Correlation Functions at Finite Temperature (1)

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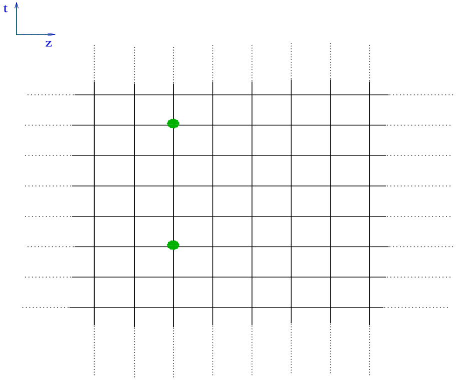
Correlator as probe

Two point functions of suitable operators:
way to find out excitations of QCD vacuum.



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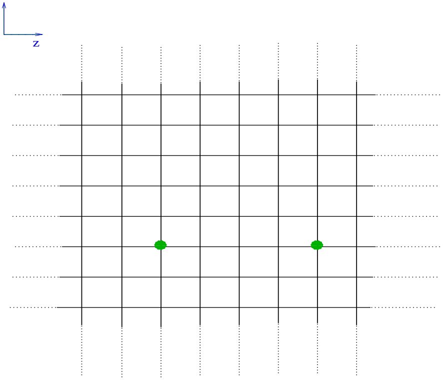


Correlators in Thermal Field Theory

Information about medium can be obtained from the set of correlators

$$G^{>}(\Phi(x)\Phi^{+}(0)) = \langle \Phi(x)\Phi^{+}(0)\rangle$$
$$G^{<}(\Phi(x)\Phi^{+}(0)) = \langle \Phi^{+}(0)\Phi(x)\rangle$$

where $\langle O\rangle = \frac{1}{Z}\mathrm{Tr}\, O e^{-H/T}, \qquad Z = \mathrm{Tr}\, O e^{-H/T}.$
The Feynman propagator is then

$$G^{F}(\Phi(x)\Phi^{+}(0)) = \theta(x^0)G^{>}(\Phi(x)\Phi^{+}(0)) \pm \theta(-x^0)G^{<}(\Phi(x)\Phi^{+}(0))$$

And the retarded propagator is defined as

$$G^{R}(\Phi(x)\Phi^{+}(0)) = \theta(x^0)\langle[\Phi(x), \Phi^{+}(0)]\rangle$$

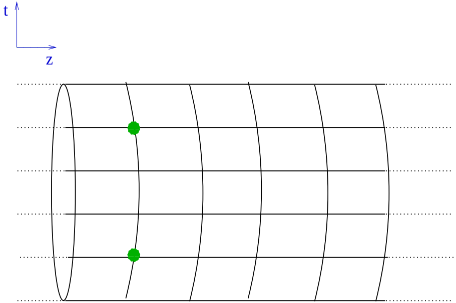
Under a small perturbation $\int d^3x J_{\mathrm{ext}}(x,t)\Phi(x,t)$ added at $t = 0$

$$\delta\langle\Phi(x,t)\rangle = \int dt' d^3x' G^{R}(x,t;x',t') J_{\mathrm{ext}}(x',t')$$

Thermal field theory on lattice

Partition function can be rewritten as

$$Z = \int_{(a)pbc} \mathcal{D}U\mathcal{D}(\psi, \bar{\psi})\ e^{-\int_0^{1/T} \mathcal{L}(U,\psi,\bar{\psi})}$$



$$G^E(\Phi(\tau, \vec{x})\Phi^+(0, \vec{0})) = \langle \Phi(\tau, \vec{x})\Phi^+(0, \vec{0}) \rangle$$

periodicity: $G^E(\tau, 0) \ = \ G^E(\beta - \tau, 0)$

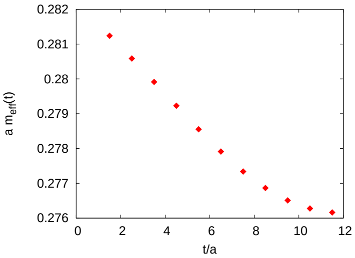
Matsubara and screening correlator

In principle, Matsubara correlators can be analytically continued to G^F

Hard problem for numerical analysis.

