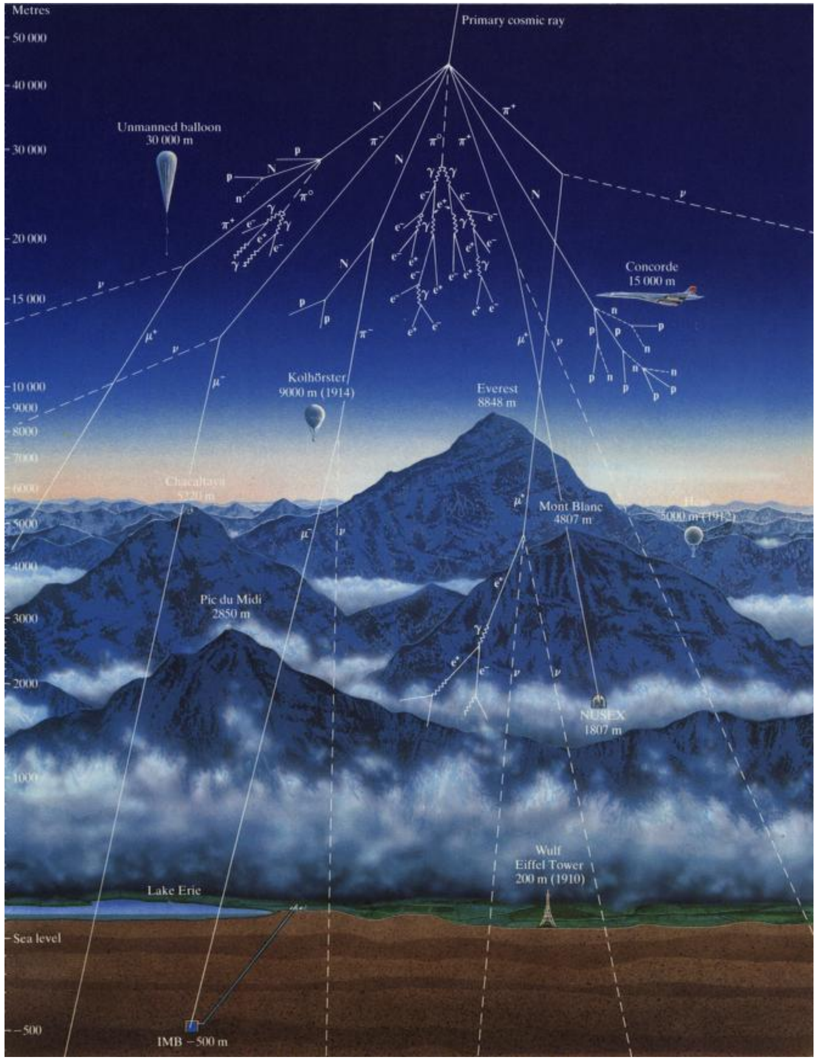
(stream of protons a.k.a. solar wind)

(b) violent events in the Universe

e.g. supernovae, AGNs, quasars,...

Secondary cosmic ray showers are created when these hit the upper atmosphere, and create an avalanche of new particles (Bhabha-Heitler theory 1934)

Perfect particle physics laboratory



Strange particles

One class of these new particles showed 'strange' behaviour...

...they were created in strong interaction scattering processes,

but decayed through weak interactions (i.e. left tracks)...

e.g. scattering process:

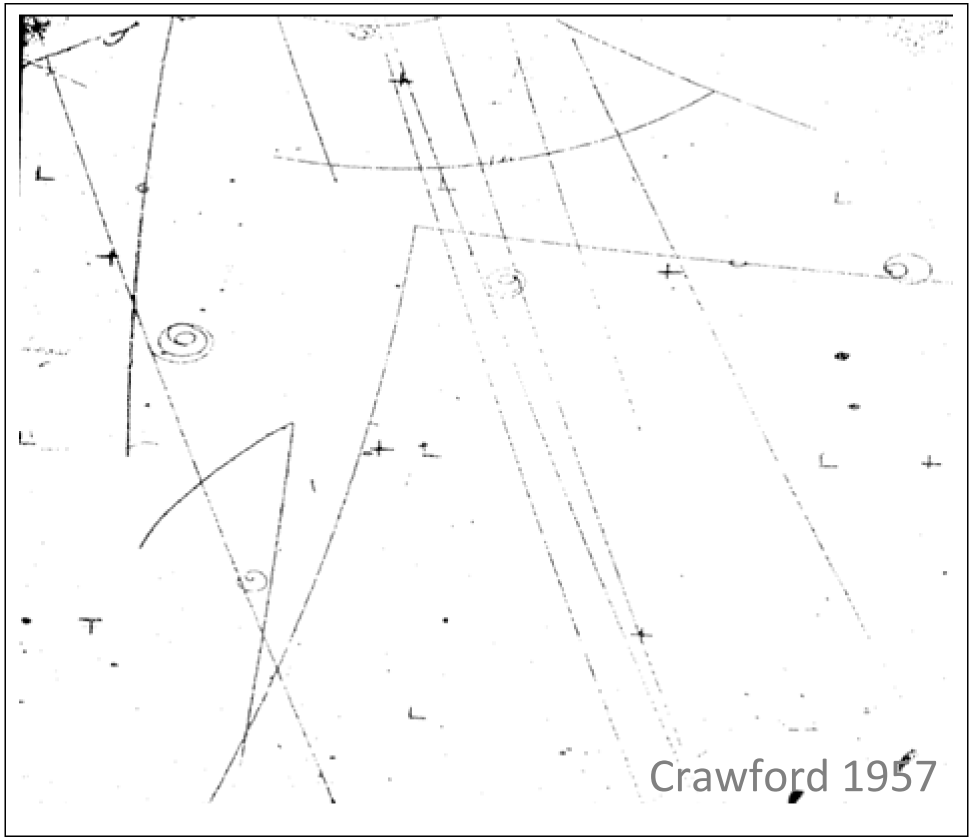
$$\pi^- + p^+ \rightarrow K^0 + \Lambda^0$$

(must be strong, else too rare to be seen with cosmic ray flux)

Decay processes:

$$\Lambda^0 \rightarrow p^+ + \pi^-$$
$$K^0 \rightarrow \pi^+ + \pi^-$$

(must be weak, for decay vertices are observably displaced)



