Correlation Functions at Finite Temperature(2)

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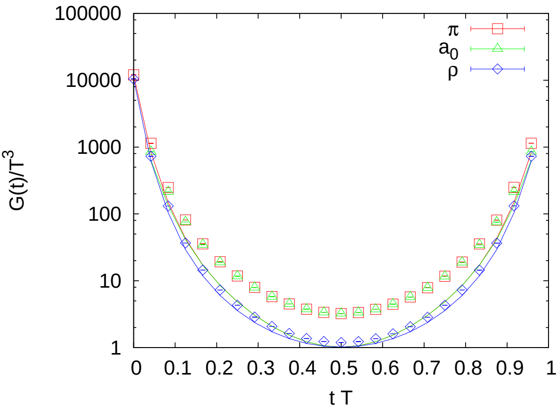
Matsubara correlator for mesons



- 1.5 T_c, $N_t = 24$, $m_q = 0$
- Chiral symmetry restoration seen
- ρ correlator $< 10\%$ away from free correlator
- More discrepancy in pion and scalar channel

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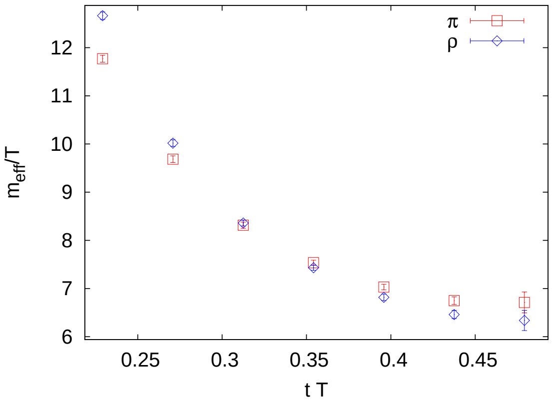
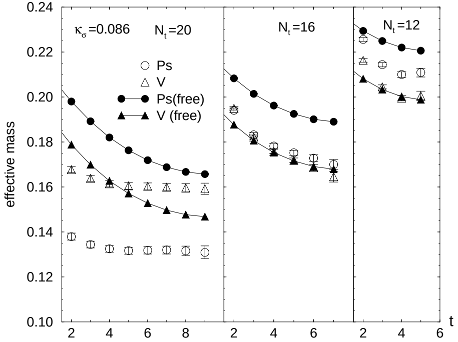


- 1.5 T_c, $N_t = 24$, $m_q = 0$
- Chiral symmetry restoration seen
- ρ correlator $< 10\%$ away from free correlator
- More discrepancy in pion and scalar channel
- No plateau in effective mass

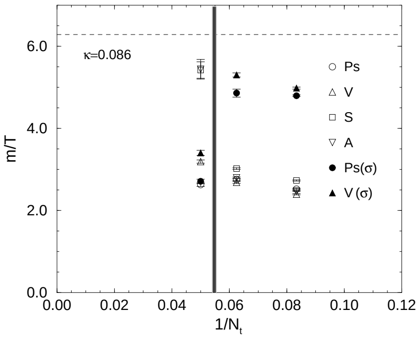
Matsubara correlators for smeared operators

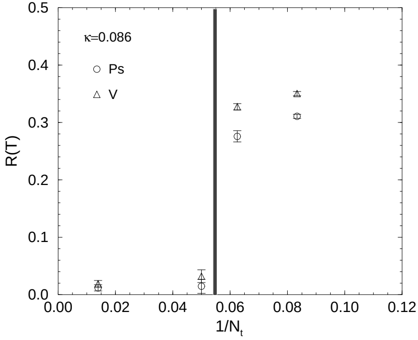
Take smeared operators optimized at T=0 for meson states.
Look at effect of increasing temperature.
Anisotropic lattice, $N_t = 20, 16, 12$ correspond to 0.93, 1.15, 1.5 T_c.



QCD-TARO, PRD63('01)054501

Pole and screening masses





$$R = \frac{m_\sigma - m_\tau}{m_\sigma + m_\tau} \sim 1 - \frac{2m_q}{\pi T} + \ldots$$

Spectral function

To connect the Matsubara correlator with real time observables, we define the spectral function

$$\rho(q) = \int dt d^3x e^{iq.x} \langle \frac{1}{2}[\phi(x), \phi^+(0)] \rangle$$

Clearly, $\rho(q) = \frac{1}{2}(G^>(q) - G^<(q))$
Exercise. Show that $G^<(q) = e^{-\beta q^0} G^>(q)$
Therefore $G^<(q) = 2n_B(q^0)\rho(q)$
Exercise. Show that

$$G^R(q) = \int \frac{dp_0}{\pi} \frac{\rho(p_0, \vec{q})}{p_0 - q_0 - i\epsilon}$$