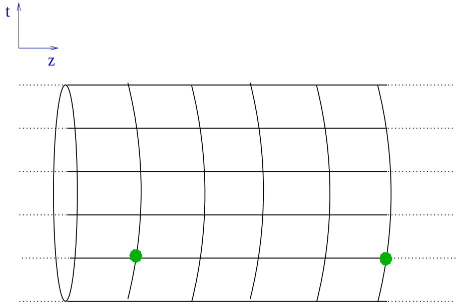
Also, short temporal extent makes analysis nontrivial, even in the simplest case.

Example: two noninteracting particles,
mass 0.25/a and 0.3/a,
T=1/(24 a)



Screening correlator

Techinques very similar to spectrum calculation can be used to study correlations in the spatial direction (take z, e.g.).



Similar to T=0 case, transfer matrix defined on z-slices

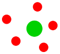
Screening masses give eigenvalues of z-slice transfer matrix.

Important informations about properties of the finite temperature theory.

In the equilibrium system, how is a static charge screened?

Debye screening in QED

T=0  V(r) ~ 1/r

T > 0  V(r) ~ 1/r $e^{-m_D r}$

$$V(r) = Q \int \frac{d^3p}{(2\pi)^3} \frac{e^{ip.x}}{p^2 + \Pi_{00}(0,p)}$$

$$\lim_{x\to\infty} \langle E_i(\vec{x}) E_j(\vec{0}) \rangle \sim e^{-m_D x}$$

One gets $m_D = \frac{eT}{\sqrt{3}} + O(e^2)$
In QCD, color electric field not gauge invariant; but correlators can be defined that give equivalent of Debye mass.

Polyakov loop correlator

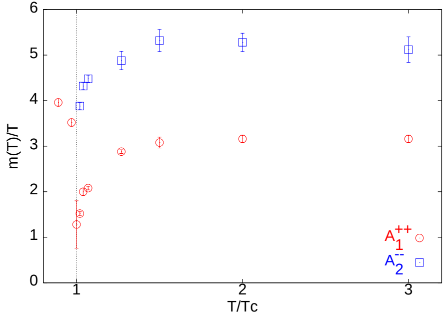
$$\langle \mathrm{ReTr} L(\vec{x}) \mathrm{ReTr} L^{+}(\vec{0}) \rangle_c \sim \frac{c(T)}{(rT)^d} exp(-2m_D r)$$

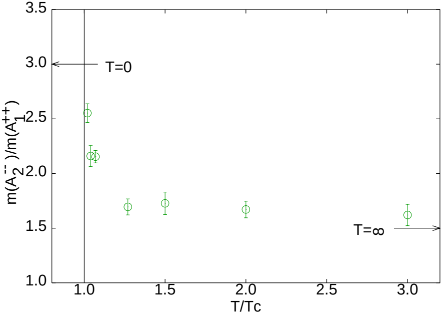
- At very high temperatures, we may instead get $m_G(3D) \sim g^2 T$

Nadkarni, PRD 33 ('86) 3738

- Similarly, correlator of the imaginary part may decay with an exponent $3m_D$ or $m_D + m_G(3D)$
- Close to T_c we may have something more complicated

Polyakov loop and electric gluon quasiparticle





S. Datta & S. Gupta, PRD 67('03)054503

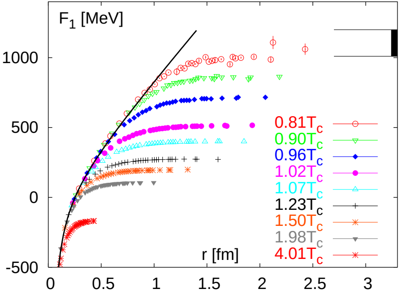
Electric gluon pseudoparticles dominate at $T \geq 1.5\,T_c$

