

Some (more) High(ish)-Spin Nuclear Structure

Lecture 3

Deformed States and EM transition rates.

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Outline of Lectures

- 1) Overview of nuclear structure 'limits'
 - Some experimental observables, evidence for shell structure
 - Independent particle (shell) model
 - Single particle excitations and 2 particle interactions.
- 2) Low Energy Collective Modes and EM Decays in Nuclei.
 - Low-energy Quadrupole Vibrations in Nuclei
 - Rotations in even-even nuclei
 - Vibrator-rotor transitions, E-GOS curves
- 3) EM transition rates. what they mean and how you measure them
 - Deformed Shell Model: the Nilsson Model, K-isomers
 - Definitions of $B(ML)$ etc. ; Weisskopf estimates etc.
 - Transition quadrupole moments (Q_0)
 - Electronic coincidences;
 - Doppler Shift methods (RDM, DSAM)
 - Yrast trap isomers
 - Magnetic moments and g-factors

Unpaired Particles in Deformed Nuclei: The Nilsson Model

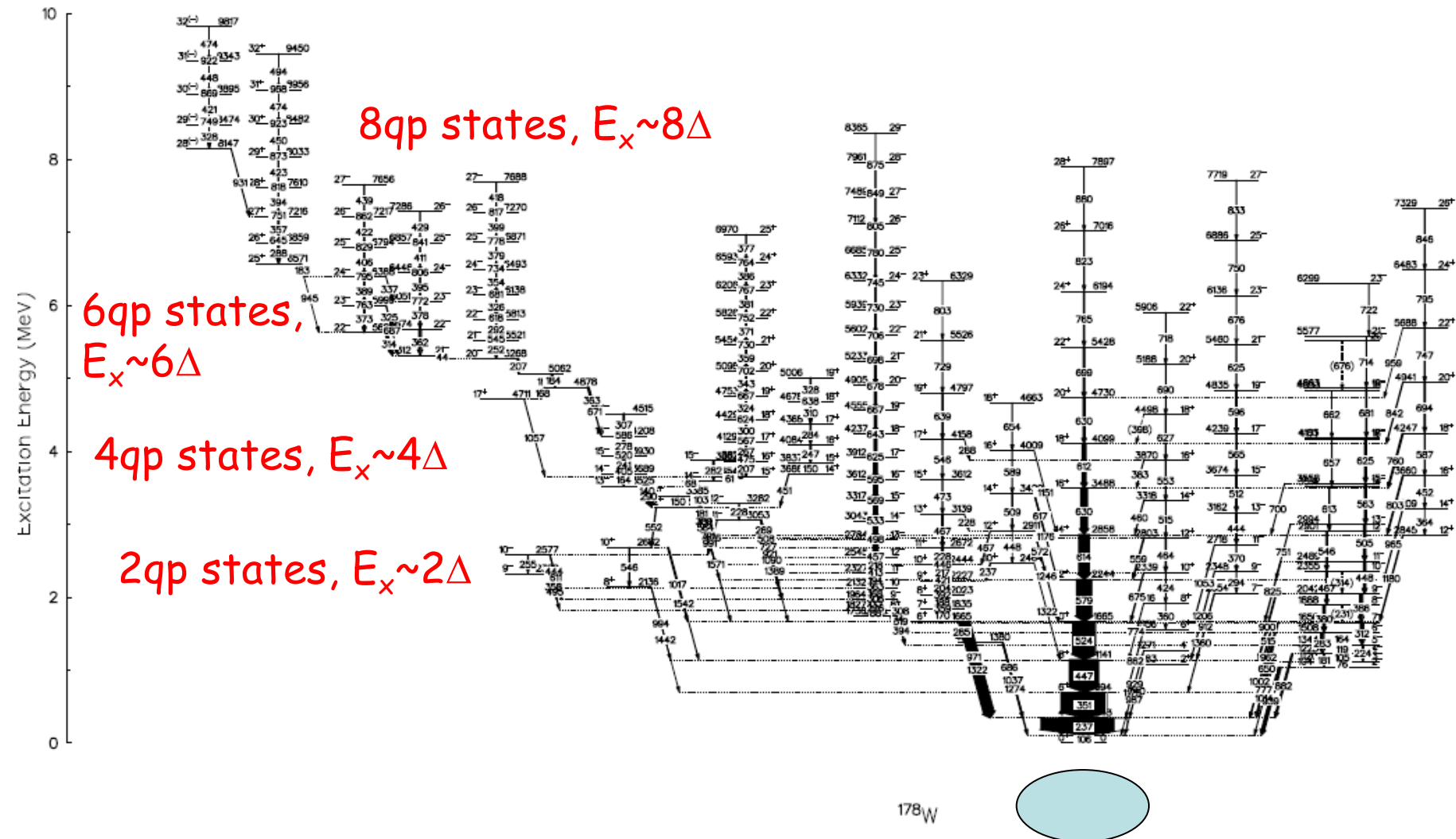


Figure 2.21: Partial decay scheme for ^{178}W showing 0, 2, 4, 6 and 8 quasi-particle structures [9].

Some of these 'isomers' can be VERY long lived, e.a., $E_x \sim 1$ MeV β -decaying K-'driven' isomer in ^{177}Lu .

$K^\pi = 23/2^-$, $T_{1/2} = 160$ d

$E_\beta(\text{max}) = 152$ keV

$K^\pi = 23/2^+$, $T_{1/2} = 1$ s

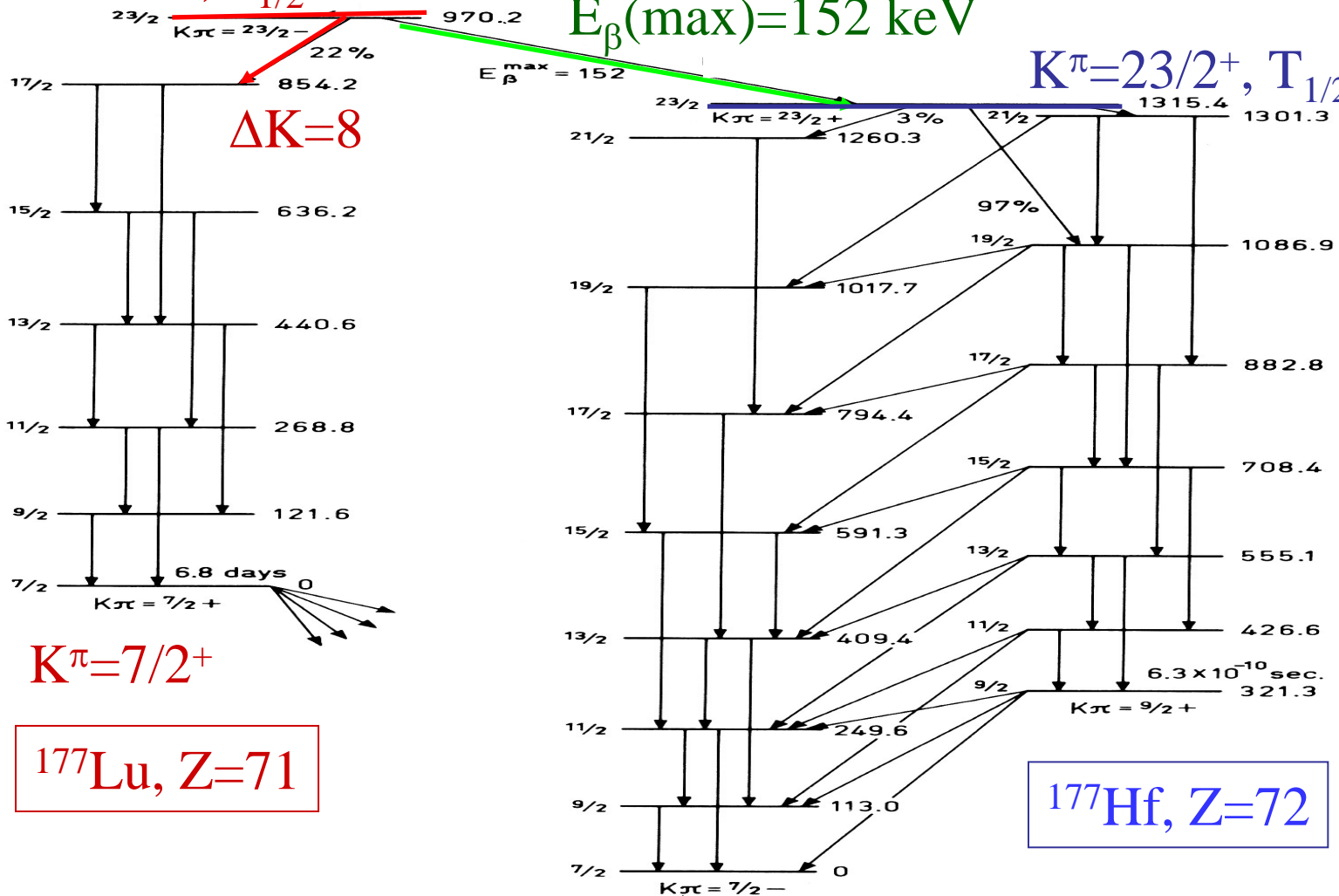
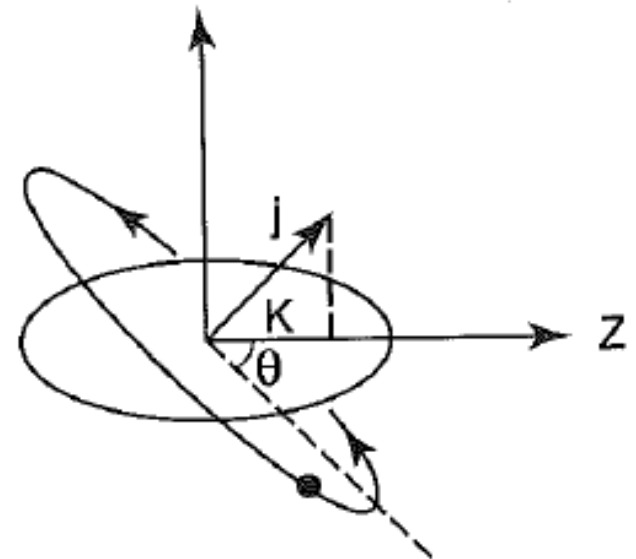
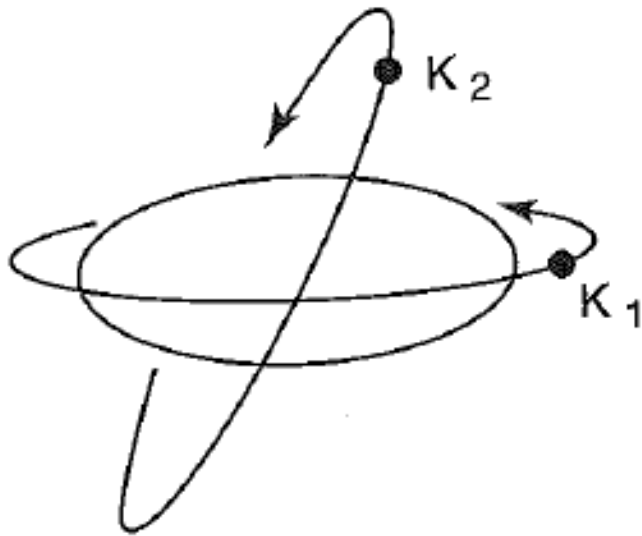


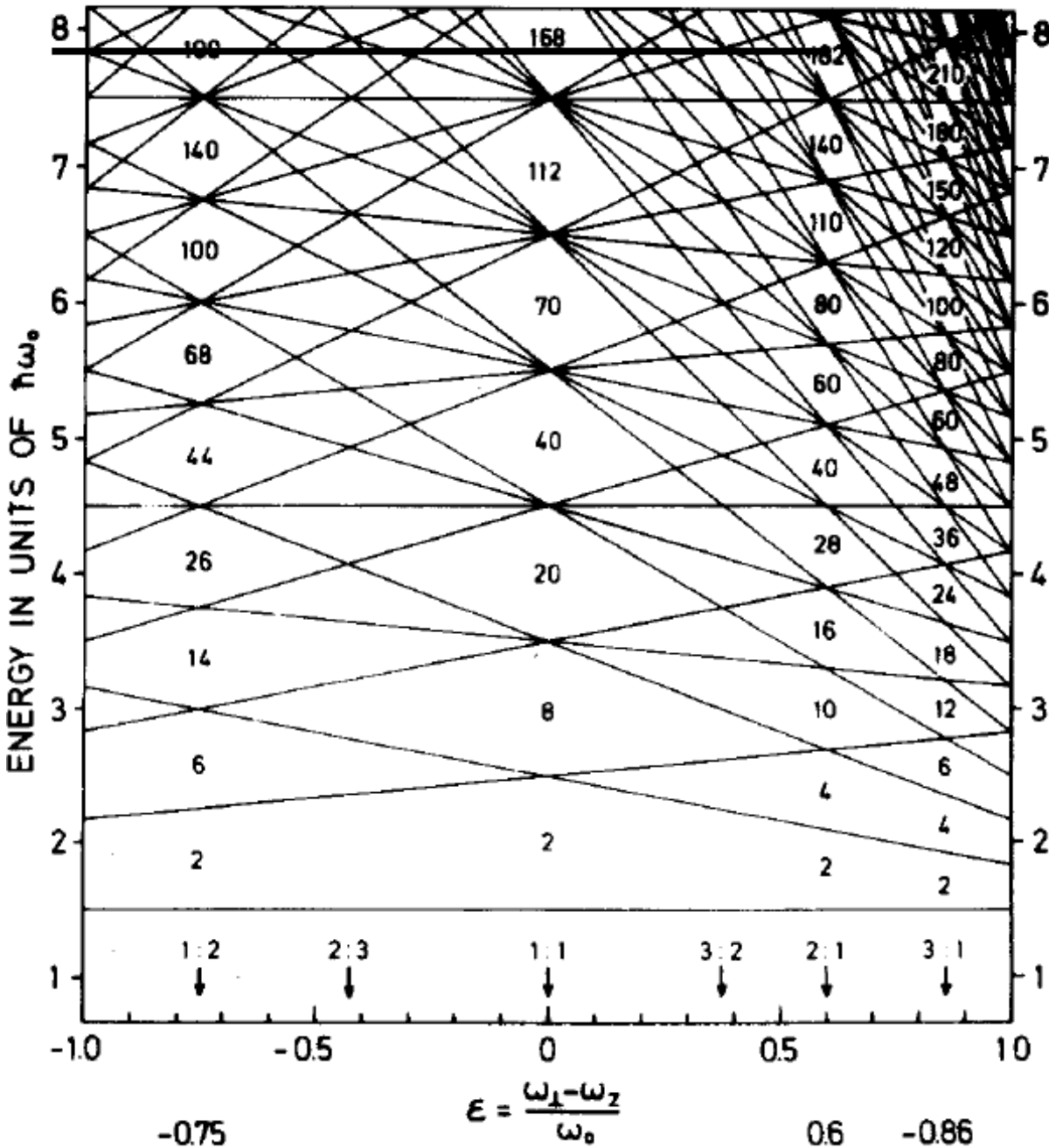
Figure 4-16 Decay scheme for $^{177}\text{Lu}^m$. The data are taken from A. J. Haverfield, F. M. Bernthal, and J. M. Hollander, *Nuclear Phys.* **A94**, 337 (1967), which also includes results of previous investigations. Additional transitions reported by F. Bernthal (private communication) have also been included. The energies in the figure are in keV, and the decay times represent half lives.

Deformed Shell Model: The Nilsson Model



$$\sin \theta = K / j$$

$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m \left[\omega_x^2(x^2 + y^2) + \omega_z^2 z^2 \right] + C \mathbf{l} \cdot \mathbf{s} + D I^2$$



Single particle states for an elongated spheroidal harmonic oscillator.

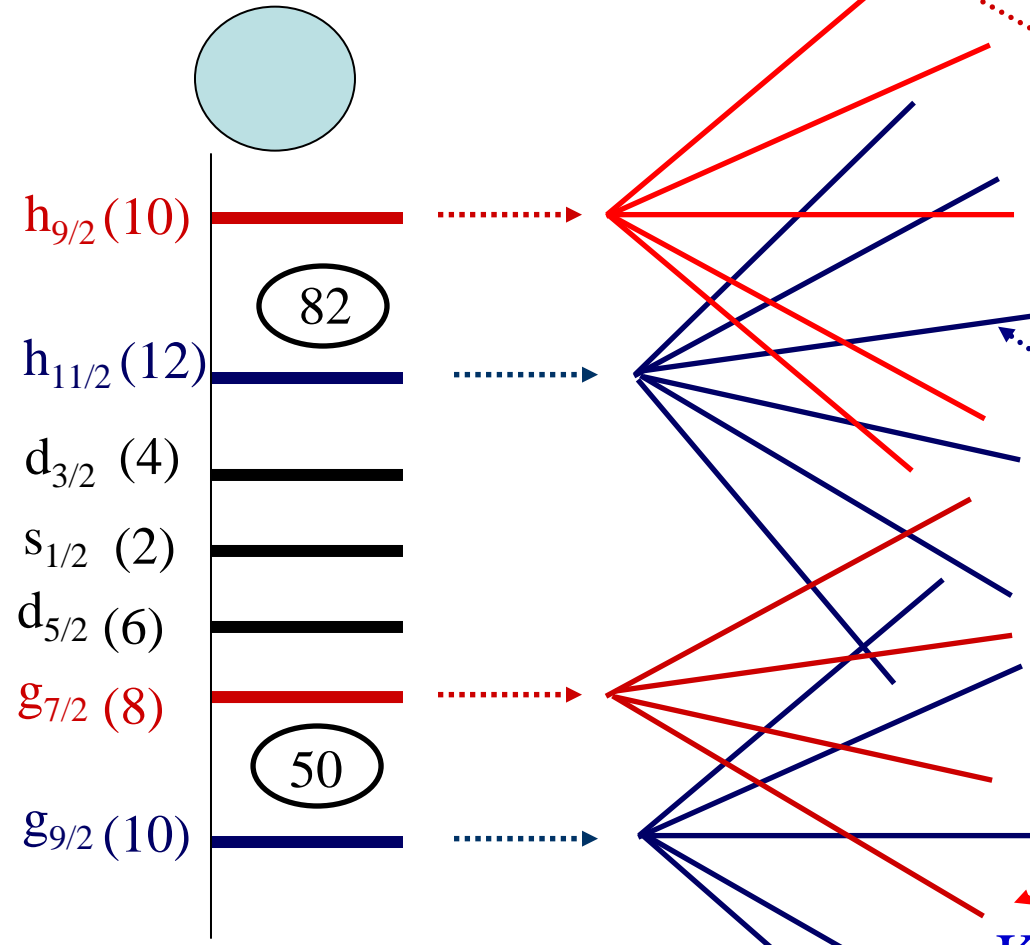
From Ragnarsson, Nilsson And Sheline, Physics Reports 45 (1978) p1

Effect of Nuclear Deformation on K-isomers

Spherical, harmon. oscillator

$$H = \hbar\omega + a\mathbf{l}\cdot\mathbf{l} + b\mathbf{l}\cdot\mathbf{s}$$

quantum numbers j^π, m_j



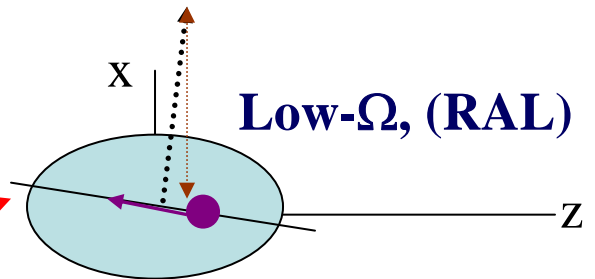
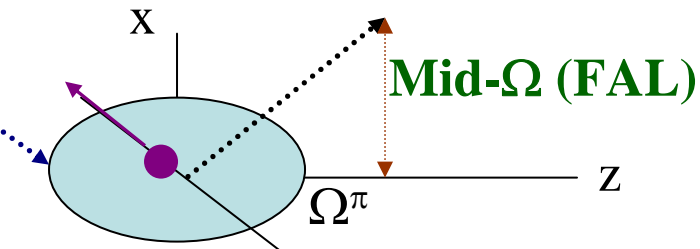
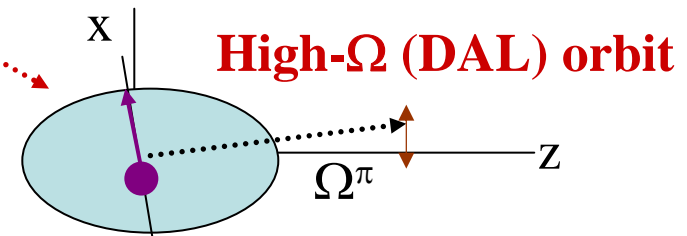
Nilsson scheme: Quadrupole deformed

3-D HO. where $\hbar\omega \rightarrow \hbar\omega_x + \hbar\omega_y + \hbar\omega_z$

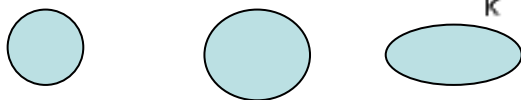
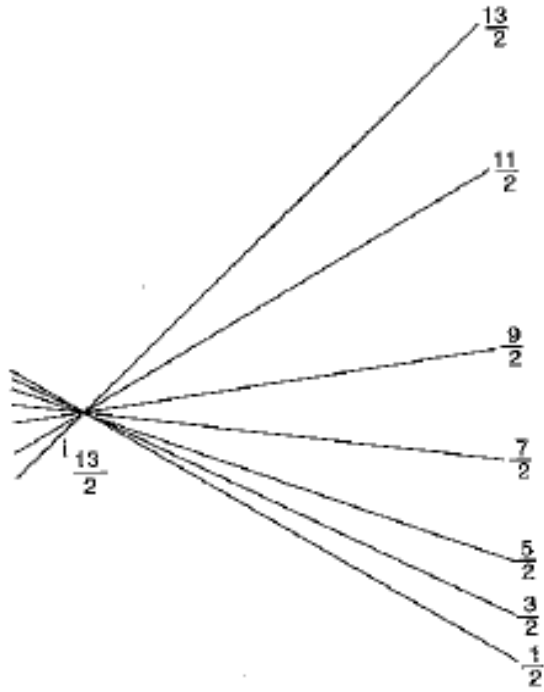
axial symmetry means $\omega_x = \omega_y$

quantum numbers $[N, n_x, \Lambda] \Omega^\pi$

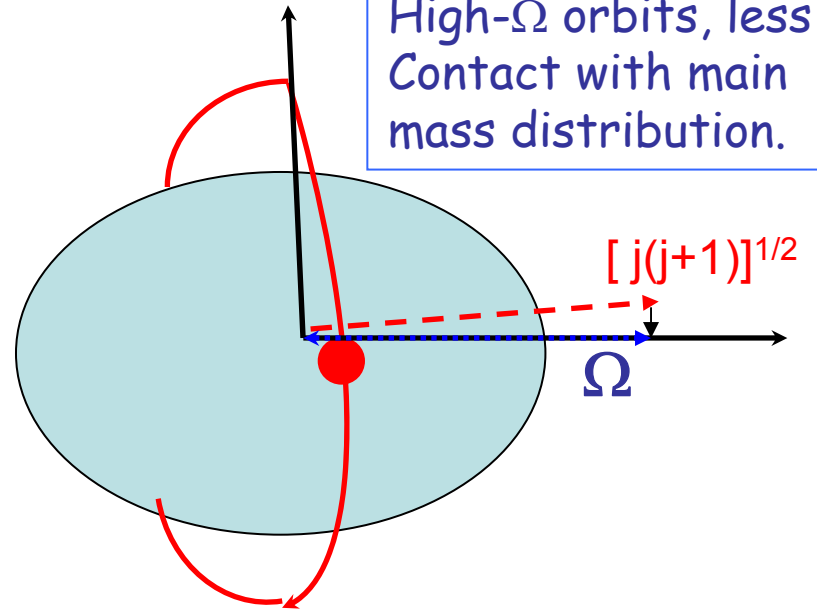
$> \text{prolate } \beta_2$



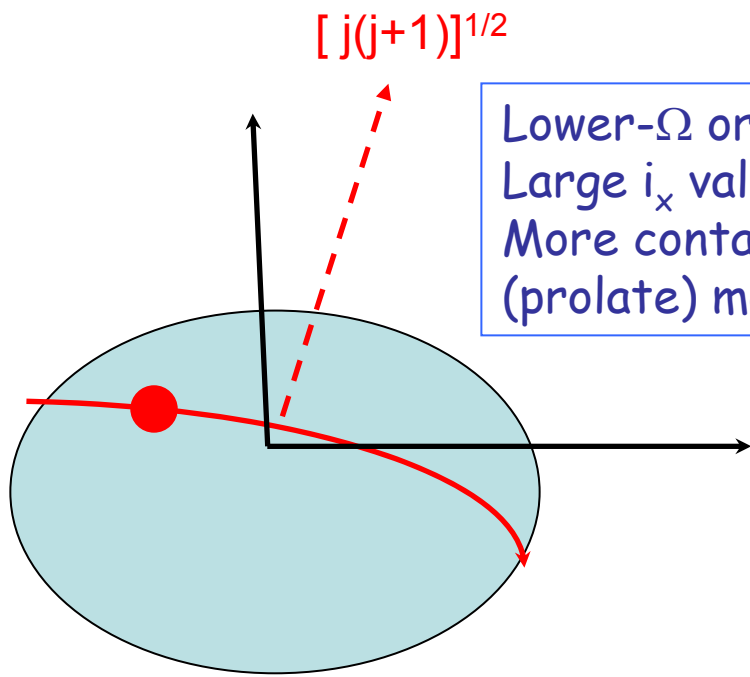
$K^\pi = \text{sum of individual } \Omega^\pi \text{ values.}$



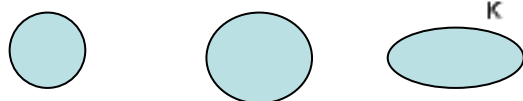
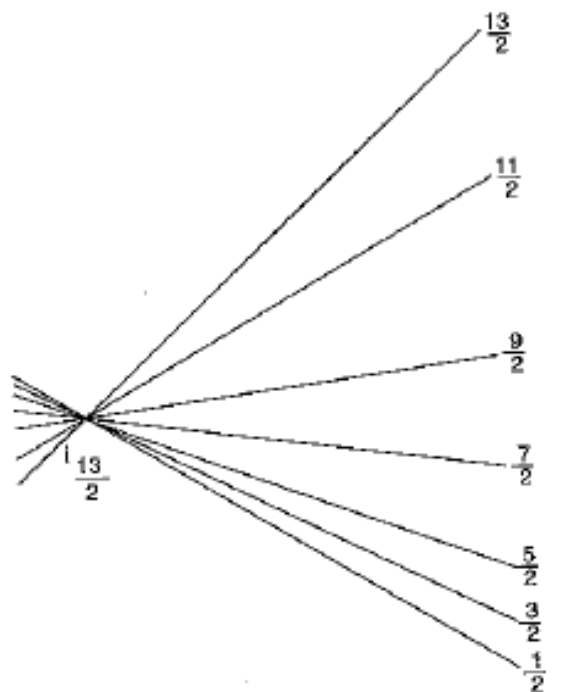
Increasing (prolate) deformation, bigger splitting.



High- Ω orbits, less Contact with main mass distribution.



Lower- Ω orbits, have Large i_x values and More contact with main (prolate) mass distribution.



Increasing (prolate) deformation, bigger splitting.

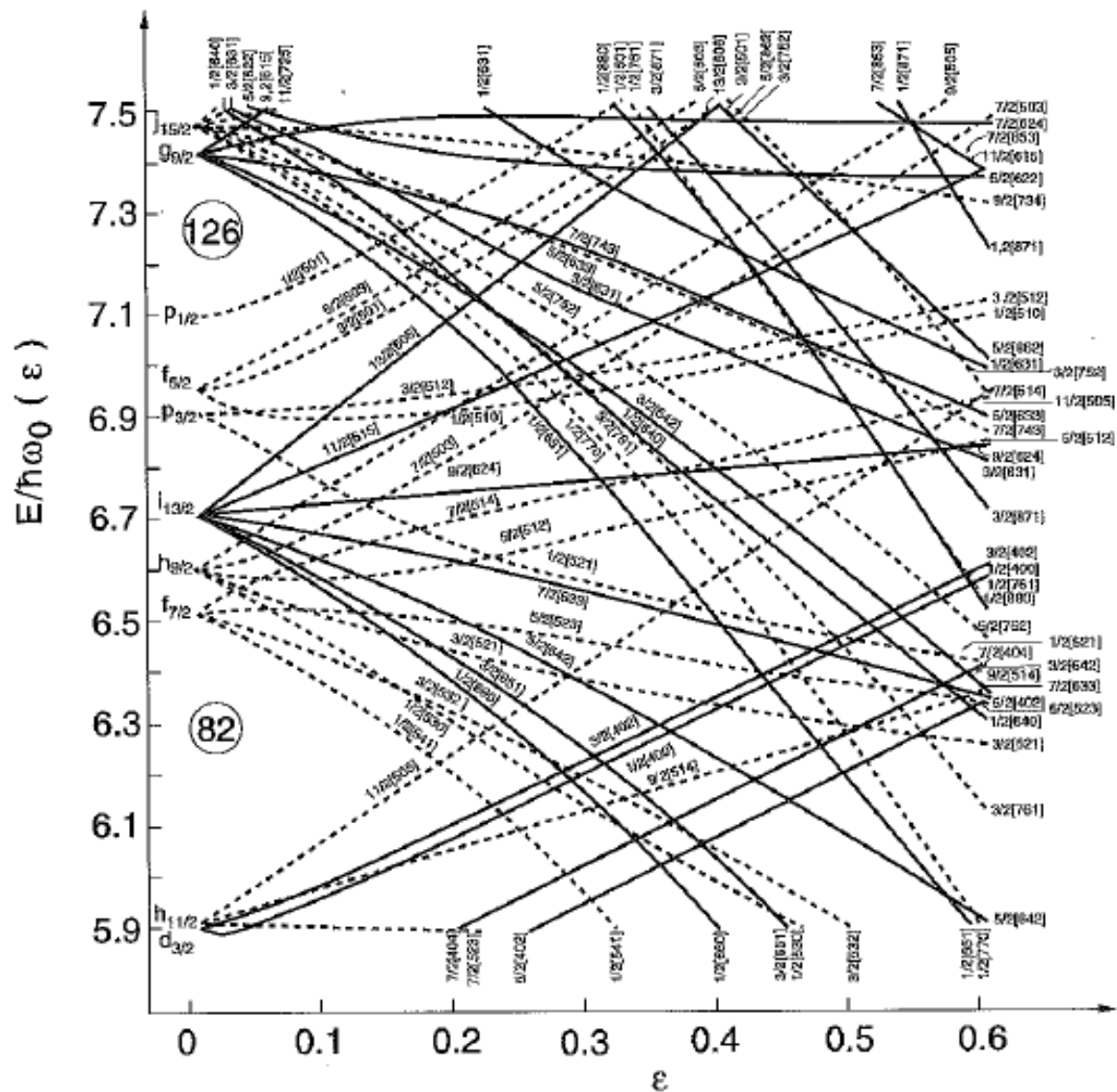
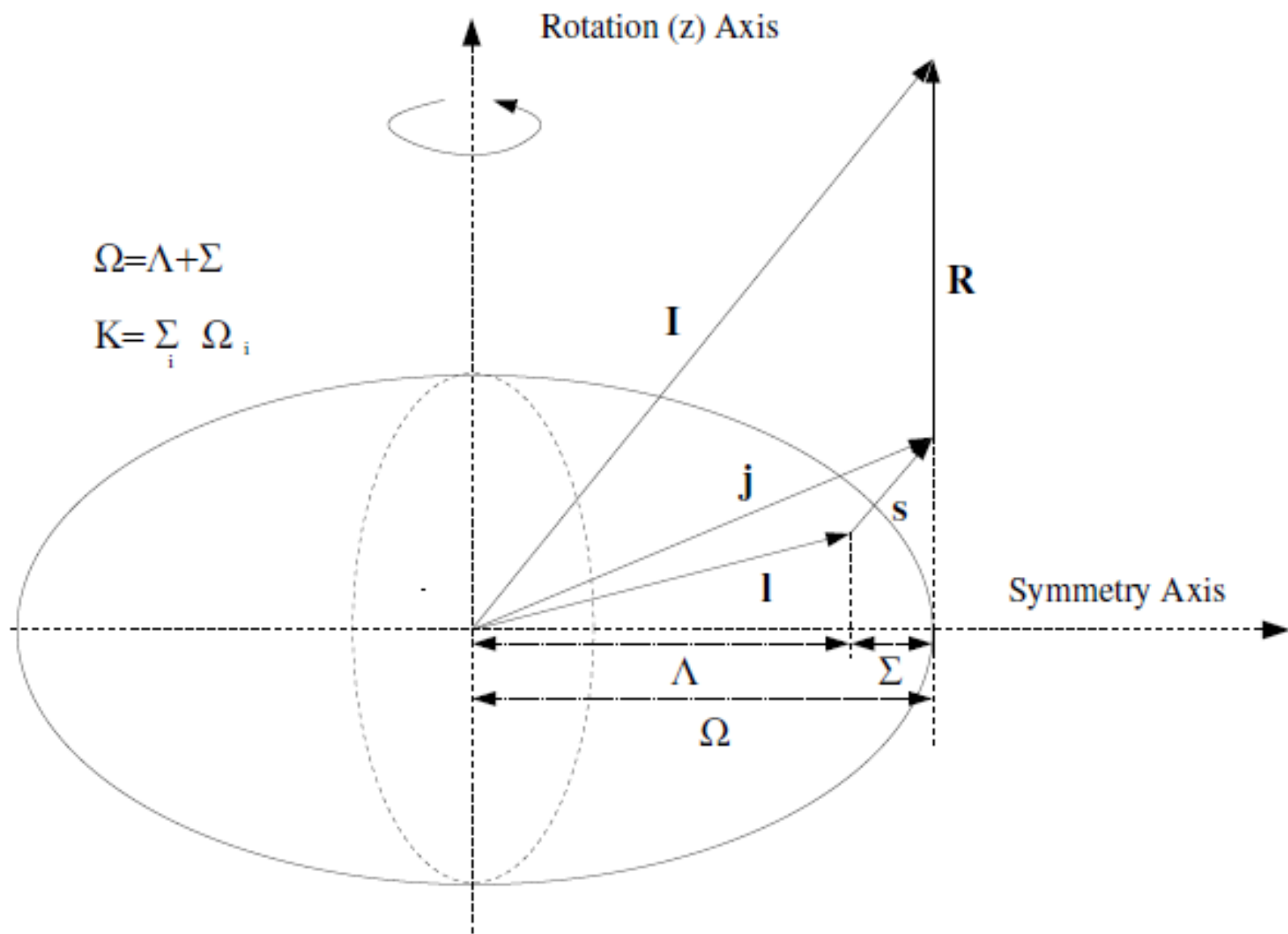


FIG. 8.4. (b) Nilsson diagram for the neutron shell 82–126. The abscissa is the deformation parameter ϵ , which is nearly the same as β . (Redrawn from Gustafson, 1967).