Therefore $\rho(q_0, \vec{q}) = \mathrm{Im}\, G^R(q_0, \vec{q})$

Spectral function and Matsubara correlator

Exercise. Show that

$$G^E(\tilde{Q}) = \int_{-\infty}^{\infty} \frac{dp_0}{\pi} \frac{\rho(p_0, \vec{q})}{p_0 - i\tilde{q}_0}$$

Then, using the summation formula

$$T \sum_{\omega_b} \frac{e^{i\omega_b t}}{\omega_b^2 + \omega^2} = \frac{n_B(\omega)}{2\omega} \left(e^{(\beta - t)\omega} + e^{t\omega} \right)$$

we get,

$$G_E(\tau) = \int_0^{\infty} \frac{dp_0}{\pi} \rho(p_0) \frac{\cosh(p_0(\beta/2 - t))}{\sinh(p_0\beta/2)}$$

Thermal photon production

Photon production processes $i \rightarrow f\gamma$

$$
\begin{aligned}
S^{\lambda}_{fi}(Q) &= -ieq_f \int d^4x e^{iQ.x} \epsilon^{\lambda}_{\mu}(Q) \langle f | j^{\mu}(x) | i \rangle \\
q^0 \frac{d\Gamma}{d^3q} &= \frac{1}{\Omega \cdot 2(2\pi)^3} \frac{1}{Z(\beta)} \sum_{if\lambda} e^{\beta E_i} |S^{\lambda}_{fi}(Q)|^2 \\
&= -\frac{e^2 q_f^2}{2(2\pi)^3} \int d^4x e^{iQ.x} g_{\mu\nu} \sum_i \langle i | j^{\nu}(0) j^{\mu}(x) | i \rangle \frac{e^{-\beta E_i}}{Z(\beta)} \\
&= -\frac{e^2 q_f^2}{2(2\pi)^3} g^{\mu\nu} G^{<}_{\mu\nu}(Q) \\
&= -\frac{\alpha q_f^2}{2\pi^2} \frac{g^{\mu\nu} \rho_{\mu\nu}(Q)}{e^{\beta q^0 - 1}}
\end{aligned}
$$

Dilepton production $i \to f l(p_1)\bar{l}(p_2 = Q - p_1)$

$$S_{fi}(p_1, p_2) = -i\frac{e^2 q_f}{Q^2}\bar{u}(p_1)\gamma^\mu v(p_2) \cdot \int d^4x e^{iQ.x}\langle f|j_\mu(x)|i\rangle$$

$$d\Gamma = \frac{1}{\Omega}\sum_{fi}\frac{e^{-\beta E_i}}{Z}|S_{fi}|^2 \cdot \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}$$

$$\frac{d\Gamma}{d^4Q} = -\frac{e^4 q_f^2}{96\pi^5 Q^4} Q^2 g^{\mu\nu} G^<_{\mu\nu}(Q)$$

$$= -\frac{\alpha^2 q_f^2}{6\pi^3 Q^2}\frac{g^{\mu\nu}\rho_{\mu\nu}(Q)}{e^\beta q^0 - 1}$$

spectral function and screening correlator

stable $\approx \delta(\omega^2 - m^2)$ resonance $\approx \frac{\omega\Gamma}{(\omega^2-m^2)^2+\Gamma^2 m^2}$ cut $\approx \omega^2$

Screening correlator $$G(z) = \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}\, e^{ip_z z} \int_0^{\infty} dp_0\, \frac{\sigma(p_0, p_z)}{2p_0}$$

Free theory
$\omega^2(\vec{p}, T) \sim m^2(T) + A^2(T)\vec{p}^2$

$m^H_{\rm scr}(T) = 2\pi T$
$m^H_{\rm scr}(T) = m^H_{\rm pole}(T)/A(T)$

T. Hashimoto *et al.*, NPB400('93)267

Need to glean $\rho(\omega)$ from Matsubara correlator

$$G(\tau_i) = \sum_{ij} K(\tau_i, \omega_j)\rho(\omega_j), \qquad K(\tau_i, \omega_j) = \frac{\cosh \omega_j(\tau_i - \frac{1}{2T})}{\sinh \frac{\omega_j}{2T}}$$

Direct inversion not possible.