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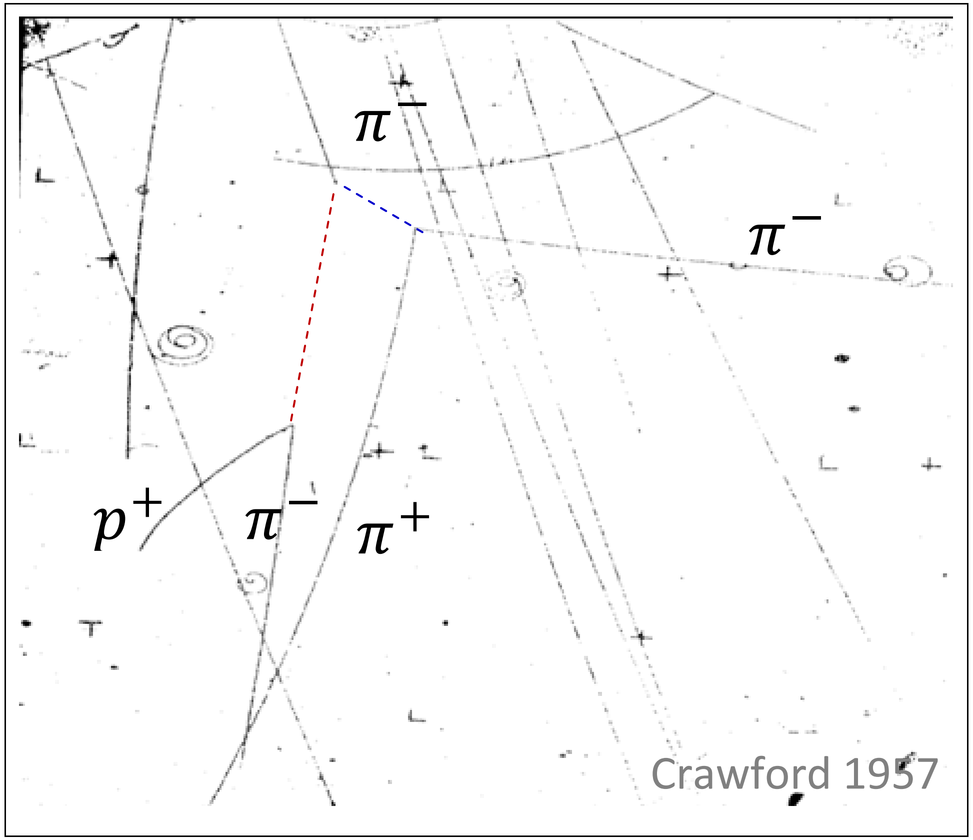


Pais (1952) :

maybe the K^0 and the Λ^0 carry some conserved quantum number which the proton and the pions do not – strangeness (S)

Strangeness is conserved in strong and electromagnetic interactions, but is not conserved in weak interactions

Strange particles (hyperons) must always be produced in pairs

K^0 is the *lightest* strange meson

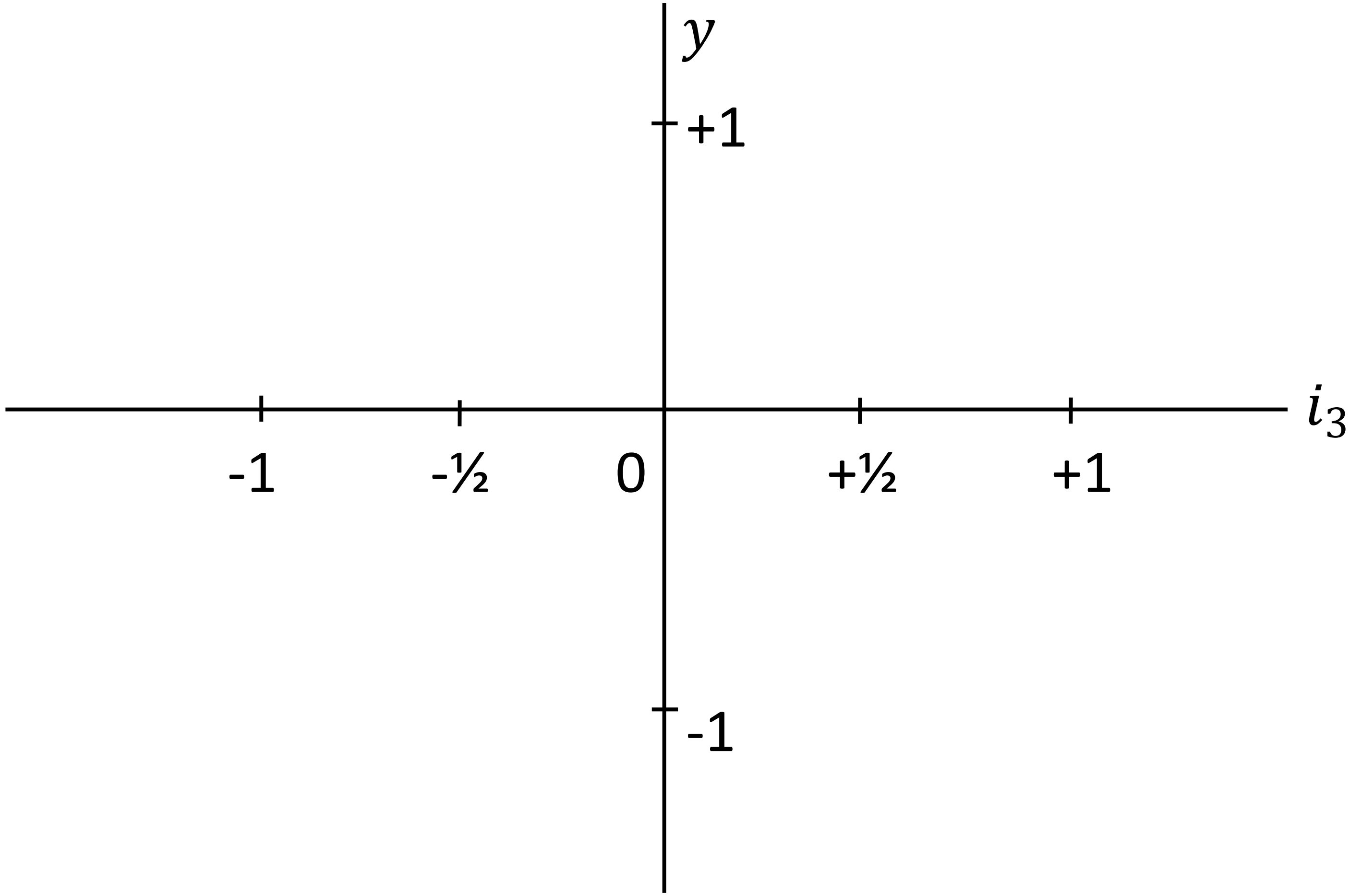
Λ^0 is the *lightest* strange baryon

Other heavier hyperons will always decay to these through strong interactions... but these can decay only through weak interactions

Thus, in addition to isospin, we have a new flavour – strangeness.

