"Non- Uniform Random Geometric Graphs with Location Dependent Radii"

Srikanth K. Iyer
Department of Mathematics
Indian Institute of Science
Bangalore

We propose a {\it distribution free} approach to the study of random geometric graphs. The distribution of vertices follows a Poisson point process with intensity function $nf(\cdot)$, where $n \in \mN$, and f is a probability density function on $\mR^d$. A vertex located at x connects via directed edges to other vertices that are within a {\it cut-off} distance $r_n(x)$. We prove strong law results for, (i) the critical cutoff function so that almost surely, the graph does not contain any node with out-degree zero for sufficiently large n, (ii) the maximum and minimum vertex degrees. We also provide a characterization of the cut-off function for which the number of nodes with out-degree zero converges in distribution to a Poisson random variable. We illustrate this result for a class of densities with compact support that have at most polynomial rates of decay to zero. Finally, we state a sufficient condition for an enhanced version of the above graph to be almost surely connected eventually.