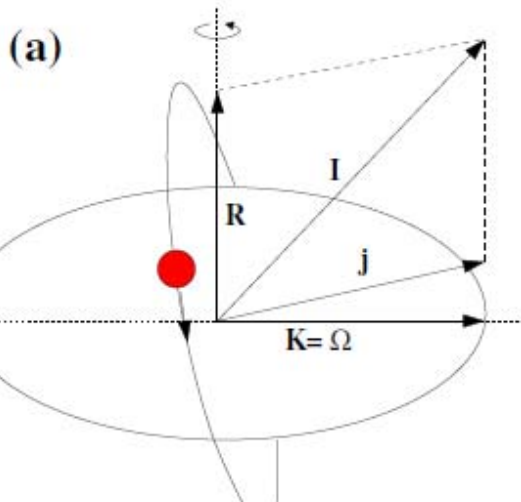
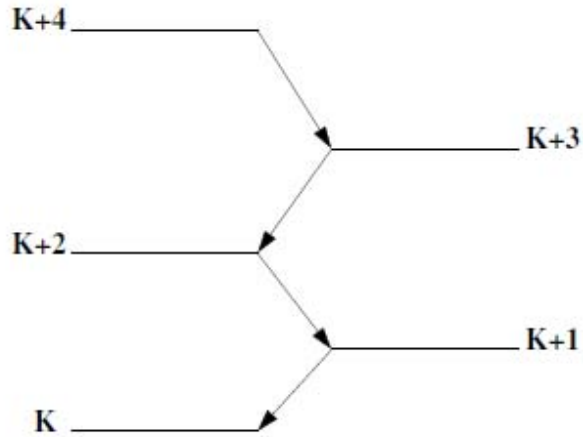
'Strongly Coupled' configuration.

Also called **'deformation aligned'**

Observe both 'signature partners',

Bandhead has $I=K$ and see $M1/E2$

Transitions between states of $I \rightarrow I-1$

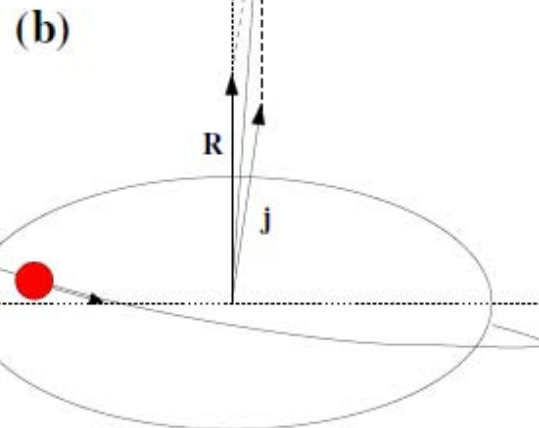
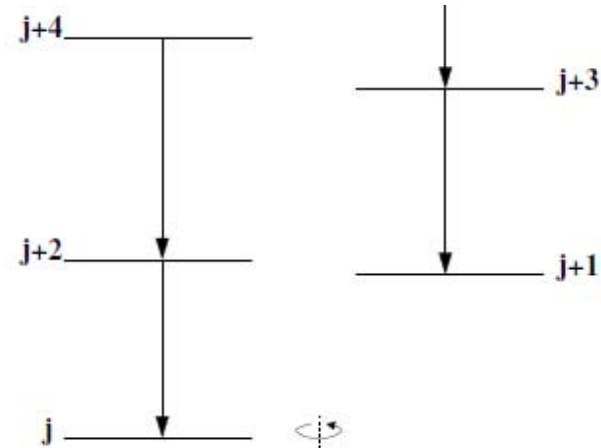


'Weakly Coupled' configuration.

Also called **'rotation aligned'**

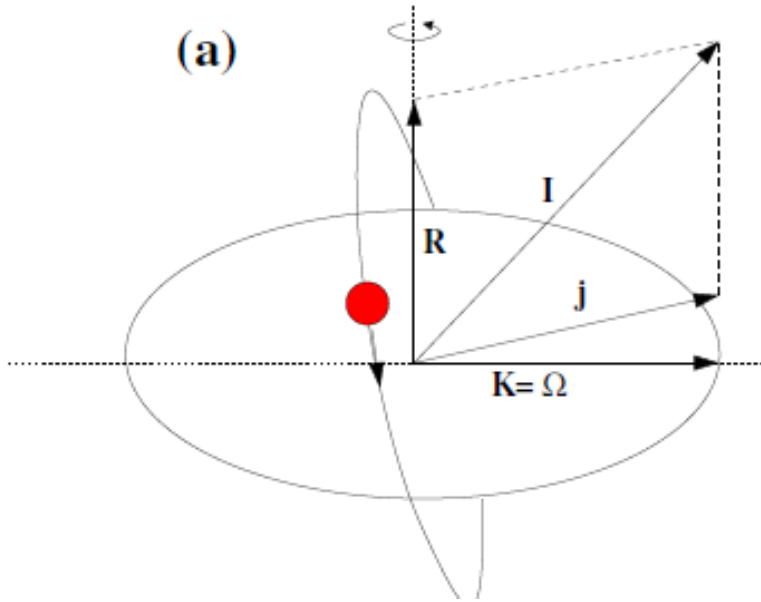
Signature partners offset due to Coriolis effects (for $\Omega=1/2$ orbits).

Bandhead has $I=j$; decoupled, E2 bands look similar to the even-even GSB.



K isomers

(a)

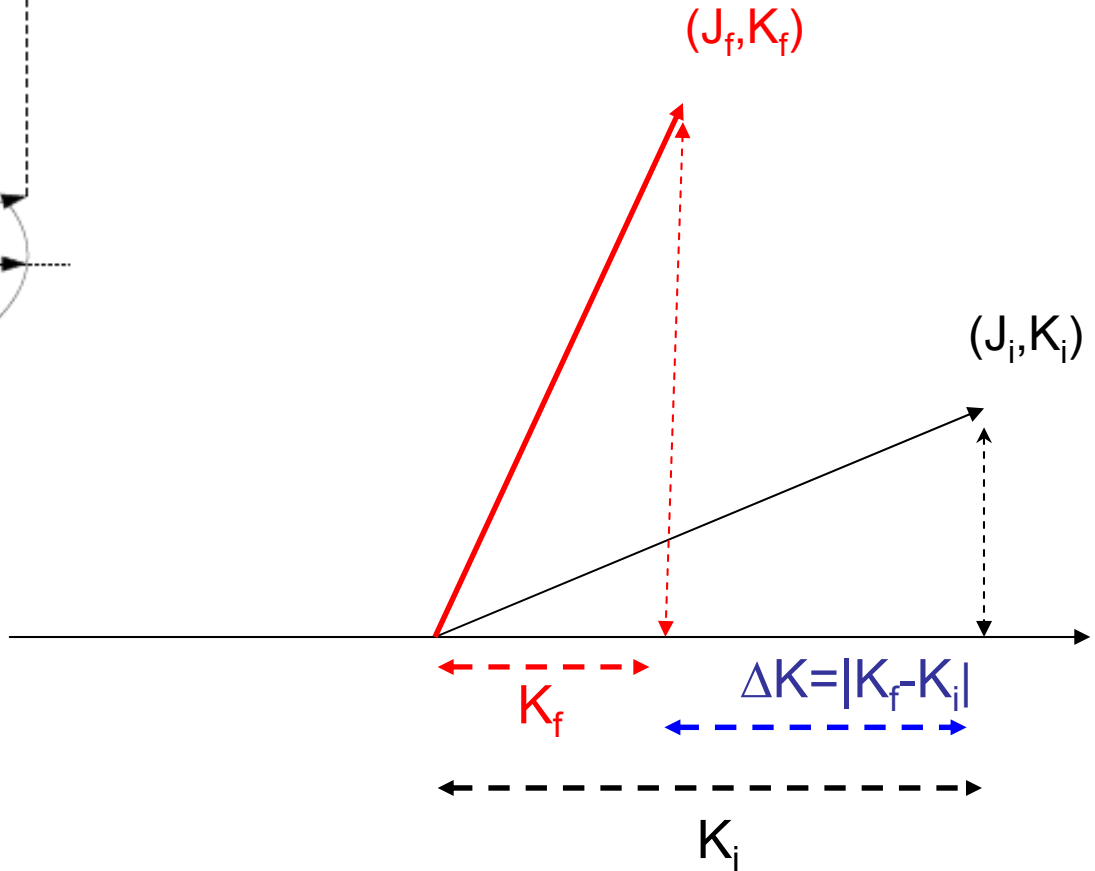


$$|K_i - K_f| = \Delta K \leq \lambda$$

$$\nu = \Delta K - \lambda$$

$$f_\nu = \left[\frac{T_{1/2}^\gamma}{T_{1/2}^W} \right]^{1/\nu}$$

= reduced hindrance for a K-isomeric decay transition.



'Forbiddenness' in K isomers

We can use single particle ('Weisskopf') estimates for transitions rates for a given multipolarity. (E_γ (keV), $T_{1/2}$ (s), Firestone and Shirley, *Table of Isotopes* (1996).

Weisskopf Estimates for $T^{1/2}$ $A = 180$, $E_\gamma = 500$ keV

$$E1 \rightarrow T_W^{1/2} = 6.76 \times 10^{-6} E_\gamma^{-3} A^{-2/3} \rightarrow 1.6 \times 10^{-15} \text{ s}$$

$$M1 \rightarrow T_W^{1/2} = 2.20 \times 10^{-5} E_\gamma^{-3} \rightarrow 1.8 \times 10^{-13} \text{ s}$$

$$E2 \rightarrow T_W^{1/2} = 9.52 \times 10^6 E_\gamma^{-5} A^{-4/3} \rightarrow 3.0 \times 10^{-10} \text{ s}$$

$$M2 \rightarrow T_W^{1/2} = 3.10 \times 10^7 E_\gamma^{-5} A^{-2/3} \rightarrow 3.1 \times 10^{-8} \text{ s}$$

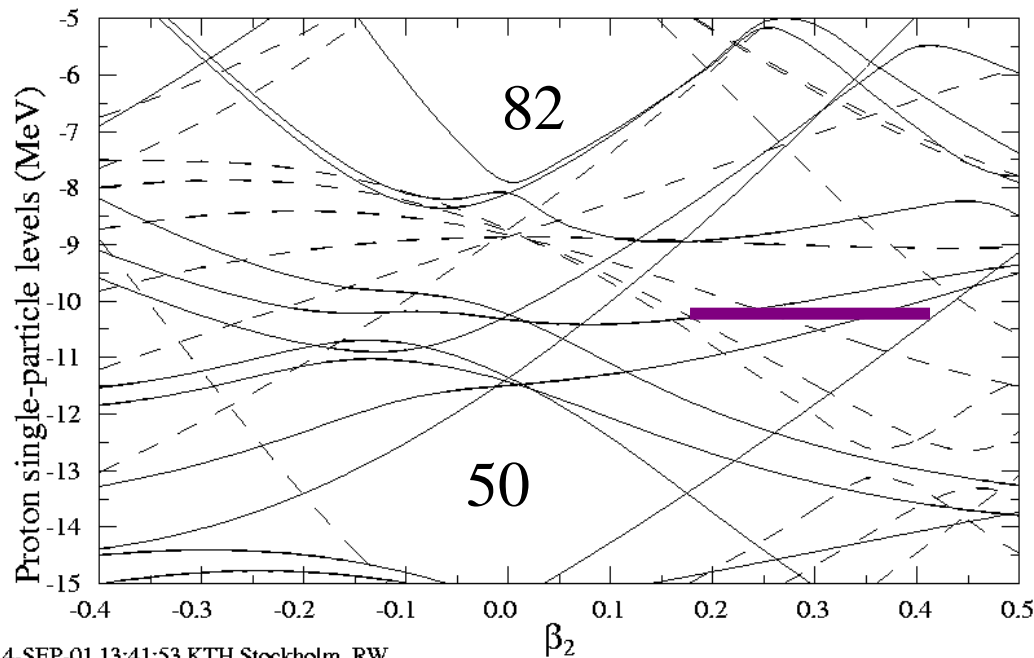
Hindrance (F) (removing dependence on multipolarity and E_γ) is defined by

$$F = \left(\frac{T_{1/2}^\gamma}{T_{1/2}^W} \right) = \text{ratio of expt. and Weisskopf trans. rates}$$

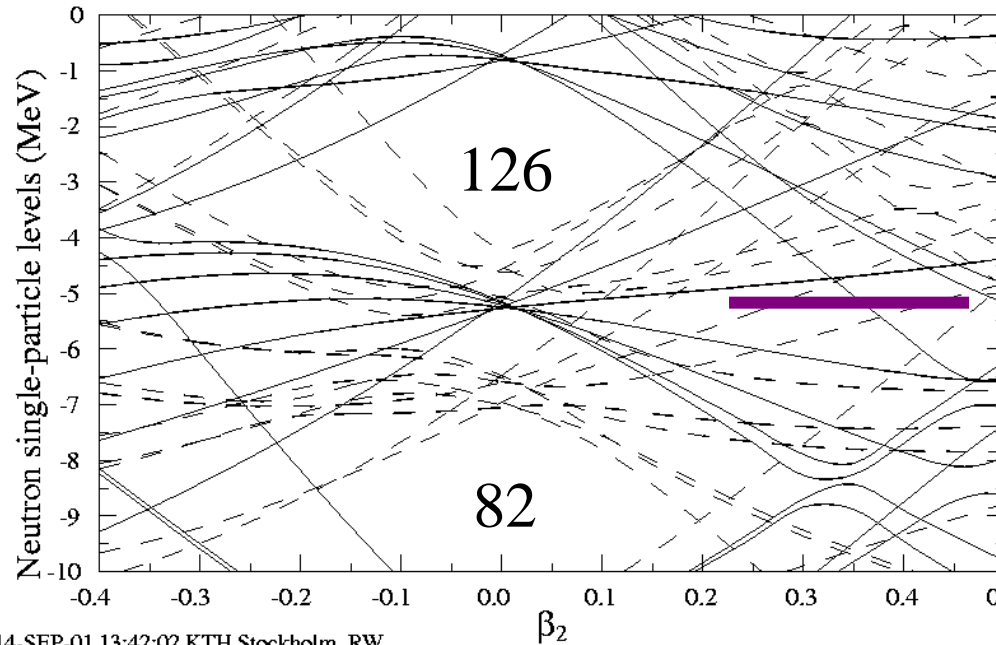
Reduced Hindrance (f_ν) gives an estimate for the 'goodness' of K- quantum number and validity of K-selection rule (= a measure of axial symmetry).

$$f_\nu = F^{1/\nu} = \left(\frac{T_{1/2}^\gamma}{T_{1/2}^W} \right)^{1/\nu}, \quad \nu = \Delta K - \lambda$$

$f_\nu \sim 100$ typical value for 'good' K isomer (see Lobner Phys. Lett. **B26** (1968) p279)



14-SEP-01 13:41:53 KTH Stockholm, RW



14-SEP-01 13:42:02 KTH Stockholm, RW

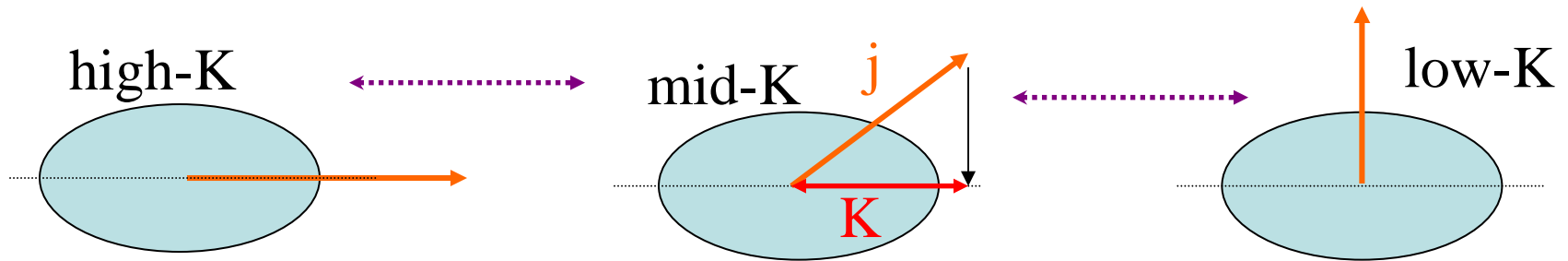
Expect to find K-isomers in regions where high-K orbitals are at the Fermi surface.

Also need large, axially symmetric deformation
 $(\beta_2 > 0.2, \gamma \sim 0^\circ)$

Conditions fulfilled at $A \sim 170-190$ rare-earth reg.

High- Ω single particle orbitals from eg. $i_{13/2}$ neutrons couple together to give energetically favoured states with high-K ($=\sum \Omega_i$).

Search for long (>100ms) K-isomers in neutron-rich(ish) A~180 nuclei.

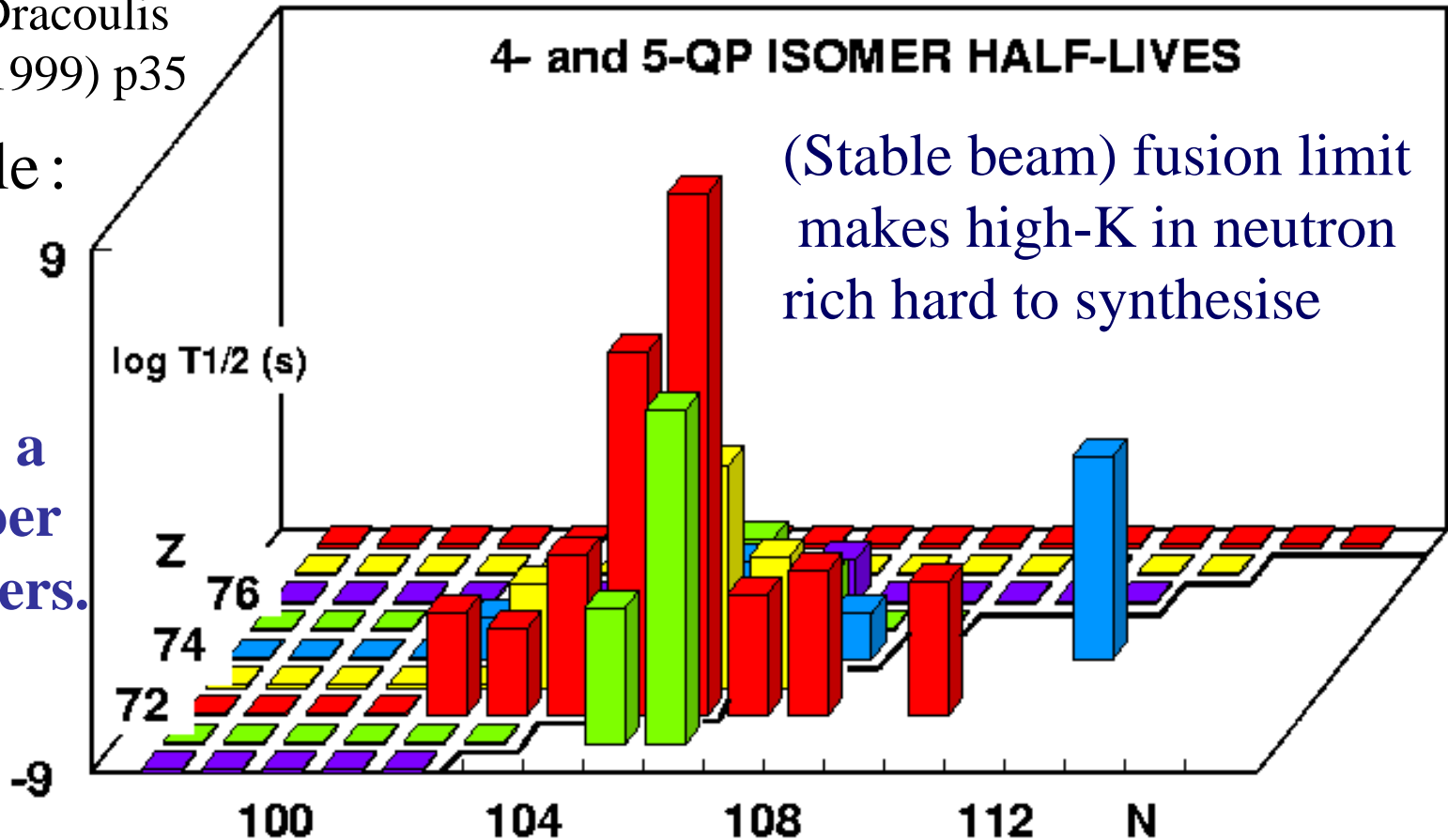


Walker and Dracoulis
Nature **399** (1999) p35

K - sel. rule :

$$\Delta\lambda \geq \Delta K \quad 9$$

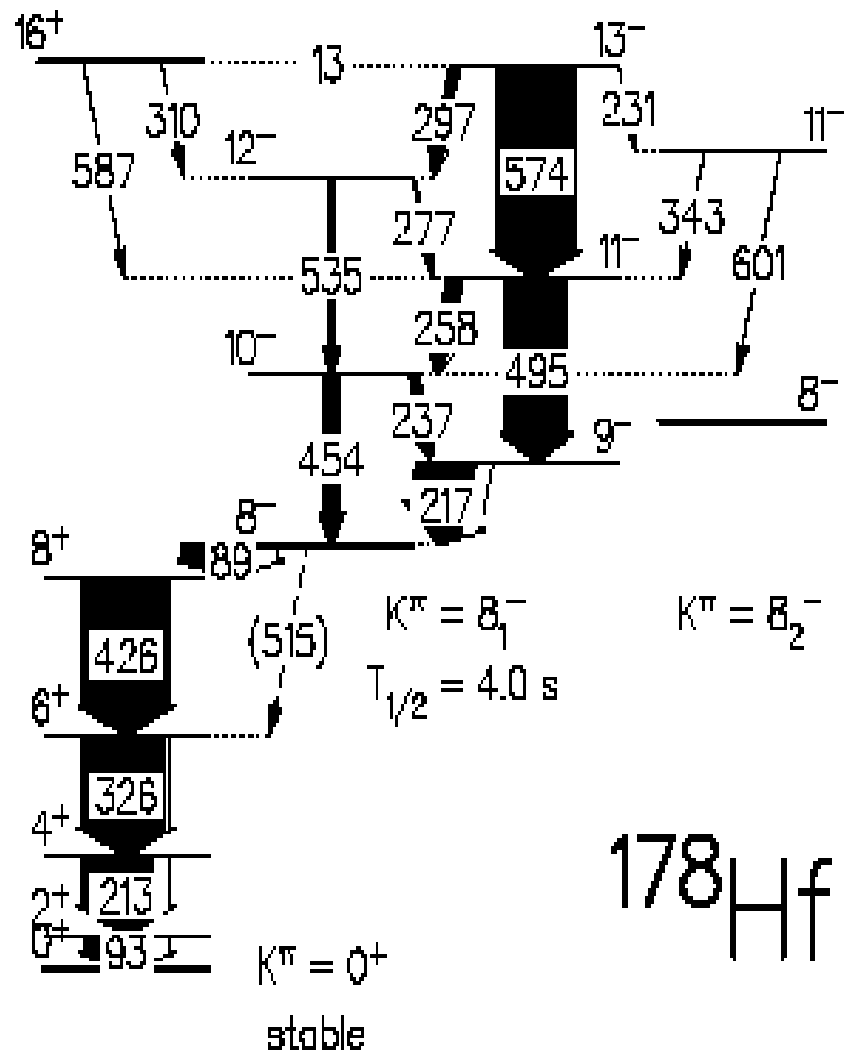
N=104 also a
good number
for K-isomers.



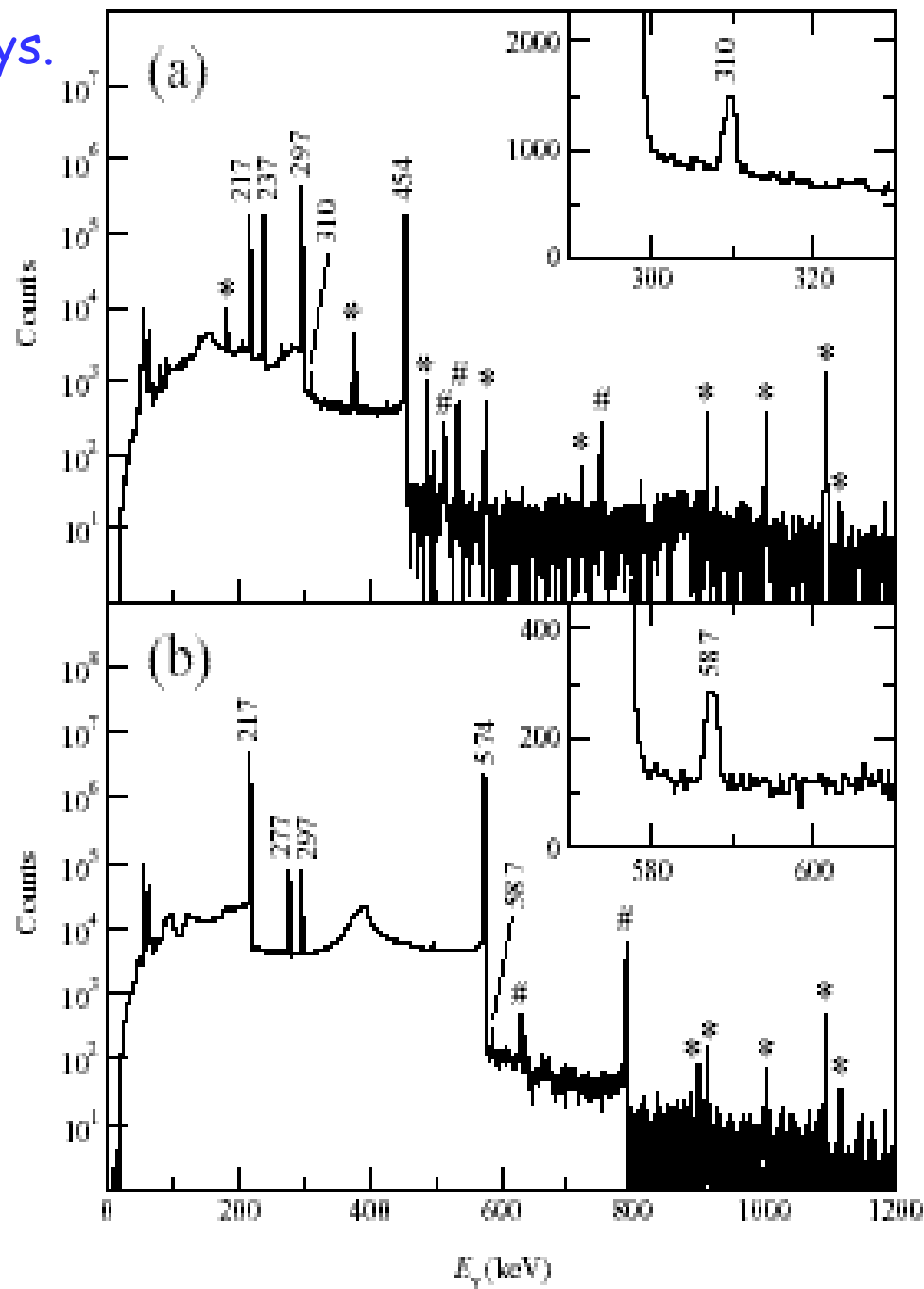
Smith, Walker et al., Phys. Rev. C68 (2003) 031302