Nishijima (1954), Gell-Mann and Pais (1955):

classify particles according to isospin (i_3) and hypercharge ($y = B + S$)



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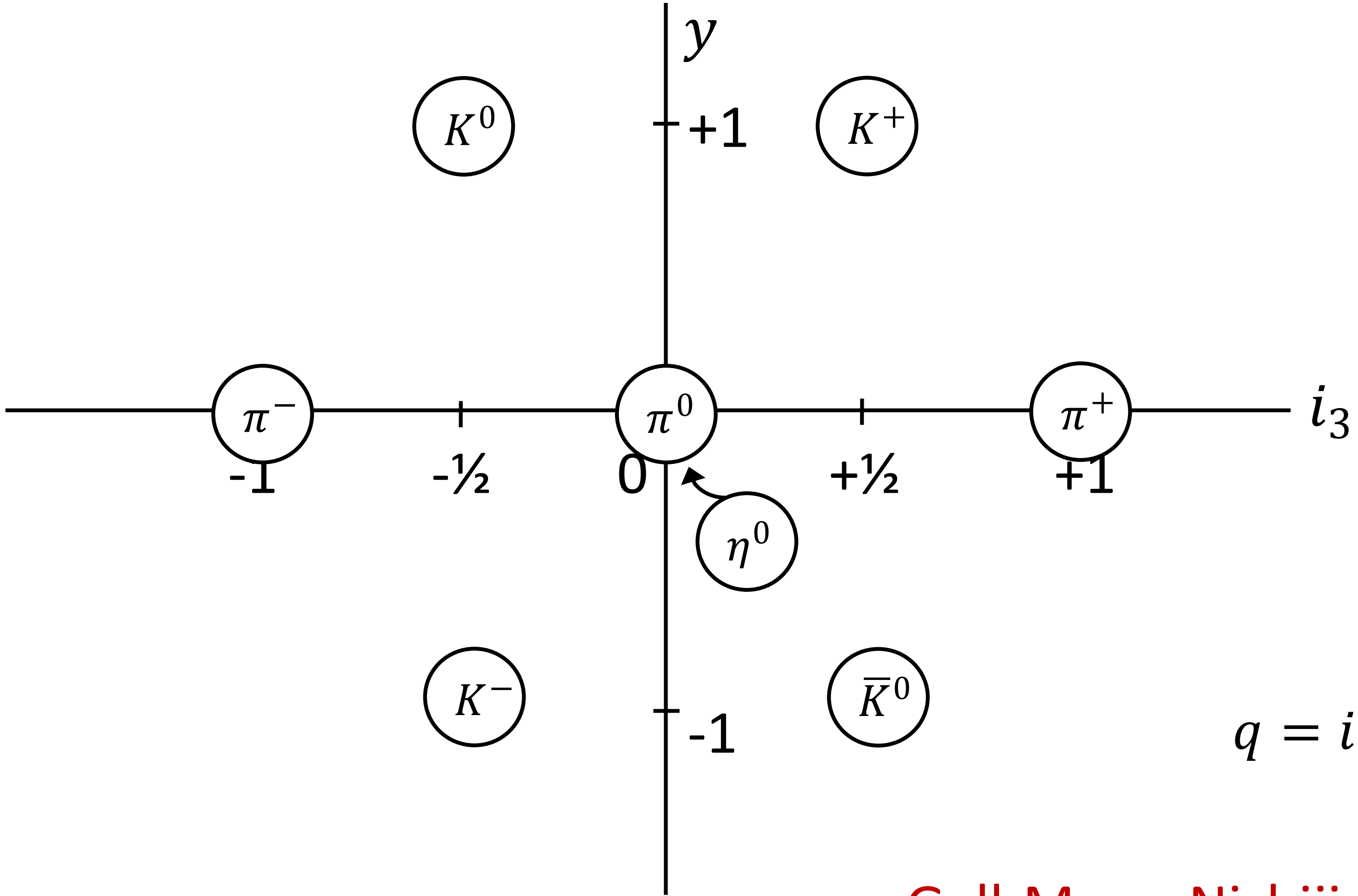

$$q = i_3 + \frac{y}{2}$$

Gell-Mann-Nishijima relation

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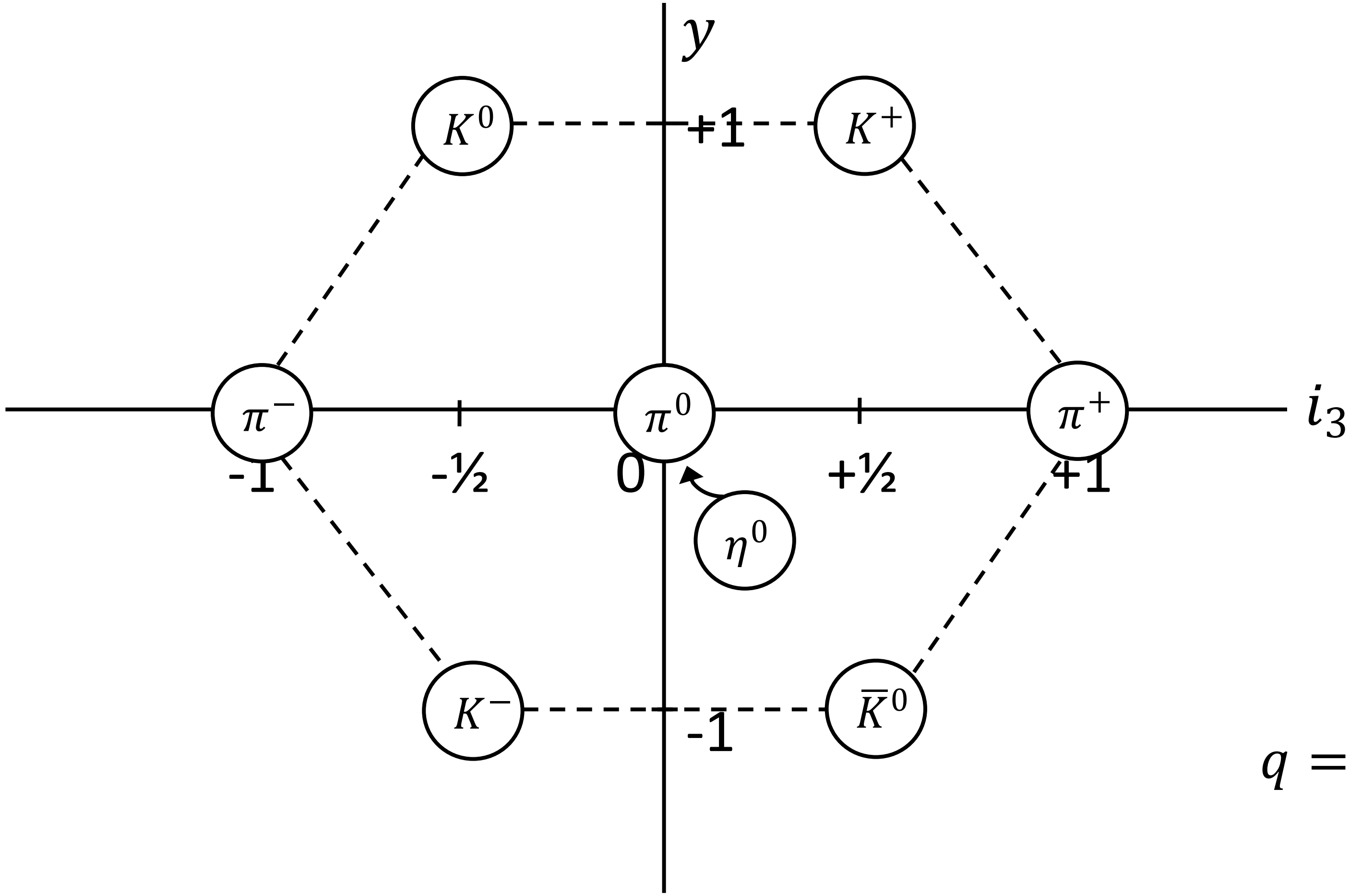