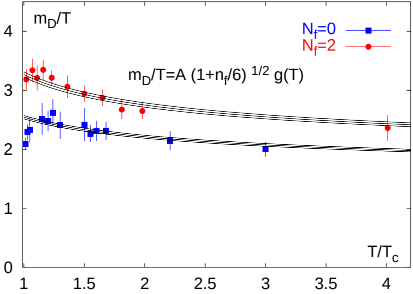
Perturbative massscale separation $m_G(3D) << m_D$ does nto occur for temperatures below $\sim 10^7\,T_c$

Hart, Laine & Philipsen, NPB 586('00)443

Polyakov loop correlator and $Q\bar{Q}$ interaction

$$e^{-F_1(T)} = \langle \mathrm{Tr} L(\vec{x}) L^{+}(\vec{0}) \rangle_c$$





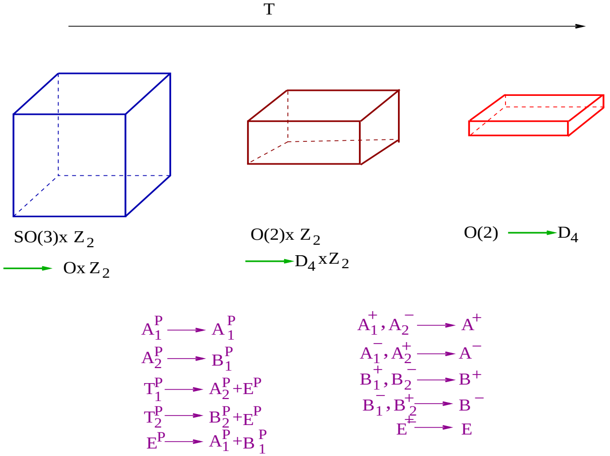
Here A=1.42(2) for 2-flavor QCD (1.52(2) for quenched)

Kaczmarek & Zantow, PRD71 ('05) 114510

Correlators for $L_{\mathrm{Adj,Octet,\ldots}}$ have been studied

Gupta, Hübner, Kaczmarek, PRD77('08)034503

Screening masses of Glueball-type operators



For SU(2) and SU(3): T=0 symmetry satisfied till $\sim T_c$
lightest screening mass $\approx m_{A_1^+}(T=0)$ till T_c
Dimensional reduction good approximation at 2 T_c

S. Datta & S. Gupta, NPB 534 ('98) 392

Screening masses of Meson-type correlators

Widely studies since pioneering works of detar(1985).

deTar & deGrand, deTar & Kogut, ... Grossman, et al., ...

Exercise. Show that for the free theory, screening masses of mesonic operators go to 2 π T.
Leading perturbative correction to this is known

$$\frac{m_{\rm scr}}{T} = 2\pi + \frac{C_F}{4\pi} g^2(6.7T)(1 + 0.327)$$

Laine & Vepsalainen, JHEP0402,004

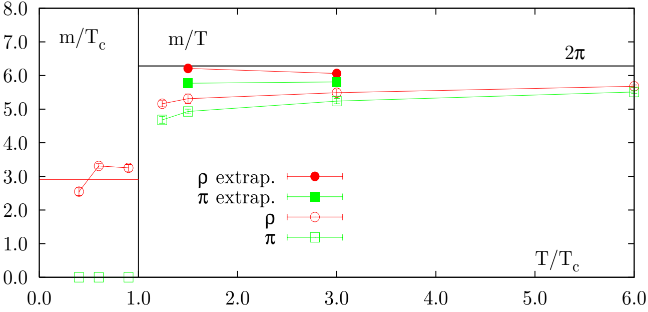
To compare with NP result: need continuum and infinite volume extrapolation