$$K^\pi = 16^+$$

$$T_{1/2} = 31 \text{ y}$$



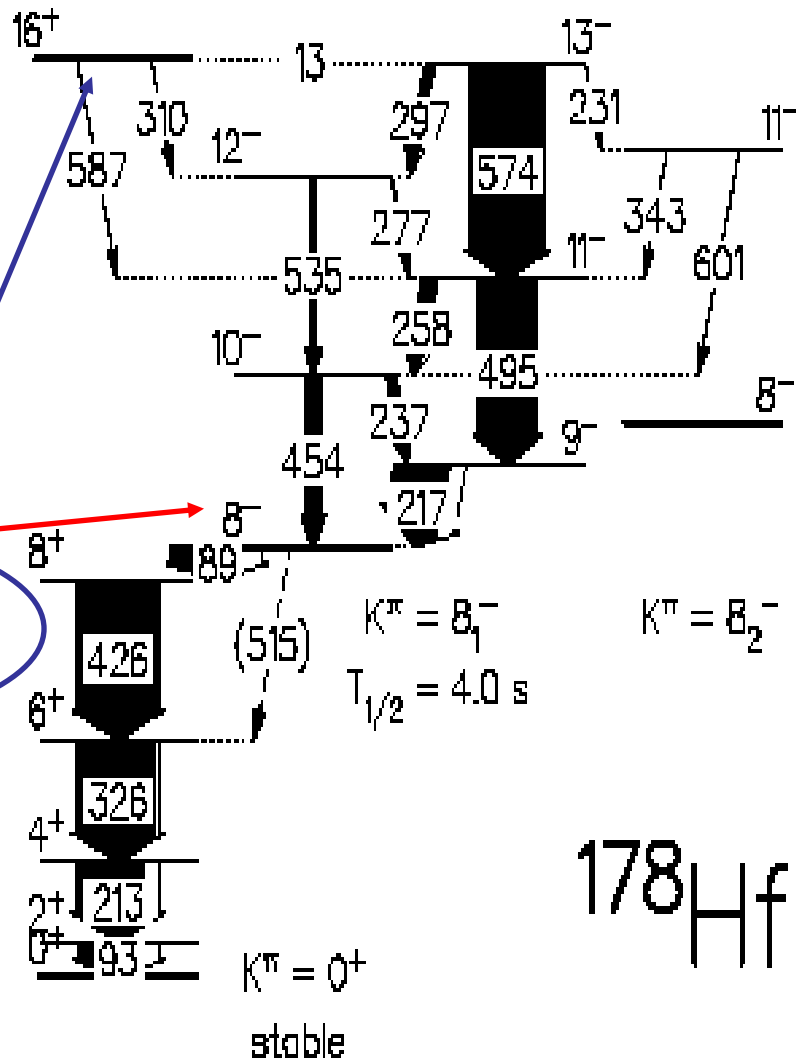
^{178}Hf



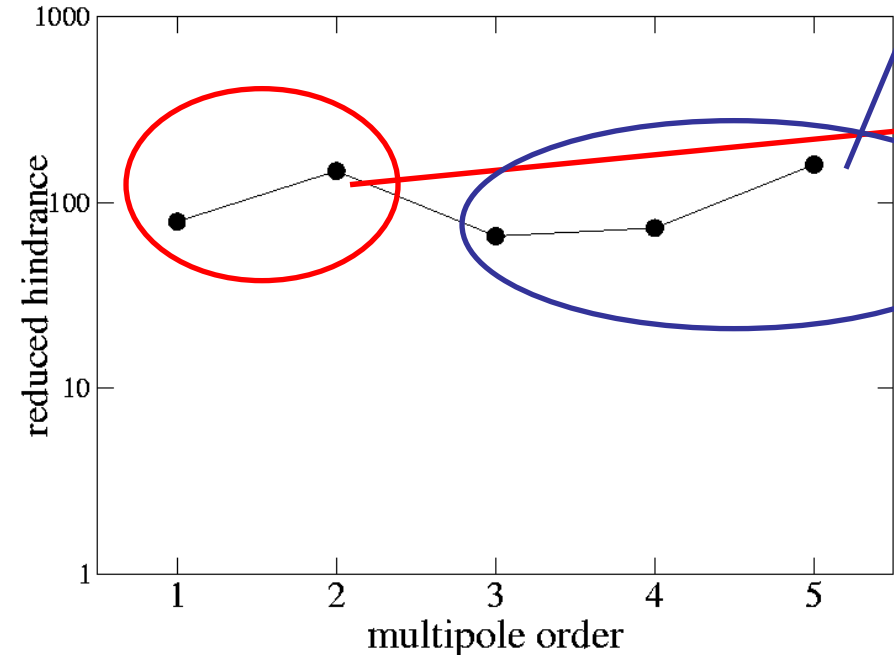
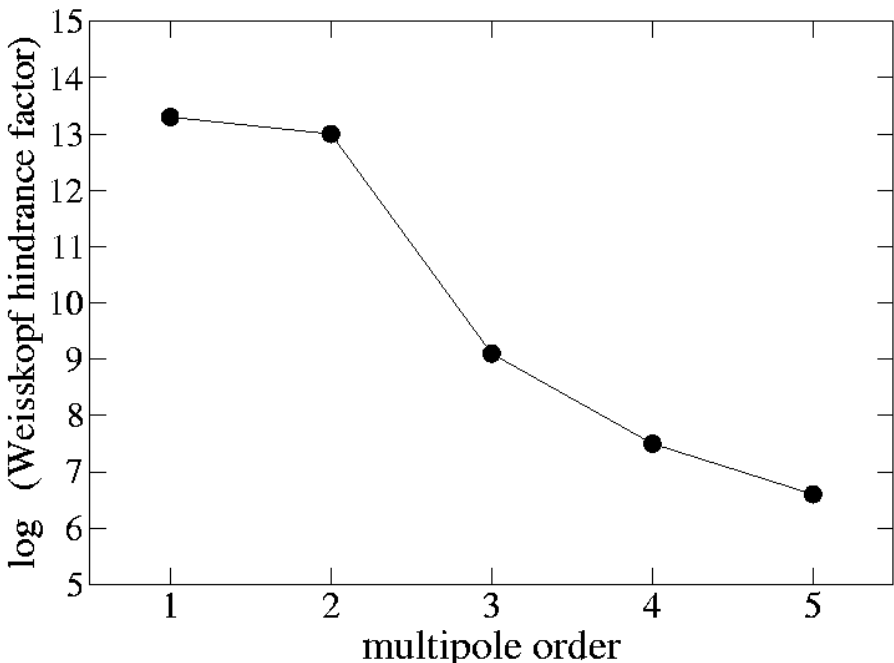
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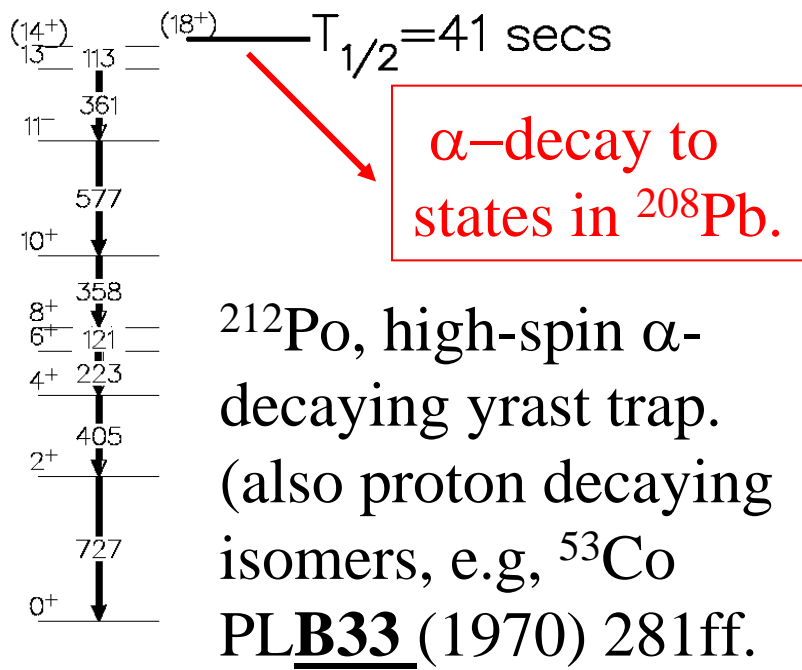
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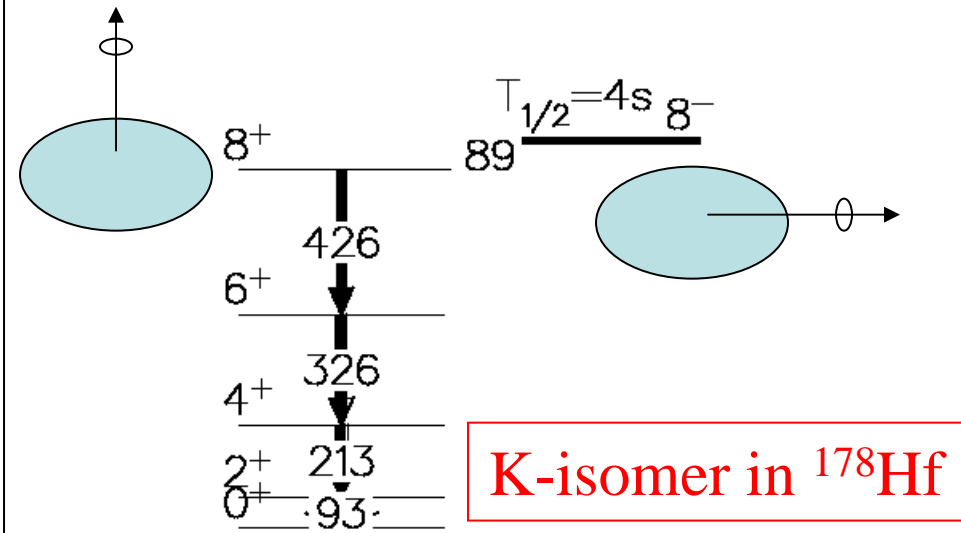
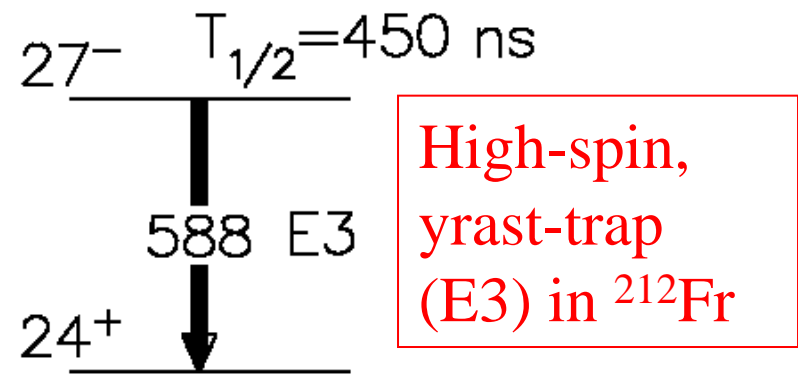
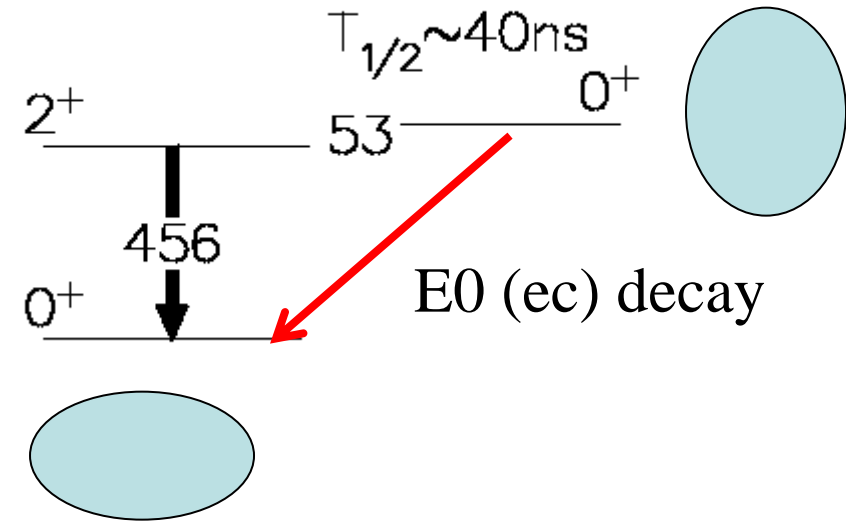


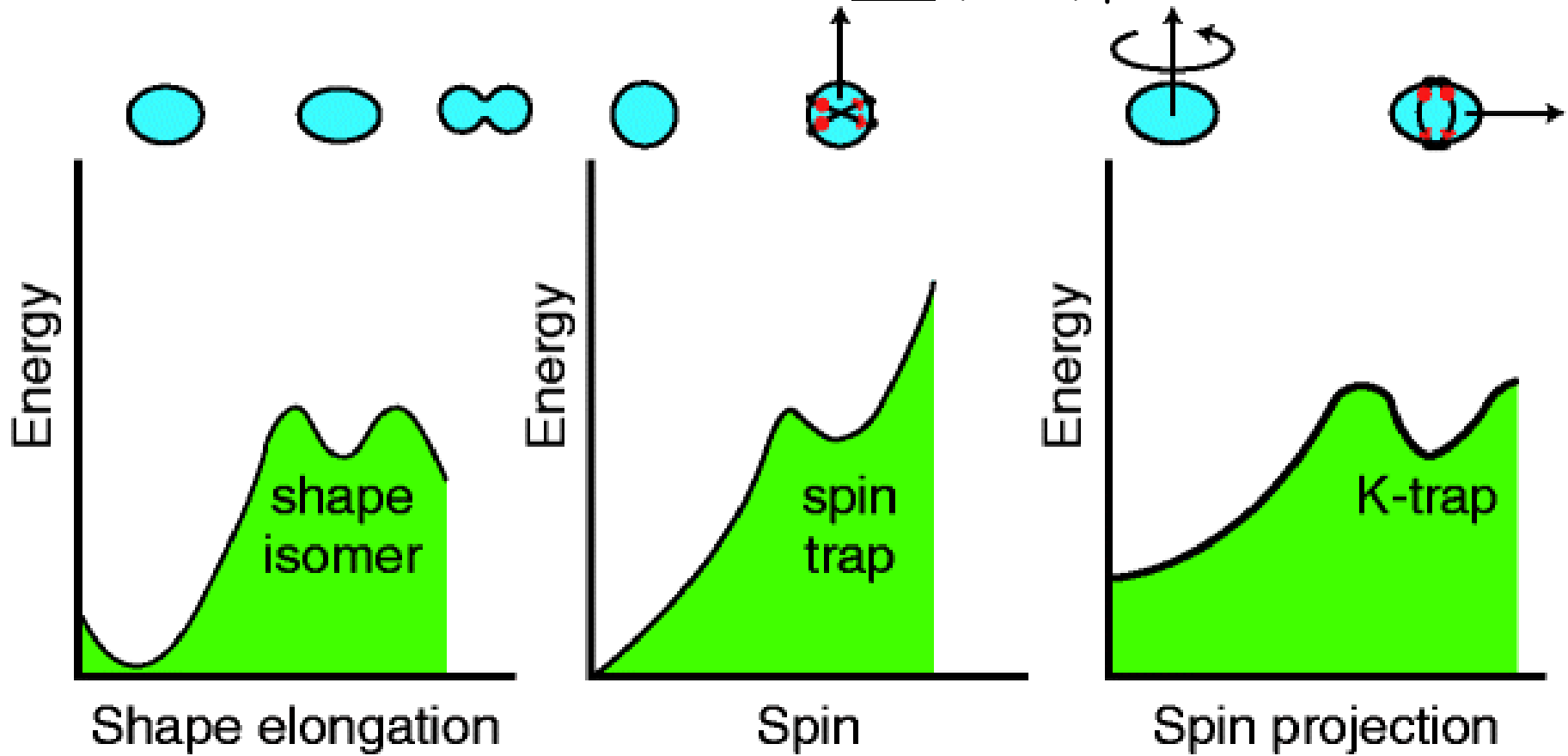
^{178}Hf





^{74}Kr , shape isomer





What about EM transition rates between low-energy states in nuclei ?

EM Transition Rates

Classically, the average power radiated by an EM multipole field is given by

$$P(\sigma L) = \frac{2(L+1)c}{\epsilon_0 L [(2L+1)!!]^2} \left(\frac{\omega}{c} \right)^{2L+2} [m(\sigma L)]^2$$

$m(\sigma L)$ is the time-varying electric or magnetic multipole moment.
 ω is the (circular) frequency of the EM field

For a quantized (nuclear) system, the decay probability is determined by the MATRIX ELEMENT of the EM MULTIPOLE OPERATOR, where

$$m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L) \psi_i dv \quad \text{i.e., integrated over the nuclear volume.}$$

We can then get the general expression for the probability per unit time for gamma-ray emission, $\lambda(\sigma L)$, from:

$$\lambda(\sigma L) = \frac{1}{\tau} = \frac{P(\sigma L)}{\hbar \omega} = \frac{2(L+1)}{\epsilon_0 \hbar L [(2L+1)!!]^2} \left(\frac{\omega}{c} \right)^{2L+1} [m_{fi}(\sigma L)]^2$$

(see Introductory Nuclear Physics, K.S. Krane (1988) p330).

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L((2L+1)!!)^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L : J_i \rightarrow J_f) \quad (5.0.3)$$

where $B(\lambda L : J_i \rightarrow J_f)$ is called the *reduced matrix element*.

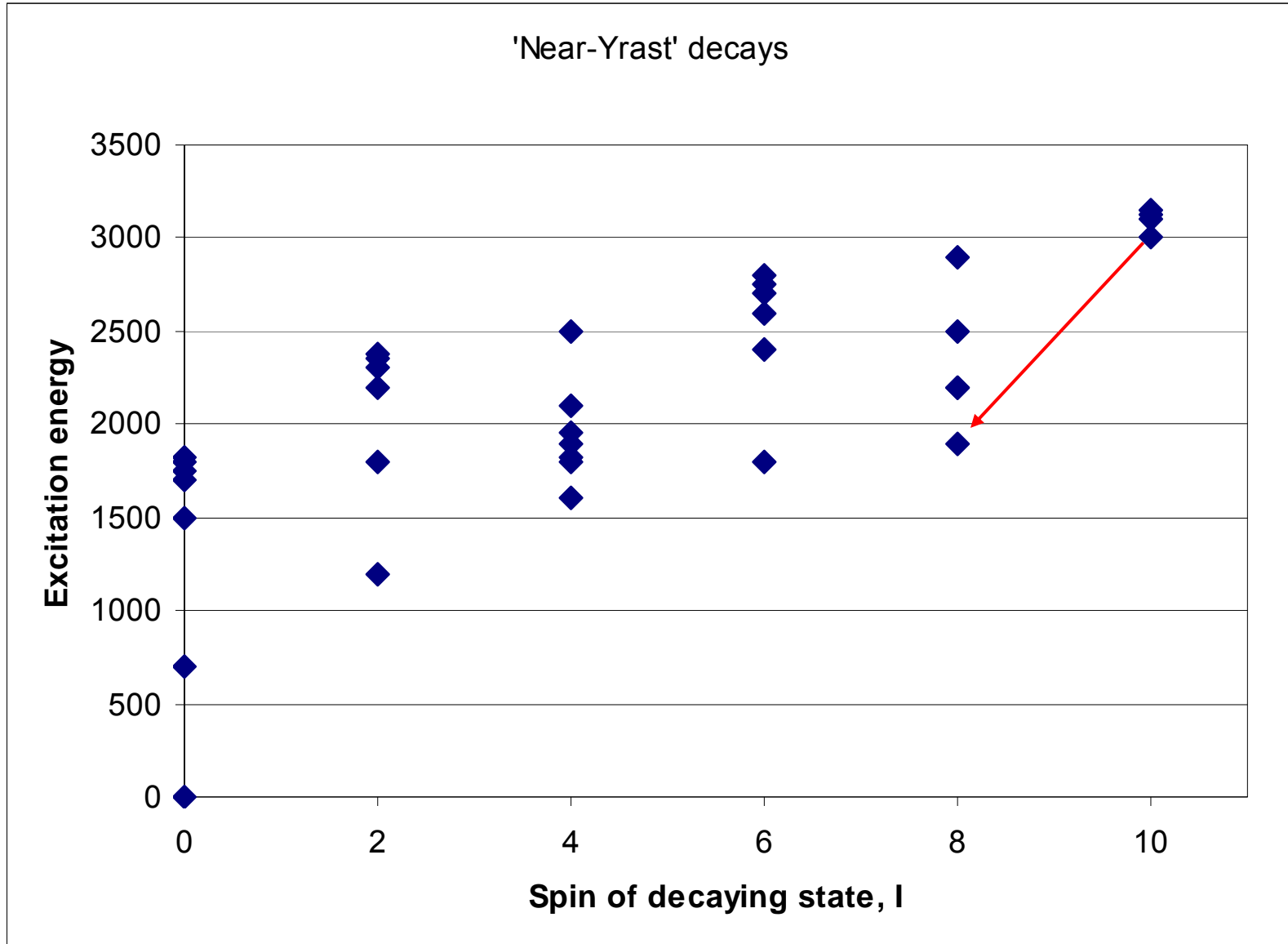
Measuring the lifetime (decay probability) of a nuclear state thus gives a value for the $B(\lambda L : J_i \rightarrow J_f)$.

For lifetimes, τ in units of seconds where the transition *probability* per unit second, $T = \frac{1}{\tau}$, (E_γ in MeV),

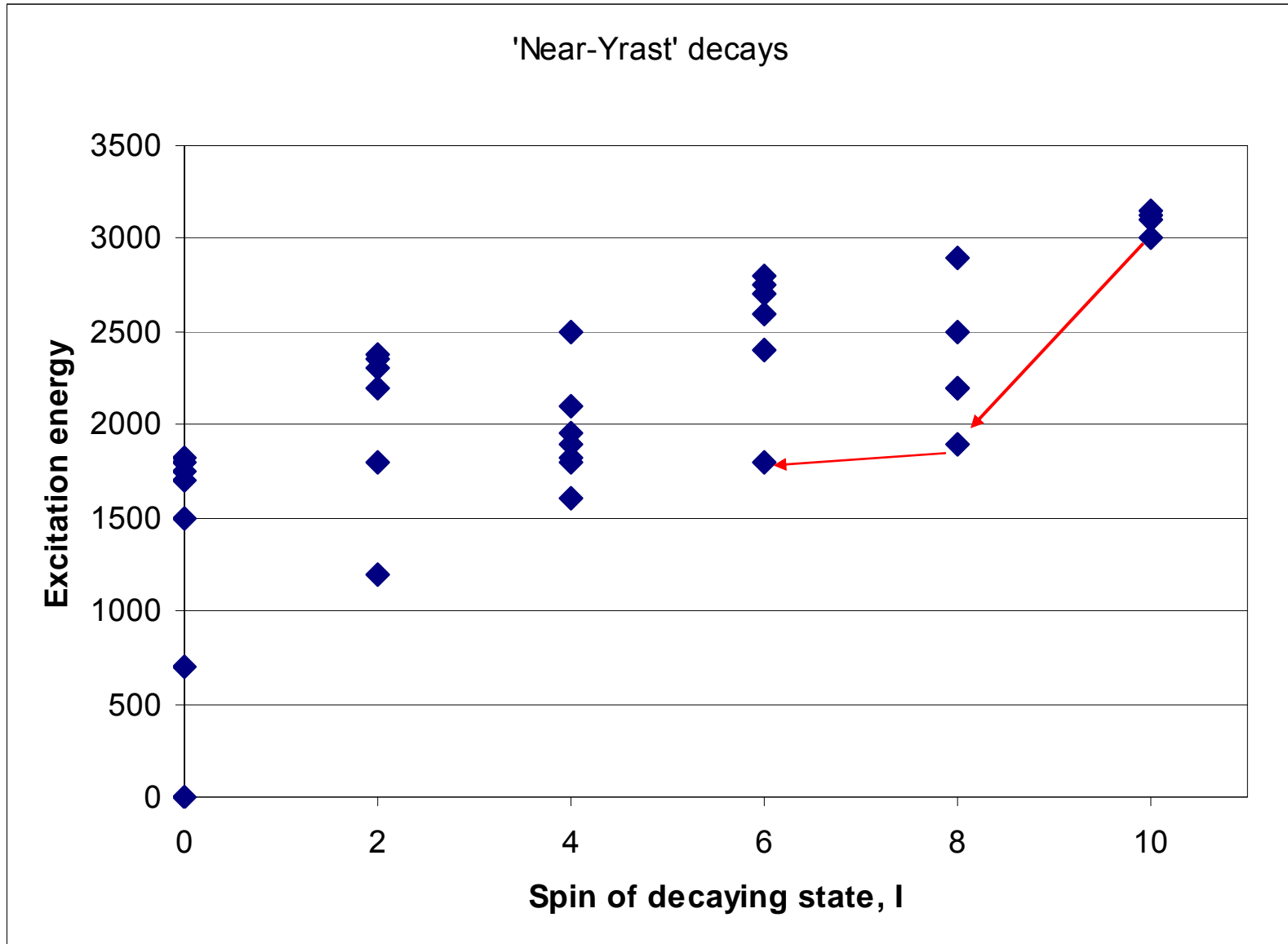
Multipolarity	Electric Transition Rate (s^{-1})	Magnetic Transition Rate (s^{-1})
1	$1.587 \times 10^{15} E_\gamma^3 B(E1)$	$1.779 \times 10^{13} E_\gamma^3 B(M1)$
2	$1.223 \times 10^9 E_\gamma^5 B(E2)$	$1.371 \times 10^7 E_\gamma^5 B(M2)$
3	$5.689 \times 10^2 E_\gamma^7 B(E3)$	$6.387 \times 10^0 E_\gamma^7 B(M3)$
4	$1.649 \times 10^{-4} E_\gamma^9 B(E4)$	$1.889 \times 10^{-6} E_\gamma^9 B(M4)$
5	$3.451 \times 10^{-11} E_\gamma^{11} B(E5)$	$3.868 \times 10^{-13} E_\gamma^{11} B(M5)$

Table 2.2: Transition probabilities $T(s^{-1})$ expressed by $B(EL)$ in $(e^2(fm)^{2L})$ and $B(ML)$ in $(\frac{eh}{2mc}(fm)^{2L-2})$. E_γ is the γ -ray energy, in MeV. (Taken from ref [69]).

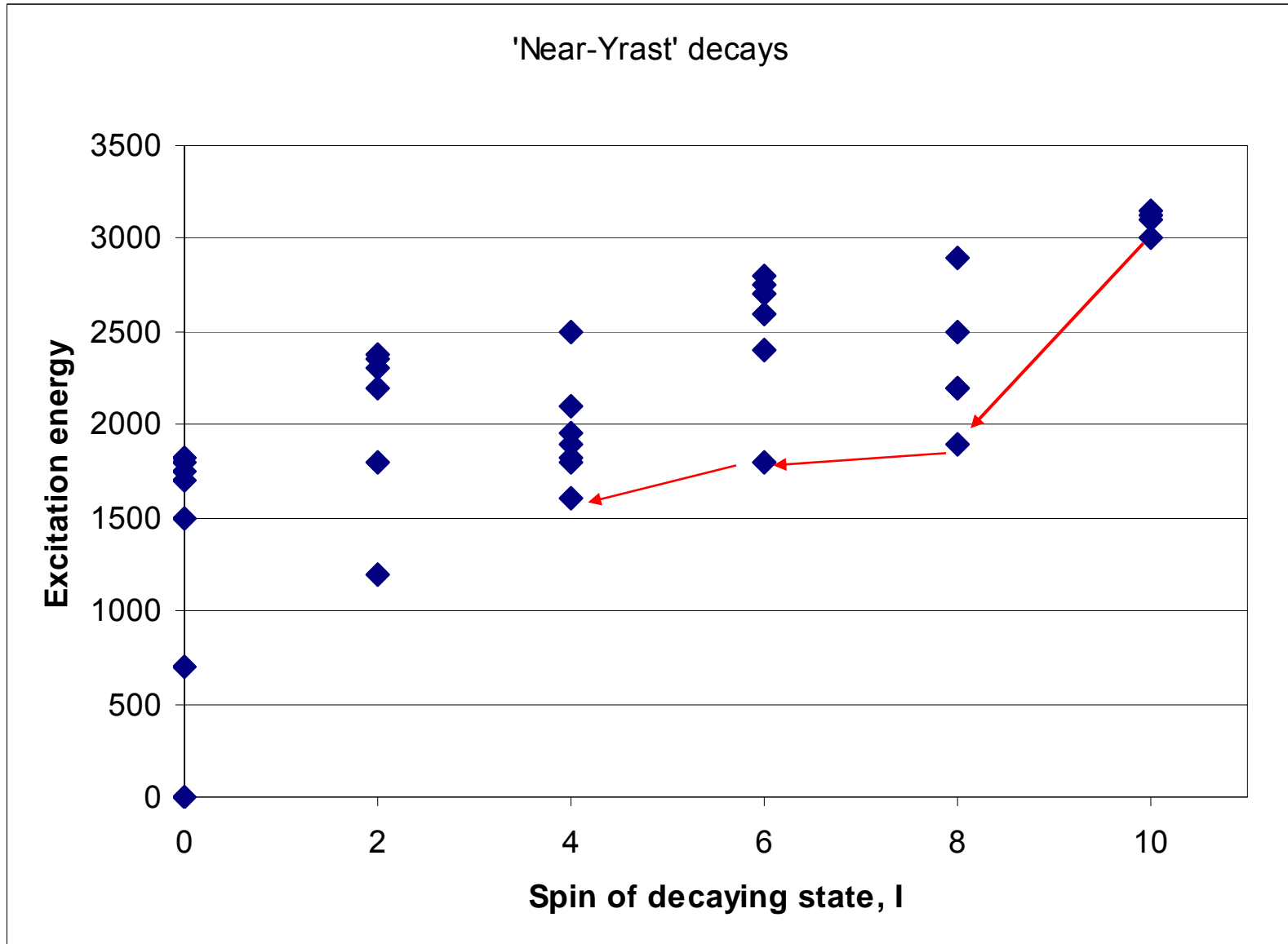
The EM transition rate depends on $E_\gamma^{2\lambda+1}$, the highest energy transitions for the lowest λ are (generally) favoured. This results in the preferential population of yrast and near-yrast states.