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Octet of pseudocalar mesons

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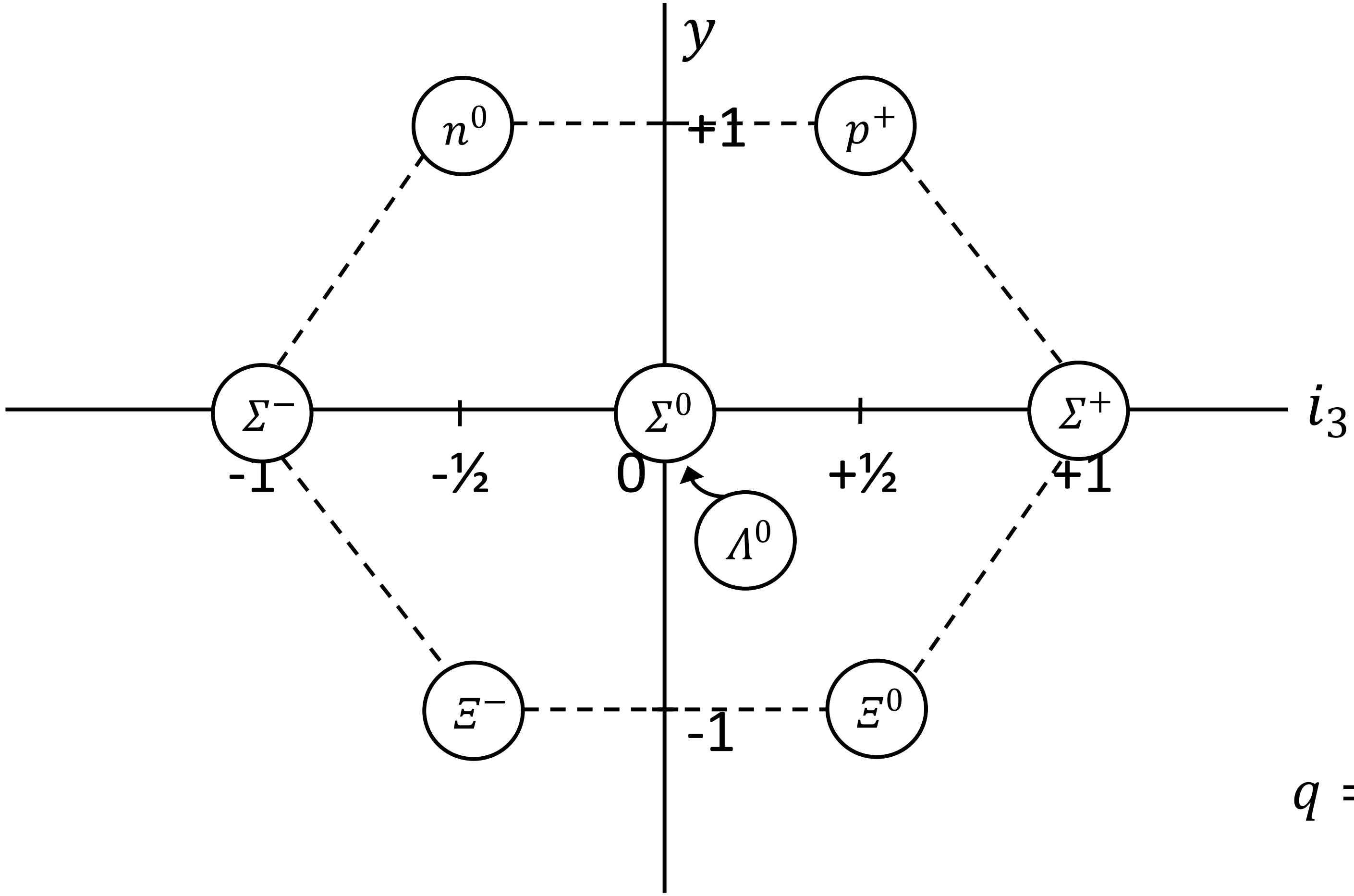


$$q = i_3 + \frac{y}{2}$$

Octet of baryons

The Eightfold Way (Gell-Mann and Ne'eman 1961)

Such a repeated pattern cannot be an accident...

Must reflect a (approx) symmetry of the strong interaction Hamiltonian

Extension of Heisenberg's idea: Maybe all the particles in an octet are different states of the same particle...

Can we find the group of transformations which is the analogue of $SU(2)_I$ in this case, i.e. which mixes the different states in an octet together?

- This group must have 2 conserved quantum numbers i_3, y
- This group must have $SU(2)_I$ as a subgroup
- This group must have octet ($\underline{8}$) representations

The only group which readily satisfies all this is SU(3)

SU(3)

Consider a triplet 3 which undergoes a unitary transformation

$$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \text{and} \qquad v \to v' = Uv$$

where $U^\dagger U = \mathbb{1}$ and $\det U = +1$.

This makes U an SU(3) transformation, i.e. it mixes up

$$a' = U_{11}a + U_{12}b + U_{12}c \quad \text{etc.}$$

Just as we label the 2 of SU(2) states by the eigenvalues i_3 of the I_3 operator, i.e.

$$I_3 = \begin{pmatrix} ½ & 0 \\ 0 & -½ \end{pmatrix}$$

similarly, for SU(3) we have two such operators

$$I_3 = \begin{pmatrix} ½ & 0 & 0 \\ 0 & -½ & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad Y = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$$

so that, just as in <u>2</u> of SU(2), we have two basis vectors

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$i_3 = ½ \quad \text{and} \quad i_3 = -½$$