Analysis techniques

- Make a guess of form of the spectral function, with few unknown parameters. Fit the parameters (possibly with prior information).
- Incorporate prior information into the problem using Bayesian technique.
  - $P(\rho|D;\alpha,m) \propto P(D|\rho;\alpha,m)P(\rho|\alpha,m)$
  - $P(D|\rho;\alpha,m) \sim e^{-L=\chi^2/2}$ has zero modes if there are flat directions in parameter space not determined by data
  - if $P(\rho|\alpha,m)$ suitably chosen, the problem is regulated
  - Maximum entropy method: $P(\rho|\alpha,m) = e^{\alpha S}$ where

$$S = \sum_{i=1}^{N_\omega} \rho(\omega_i) - m(\omega_i) - \rho(\omega_i)\log\left(\frac{\rho(\omega_i)}{m(\omega_i)}\right)$$

Asakawa, Nakahara & Hatsuda, Prog. Part. Nucl. Phys. 46('01) 459

Maximum Entropy Method

- find $\rho_\alpha(\omega)$ such that

$$\frac{\delta(\alpha S - L)}{\delta\rho}|_{\rho=\rho_\alpha(\omega)} = 0$$

- Search for solution in the singular space of the kernel $K(\tau_i, \omega_j)$
-

$$K^T(\tau_i, \omega_j) = U_{jk}\sigma_k V_{ki}, \qquad i, k = 1, N_\tau, \; j = 1, N_\omega$$

- Writing $\rho(\omega_i) = m(\omega_i)\exp(\sum_l a_l u_l(\omega_i))$ where u_l are the eigenvectors of K in the singular space, one needs to solve for b_l
- Calculate $P(\alpha|Dm)$ and integrate over α

R. K. Bryan, Eur. Biophys. J. 18 ('90) 165

Charge current from lattice

Measurement of electric conductivity in gluon plasma: connected to low ω limit of emission rate of soft photons

$$\sigma(T) = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, T)}{\omega}$$

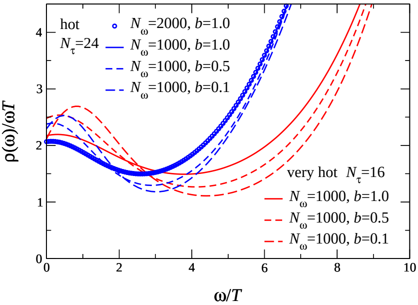
S. Gupta, PLB 597('04)57; Aarts, et al., PRL 99 ('07)022002; Ding et al., arXiv:1012.4963

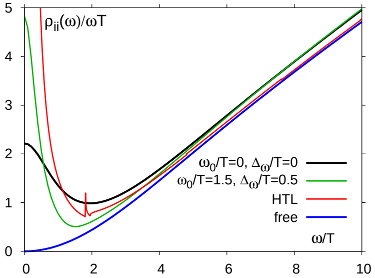
First two references use staggered valence quark, employ Bayesian analysis.

Ding et al use clover valence quark,assume a form for the spectral function, and do a fit.

$$\begin{aligned}\rho_{ii}(\omega) &= 2\chi_q Z \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} \\ &+ \frac{3}{2\pi}(1 + k(T))\omega^2 \tanh\frac{\omega}{4T}(1 + \exp((\omega_0^2 - \omega^2)/\omega\Delta_\omega))^{-1}\end{aligned}$$

Electric conductivity





Aarts et al.

T=1.5 and 2.25 T_c resp.

Maximum entropy analysis with default model

$m(\omega) = m_0(b + a\omega)$

$$\frac{\sigma}{C_{em}T} = 0.4 \pm 0.1$$

Ding et al.