Mesonic correlators in the quenched theory

Careful analysis of volume and cutoff dependence in quenched theory with NP clover fermions

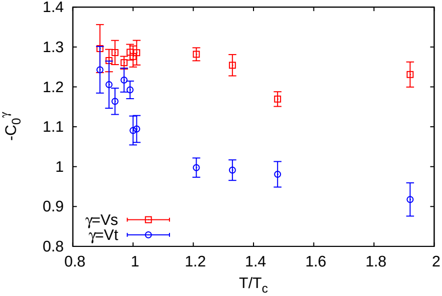
$$\frac{m(L,a)}{T} = \frac{m(a)}{T}\left(1 - \gamma(N_\tau/N_\sigma)^p\right) \qquad \frac{m(a)}{T} = \frac{m_{\rm scr}}{T} + \lambda \cdot (1/N_\tau)^2$$

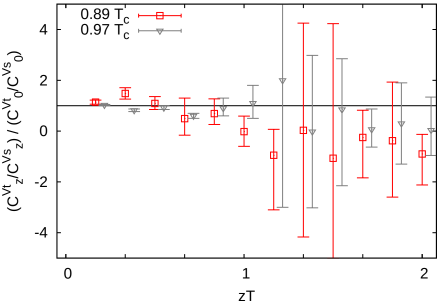
p $\sim$ 2.2 (1.5) at 1.5(3) T_c (p=1 for free theory)



S. Wissel et al., PoS LAT2005,164.

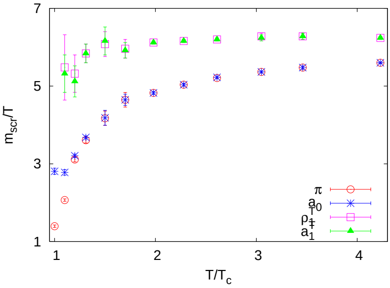
Mesonic correlator in 2 flavor QCD

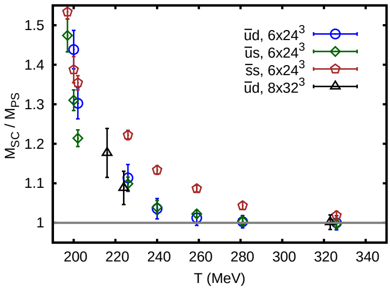




Banerjee et al., arXiv:1102.4465

Mesonic correlator in 2+1 flavor QCD





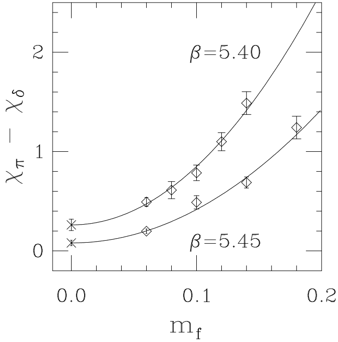
M. Cheng et al. (RBC-Bielefeld), Eur.Phys.J.C71('11)1564

Similar results in 2 flavor QCD

PS-S difference: finite quark mass? $U_A(1)$ breaking?

$U_A(1)$ symmetry breaking

2-flavor Domain wall fermion on 16^3x4 lattices; $\beta_\chi = 5.325$



P. Vranas, Nucl. Phys. Proc. Suppl. 83 ('00)

Discussion on screening masses

- Screening masses are easy to study. All the machinary of T=0 spectrum calculations can be employed.
- Widely studied since the works of DeTar (1985).
- Only a subset of available techniques have been employed sofar, though. Scope for improvement.
  For example, one has the technology now for calculating the disconnected part and investigating carefully the $U_A(1)$ symmetry restoration.
- Gives information about symmetries of the system, e.g., chiral symmetry restoration, dimensional reduction, etc.
- Also can give indirect insight into degrees of freedom, applicability of perturbation theory, etc.
- Practically impossible to connect with real time correlators.
- Therefore we look at Matsubara correlators, which at least in principle can be connected to real time correlators.