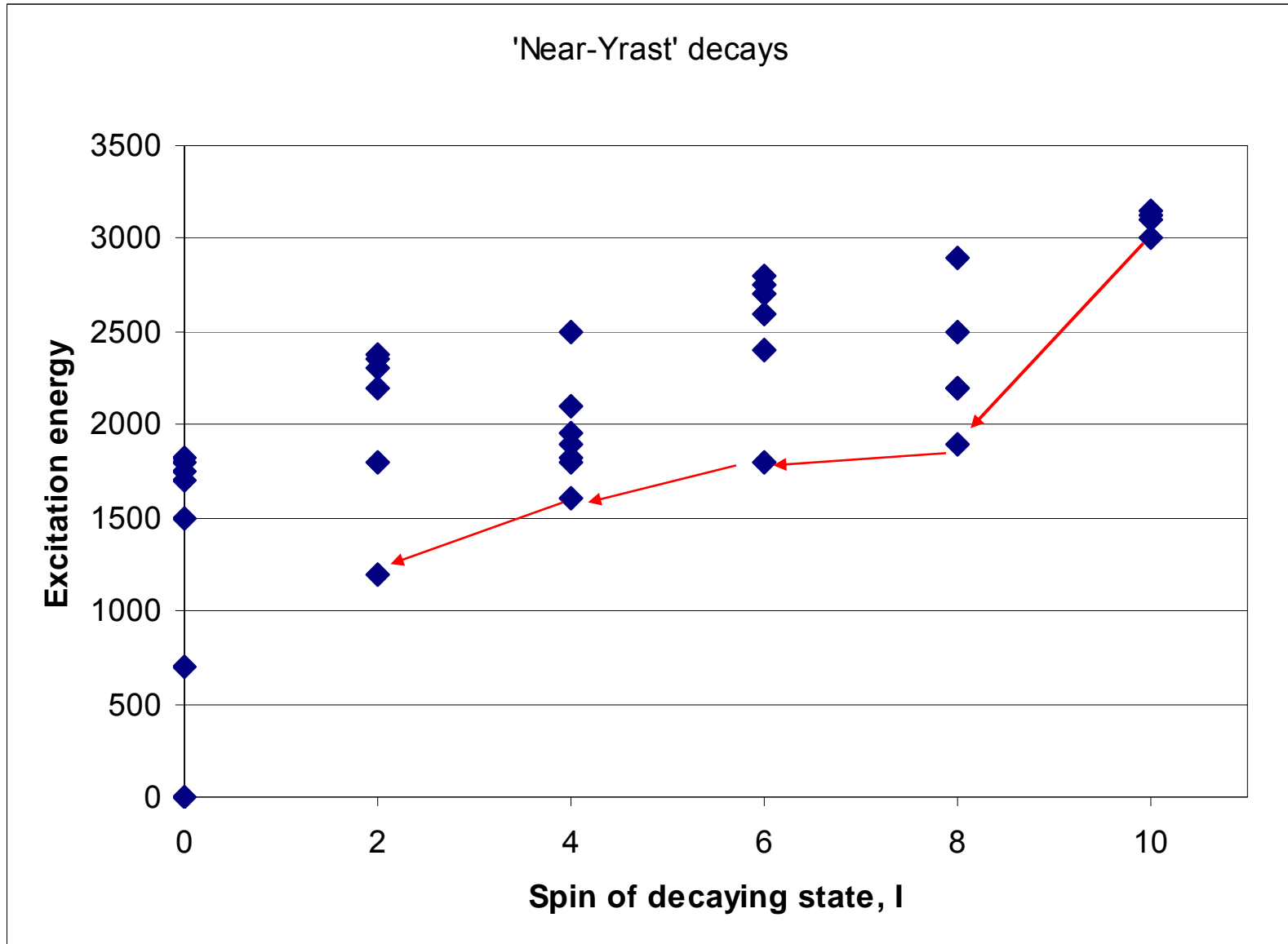
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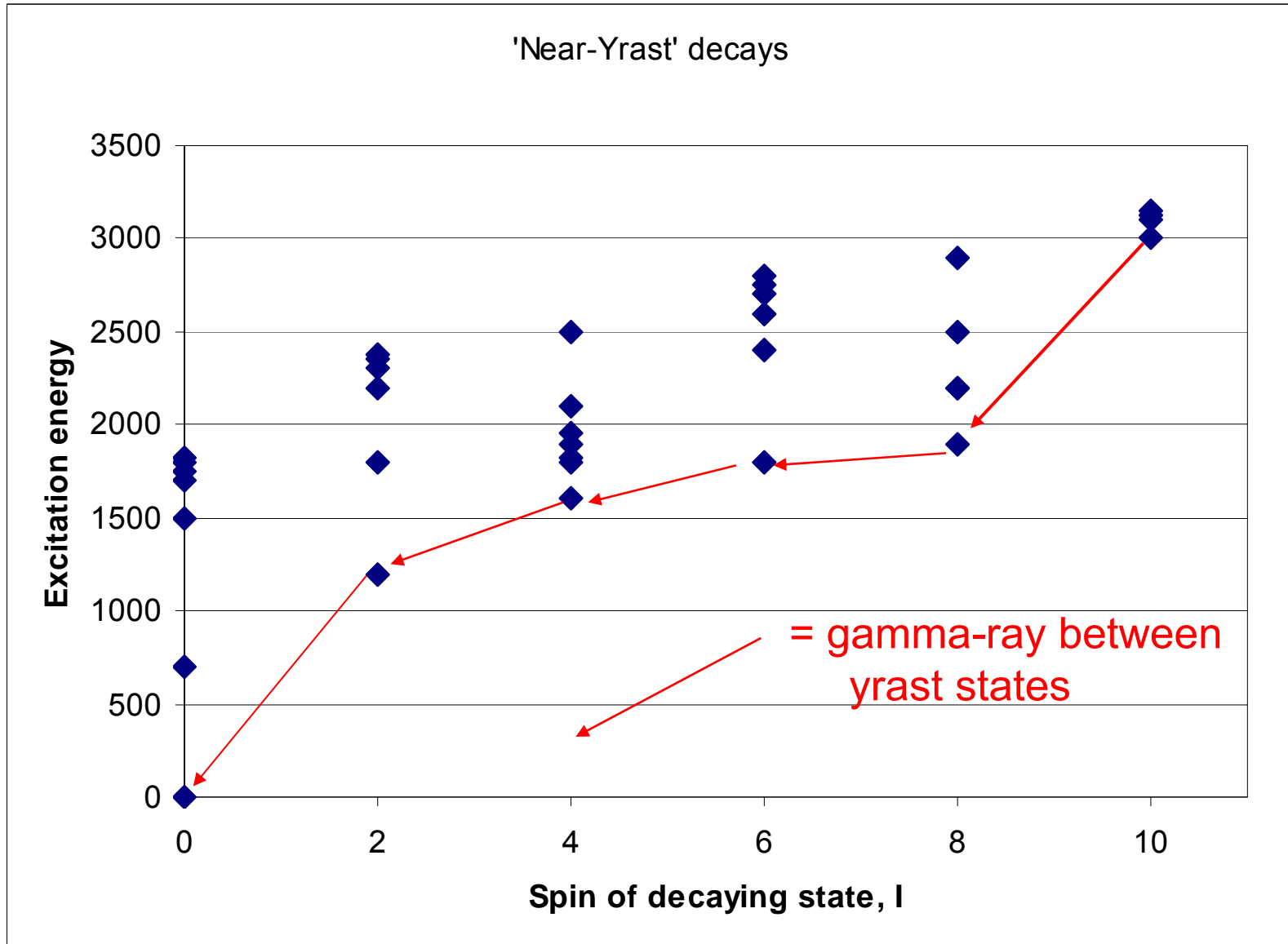
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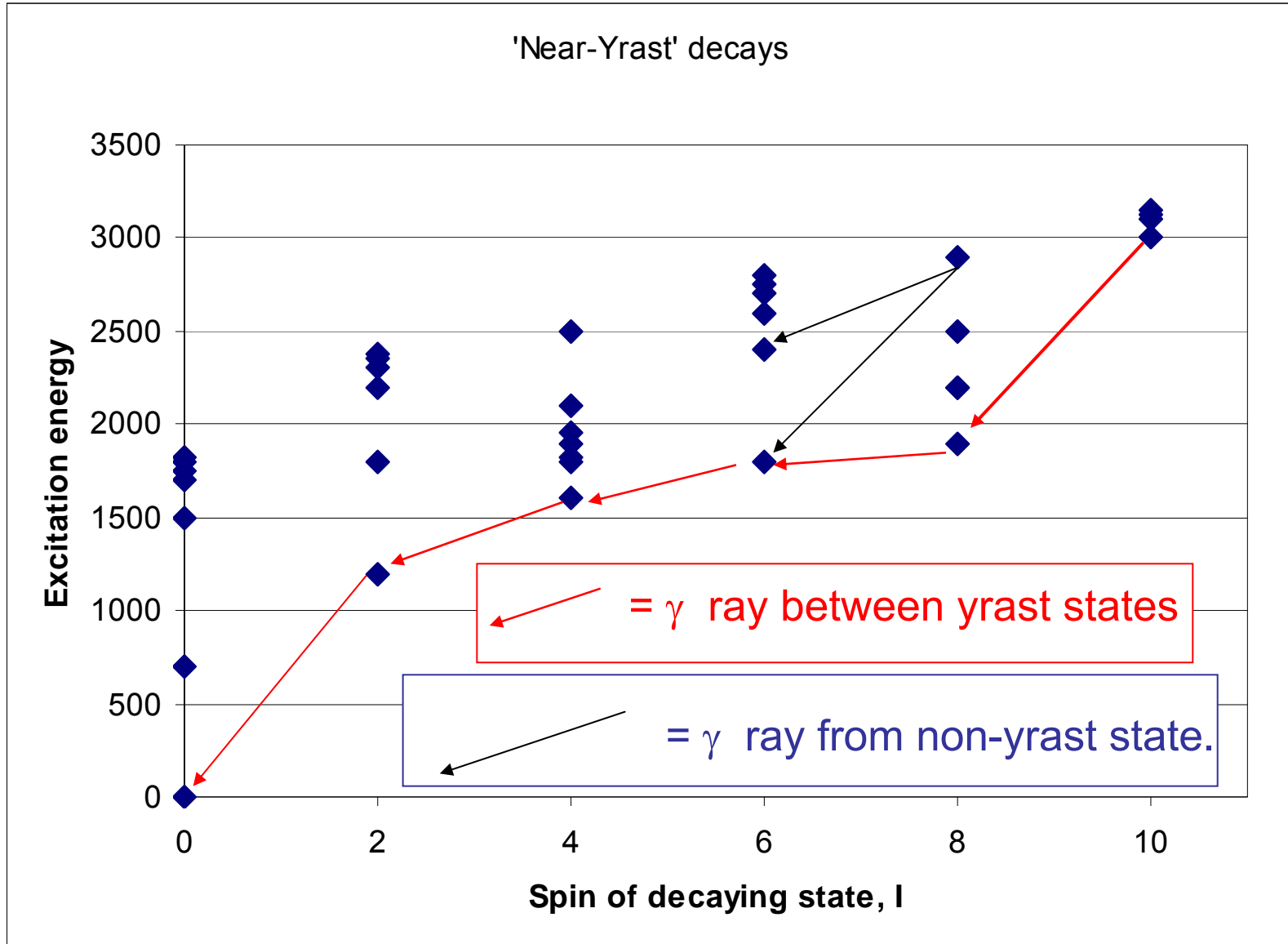
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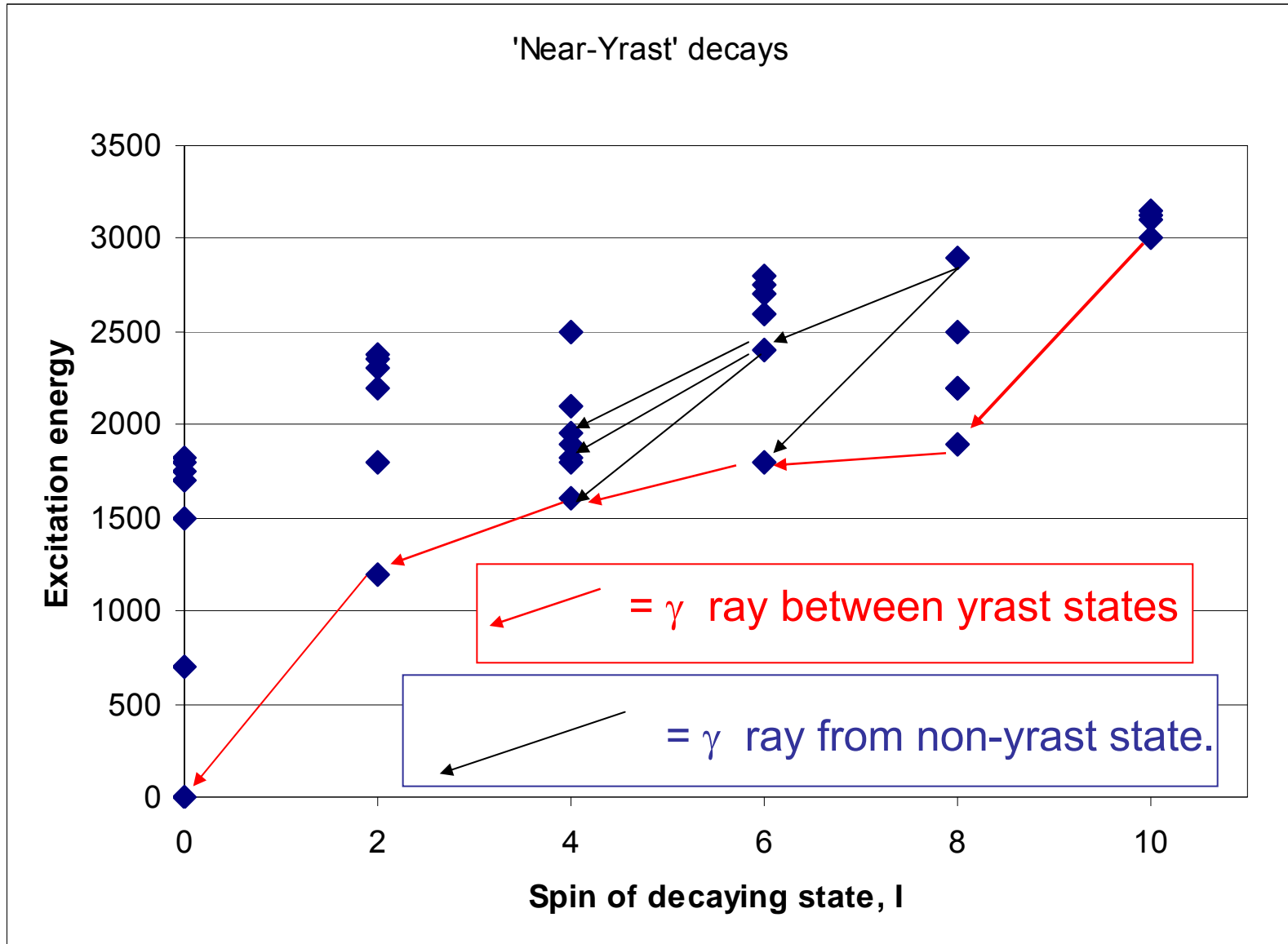
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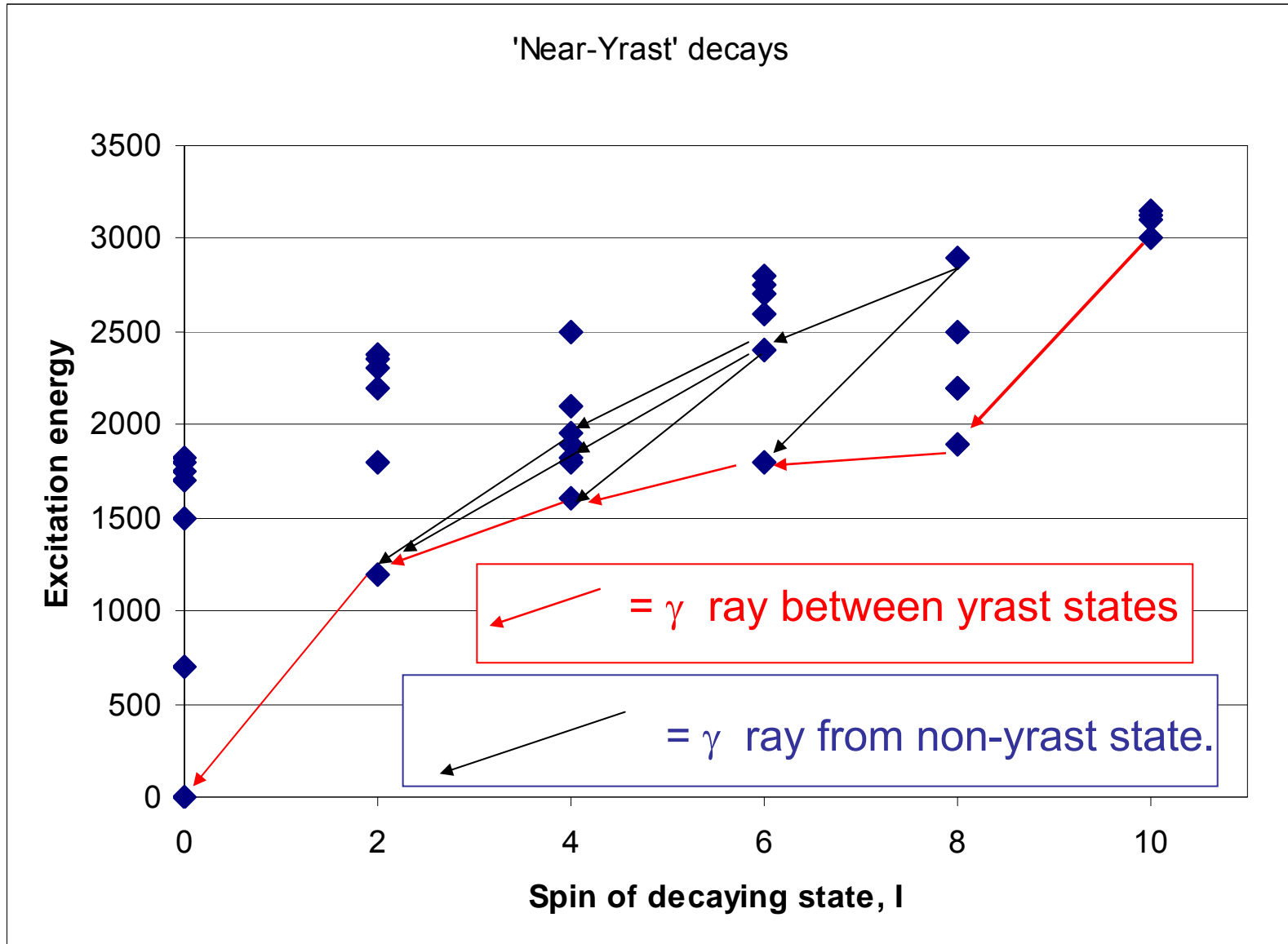
The EM transition rate depends on $E_\gamma^{2\lambda+1}$, (for E2 decays E_γ^5)
 Thus, the highest energy transitions for the lowest λ are usually favoured.
 Non-yrast states decay to yrast ones (unless very different wfs, K-isomers)



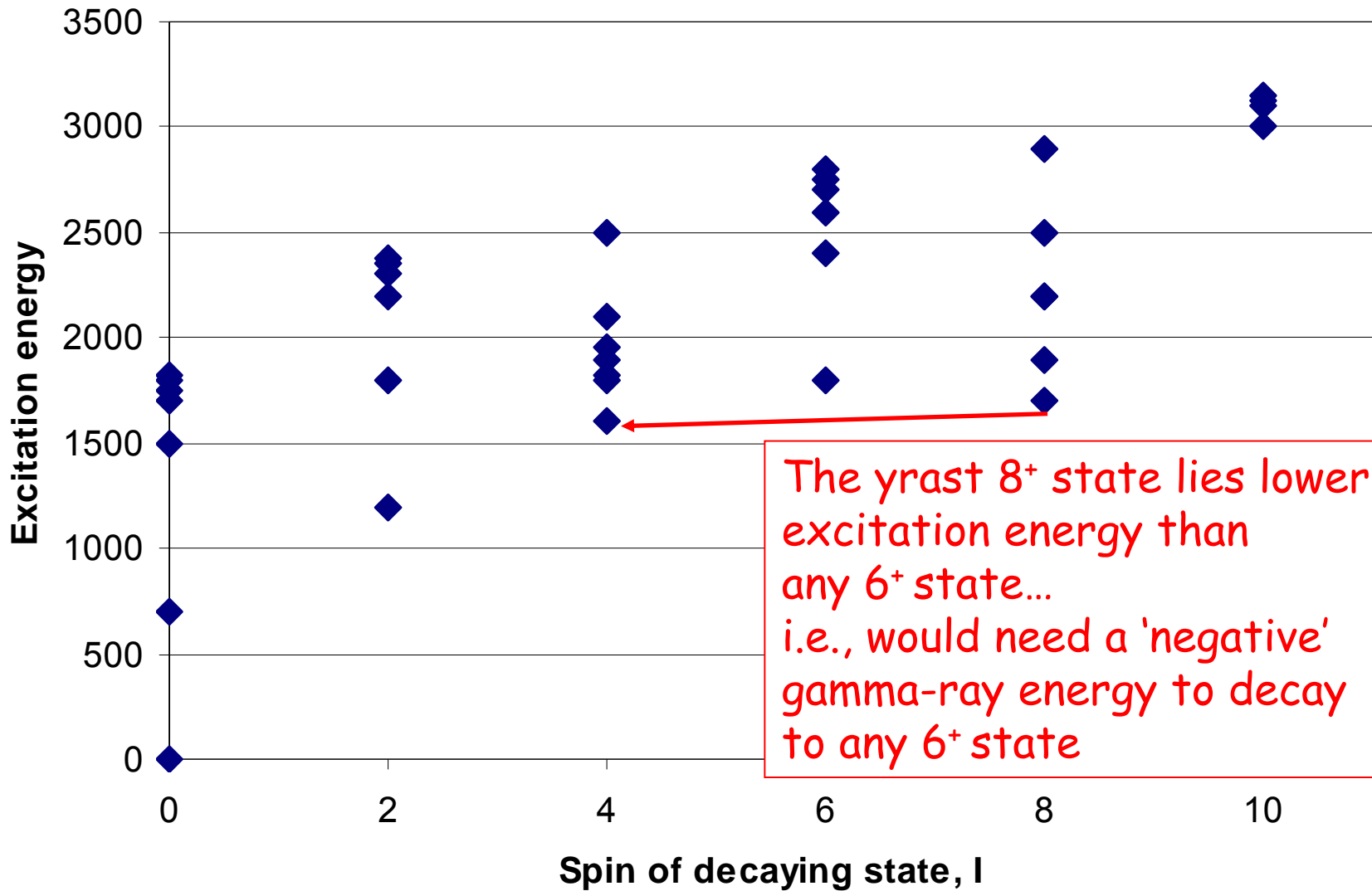
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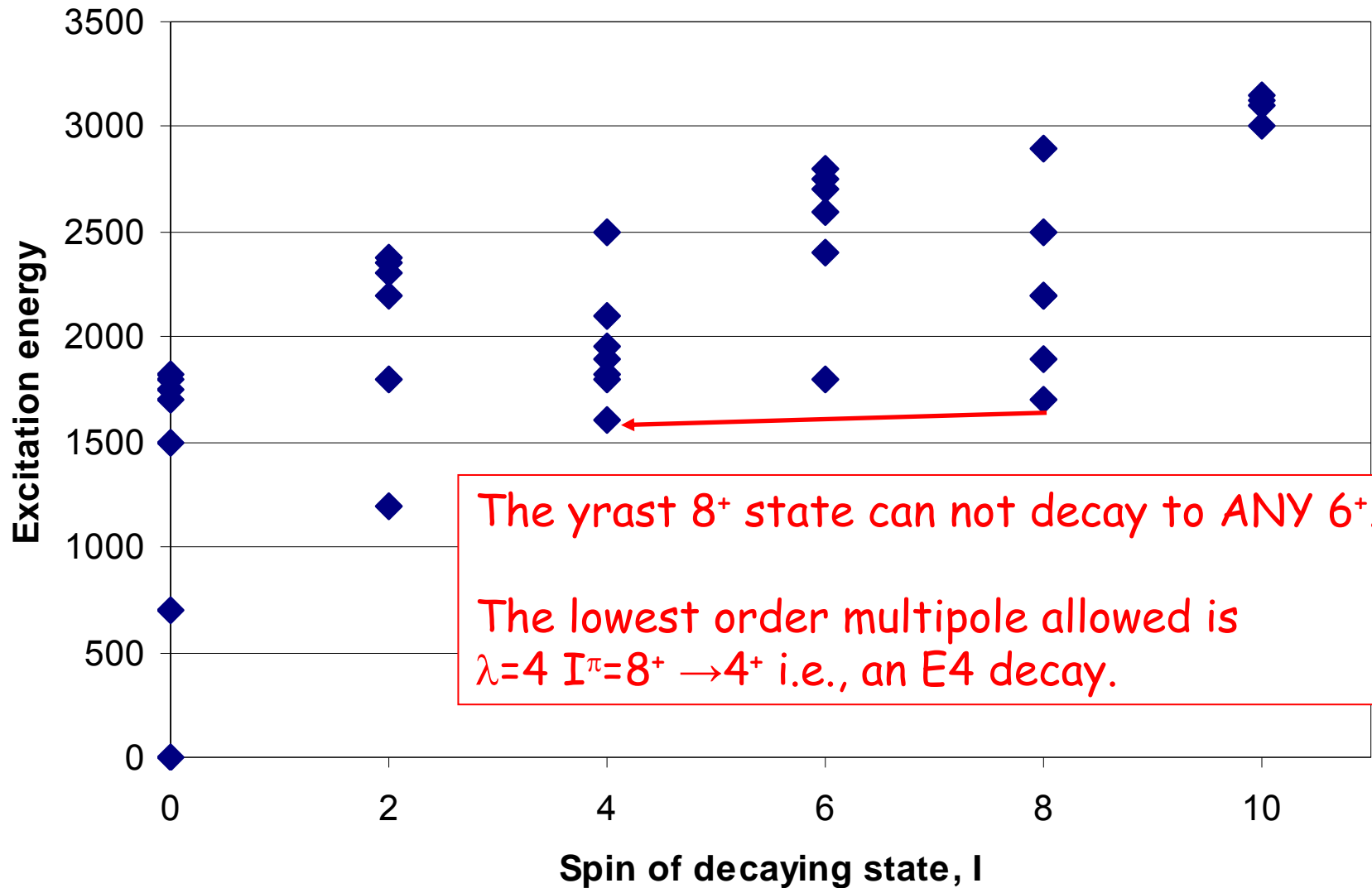


Yrast Traps



The yrast 8^+ state lies lower in excitation energy than any 6^+ state...
i.e., would need a 'negative' gamma-ray energy to decay to any 6^+ state

Yrast Traps



The yrast 8^+ state can not decay to ANY 6^+ .

The lowest order multipole allowed is $\lambda=4$ $I^\pi=8^+ \rightarrow 4^+$ i.e., an E4 decay.

Weisskopf Single Particle Estimates:

These are 'yardstick' estimates for the speed of electromagnetic decays for a given electromagnetic multipole.

They depend on the size of the nucleus (i.e., A) and the energy of the photon (E_γ^{2L+1})

They estimates using of the transition rate for spherically symmetric proton orbitals for nuclei of radius $r=r_0A^{1/3}$.

Multipolarity	Electric Transition Rate (s^{-1})	Magnetic Transition Rate (s^{-1})
1	$1.0 \times 10^{14} A^{2/3} E_\gamma^3$	$3.1 \times 10^{13} E_\gamma^3$
2	$7.3 \times 10^7 A^{4/3} E_\gamma^5$	$2.2 \times 10^7 A^{2/3} E_\gamma^5$
3	$3.4 \times 10^1 A^2 E_\gamma^7$	$1.0 \times 10^1 A^{4/3} E_\gamma^7$
4	$1.1 \times 10^{-5} A^{8/3} E_\gamma^9$	$3.3 \times 10^{-6} A^2 E_\gamma^9$
5	$2.4 \times 10^{-12} A^{10/3} E_\gamma^{11}$	$7.4 \times 10^{-13} A^{8/3} E_\gamma^{11}$

Table 2.3: Single-particle Weisskopf transition probability estimates as a function of the atomic mass measured in atomic mass units A and the γ -ray energy E_γ in MeV.

$$(2.15) \quad B_{sp}(EL) = \frac{1.2^{2L}}{4\pi} \left(\frac{3}{L+3} \right)^2 A^{\frac{2L}{3}} e^2 fm^{2L}$$

$$(2.16) \quad B_{sp}(ML) = \frac{10}{\pi} 1.2^{2L-2} \left(\frac{3}{L+3} \right)^2 A^{2L-2} 2 \left(\frac{e\hbar}{2Mc} \right)^2 fm^{2L-2}$$

Weisskopf Estimates

τ_{sp} for $1Wu$ at $A \sim 100$ and $E_g = 200$ keV

M1 2.2ps	M2 4.1 μ s	M3 36 s	M4 43Ms
E1 5.8 fs	E2 92 ns	E3 0.2s	E4 66Ms

i.e., lowest multipole decays are favoured...but need to conserve angular momentum so need at least $\lambda = I_i - I_f$ for decay to be allowed.

Note, for low E_γ and high- l , internal conversion also competes/dominates.

EM Selection Rules and their Effects on Decays

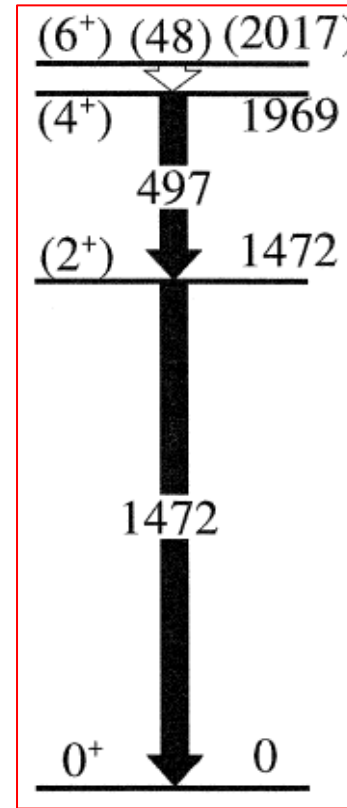
- Allows decays have:

$$|I_i - I_f| \leq \lambda \leq |I_i + I_f|$$

e.g., decays from $I^\pi = 6^+$ to $I^\pi = 4^+$ are allowed to proceed with photons carrying angular momentum of $\lambda = 2, 3, 4, 5, 6, 7, 8, 9$ and $10 \hbar$.

Need also to conserve parity between initial and final states, thus, here the transition can not change the parity.

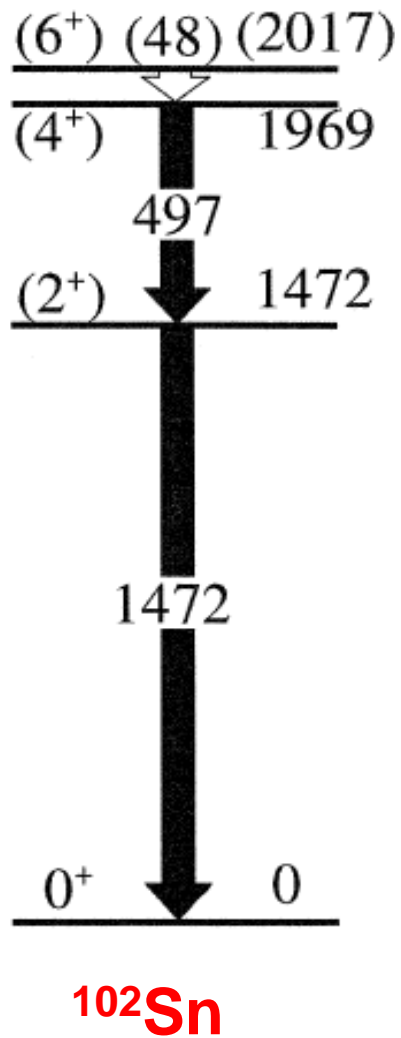
This adds a further restriction : Allowed decays now restricted to E2, E4, E6, E8 and E10 ; and M3, M5, M7, M9



e.g., $^{102}\text{Sn}_{52}$

Why do we only observe the E2 decays ?

Are the other allowed decays present ?



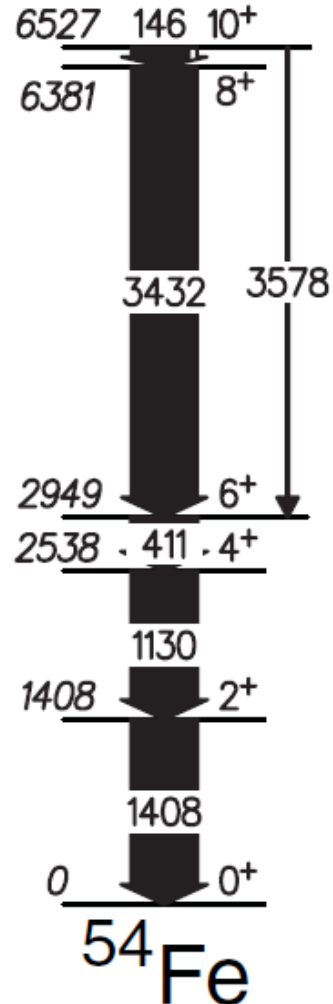
E_γ	E2 (1Wu)	M3 (1Wu)	E4 (1Wu)
48 ($6^+ \rightarrow 4^+$)	112 μs	782,822 s	2.5E+14s
555 ($6^+ \rightarrow 2^+$)			66,912s

497 ($4^+ \rightarrow 2^+$)	0.9ns	61ms	180,692s
1969 ($4^+ \rightarrow 2^+$)			751ms

Conclusion, in general see a cascade of (stretched) E2 decays in near-magic even-even nuclei.

What about core breaking?

We can have cases where low-energy (~ 100 keV) E2 decays competing with high-energy (~ 4 MeV) E4 transitions across magic shell closures, e.g. $^{54}\text{Fe}_{28}$.



Z=26; N=28 case.

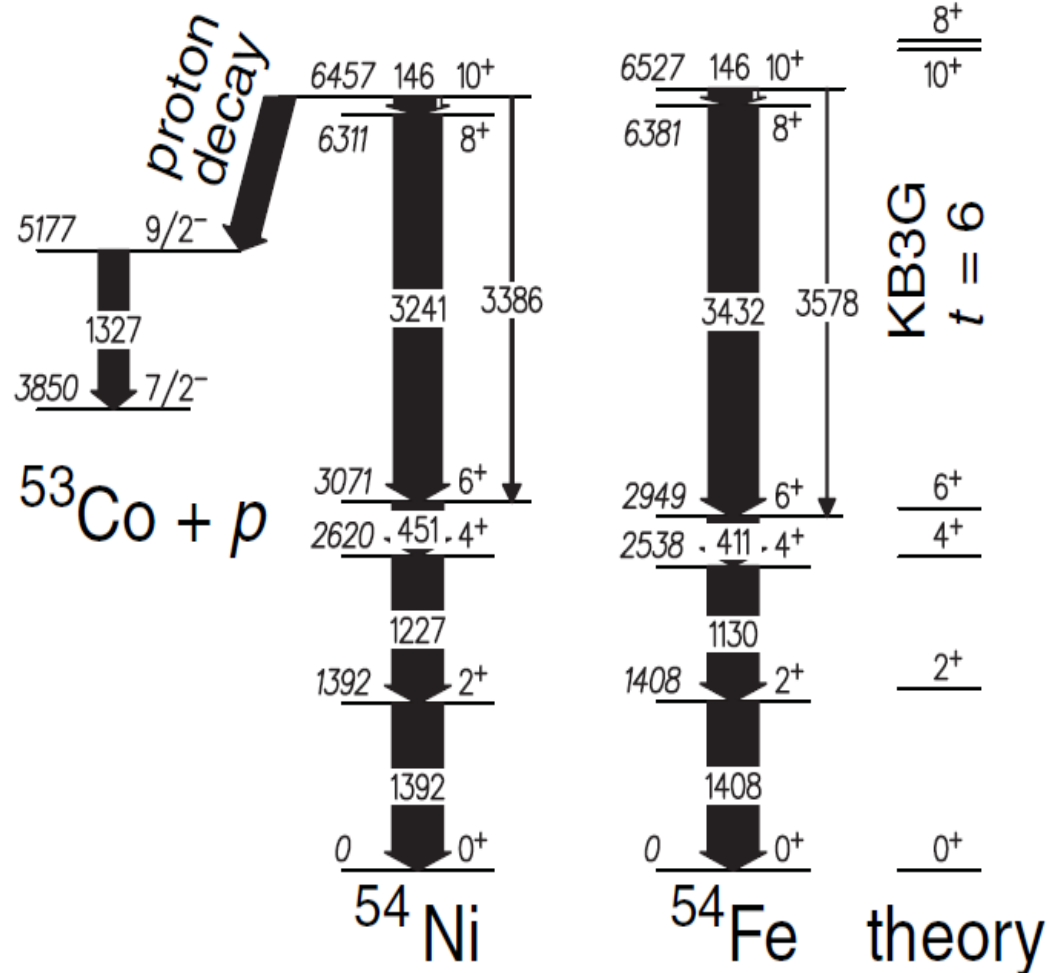
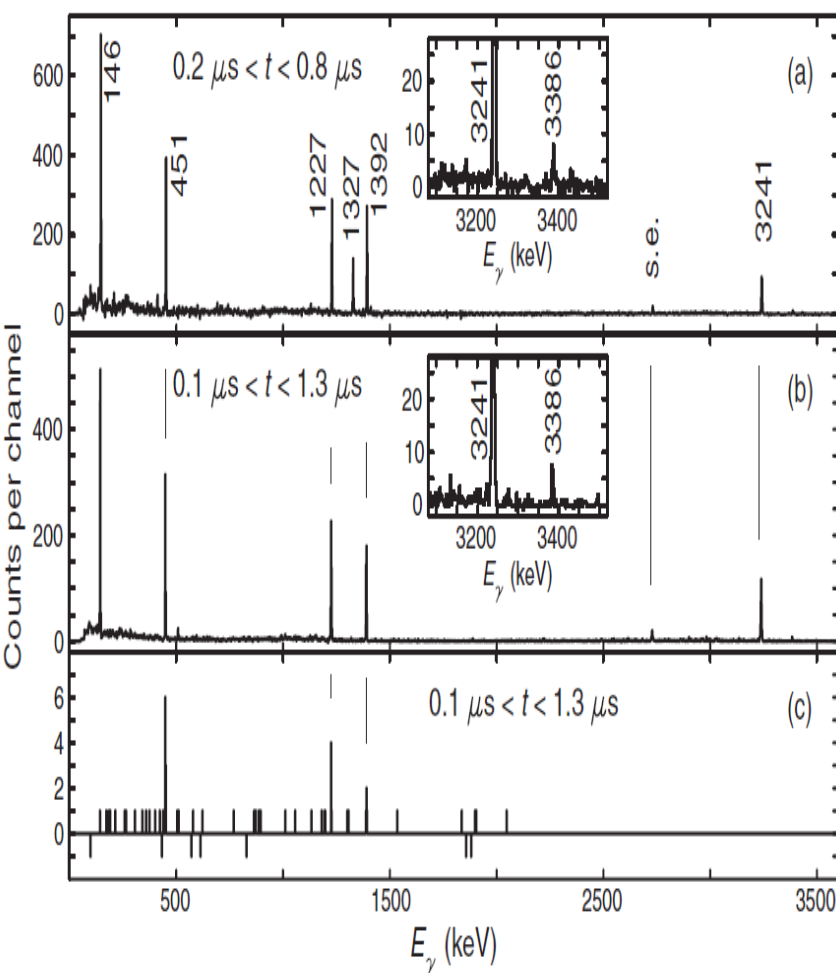
- 2 proton holes in $f_{7/2}$ shell.
- Maximum spin in simple valence space is $I^\pi=6^+$.
- i.e., $(\pi f_{7/2})^{-2}$ configuration coupled to $I^\pi=6^+$

Additional spin requires exciting (pairs) of nucleons across the N or Z=28 shell closures into the $f_{5/2}$ shell.