Clover fermion

N_t = 16-48 at 1.46 T_c

N_s/N_t = 2-8

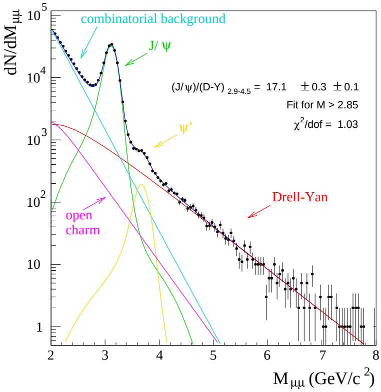
Fit with varying parameters

$$\frac{\sigma}{C_{em}T} = \frac{1}{3} - 1$$

Charmonia in QGP

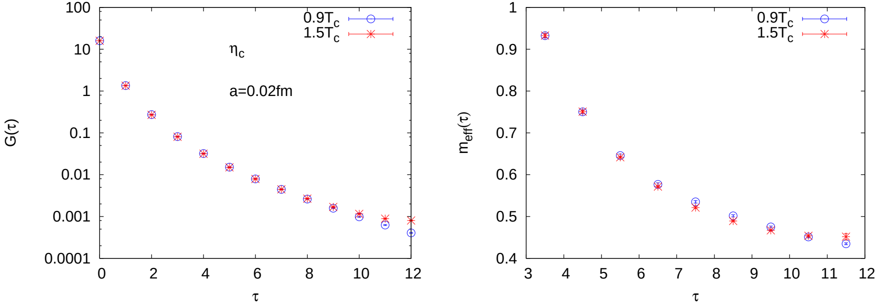
J/ψ as probe of deconfinement

Matsui & Satz, PLB178('86)416



- Screening in plasma $\longrightarrow$ reduced binding between $\bar{c}c$
- J/ψ was estimated to dissolve at around 1.1 T_c
- large branching into dileptons $\longrightarrow$ suitable as probe for deconfinement transition

Direct lattice study

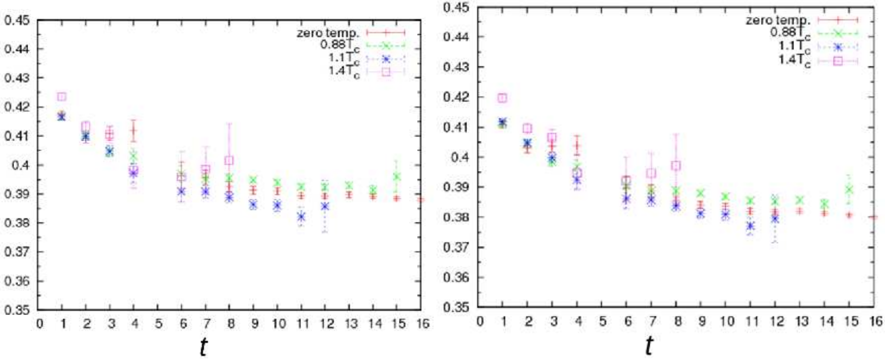


Point-point operators.
Smearing can help in isolating ground state
compare smeared correlators below and above T_c

Umeda et al., IJMP A16 ('01) 2216; Ohno et al.(WHOT-QCD), Lattice2010(arXiv:1011.1728)

Smeared operators

Basis of Gaussian smeared operators with various smearing radii
Variational analysis to isolate ground state



"No clear evidence of dissociation of J/ψ (left) and η_c (right) up to 1.4 T_c"
Smeared operators: difficult to connect to a thermal observable!

Spectral function based analysis

Use point-point operator
Maximum entropy analysis to extract spectral function
by giving information about the high energy structure of $\sigma_H(\omega)$
Datta, et al., NP(PS)119('03)487; Isotropic lattice (PRD 69,094507('04))
Asakawa and Hatsuda, Anisotropic lattice (PRL92,012001 ('04))
Jakovác et al., Anisotropic lattice, improved action (PRD75,014506('07))
Ding et al, Isotropic lattice(Lattice 2010 (arXiv:1011.0695))
Aarts et al., Anisotropic lattice, 2-flavor QCD (PRD76, 094513('07))

Analysis tools

At high temperatures, low ω part sensitive to the prior information about high ω part

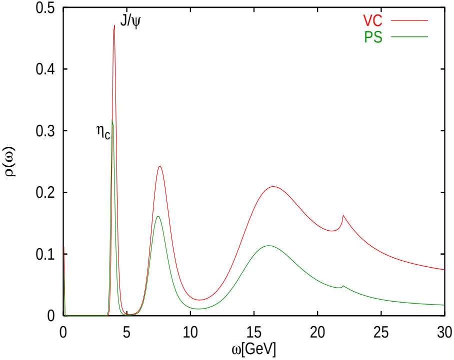
- Supply high ω information from low temperature studies
- Compare high temperature correlators with correlators "reconstructed" from spectral function at low temperature

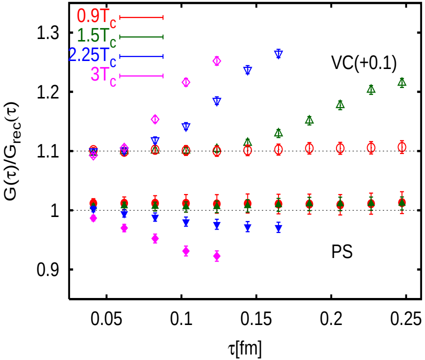
Reconstructed correlator

$$G_{\mathrm{recon,T^*}}(\tau, T) = \int d\omega\, \sigma(\omega, T^*)\, K(\omega, \tau, T)$$