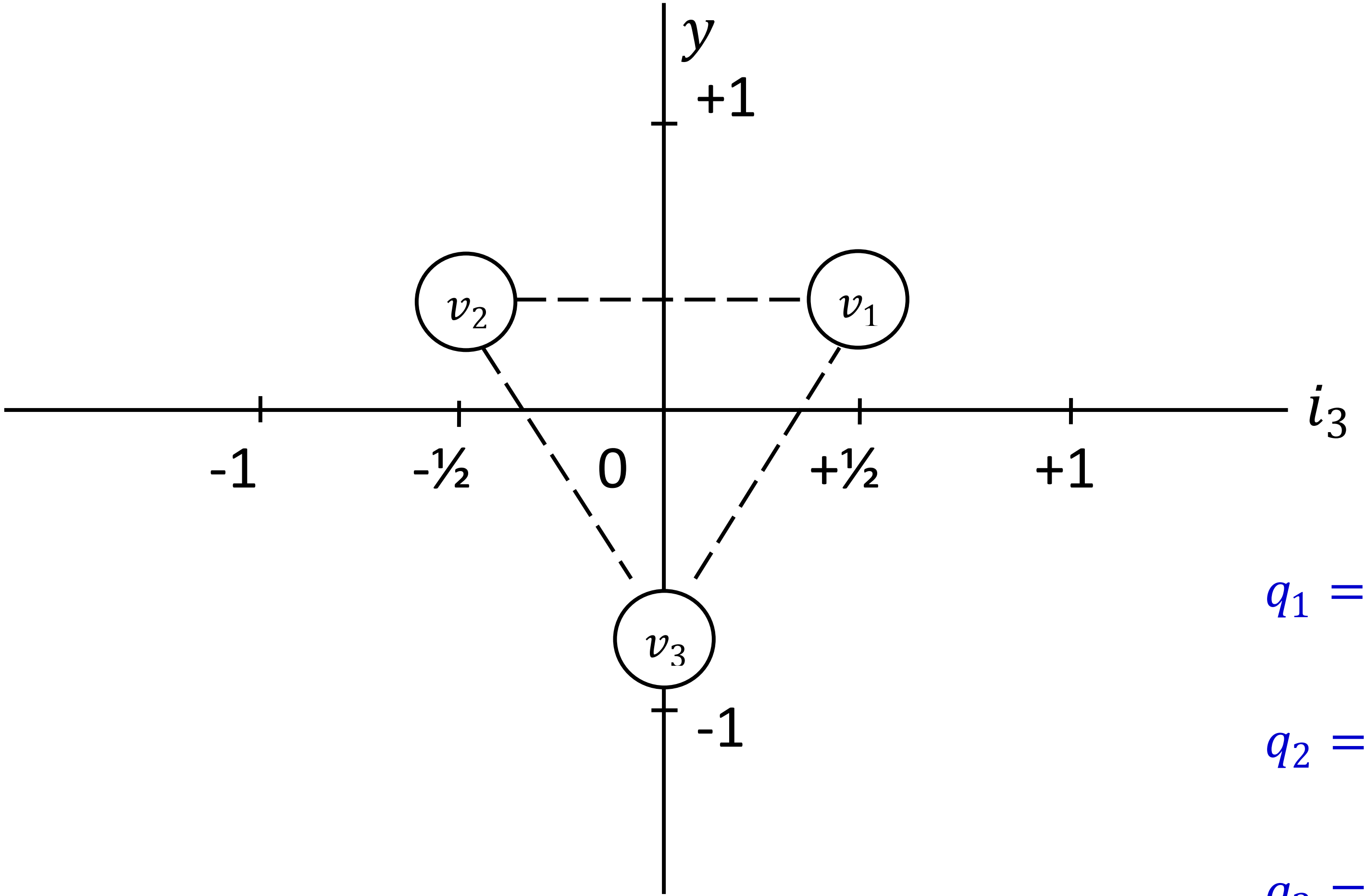
in <u>3</u> of SU(3), we have three basis vectors

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$i_3 = 1/2 \qquad i_3 = -1/2 \qquad i_3 = 0$$

$$y = 1/3 \qquad y = 1/3 \qquad y = -2/3$$

Put these on the $i_3 - y$ plot:



$$q = i_3 + \frac{y}{2}$$

$$q_1 = +\frac{1}{2} + \frac{1}{6} = +\frac{2}{3}$$

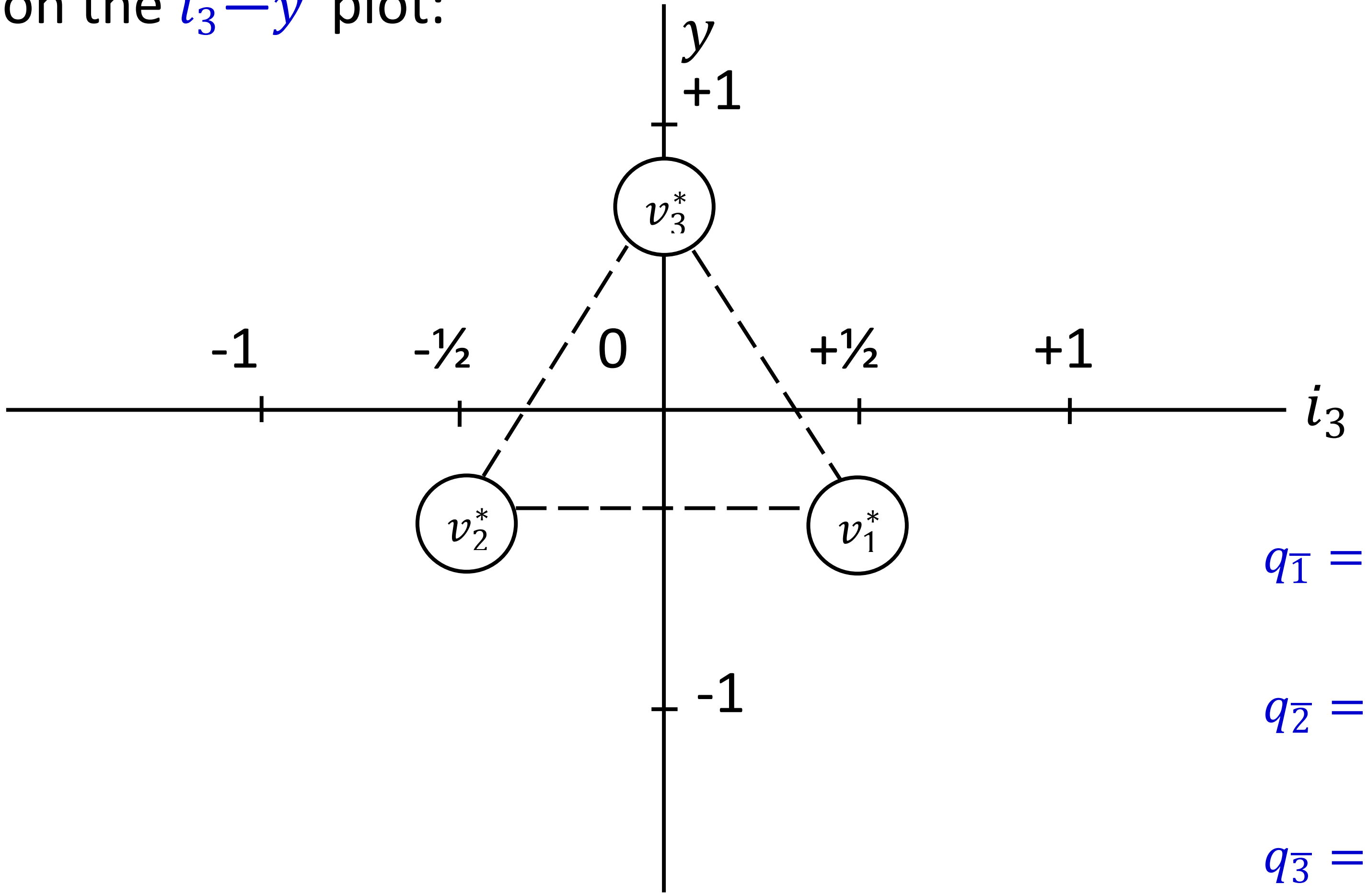
$$q_2 = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$$

$$q_3 = 0 - \frac{2}{6} = -\frac{1}{3}$$

Such fractional charges are not found in nature...