E_γ	E2 (1Wu)	M3 (1Wu)	E4 (1Wu)
146 keV ($10^+ \rightarrow 8^+$)	1.01 μs	613s	20.9E+6s
3578 keV ($10^+ \rightarrow 6^+$)			6.5ms

Isospin symmetry and proton decay: Identification of the 10^+ isomer in ^{54}Ni

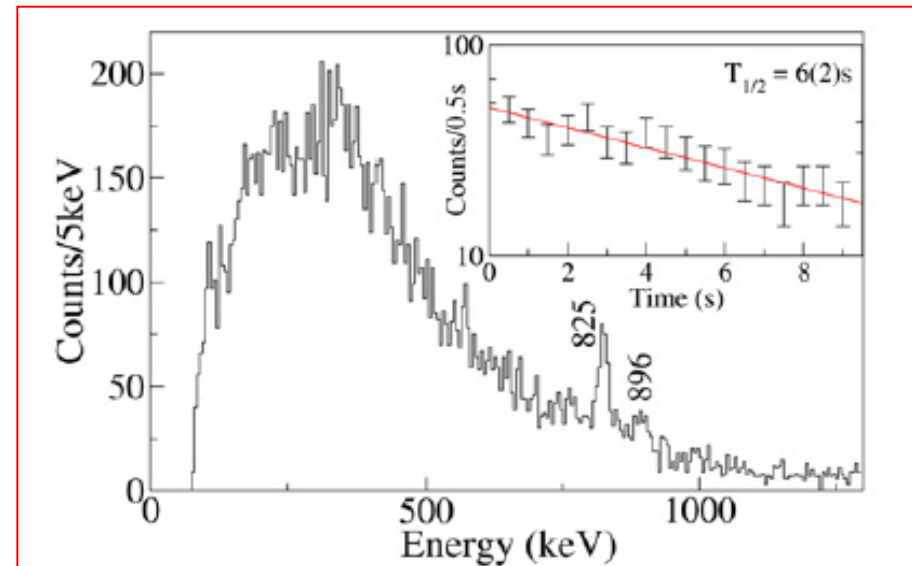
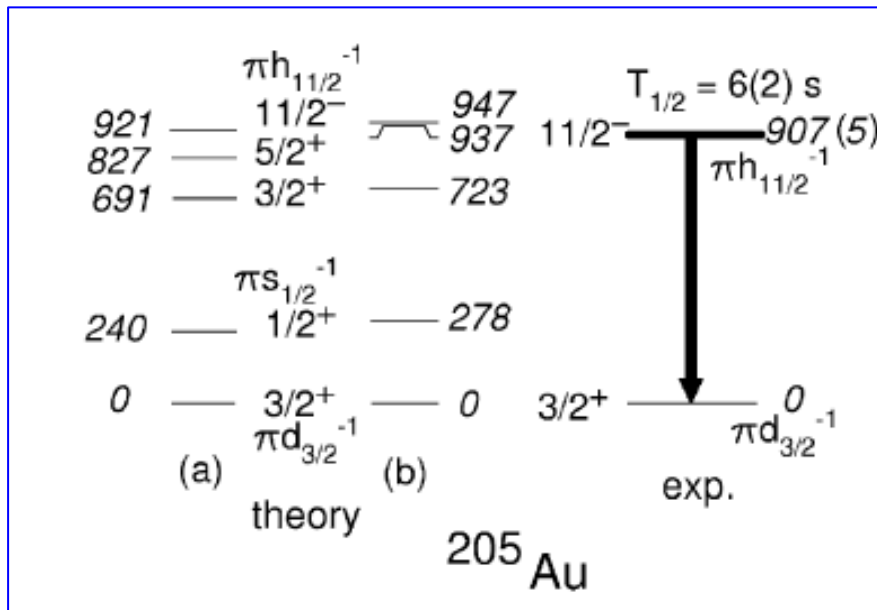
D. Rudolph,¹ R. Hoischen,^{1,2} M. Hellström,¹ S. Pietri,³ Zs. Podolyák,³ P. H. Regan,³ A. B. Garnsworthy,^{3,4} S. J. Steer,³ F. Becker,^{2,*} P. Bednarczyk,^{2,5} L. Cáceres,^{2,6} P. Doornenbal,^{2,7,†} J. Gerl,² M. Górska,² J. Grębosz,^{2,5} I. Kojouharov,² N. Kurz,² W. Prokopowicz,^{2,5} H. Schaffner,² H. J. Wollersheim,² L.-L. Andersson,¹ L. Atanasova,⁸ D. L. Balabanski,^{8,9} M. A. Bentley,¹⁰ A. Blazhev,⁷ C. Brandau,^{2,3} J. R. Brown,¹⁰ C. Fahlander,¹ E. K. Johansson,¹ A. Jungclaus,⁶ and S. M. Lenzi¹¹

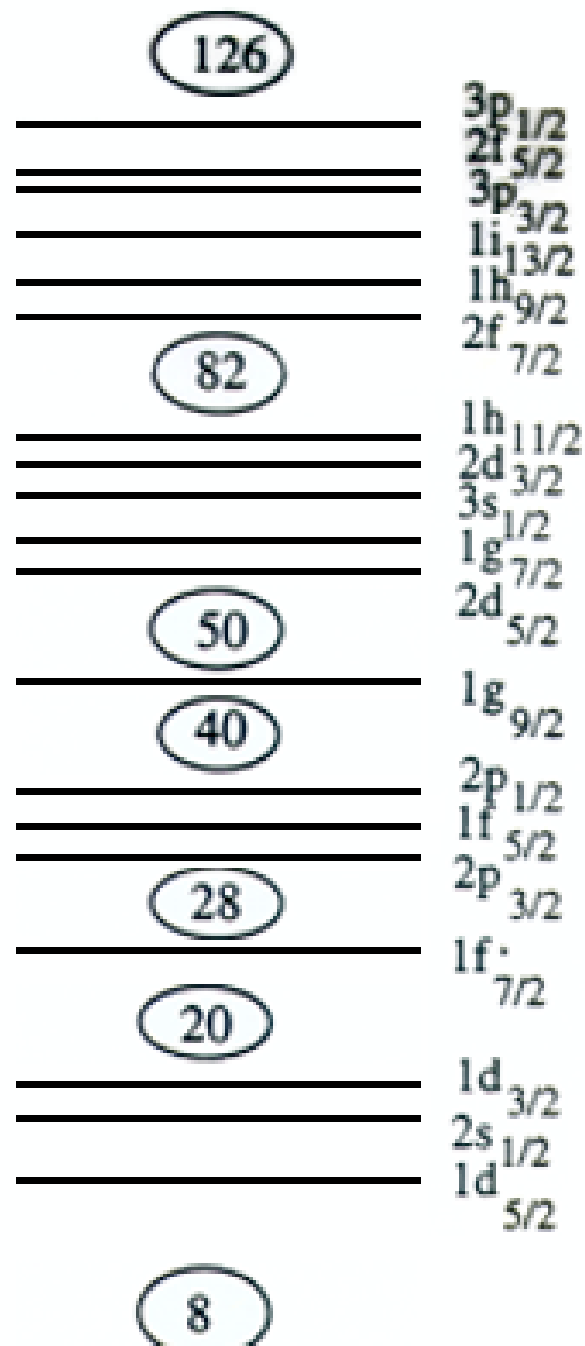
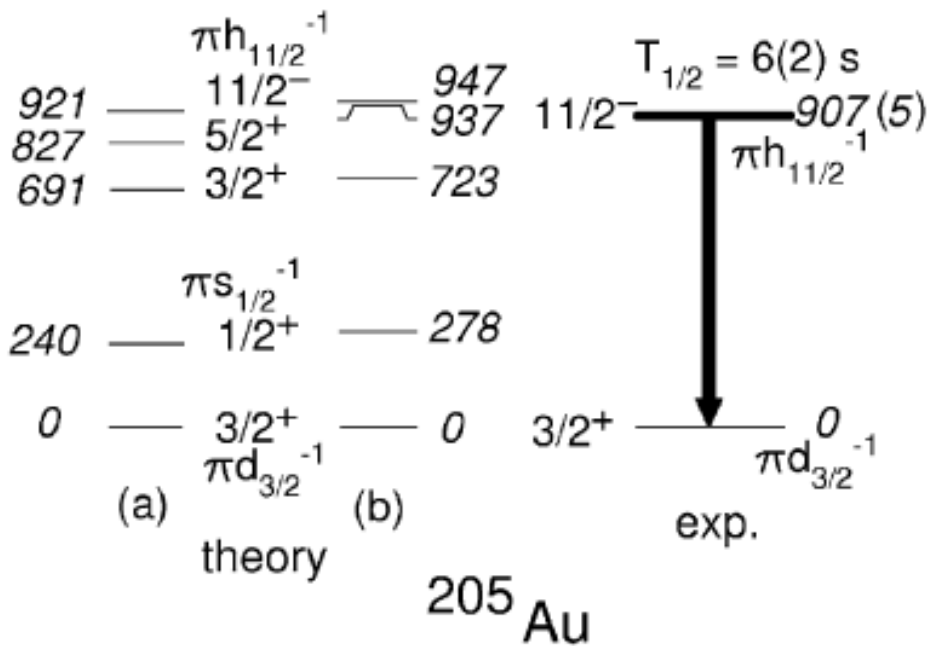


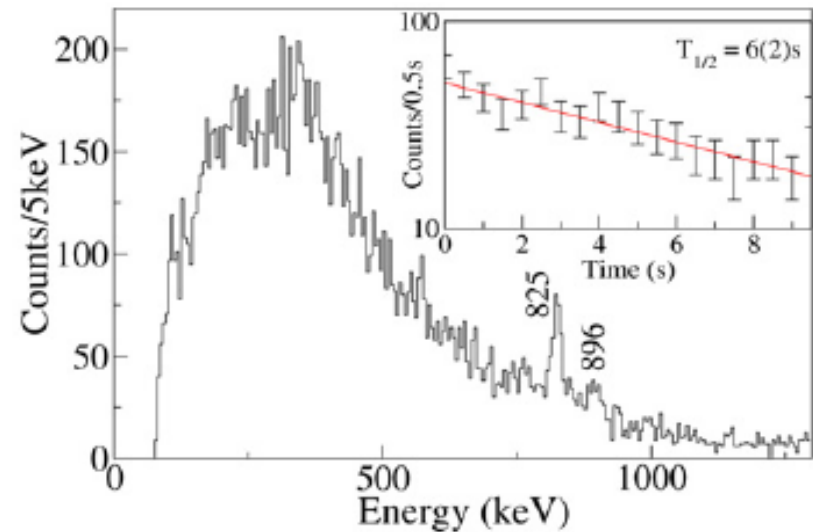
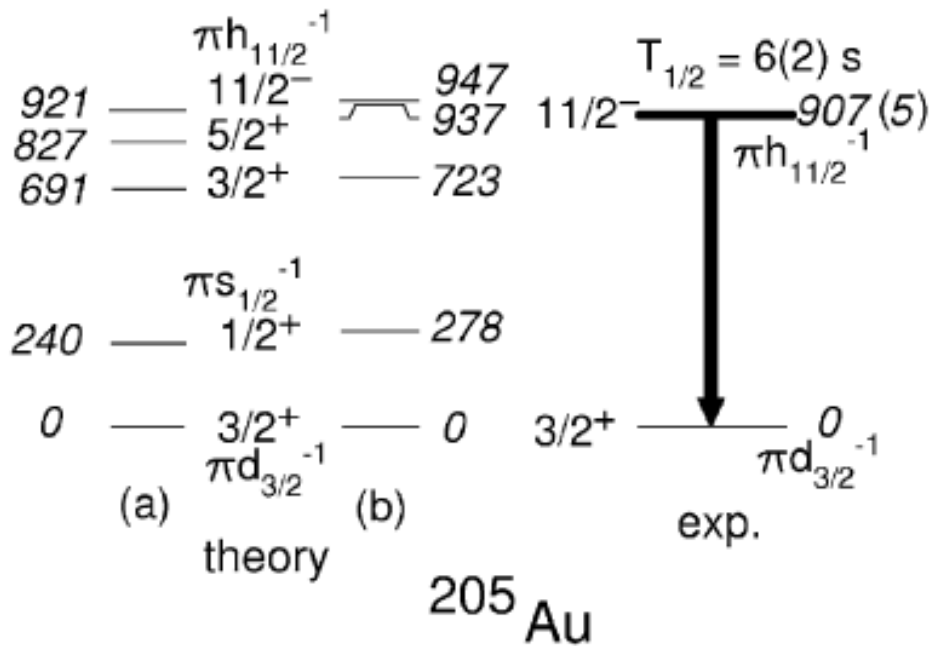


Proton–hole excitation in the closed shell nucleus ^{205}Au

Zs. Podolyák^{a,*}, G.F. Farrelly^a, P.H. Regan^a, A.B. Garnsworthy^a, S.J. Steer^a, M. Górska^b, J. Benlliure^c, E. Casarejos^c, S. Pietri^a, J. Gerl^b, H.J. Wollersheim^b, R. Kumar^d, F. Molina^e, A. Algora^{e,f}, N. Alkhomashi^a, G. Benzoni^g, A. Blazhev^h, P. Boutachkov^b, A.M. Bruceⁱ, L. Caceres^{b,j}, I.J. Cullen^a, A.M. Denis Bacelarⁱ, P. Doornenbal^b, M.E. Estevez^c, Y. Fujita^k, W. Gelletly^a, H. Geissel^b, H. Grawe^b, J. Grębosz^{b,l}, R. Hoischen^{m,b}, I. Kojouharov^b, S. Lalkovskiⁱ, Z. Liu^a, K.H. Maier^{n,1}, C. Mihai^o, D. Mücher^h, B. Rubio^e, H. Schaffner^b, A. Tamii^k, S. Tashenov^b, J.J. Valiente-Dobón^p, P.M. Walker^a, P.J. Woods^q







$N=126$; $Z=79$. Odd, single proton transition;
 $h_{11/2} \rightarrow d_{3/2}$ state (holes in $Z=82$ shell).

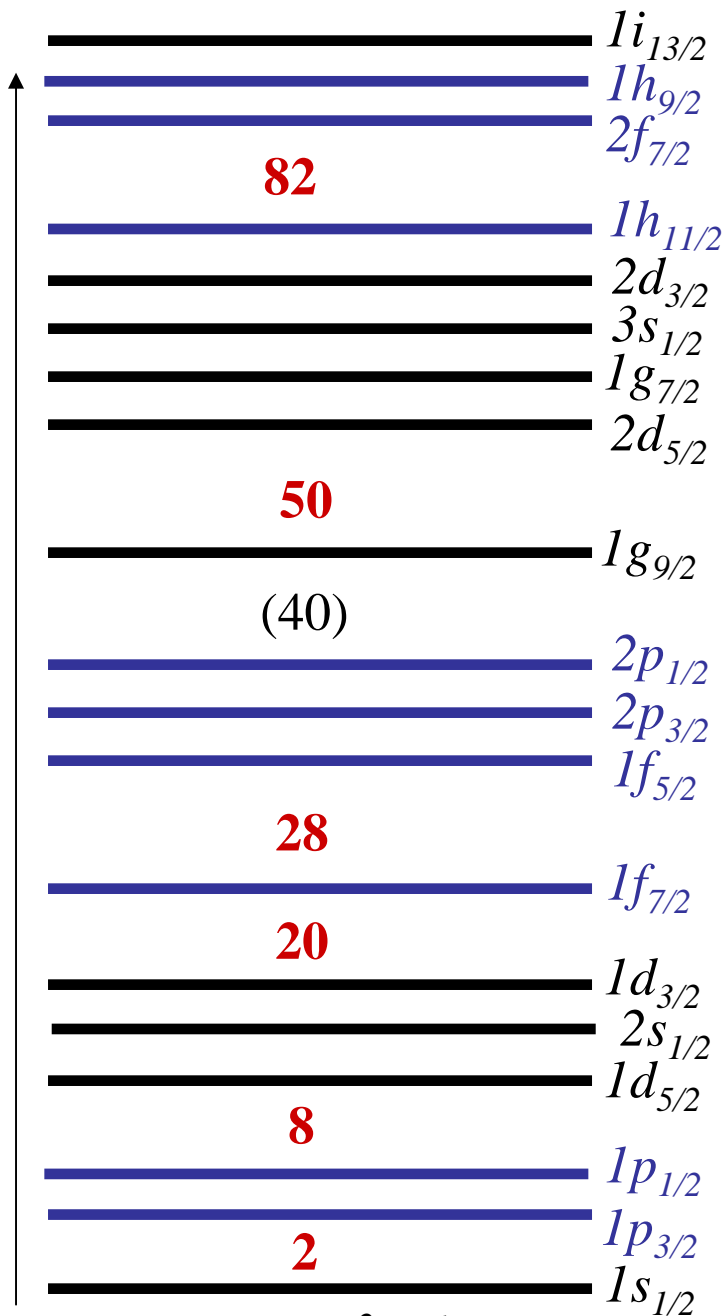
Selection rule says lowest multipole decay allowed is $\lambda=11/2 - 3/2 = 4$.
 Change of parity means lowest must transition be M4.

1Wu 907 keV M4 in ^{205}Au has $T_{1/2} = 8 \text{ secs}$.

'Pure' single particle (proton) transition from 11/2⁻ state to 3/2⁺ state.

(note, decay here is observed following INTERNAL CONVERSION).

These competing decays (to gamma emission) are often observed in isomeric decays



$$V = \text{SHO} + l^2 + l.s.$$

ASIDE:

Why are E1 s isomeric?

E1s often observed with decay probabilities
Of $10^{-5} \rightarrow 10^{-9}$ Wu

E1 single particle decays need to proceed
between orbitals which have $\Delta l = 1$ and
change parity, e.g.,

$f_{7/2}$ and $d_{5/2}$

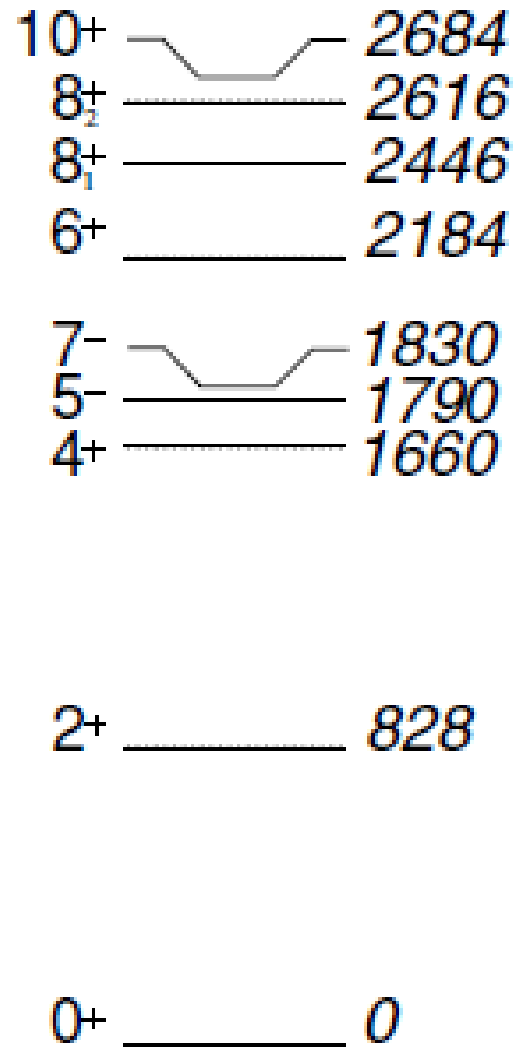
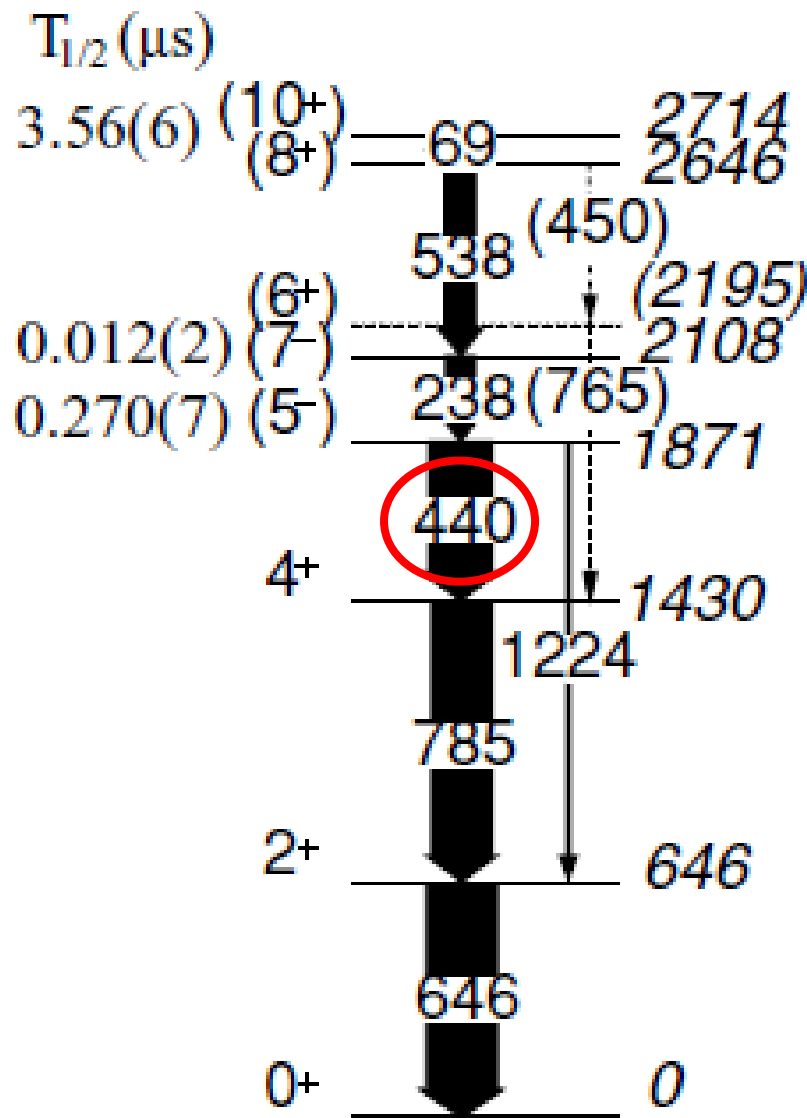
or $g_{9/2}$ and $f_{7/2}$

or $h_{11/2}$ and $g_{9/2}$

or $i_{13/2}$ and $h_{11/2}$

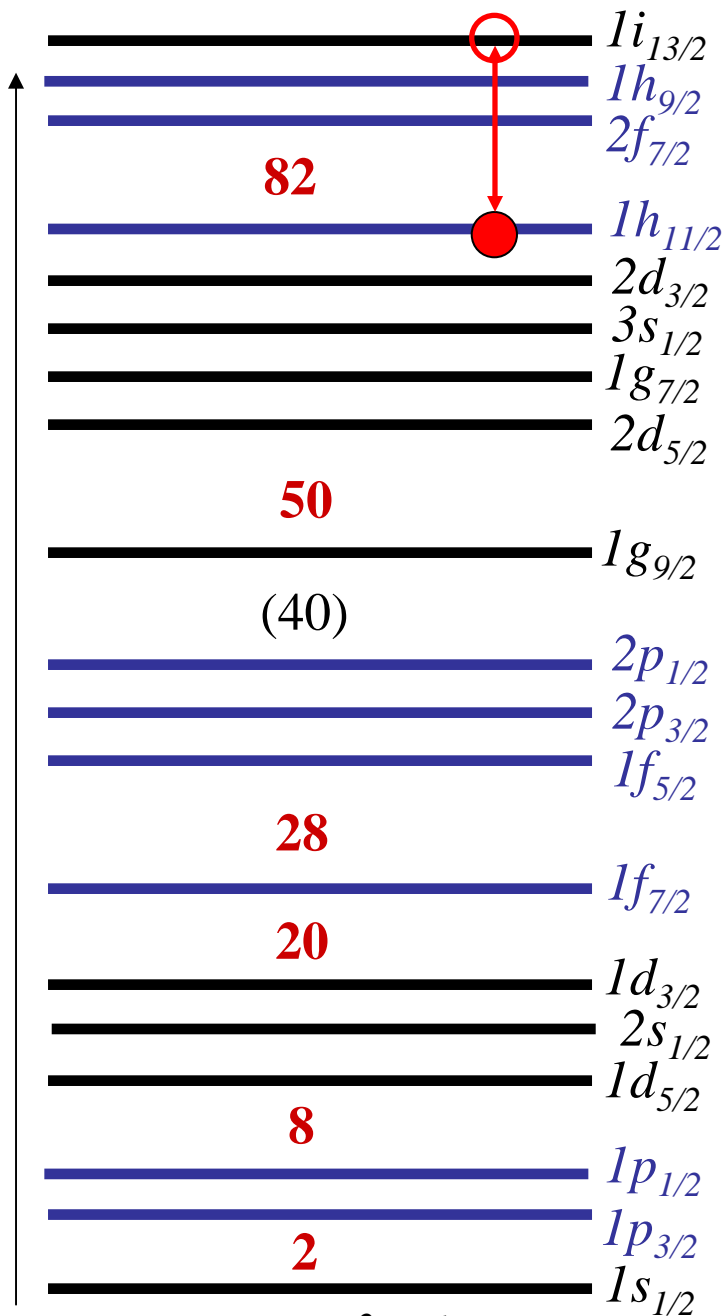
or $p_{3/2}$ and $d_{5/2}$

BUT these orbitals are along way from each
other in terms of energy in the mean-field
single particle spectrum.



e.g., ^{128}Cd , isomeric 440 keV E1 decay.
 1 Wu 440 keV E1 should have $\sim 4 \times 10^{-15}\text{s}$;
 Actually has $\sim 300\text{ ns}$ (i.e hindered by $\sim 10^8$!!)

L. Cáceres,^{1,2,*} M. Górska,¹
 PHYSICAL REVIEW C 79, 011301(R) (2009)



$$V = \text{SHO} + l^2 + l.s.$$

Why are E1 s isomeric?

E1 single particle decays need to proceed between orbitals which have $\Delta L=1$ and change parity, e.g.,

What about typical 2-particle configs.
e.g.,

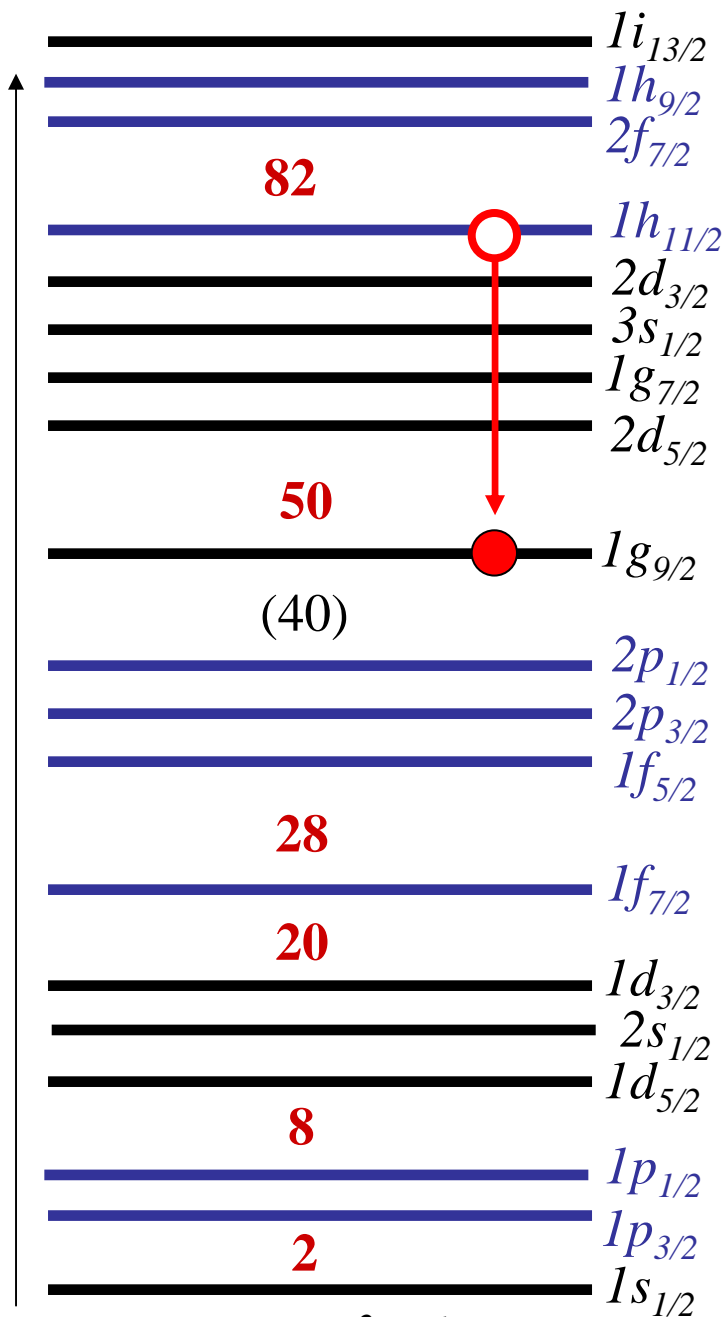
$$I^\pi=5^- \text{ from mostly } (h_{11/2})^{-1} \times (s_{1/2})^{-1}$$

$$I^\pi=4^+ \text{ from mostly } (d_{3/2})^{-1} \times (s_{1/2})^{-1}$$

No E1 'allowed' between such orbitals.

E1 occur due to (very) small fractions of the wavefunction from orbitals in higher shells.

Small overlap wavefunction in multipole Matrix element causes 'slow' E1s



$$V = \text{SHO} + l^2 + l.s.$$

Why are E1 s isomeric?

E1s often observed with decay probabilities Of $10^{-5} \rightarrow 10^{-8}$ Wu

E1 single particle decays need to proceed between orbitals which have $\Delta L=1$ and change parity, e.g.,

$f_{7/2}$ and $d_{5/2}$

or $g_{9/2}$ and $f_{7/2}$

or $h_{11/2}$ and $g_{9/2}$

or $p_{3/2}$ and $d_{5/2}$

BUT these orbitals are along way from each other in terms of energy in the mean-field single particle spectrum.