Deviation of $G(\tau, T)$ from $G_{\mathrm{recon}}(\tau, T)$ indicate medium modification

Results for J/ψ and η_c





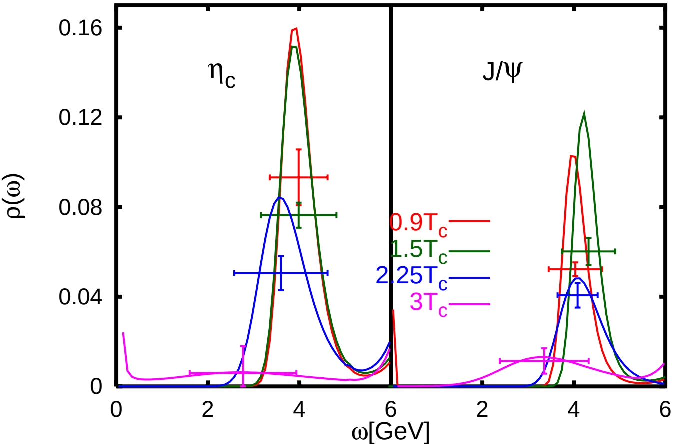
1S charmonia survive till high temperatures

Datta et al., PRD 2004

The small change in J/ψ associated with transport

Umeda, PRD 75('07) 094502; Datta & Petreczky, J.Phys.G

Spectral function



η_c and J/ψ show no weakening upto 1.5 T_c

Spread of Results

- Introduction of the maximum entropy method has led to insights into behavior of charmonia in plasma, but uncertainties of the method, in particular dependence on the prior model, gets reflected in variation of conclusions between different groups.
- Asakawa & Hatsuda found essentially no change upto 1.67 T_c, but no sign of a peak for $> 1.78 T_c$.
- Mocsy & Petreczky pointed out that small change in correlator does not necessarily mean small change in the low ω structure.
- Ding et al. use finer lattices, find very small change between the finite temp. correlator and reconstructed correlator. They, however, include structure in default model at very low ω and find considerably weaker peak at 1.46 T_c.
- Skullerud et al found results similar to ours for 2-flavor QCD.

Other results

- Υ and η_b have been studied using clover fermions (Datta et al., hep-lat/0603002) and NRQCD (Aarts et al, PRL 106('11) 061602). Seen to exist above 2 T_c.
- Shear viscosity of gluon plasma has been measured by studying the correlators of the off-diagonal elements of the energy-momentum tensor

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho^{T_{12}, T_{12}}(\omega)}{\omega}$$

Karsch & Wyld, PRD 35 ('87) 2518; Nakamura & Sakai, PRL 94 ('05) 072305; Meyer, PRD 76('07) 101701

Meyer: write $\rho(\omega) = m(\omega) \cdot (1 + \sum_l a_l u_l(\omega))$ where u_l are the singular directions of the kernel, and iteratively solve for a_l.

$$\begin{aligned} \frac{\eta}{s} &= 0.134(33) \qquad \text{at } 1.65\, T_c \\ &= 0.102(56) \qquad \text{at } 1.24\, T_c \end{aligned}$$

Other results (contd.)

Perturbation theory gives a larger value for $\frac{\eta}{s}$, while RHIC results seem to prefer a small value.

- Bulk viscosity can be measured from correlator of the trace of the energy-momentum tensor

$$\zeta(T) = \frac{\pi}{9} \lim_{\omega \to 0} \frac{\rho^{T_{ii},T_{ii}}(\omega)}{\omega}$$

$$\begin{aligned} \frac{\zeta}{s} &= 0.008(7)_{0}^{0.15} \quad \text{at } 1.65\,T_c \\ &= 0.065(17)_{0.01}^{0.37} \quad \text{at } 1.24\,T_c \\ &= 0.73(3)_{0.5}^{2.0} \quad \text{at } 1.02\,T_c \end{aligned}$$

Meyer, PRL 100('08) 162001

Ignoring bulk viscosity justified in RHIC?
Perturbative result $\sim 0.02\alpha_s^2$

Summary

- In the last decade, there has been considerable activity in lattice study of various observables connected to hot QCD, through an analysis of the Matsubara correlators.
- Various analysis tools have been invented and adopted.
- Interesting insights have been obtained.
- But the field is still far from having reached maturity, and so far, results are more qualitative than quantitative.
- Since quark gluon plasma seems to be far from perturbative, and lattice is the only tool available for nonperturbative calculations in QCD, the activity in this field is bound to grow.