Conjugate representation (antiparticles): $v^* \rightarrow v'^* = U^* v^*$

on the $i_3 - y$ plot:



$$q = i_3 + \frac{y}{2}$$

$$q_{\bar{1}} = -\frac{1}{2} - \frac{1}{6} = -\frac{2}{3}$$

$$q_{\bar{2}} = +\frac{1}{2} - \frac{1}{6} = +\frac{1}{3}$$

$$q_{\bar{3}} = 0 + \frac{2}{6} = +\frac{1}{3}$$

Such fractional charges are not found in nature...

But we can combine the v_1, v_2 and v_3 in such a way a to get octets!

e.g. take $v_1 v_1 v_2$. This will have

$$i_3 = ½ + ½ - ½ = ½ \quad \text{and} \quad y = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \quad \text{proton!!}$$

Similarly, $v_1 v_2 v_2$ will have

$$i_3 = ½ - ½ - ½ = -½ \quad \text{and} \quad y = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \quad \text{neutron!!}$$

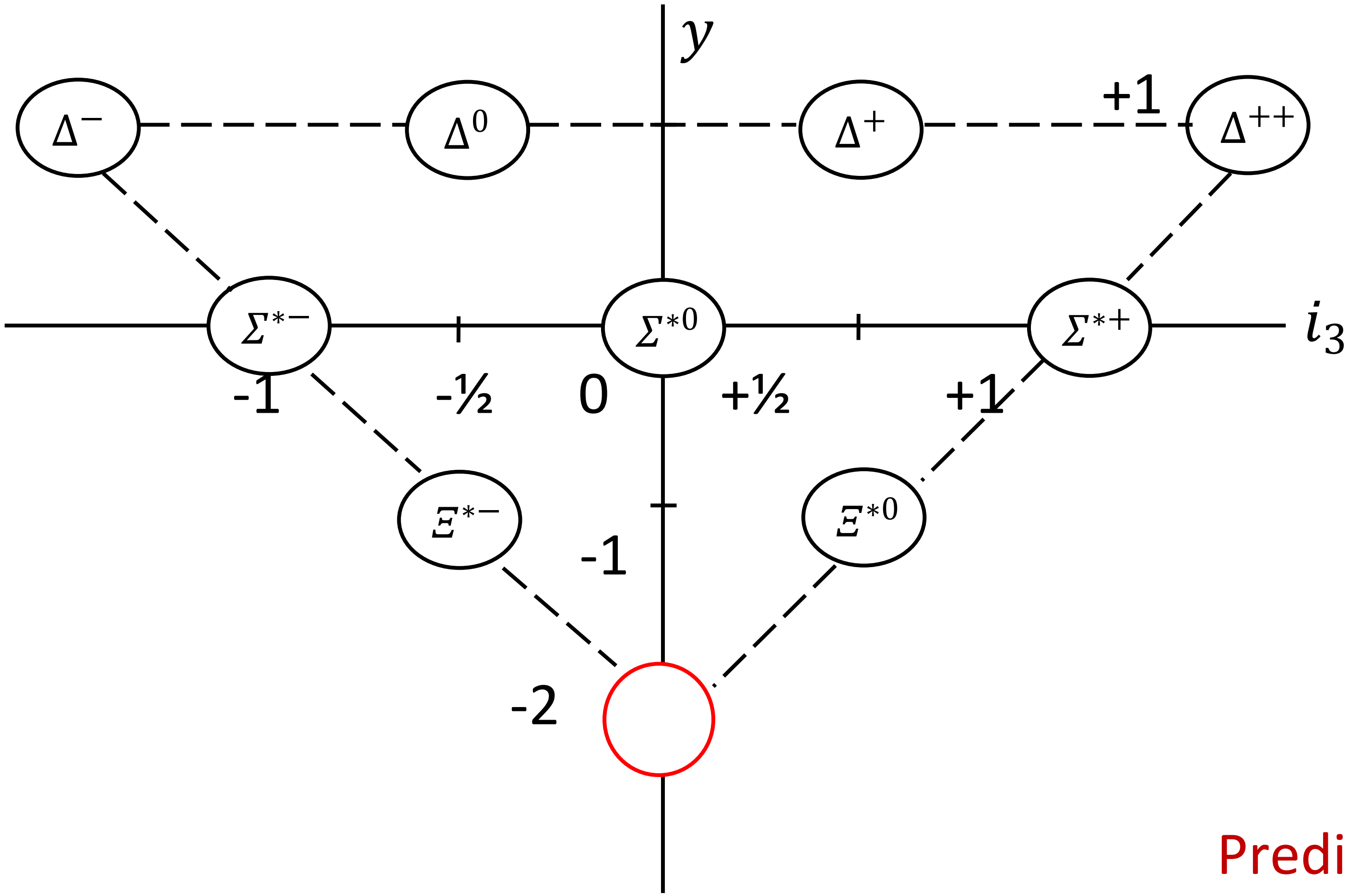
In fact, if we take all the combinations $v_i v_j v_k$ we will generate the baryon octet... $\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{10} \oplus \underline{8} \oplus \underline{8'} \oplus \underline{1}$

Similarly, if we take all the combinations $v_i v_j^*$ we will generate the meson octet... $\underline{3} \otimes \underline{3}^* = \underline{8} \oplus \underline{1}$

e.g. take $v_1 v_2^*$. This will have $i_3 = +1$ and $y = 0$, i.e. it is a π^+.

What about the 10, or baryon decuplet?

We can put suitable $v_i v_j v_k$ combinations to generate:



$\Delta(1236)$

$\Sigma^*(1385)$

$\Xi^*(1530)$

$\Omega(1680)$

Prediction of the Ω^-

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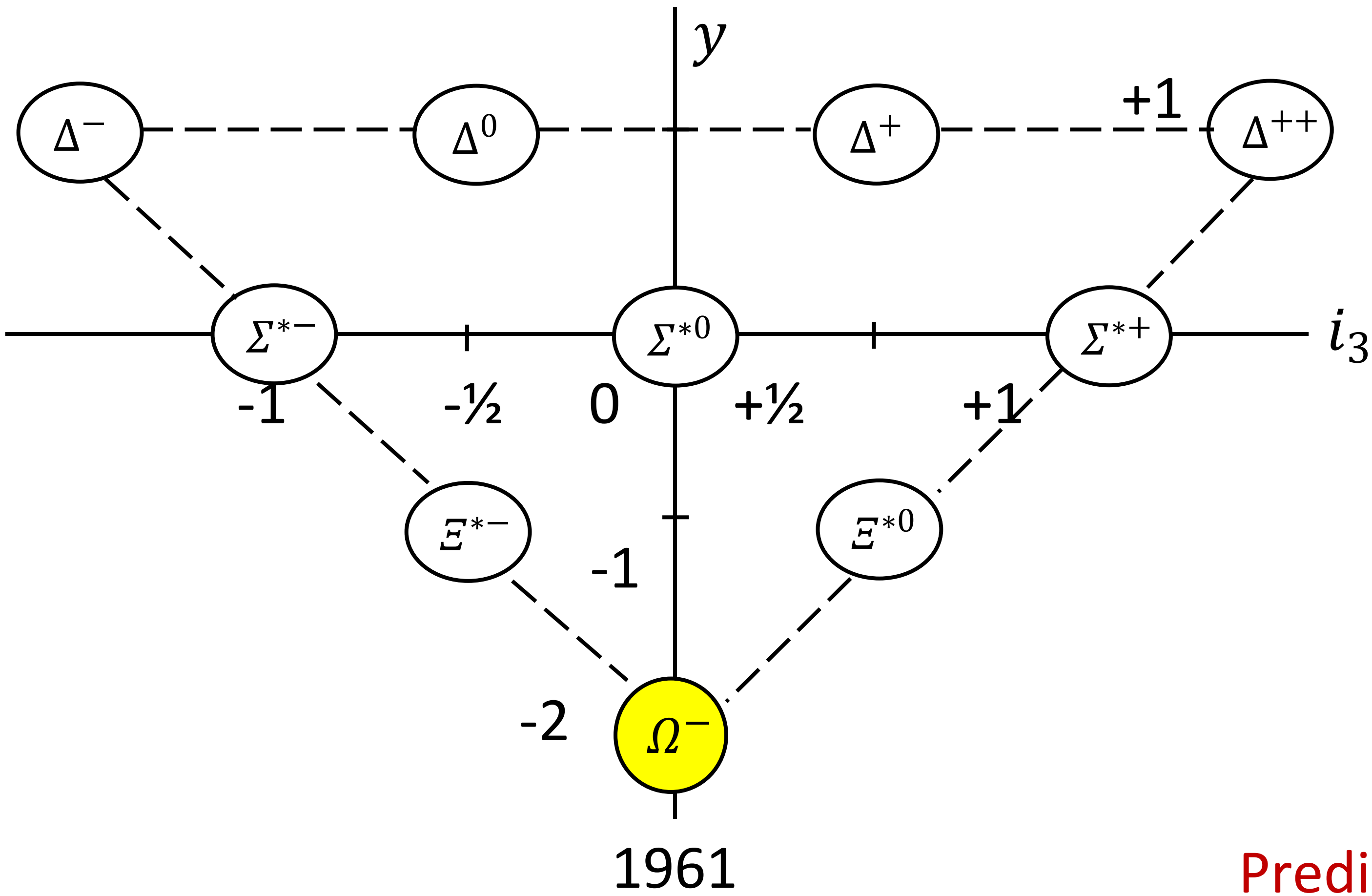

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Prediction of the Ω^-

<u>Quark Model</u> (Gell-Mann 1964, Zweig 1964)

Claim that the v_1, v_2 and v_3 are not just mathematical tricks to get octets, but are real particles $\Rightarrow$ quarks

$$v_1 = u \qquad v_2 = d \qquad v_3 = s$$

Also, mass of s is about 150 MeV more than that of u and d

Vector	Quark	j	i_3	y	B	S
v_1	u	$1/2$	$1/2$	$1/3$	$1/3$	0
v_2	d	$1/2$	$-1/2$	$1/3$	$1/3$	0
v_3	s	$1/2$	0	$-2/3$	$1/3$	-1

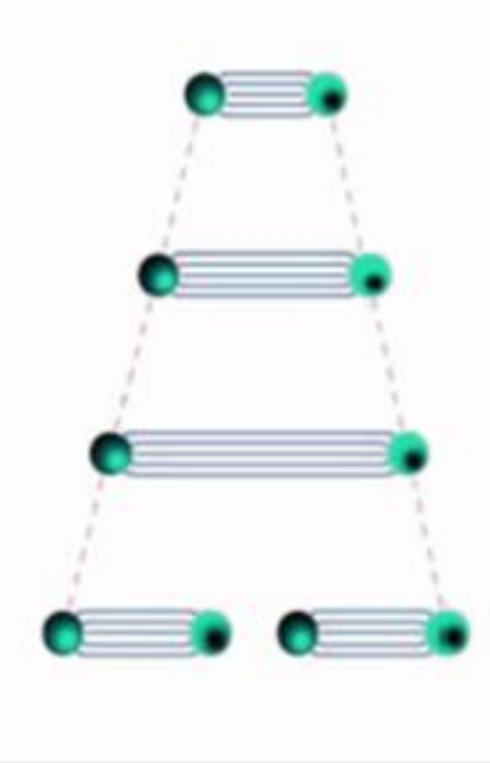
Today we know that $M_u \approx 3\text{ MeV}$ $\quad M_d \approx 5\text{ MeV}$ $\quad M_s \approx 120\text{ MeV}$

i.e. most of the mass of hadrons is of dynamic origin

Quark model also explains why the ${}^1_0n$ has a magnetic dipole moment

Quarks are never seen in the free state, i.e. they must be <u>confined</u>
e.g. by a potential which is unknown but often taken of the form

$$V(r) \approx \frac{1}{2}kr^2 \qquad \text{or} \qquad V(r) \approx \frac{a}{r} + br$$



As we pull quarks apart it becomes energetically more favourable to create a new quark-antiquark pair from the vacuum and form new meson states... hadronisation...

Analogous to the fact that we cannot separate a magnetic north pole or a south pole...

Since any pair can arise from the vacuum, this is a random process

We now understand hadronic processes as mostly quark processes...