Measurements of EM Transition Rates

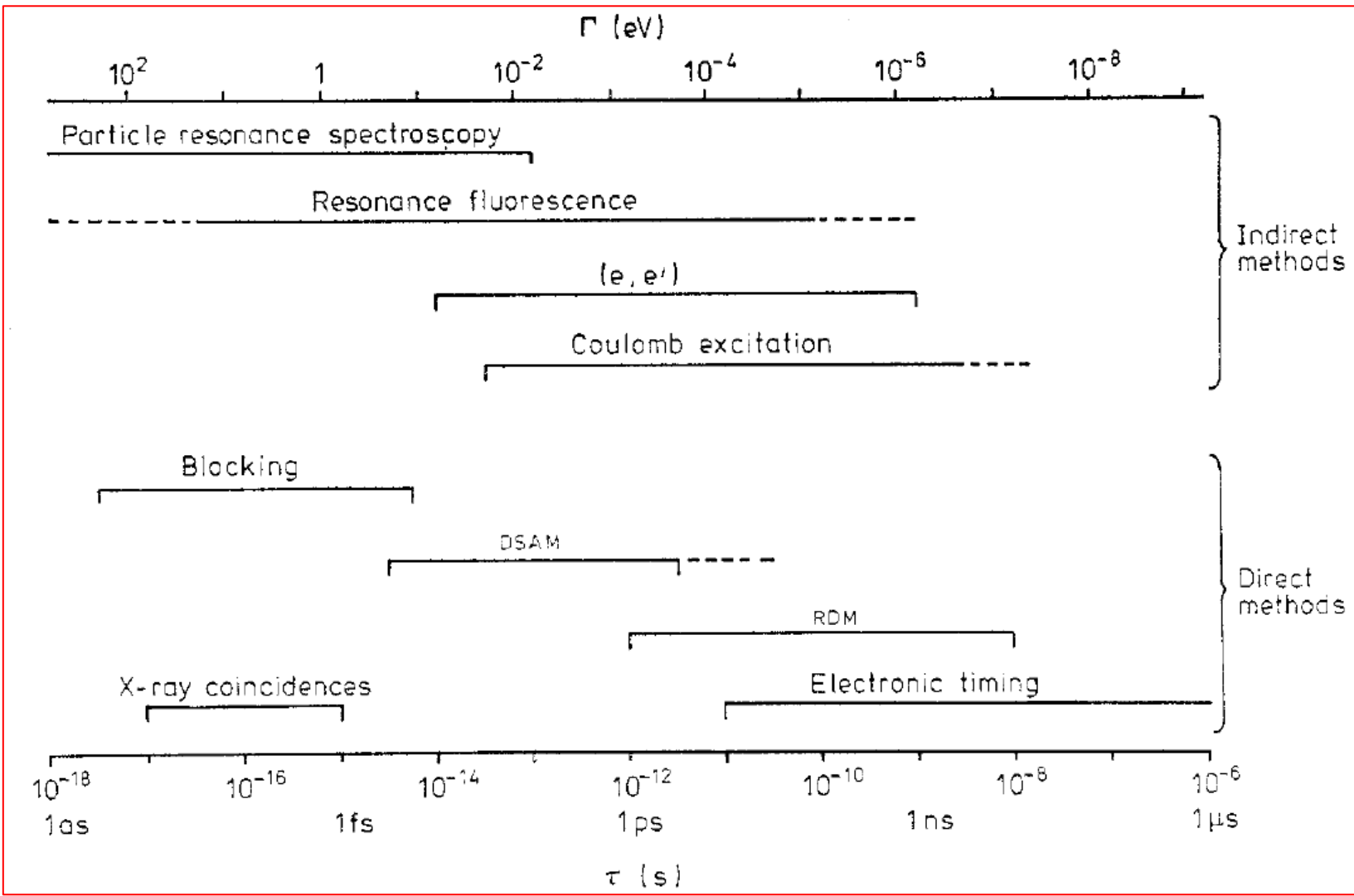
$$\Gamma \tau = \hbar.$$

The measurement of the lifetimes of excited nuclear states

$$\Gamma \propto |\langle \psi_f | \hat{O}_{\text{decay}} | \psi_i \rangle|^2$$

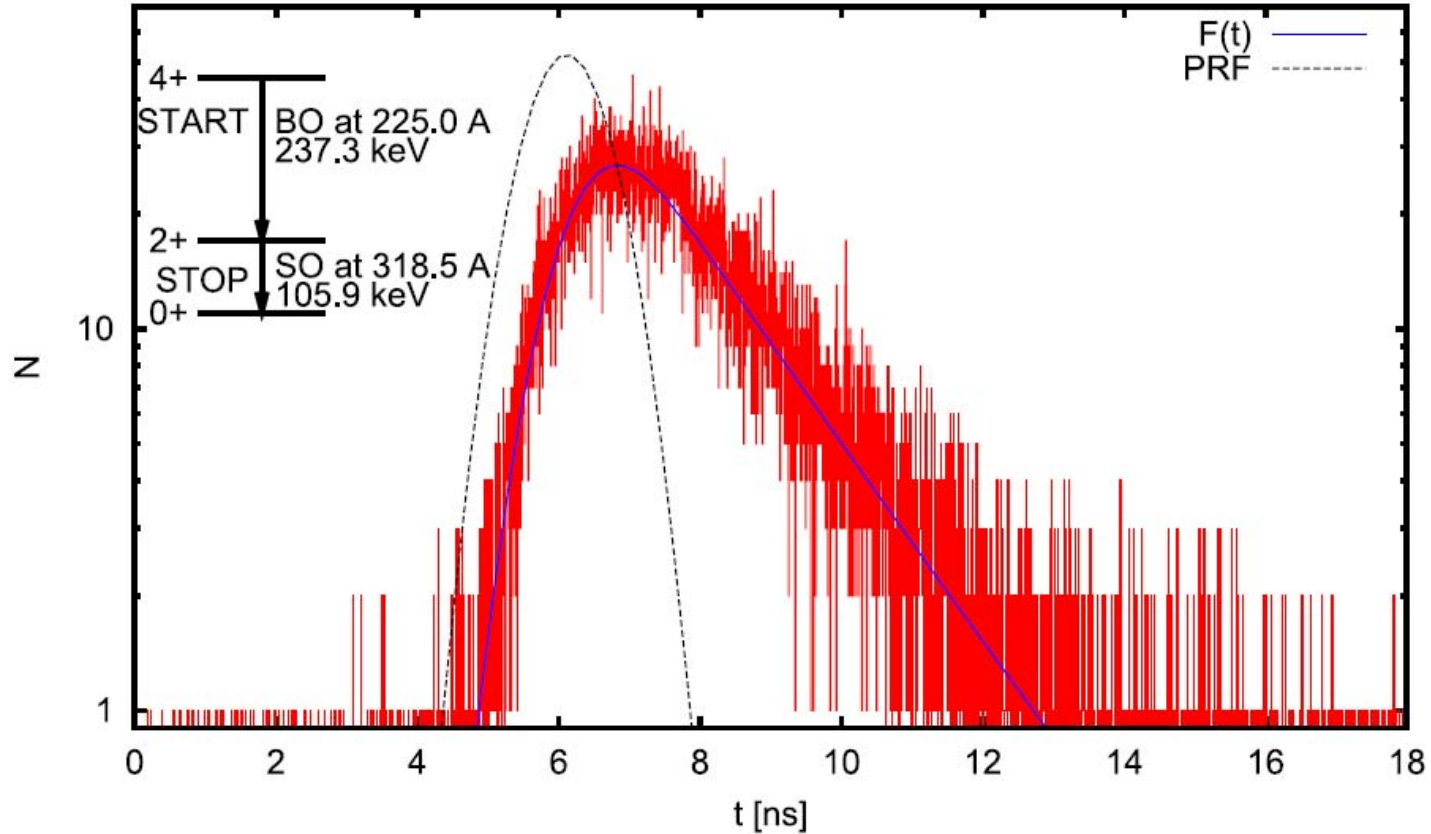
PJ NOLAN† and JF SHARPEY-SCHAFFER‡

$$\Gamma_{\text{total}} = \sum_j \Gamma_j.$$



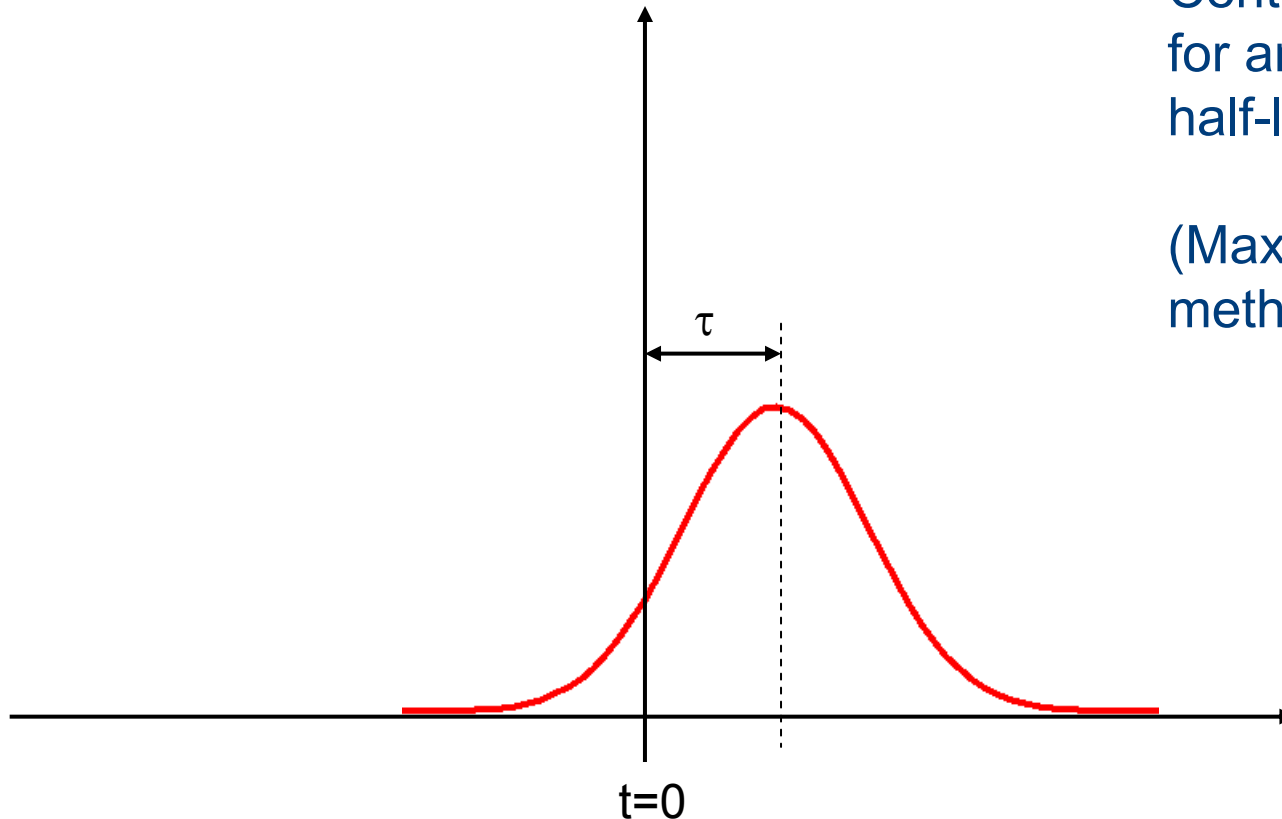
Fast-timing Techniques

M. Rudigier et al. / Nuclear Physics A 847 (2010) 89–100



Gaussian-exponential convolution to account for timing resolution

Fast-timing Techniques

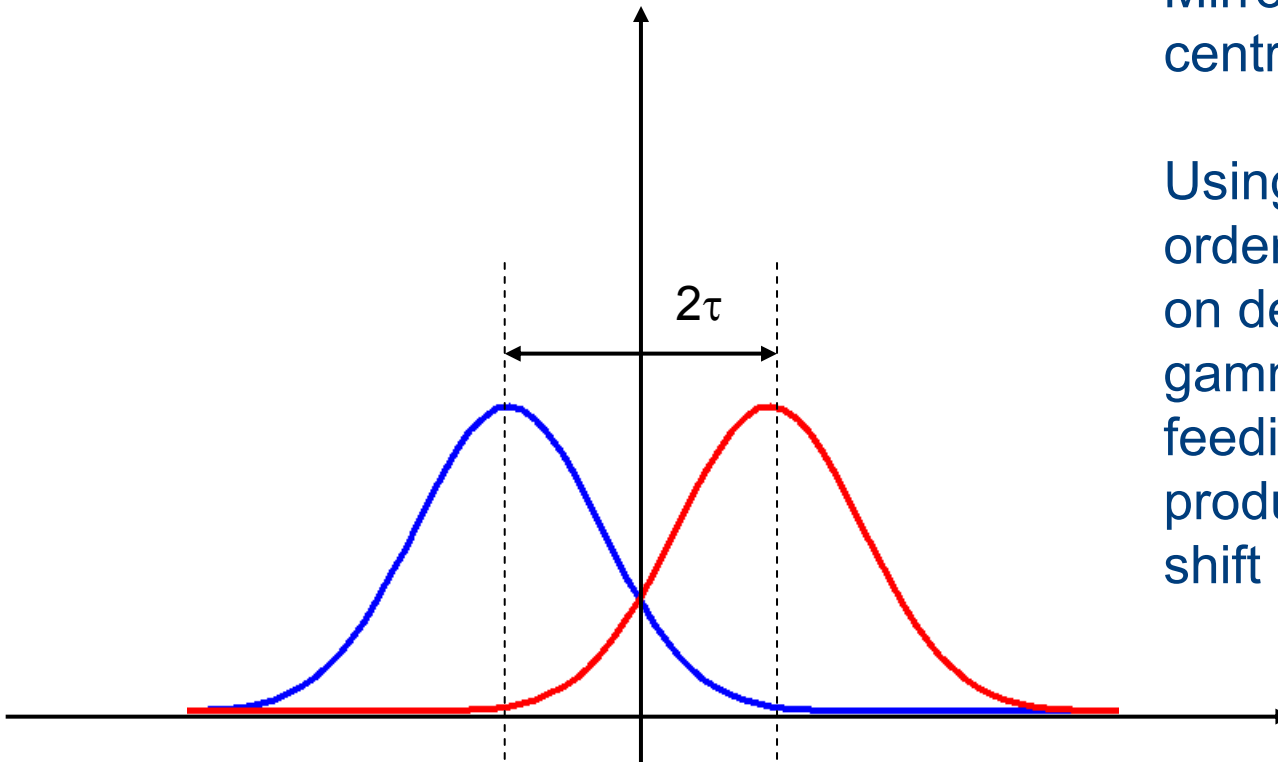


Centroid shift method
for an analysis of short
half-lives

(Maximum likelihood
method)

Difference between the centroid of observed time spectrum and the prompt response give lifetime, τ

Fast-timing Techniques

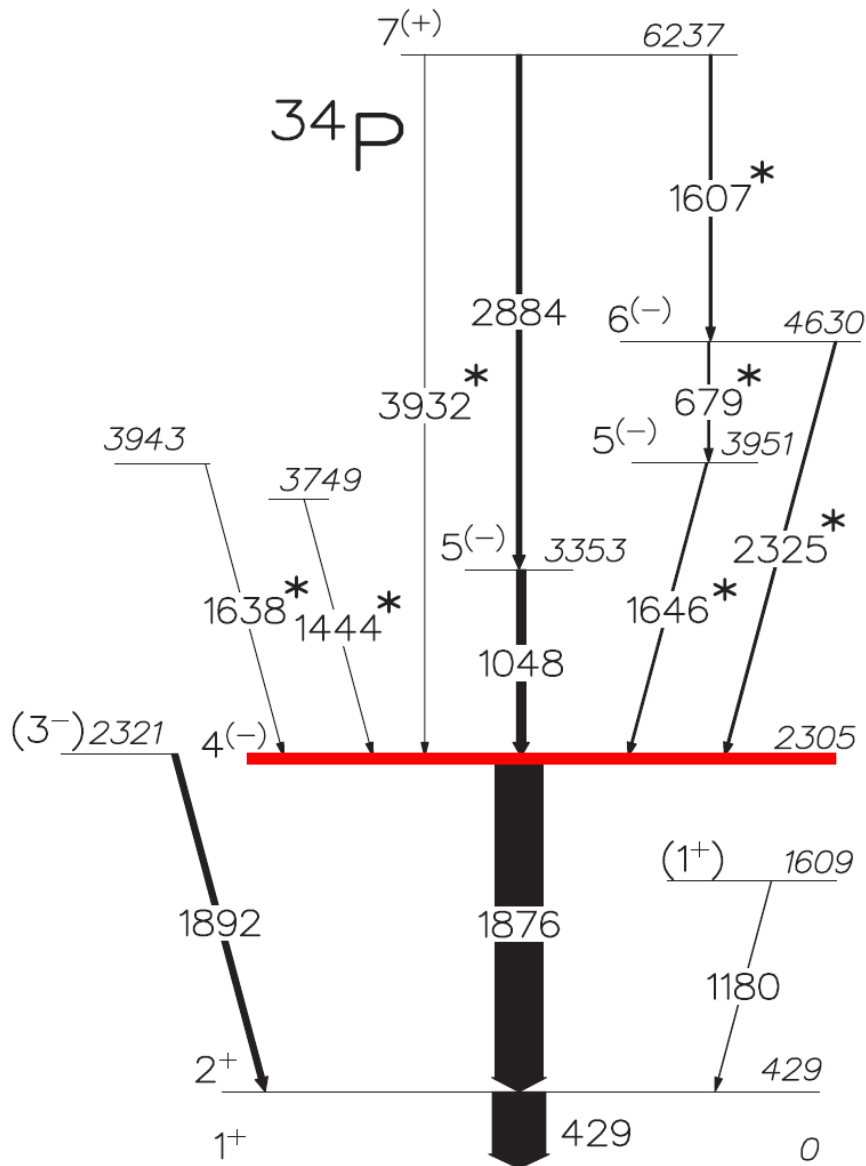


Mirror-symmetric
centroid shift method.

Using reversed gate
order (e.g. start TAC
on depopulating
gamma, stop on
feeding gamma)
produces opposite
shift

Removes the need to know where the prompt distribution is and other problems to do with the prompt response of the detectors

An example, 'fast-timing' in ^{34}P .



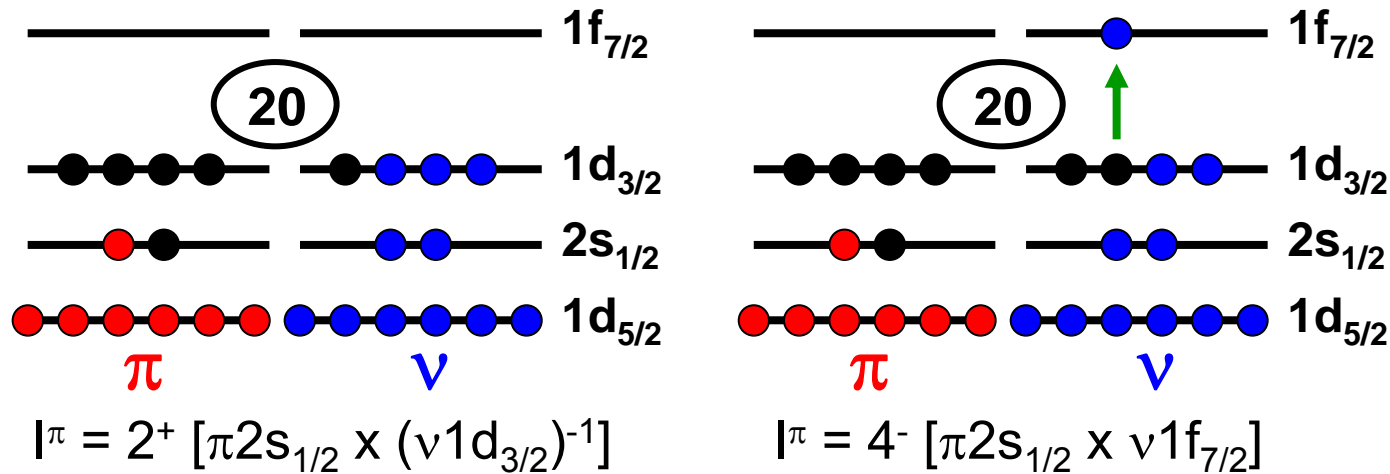
P. C. BENDER *et al.* PHYSICAL REVIEW C 80, 014302 (2009)

R. CHAKRABARTI *et al.* PHYSICAL REVIEW C 80, 034326 (2009)

- Recent study of ^{34}P identified low-lying $I^\pi=4^-$ state at $E=2305$ keV.
- $I^\pi=4^- \rightarrow 2^+$ transition can proceed by M2 and/or E3.
- Aim of experiment is to measure precision lifetime for 2305 keV state and obtain $B(M2)$ and $B(E3)$ values.
- Previous studies limit half-life to $0.3 \text{ ns} < t_{1/2} < 2.5 \text{ ns}$

Physics...which orbitals are involved?

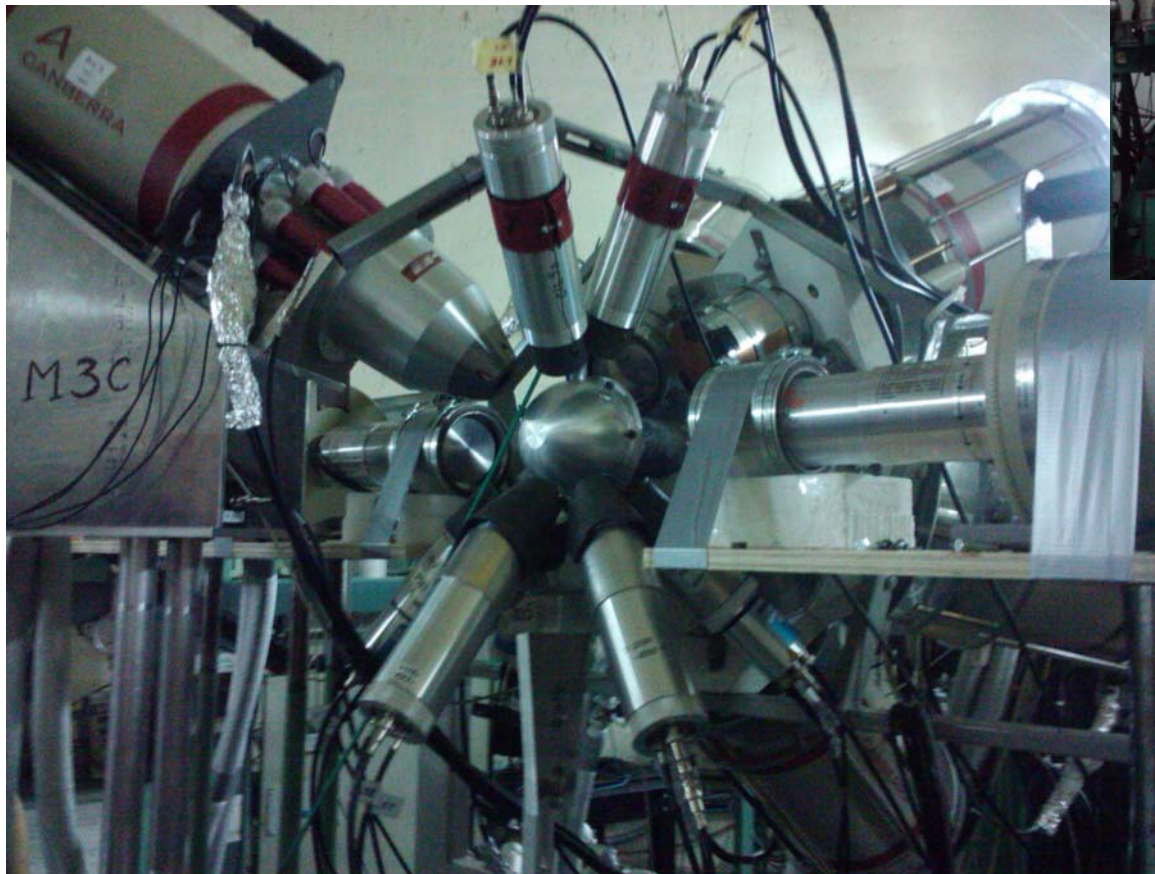
- Theoretical (shell model) predictions suggest 2^+ state based primarily on $[\pi 2s_{1/2} \times (\nu 1d_{3/2})^{-1}]$ configuration and 4^- state based primarily on $[\pi 2s_{1/2} \times \nu 1f_{7/2}]$ configuration.
- Thus expect transition to go mainly via $f_{7/2} \rightarrow d_{3/2}$, M2 transition.
- Different admixtures in 2^+ and 4^- states may also allow some E3 components (e.g., from, $f_{7/2} \rightarrow s_{1/2}$) in the decay.



Experiment to Measure Yrast 4- Lifetime in ^{34}P

$^{18}\text{O}(^{18}\text{O},\text{pn})^{34}\text{P}$ fusion-evaporation at 36 MeV $\sigma \sim 5 - 10$ mb

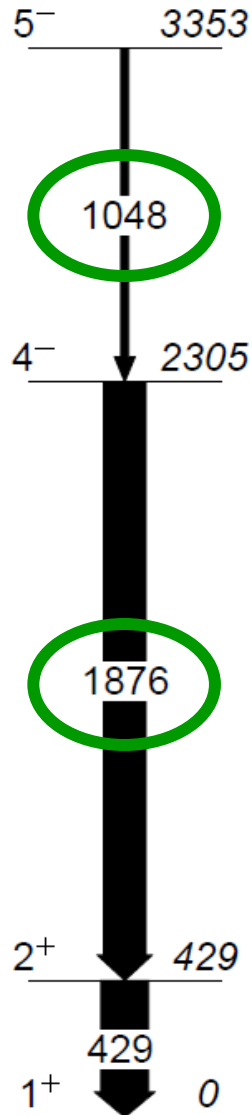
50mg/cm² Ta₂¹⁸O enriched foil; ¹⁸O Beam from Bucharest Tandem (~20pA)



Array 8 HPGe (unsuppressed) and 7 LaBr₃:Ce detectors

- 3 (2"x2") cylindrical
- 2 (1"x1.5") conical
- 2 (1.5"x1.5") cylindrical

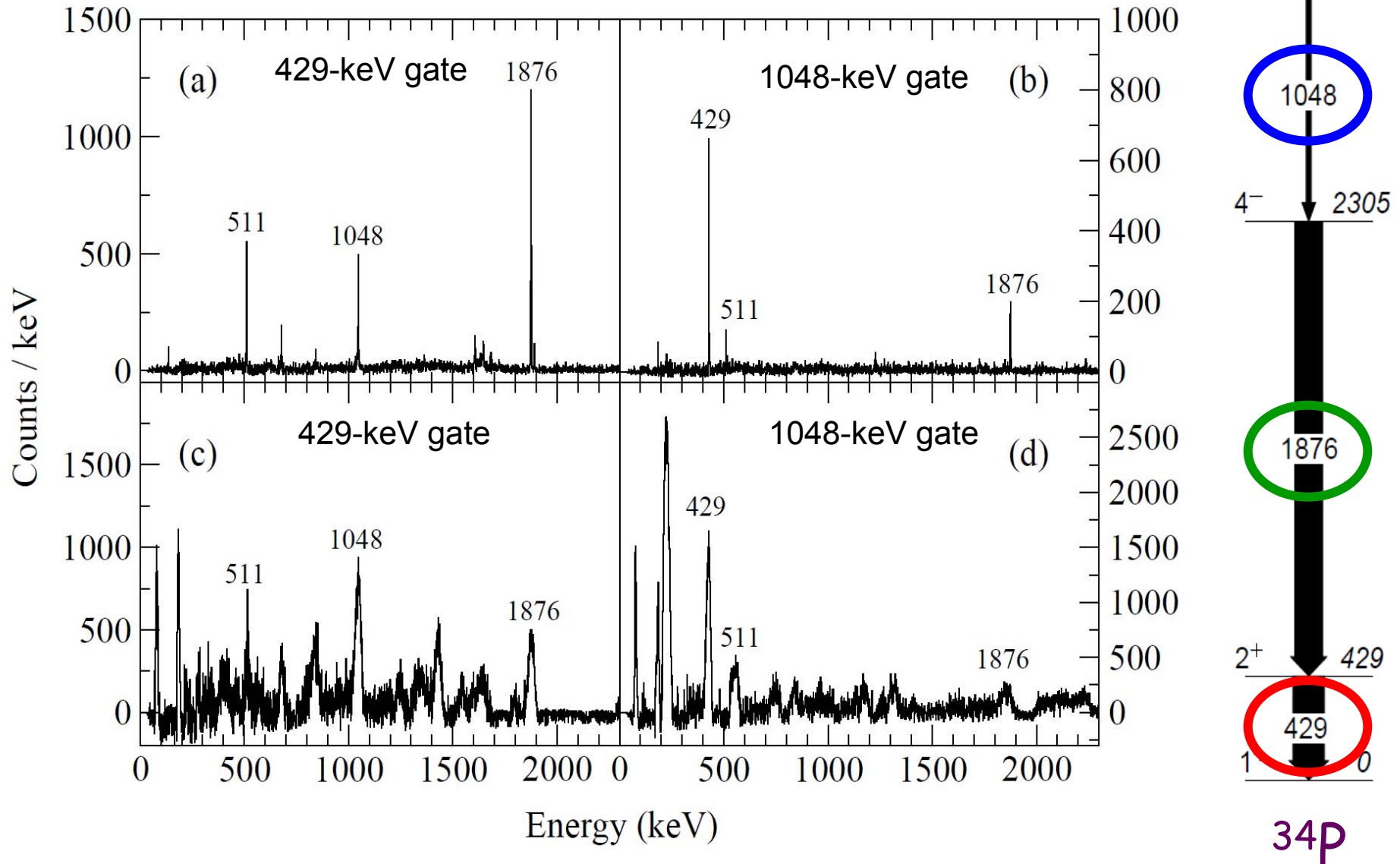
Ge-Gated Time differences



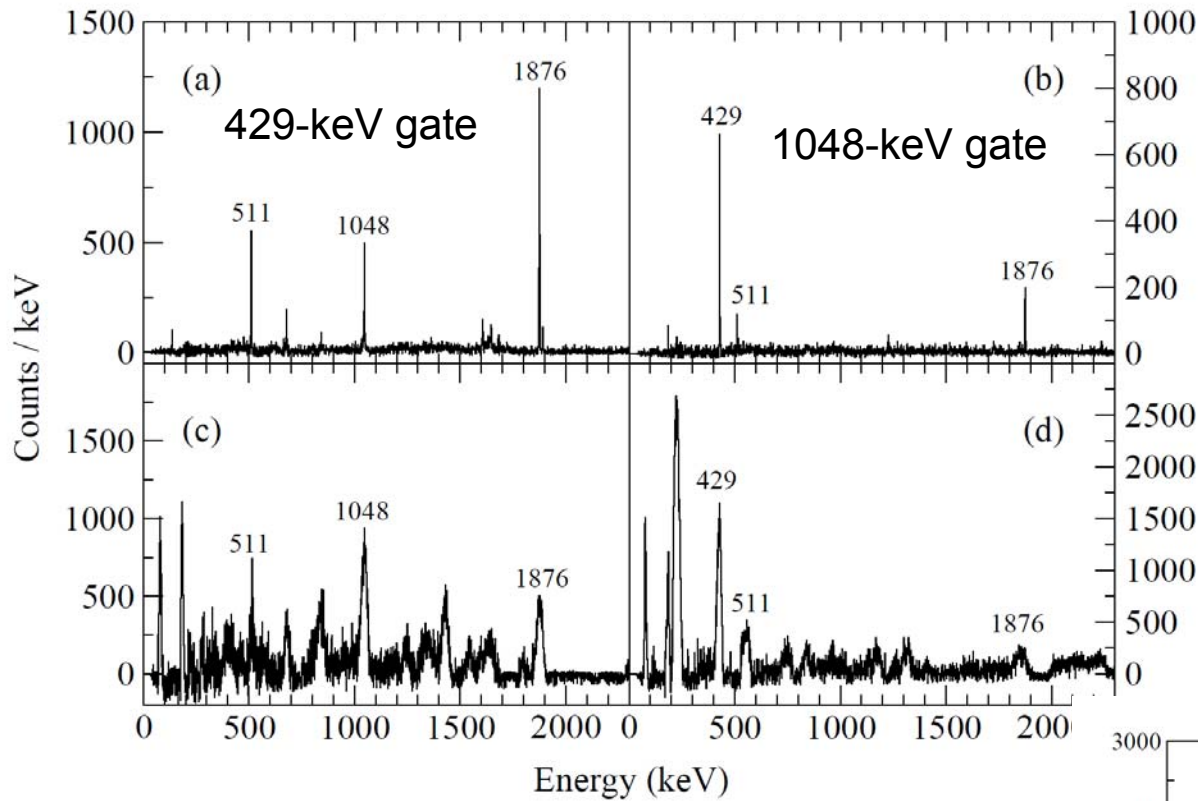
Gates in LaBr₃ detectors to observe time difference and obtain lifetime for state

Ideally, we want to measure the time difference between transitions directly feeding and depopulating the state of interest (4⁻)

Gamma-ray energy coincidences 'locate' transitions above and below the state of interest....