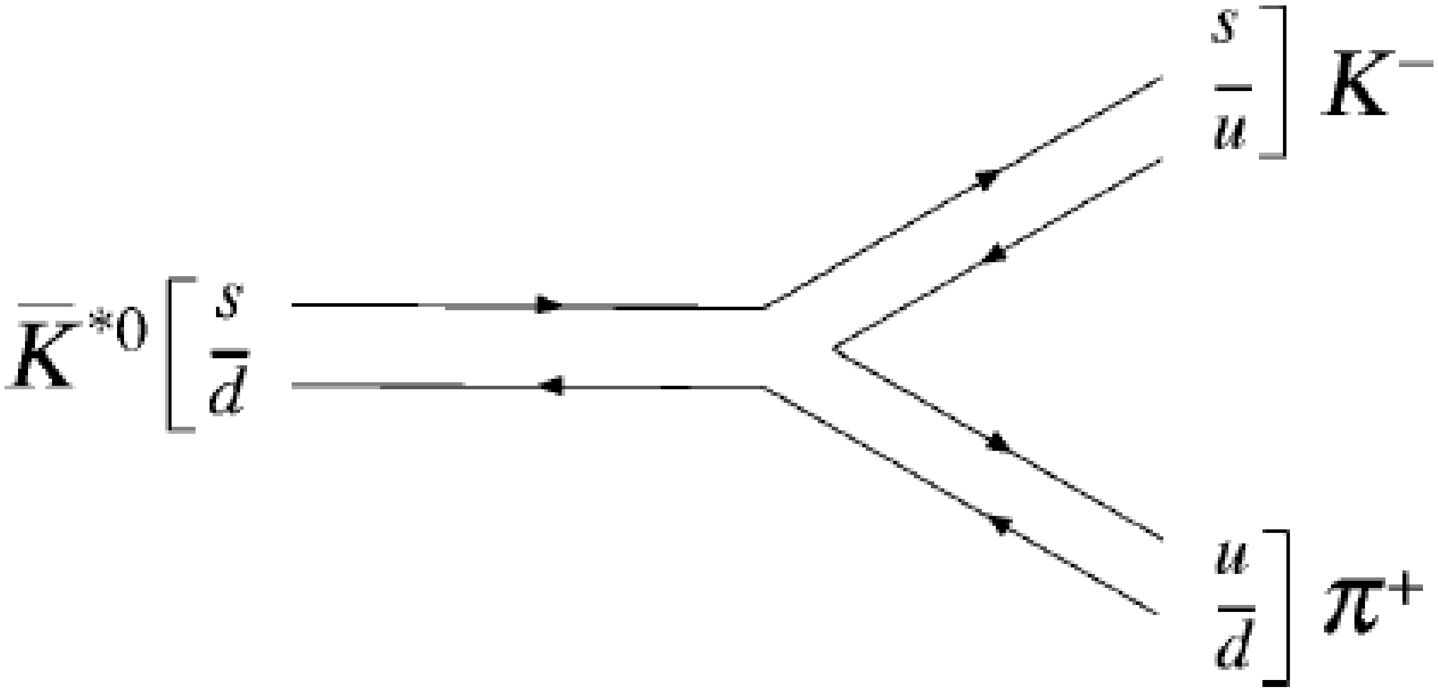


Figure 3.11 Quark diagram for the decay $\bar{K}^{*0} \to K^- + \pi^+$.

(from Martin and Shaw: Particle Physics)

Partons

Q. Why should we believe the quark model, since we cannot see or photograph free quarks?

Q. Why do we believe in the existence of the nucleus, though we cannot see or photograph nuclei?

A. Because of Rutherford scattering, i.e. we can send a small high energy probe inside an atom and infer the existence of the nucleus from its scattering pattern

Can we do the same for quarks inside a nucleon?

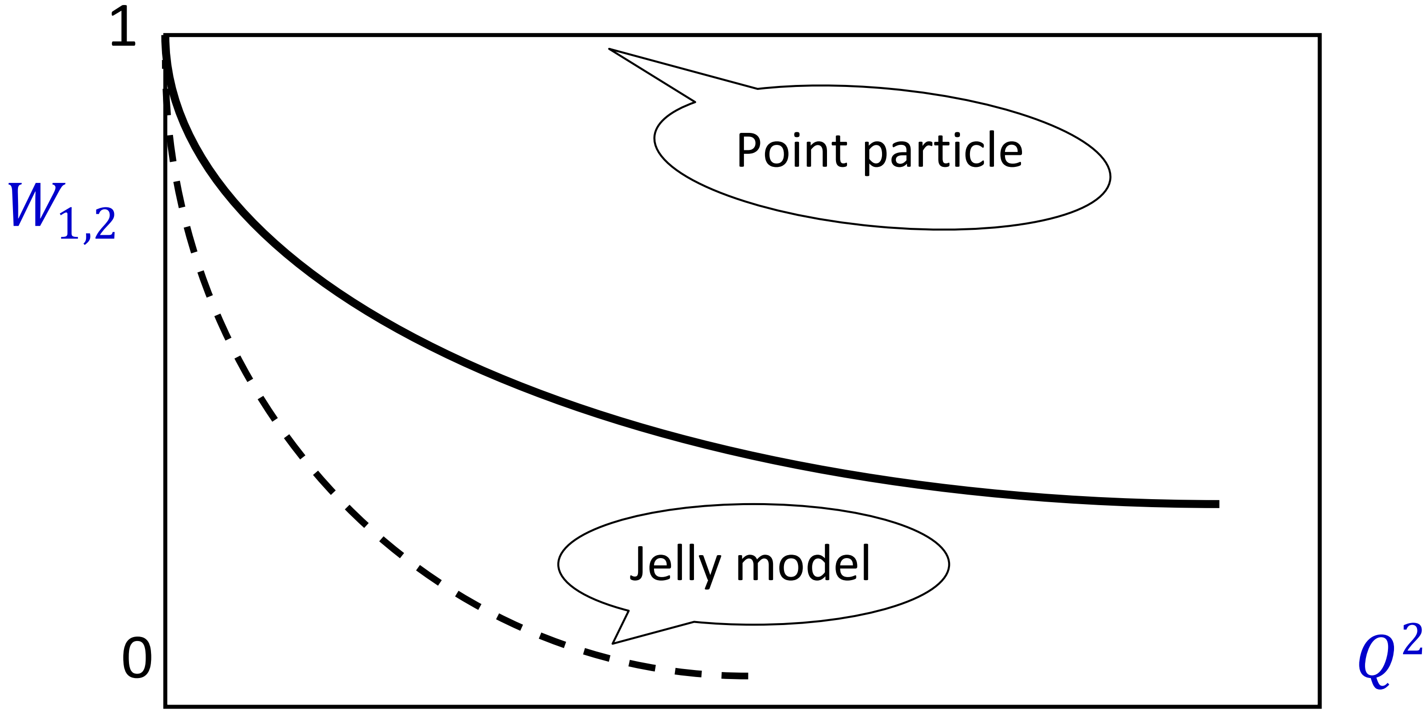
Shoot high energy electrons inside: *deep inelastic scattering* (DIS)

DIS : $e^- + p^+ \rightarrow \; e^- + X$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\cos\frac{\theta}{2}}{2E\sin^2\frac{\theta}{2}}\right)^2 \left(W_2 - 2W_1\tan^2\frac{\theta}{2}\right)$$

W_1 and W_2 are *structure functions*: if the proton is a point particle,

$$W_1 = W_2 = 1$$



Feynman interpreted this as showing that at high energies, one regains the point-particle behaviour, i.e. the electron is scattering off hard point particles (*partons*) inside the proton...

The fall to about one-third shows that there are three such partons inside the proton...

Matches perfectly with the quark model... quark-parton model

We have now discovered three more quarks...

c charmed quark .... similar to u but with mass 1.4 GeV and $C = 1$

b bottom quark .... similar to d but with mass 4.3 GeV and $\mathcal{B} = 1$

t top quark .... similar to u but with mass 172.3 GeV and $\mathcal{T} = 1$