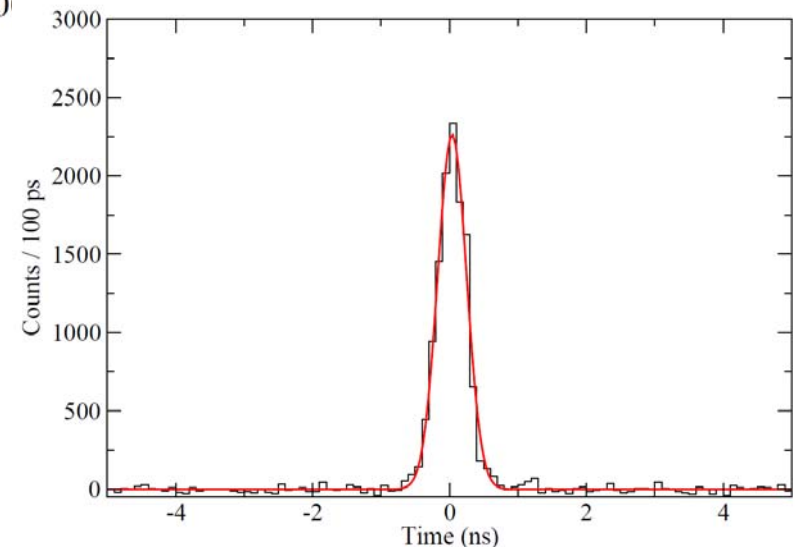
Unated LaBr_3 Time difference



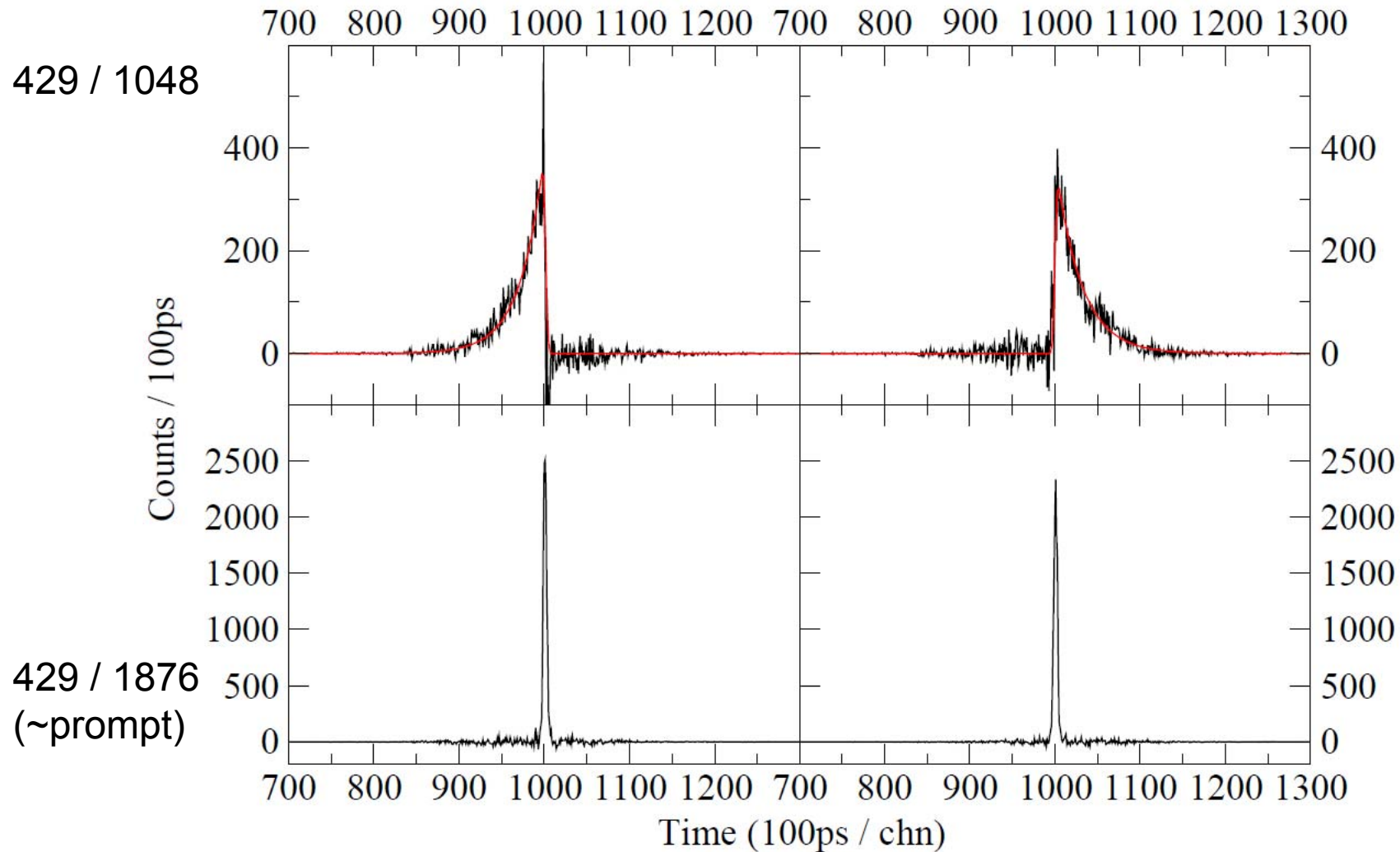
The LaBr_3 - LaBr_3 coincidences were relatively clean where it counts so try without the Ge gate...

e.g. The 1876-429-keV time difference is ^{34}P . Should show prompt distribution as half-life of 2^+ is short.

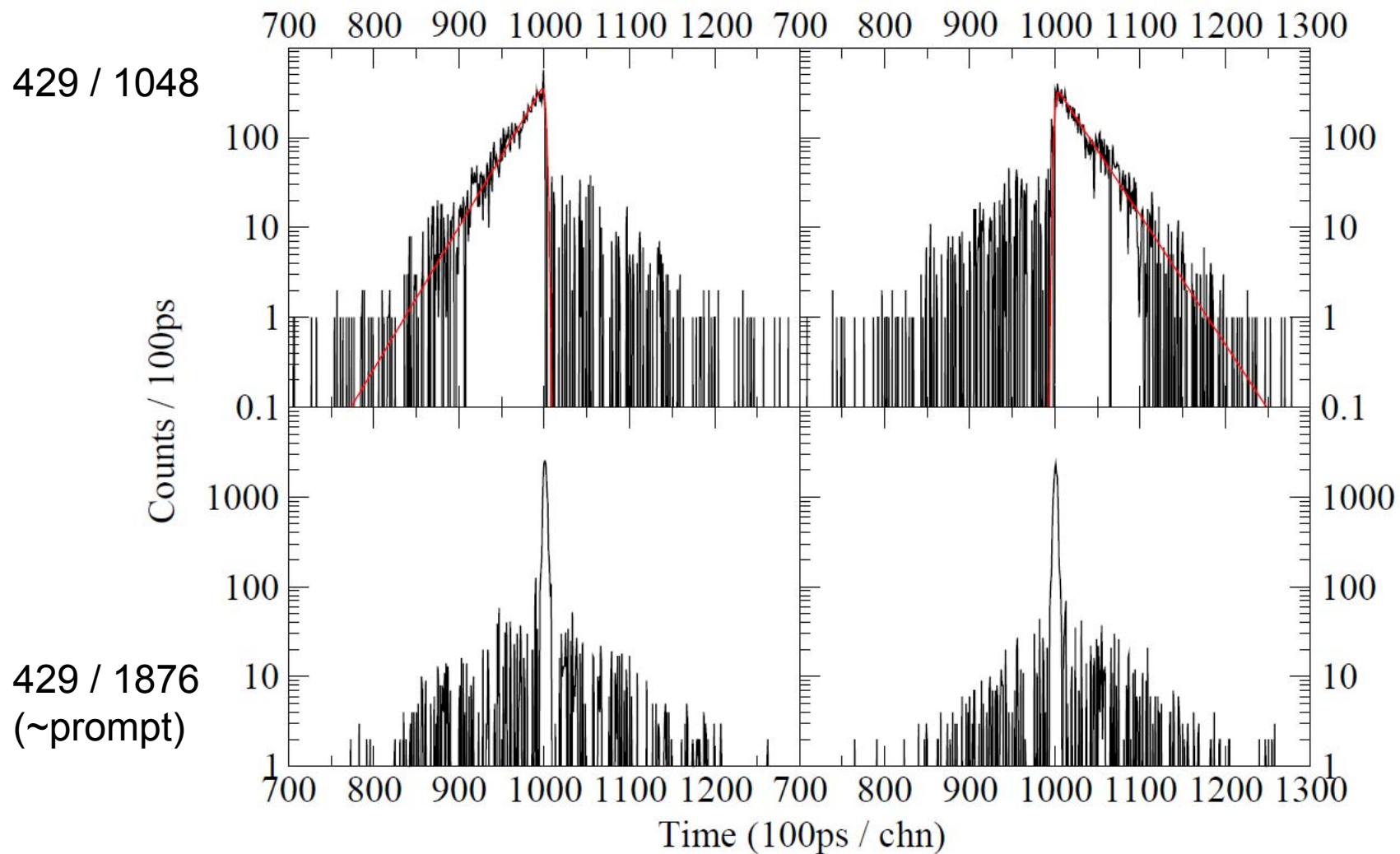
FWHM = 470(10) ps



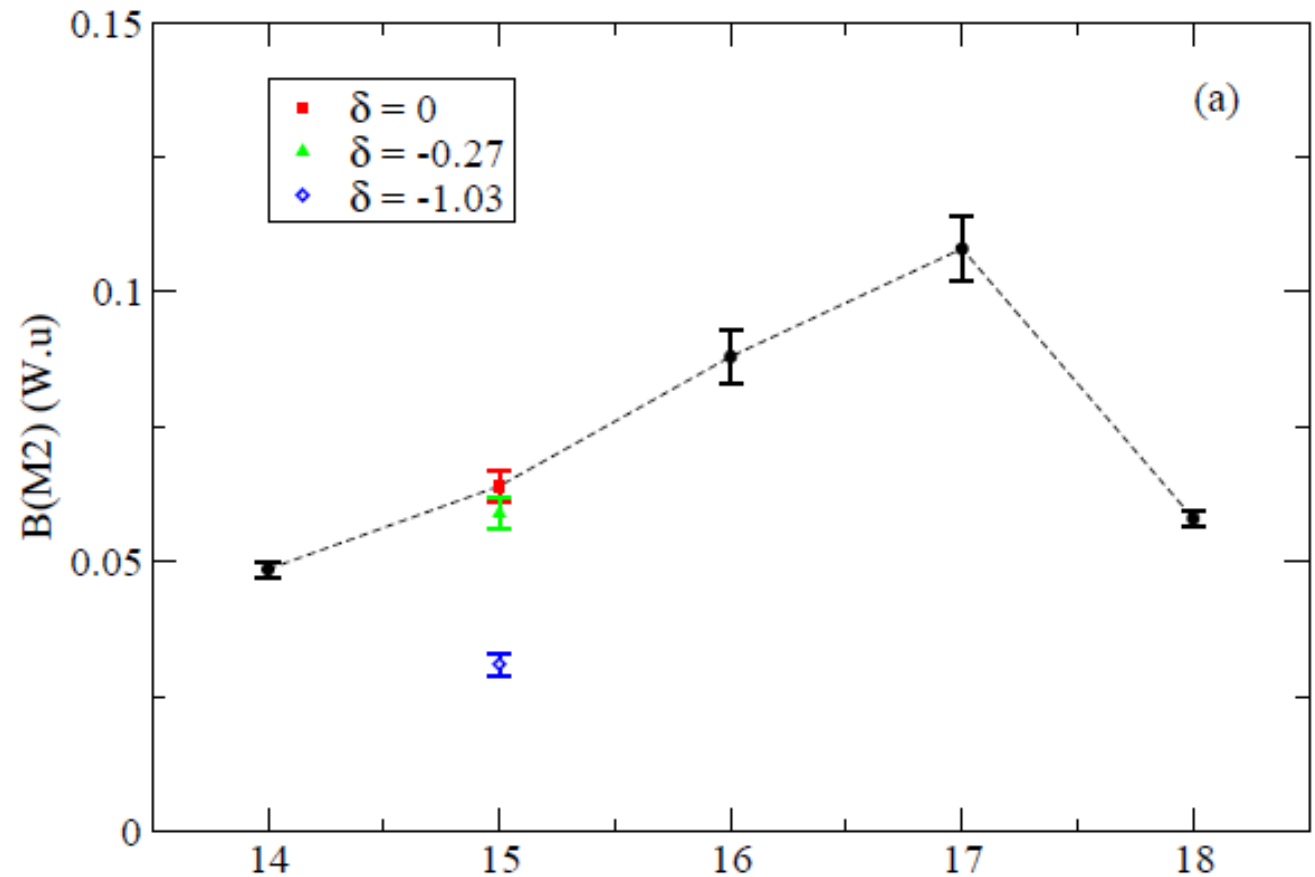
Result: $T_{1/2}$ ($I^\pi=4^-$) in $^{34}\text{P}= 2.0(1)$ ns



Results: $T_{1/2} = 2.0(1)\text{ns}$



Discussion: $M2$ Strengths



- Experimental B(M2) and Mixing ratios from N=19 nuclei approaching the island of inversion.

What about 'faster' transitions..
i.e. $< \sim 10$ ps ?

Collective Model B(E2), B(M1) values ?

The reduced in-band transition probabilities¹ are given by,

$$B(E2; I_i K \rightarrow I_f K) = \frac{5}{16\pi} e^2 Q_o^2 | \langle I_i 2K0 | I_f K \rangle |^2$$

$$B(E2; I_i K \rightarrow I_f K) = \frac{5}{16\pi} e^2 Q_o^2 | \langle I_i 1K0 | I_f K \rangle |^2 \quad (2.4.56)$$

$$B(M1; I_i K \rightarrow I_f K) = \frac{3}{4\pi} e^2 | \langle I_i 1K0 | I_f K \rangle |^2 (g_K - g_R)^2 K^2$$

where Q_o is the intrinsic quadrupole moment and g_K and g_R are the intrinsic and rotational gyromagnetic ratios respectively. The relevant Clebsch-Gordon coefficients² are given below.

$$E2(\Delta I = 2) = \left[\frac{3(I - K)(I - K - 1)(I + K)(I + K - 1)}{(2I - 2)(2I - 1)I(2I + 1)} \right]^{1/2}$$

$$E2(\Delta I = 1) = -K \left[\frac{3(I - K)(I + K)}{(I - 1)I(2I + 1)(I + 1)} \right]^{1/2} \quad (2.4.57)$$

$$M1(\Delta I = 1) = - \left[\frac{(I - K)(I + K)}{I(2I + 1)} \right]^{1/2}$$

¹K.E.G. Löbner in, The Electromagnetic Interaction in Nuclear Spectroscopy, W.D. Hamilton (Ed), North-Holland (1975) Chapter 5

²The Theory of Atomic Spectra, Condon and Shortley (1935) reprinted (1963) p76-77

THE MEASUREMENT OF SHORT NUCLEAR LIFETIMES¹

By A. Z. SCHWARZSCHILD AND E. K. WARBURTON

Brookhaven National Laboratory, Upton, New York²

Annual Review of Nuclear Science (1968) **18** p265-290

SCHWARZSCHILD & WARBURTON

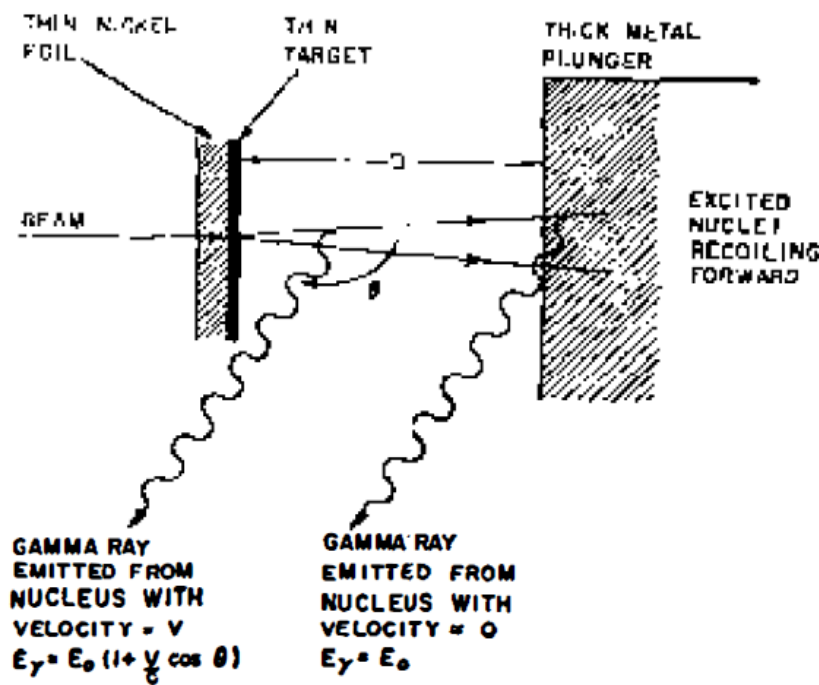
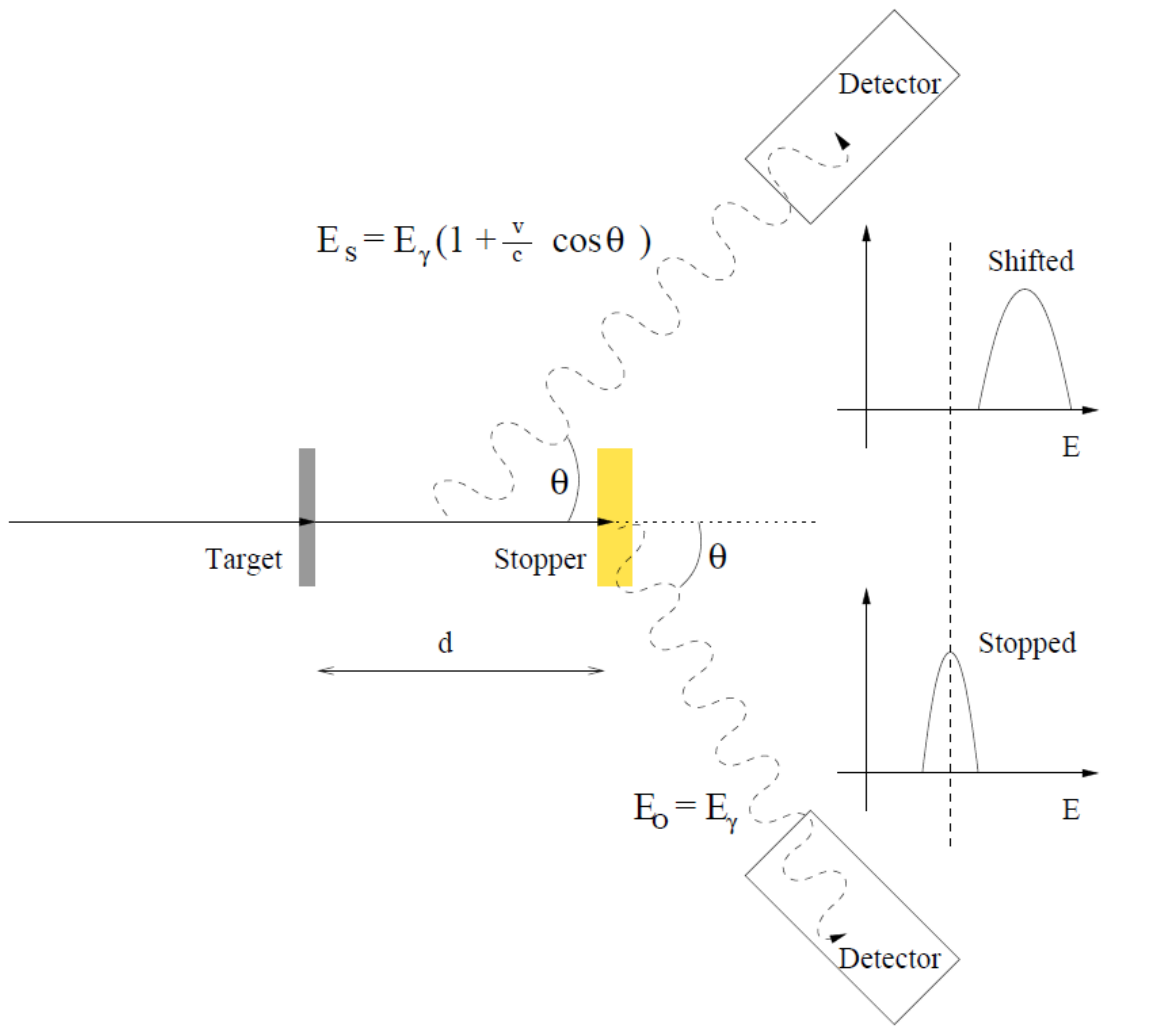


FIG. 5. Recoil method of measuring lifetimes of excited states.



$$E_0 = E_0$$

$$E_S = E_0 \left(\frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta} \right)$$

$$E_S \approx E_0(1 + \beta \cos \theta)$$

$$I_0 = N_0 \exp\left(\frac{-d}{v\tau}\right)$$

$$I_S = N_0 \left(1 - \exp\left(\frac{-d}{v\tau}\right) \right)$$

$$R = \frac{I_0}{I_0 + I_S} = \exp\left(\frac{-d}{v\tau}\right)$$

$$\tau = \frac{-d}{v \ln(R)}$$

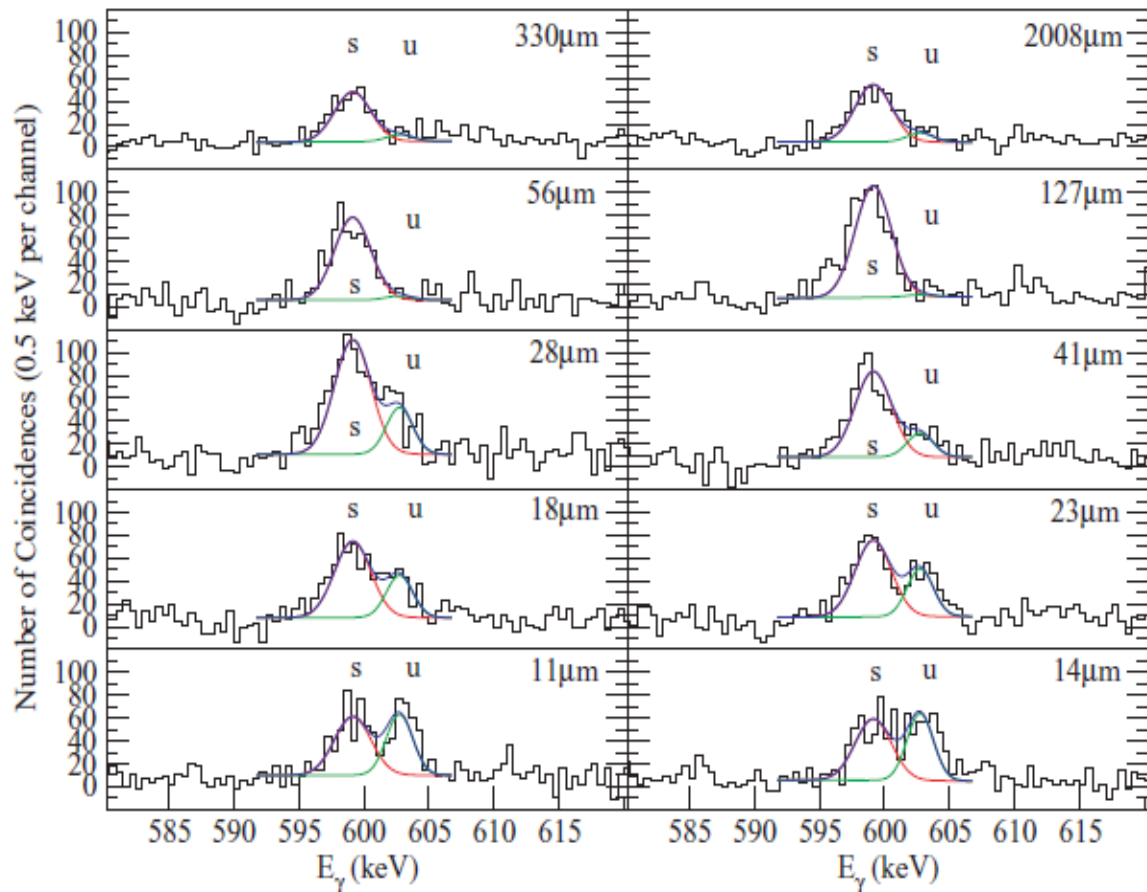
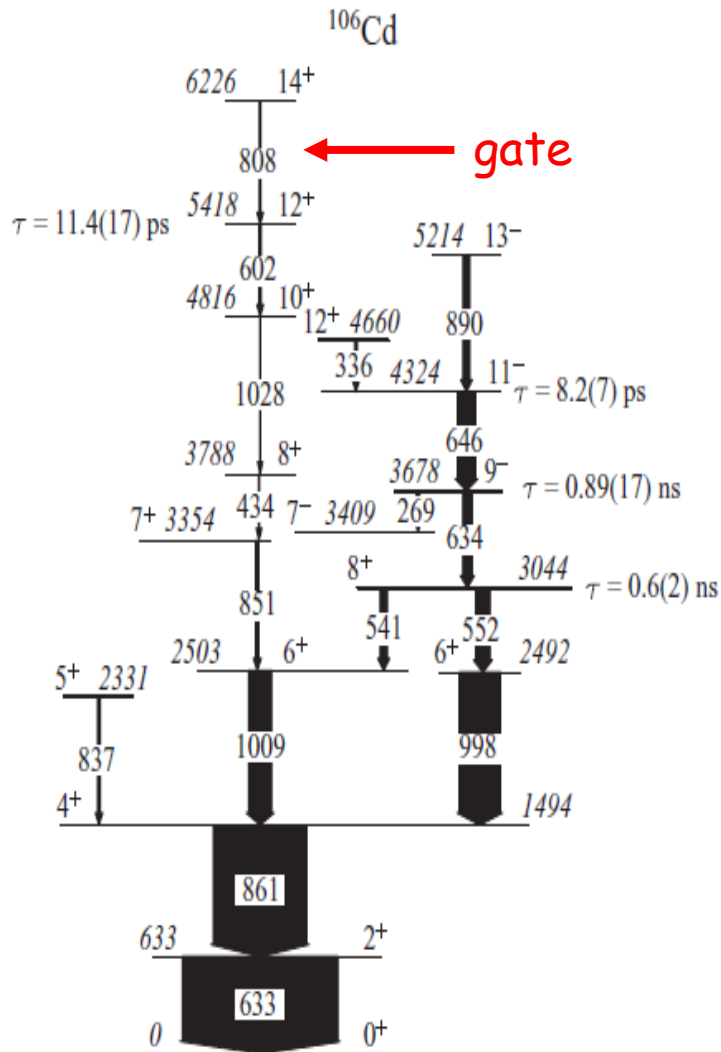
$$B(E2) = \frac{1}{1.225 \times 10^9 E_\gamma^5 \tau (1 + \alpha)}, \quad \longrightarrow$$

$$B(E2) = \frac{5}{16\pi} e^2 Q_t^2 |\langle J_i K 20 | J_f K \rangle|^2,$$

$$Q_t \approx \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2 (1 + 0.16\beta_2),$$

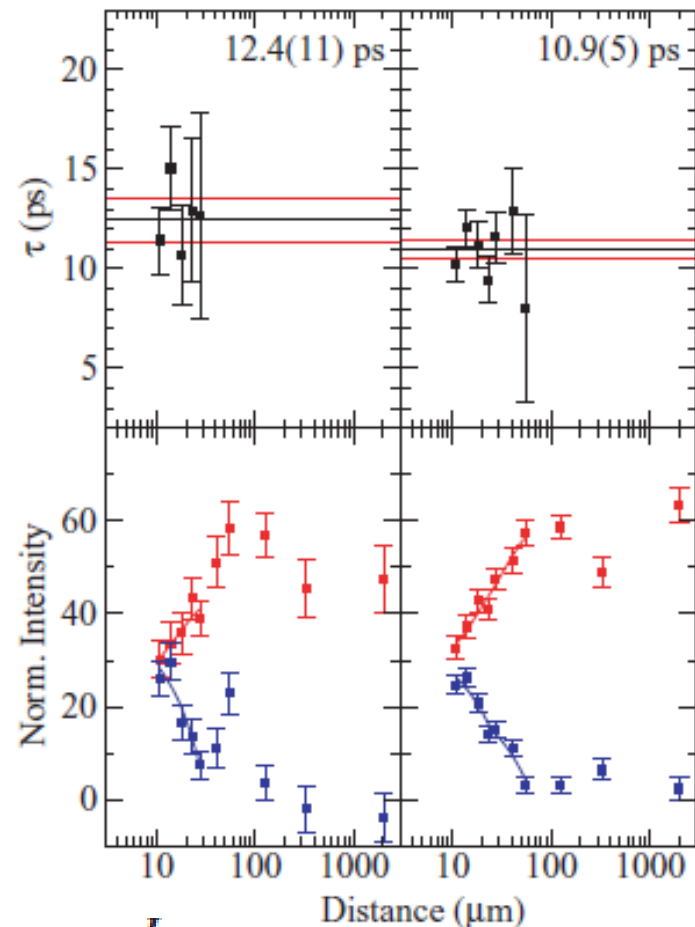
Intrinsic state lifetimes in ^{103}Pd and $^{106,107}\text{Cd}$

S. F. Ashley,^{1,2,*} P. H. Regan,¹ K. Andgren,^{1,3} E. A. McCutchan,² N. V. Zamfir,^{2,4,5} L. Amon,^{2,6} R. B. Cakirli,^{2,6} R. F. Casten,²
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 C. Plettner,² G. Rainovski,^{10,12} R. V. Ribas,¹³ N. J. Thomas,^{1,2} J. Vinson,² D. D. Warner,¹⁴ V. Werner,²
 E. Williams,² H. L. Liu,¹⁵ and F. R. Xu¹⁵

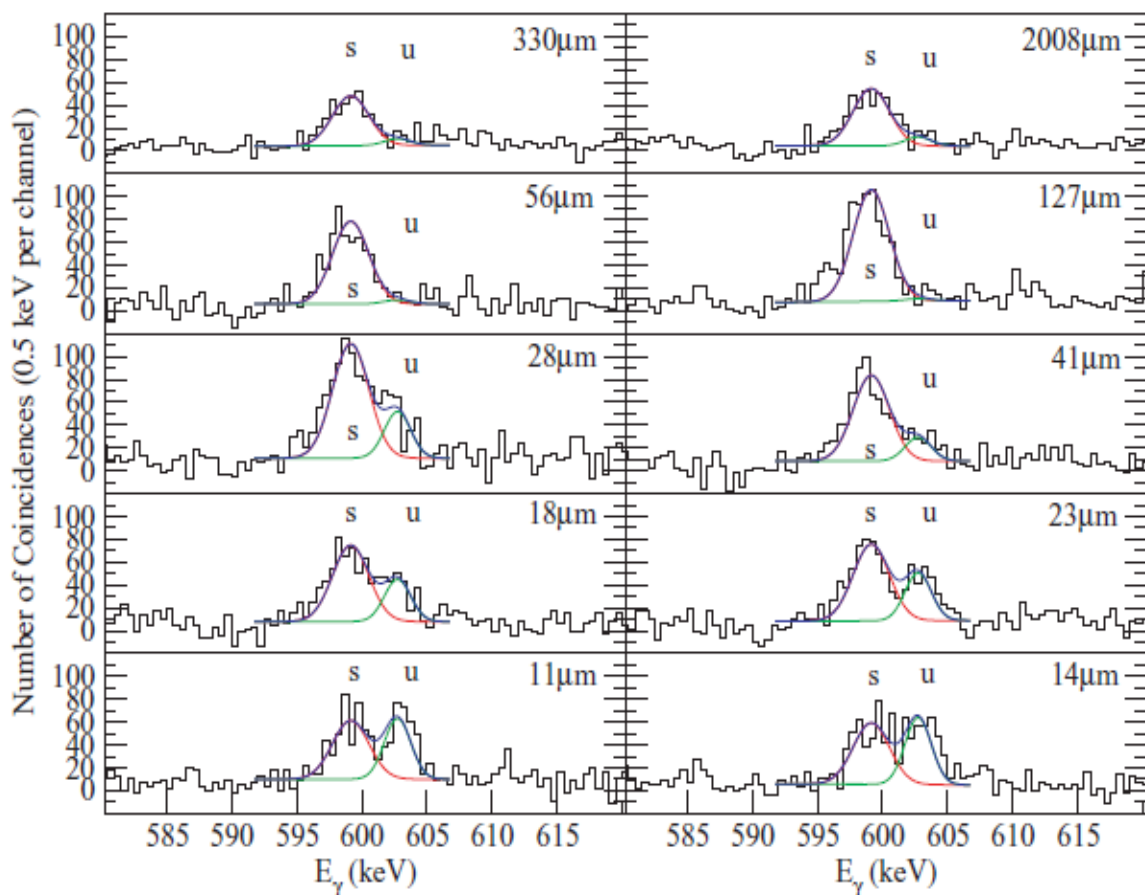


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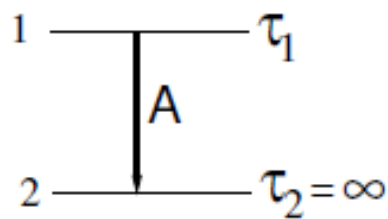


$$\tau = \frac{IU}{v \frac{dI_s}{dx}}$$



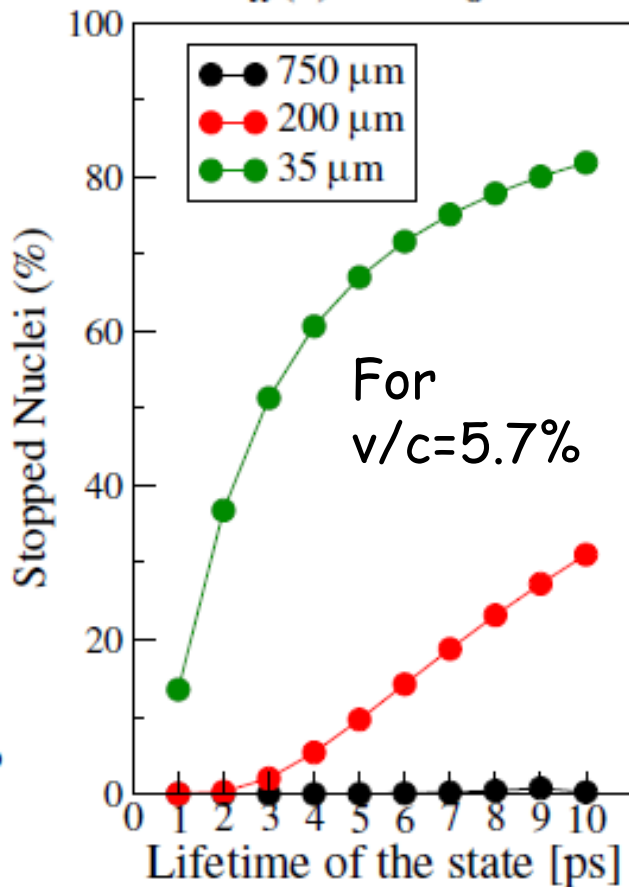
ASIDE: Multistep cascades, need to account for decay lifetimes of states feeding the state of interest....need to account for the Bateman Equations.

$$TOF = \frac{d}{v}$$

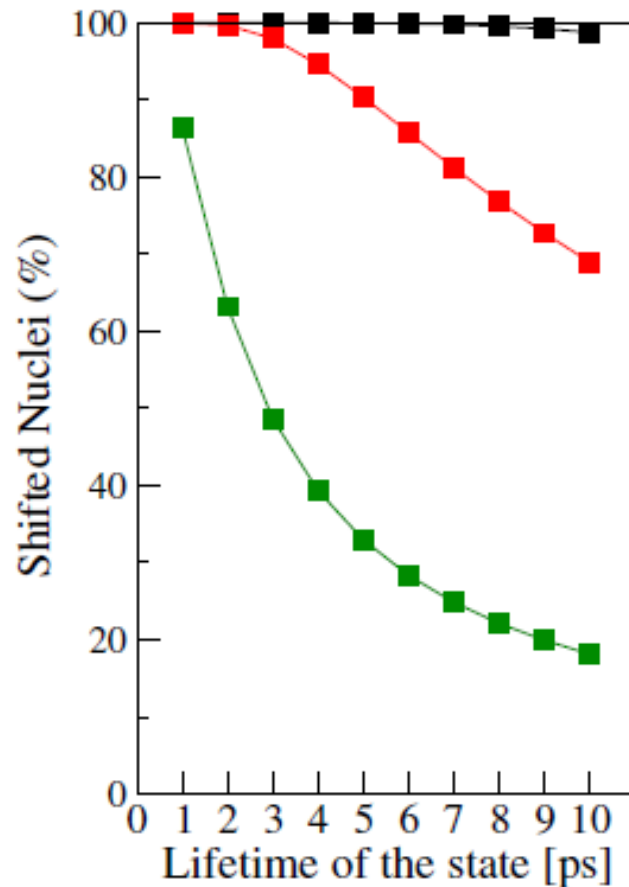


(a) Single decay

$$N_u(t) = N_0 e^{-\frac{t}{\tau}}$$

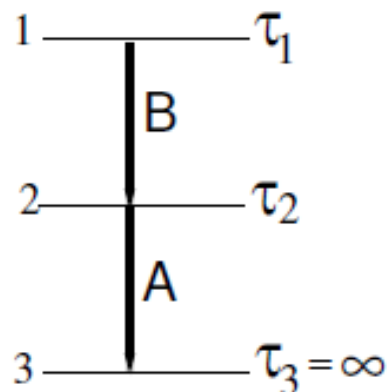


$$N_s(t) = N_0 - N_u(t) = N_0[1 - e^{-\frac{t}{\tau}}]$$

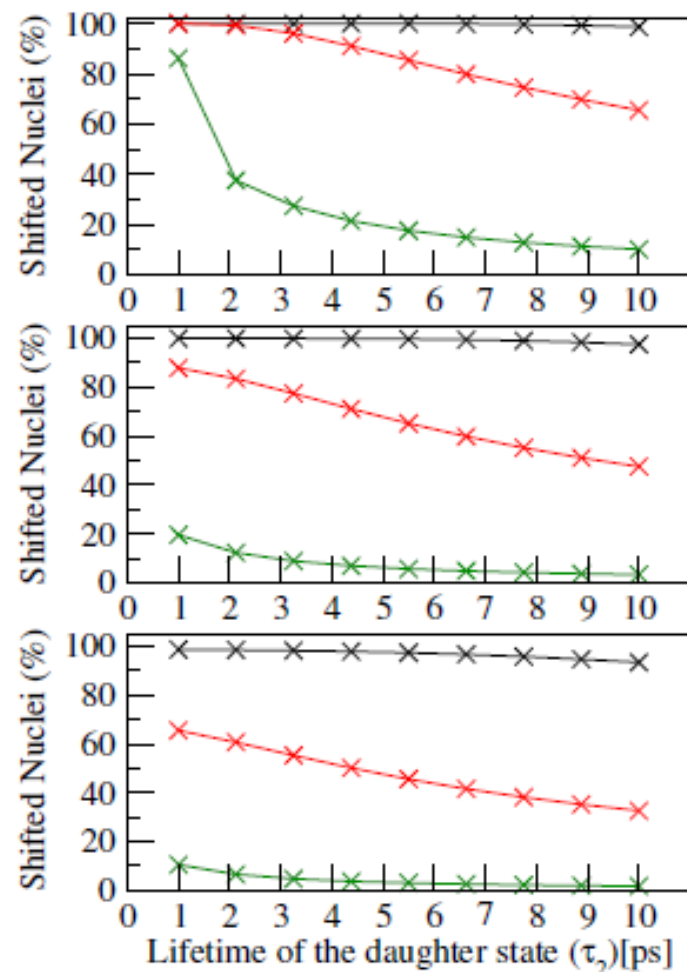
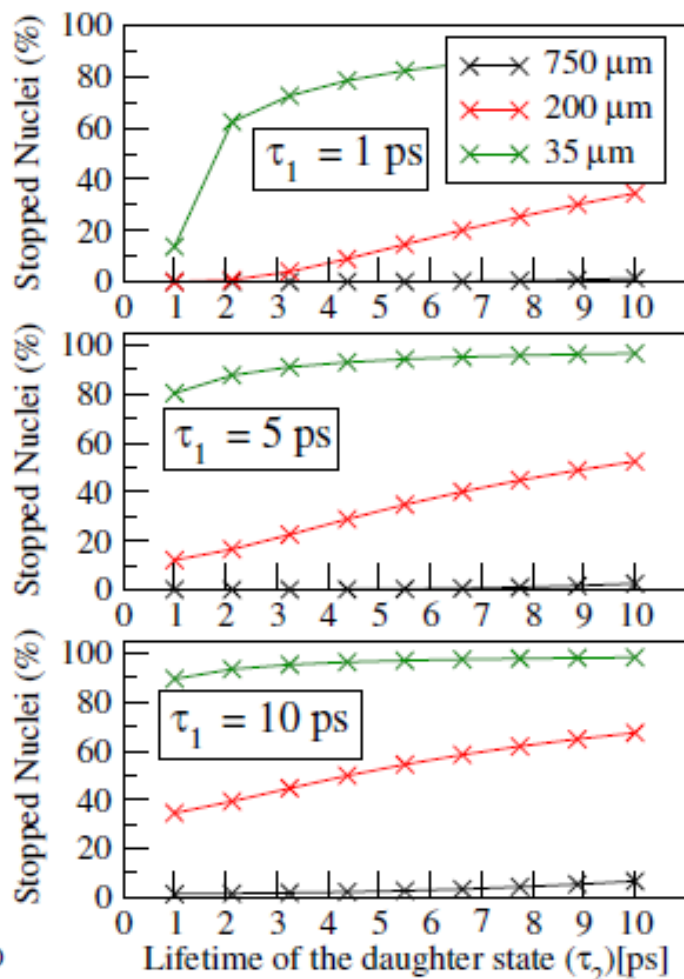


(b)

For $v/c=5.7\%$



(a) Two-step decay

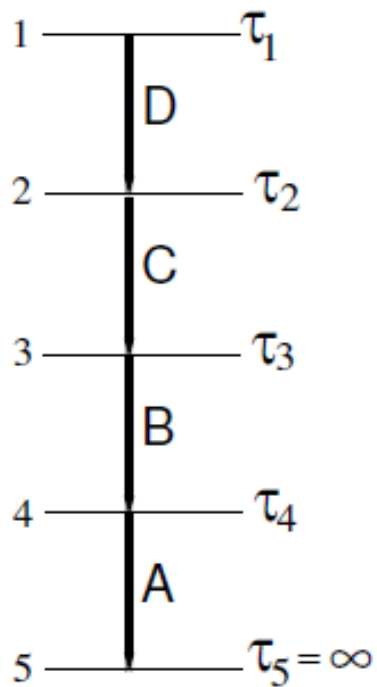


(b)

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t)$$

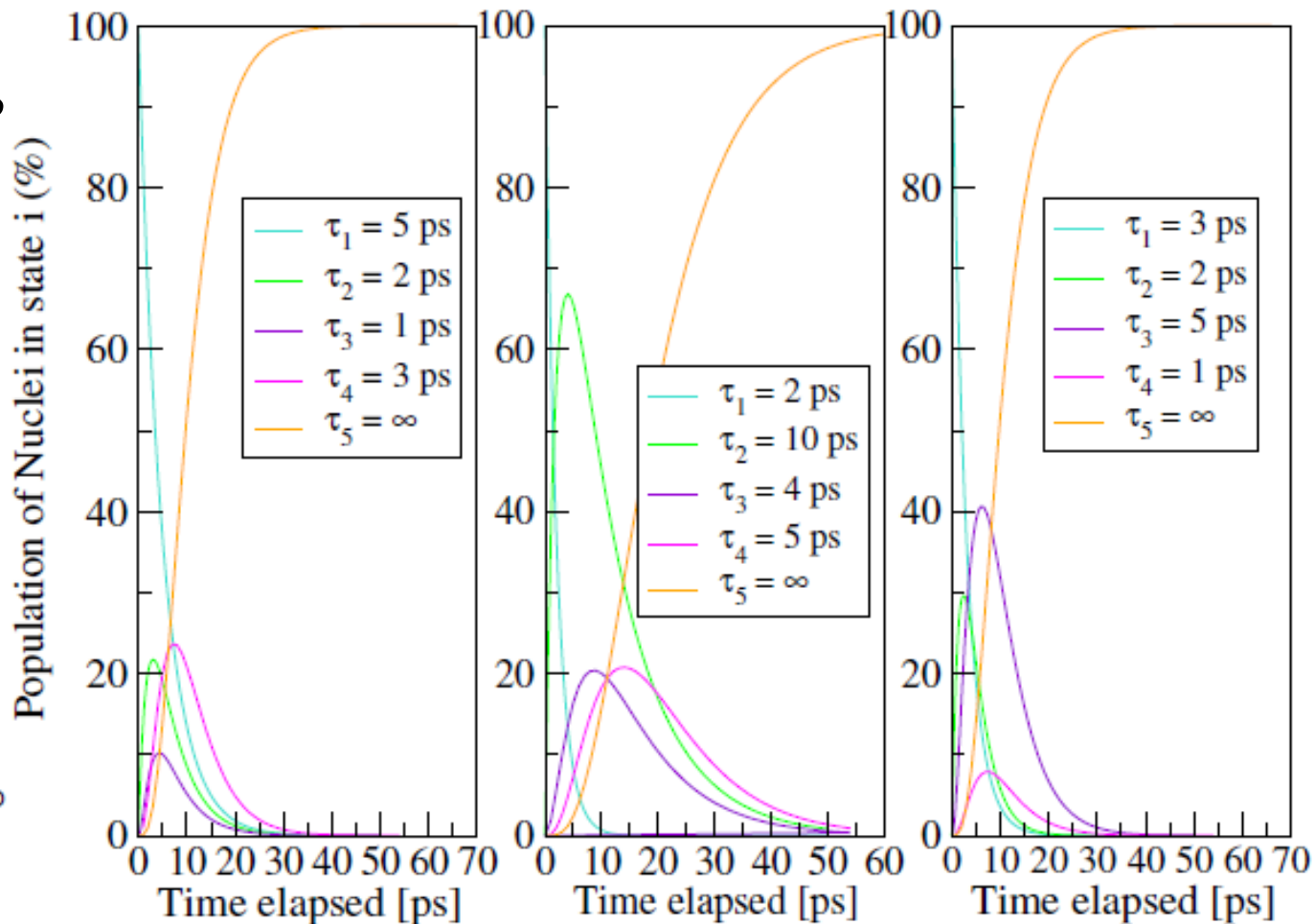
ASIDE: Multistep cascades, need to account for decay lifetimes of states feeding the state of interest....need to account for the Bateman Equations.

For $v/c=5.7\%$



(d)

(a) Four-step decay



(b)

This can be accounted for by using the 'differential decay curve method' by gating on the Doppler shifted component of the direct feeding gamma-ray to the state of interest, see G. Bohm et al., Nucl. Inst. Meth. Phys. Res. **A329** (1993) 248.

If the lifetime to be measured is so short that all of the states decay in flight, the RDM reaches a limit.

To measure even shorter half-lives ($< 1\text{ps}$).

In this case, make the 'gap' distance zero !! i.e., have nucleus slow to do stop in a backing.

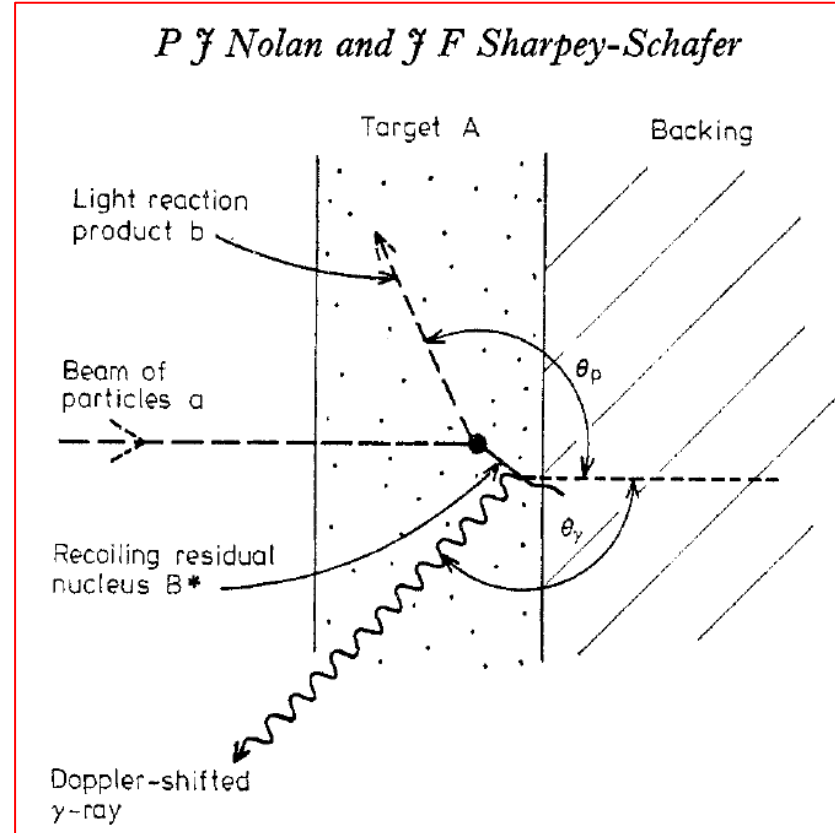
We can use the quantity

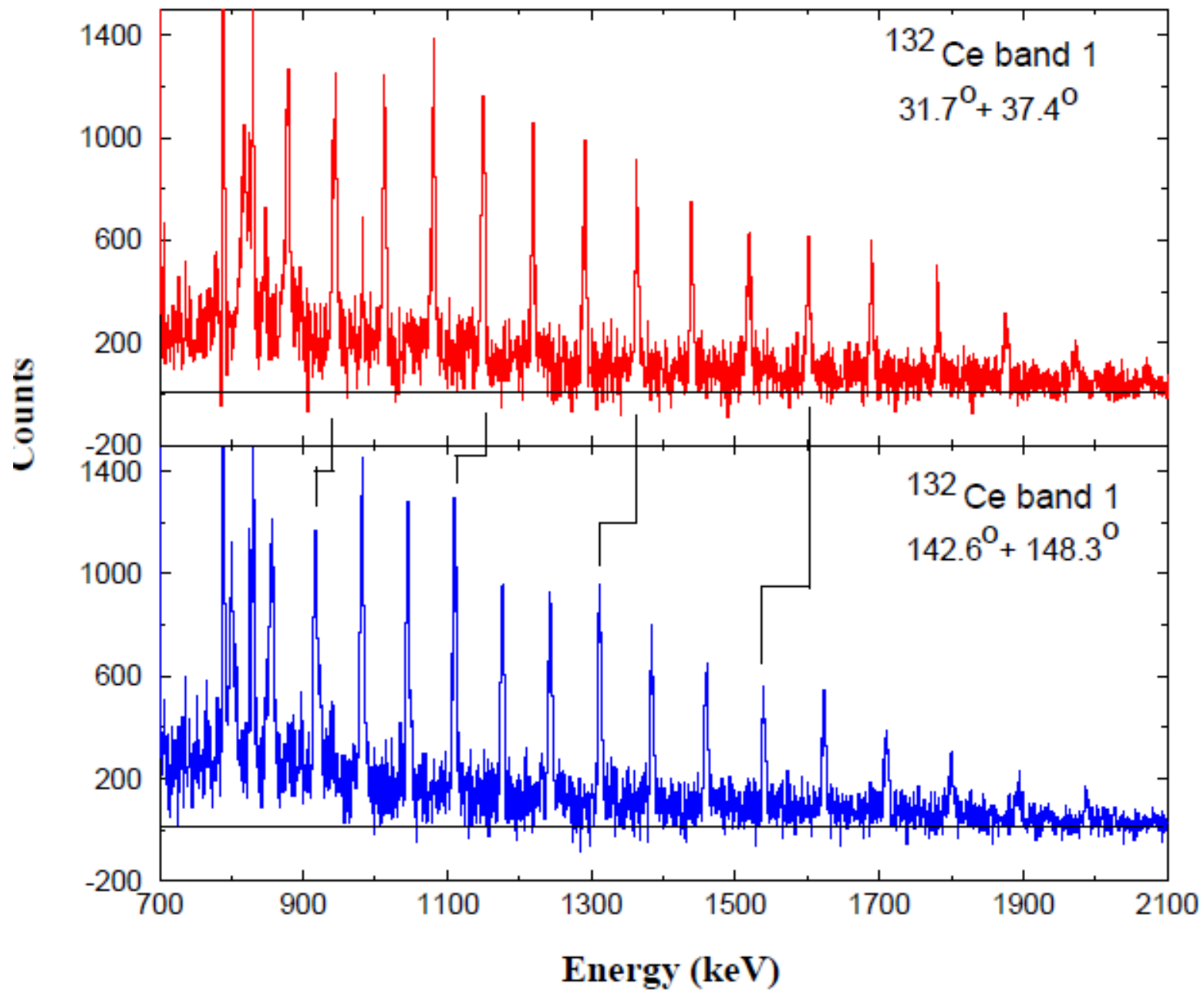
$$F(\tau) = (v_s / v_{\max}).$$

$$E_s(v, \theta) = E_0(1 + v/c \cos(\theta))$$

Measuring the centroid energy of the Doppler shifted line gives the average value for the quantity E_s (and this v) when transition was emitted.

The ratio of v_s divided by the maximum possible recoil velocity (at $t=0$) is the quantity, $F(\tau) =$ fractional Doppler shift.





In the rotational model,

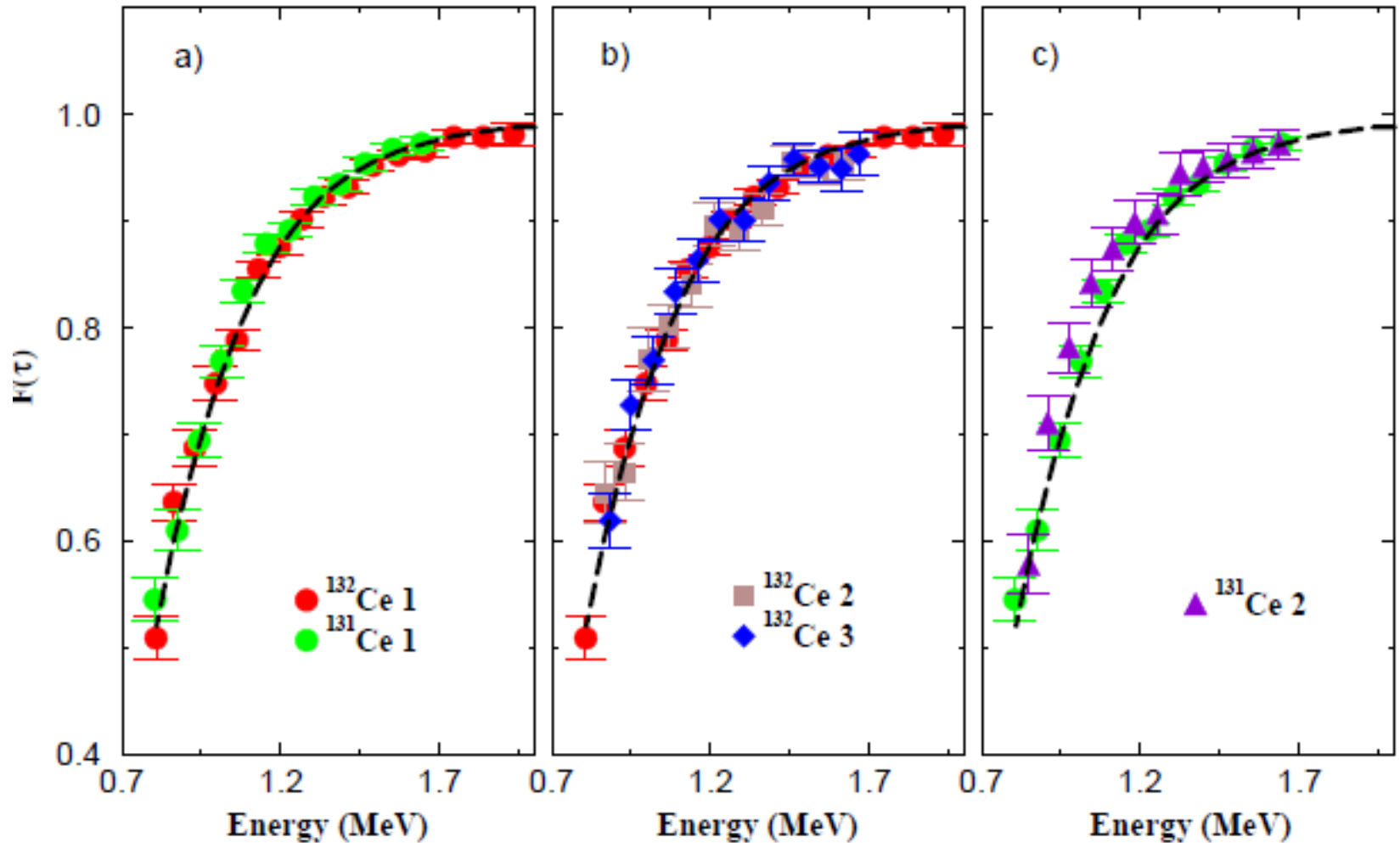
$$\frac{1}{\tau} = 1.223 E_{\gamma}^5 \frac{5}{16} Q_0^2 | \langle J_i K 20 | J_f K \rangle |^2$$

where the CG coefficient is given by,

$$\langle J_i K 20 | J_f K \rangle = \sqrt{\frac{3(J - K)(J - K - 1)(J + K)(J + K - 1)}{(2J - 2)(2J - 1)J(2J + 1)}}$$

Thus, measuring τ and knowing the transition energy, we can obtain a value for Q_0

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left(1 + \frac{1}{8} \sqrt{\frac{5}{\pi}} \beta_2 \dots \right)$$



If we can assume a constant quadrupole moment for a rotational band (Q_0), and we know the transition energies for the band, correcting for the feeding using the Bateman equations, we can construct 'theoretical' $F(\tau)$ curves for bands of fixed Q_0 values

Intrinsic Quadrupole Moment of the Superdeformed Band in ^{152}Dy

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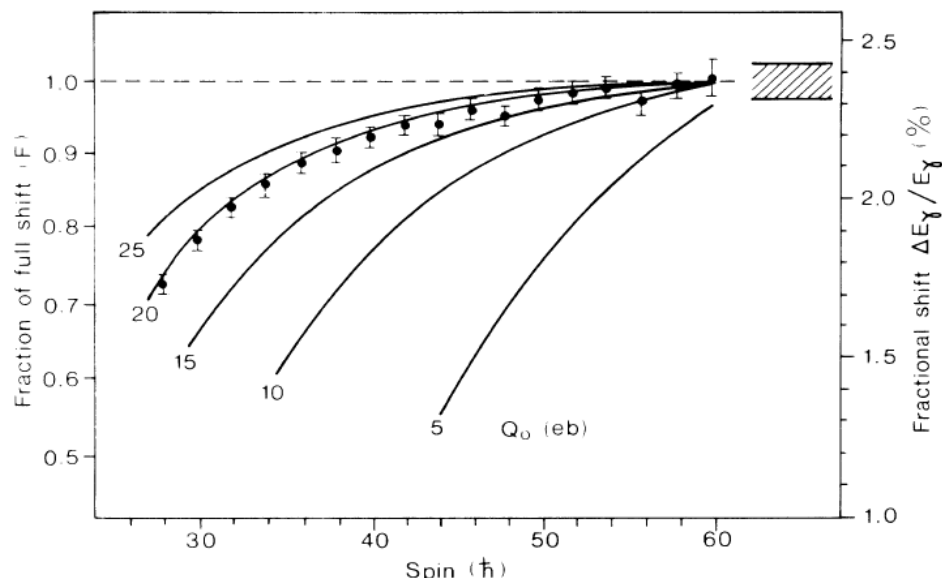
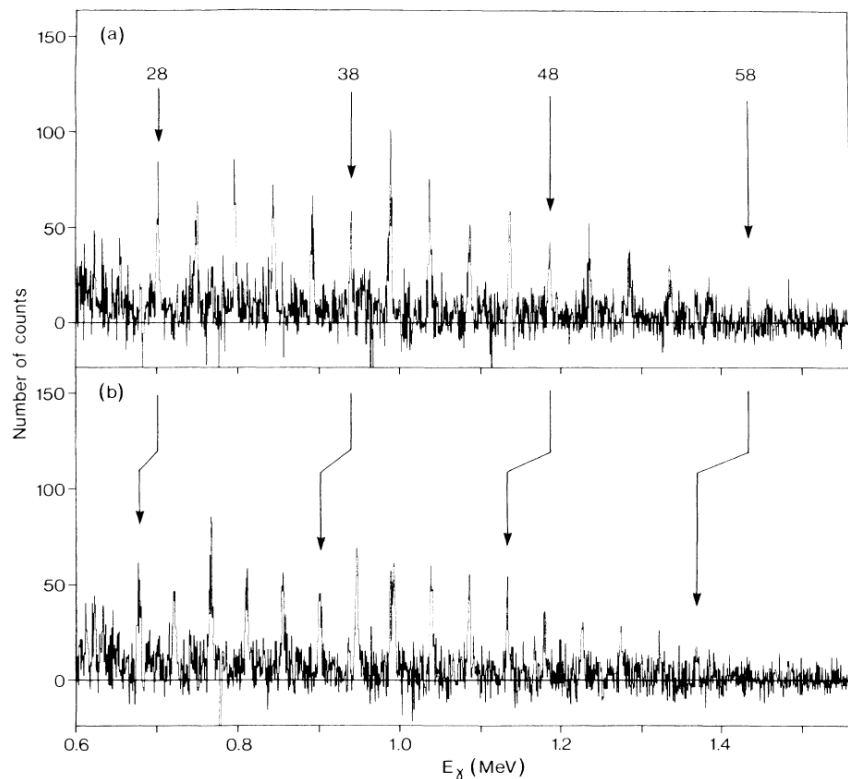
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B. Tamain, Phys. Rev. Lett. 32, 738 (1974).

⁸K. L. Wolf, J. P. Unik, J. R. Hulzenga, J. R. Birkenlund, H. Freiesleben, and V. E. Viola, Phys. Rev. Lett.

33, 1105 (1974).

⁹M. M. Fowler and R. C. Jared, Nucl. Instrum. Methods 124, 341 (1975).

Magnetic Moment of the 6^+ Isomeric State of $^{134}\text{Te}^\dagger$

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(Received 4 March 1976)

The inherent alignment of the prompt fission fragments was used to measure the g factor of the 6^+ isomeric state of ^{134}Te by the time-differential perturbed angular-correlation method. The experimental result $g_{\text{exp}} = 0.846 \pm 0.025$ is close to effective g factors of $g_{7/2}$ protons in nuclei with 82 neutrons. The deviation from the Schmidt value is discussed in terms of the polarization of the $^{132}_{50}\text{Sn}_{82}$ core by the $g_{7/2}$ protons.

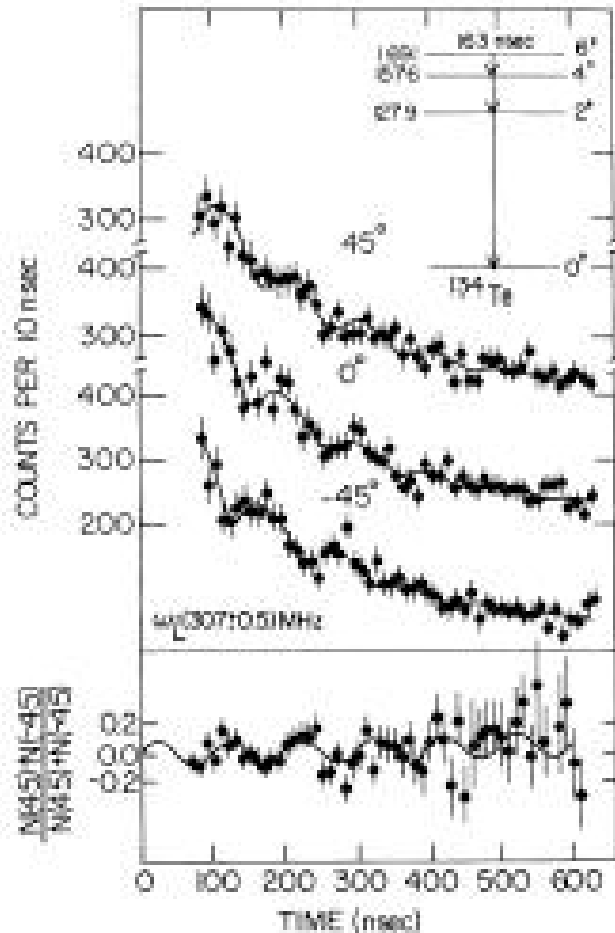


FIG. 2. Time spectra of the 115.3-keV radiation of ^{134}Te at three angles with respect to the fission direction. Normalized differences between the intensities at 45° and -45° are given in the lower part of the figure. The solid lines were obtained by a least-squares fit to the data. The decay scheme of ^{134}Te as determined in Refs. 3, 4, and 5 is also shown.

classically $\mu = I \times A$
 magnetic moment reflects
 size of current loop (i.e. ang.
 mom. of state) and direction/sign
 of current (i.e., π or ν config.)

$g = \mu / I$, can use **Schmidt model** to
 give estimates for what the g-factors
 should be for pure spherical orbits.

Can measure g directly from
 ‘twisting’ effect of putting
 magnetic dipole moment, μ ,
 in a magnetic field, B .

Nucleus precesses with the
 Larmor frequency, $\omega_L = g\mu_N B$