Some (more) High(ish)-Spin Nuclear Structure

Lecture 2

Low-energy Collective Modes and Electromagnetic Decays in Nuclei

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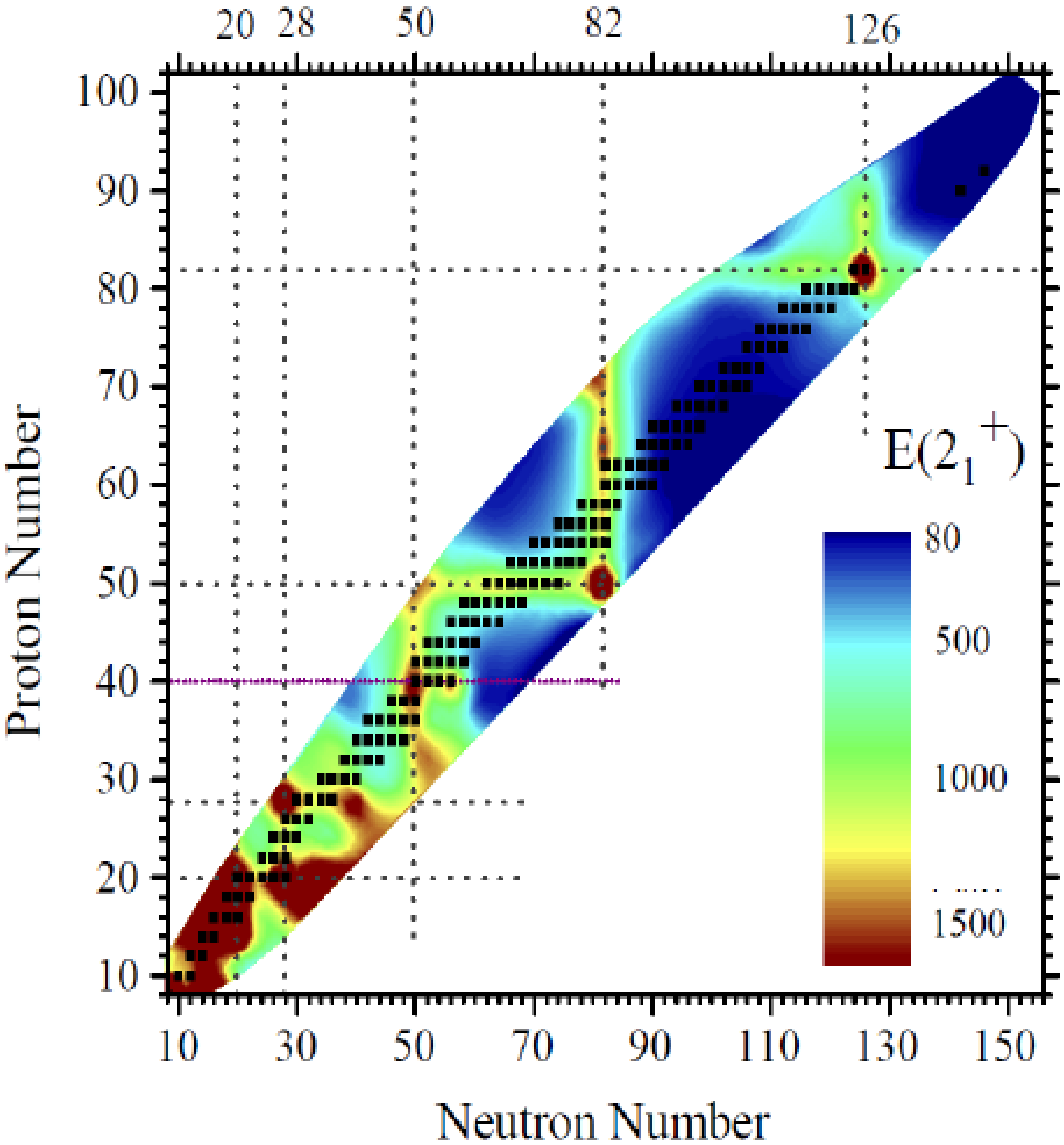
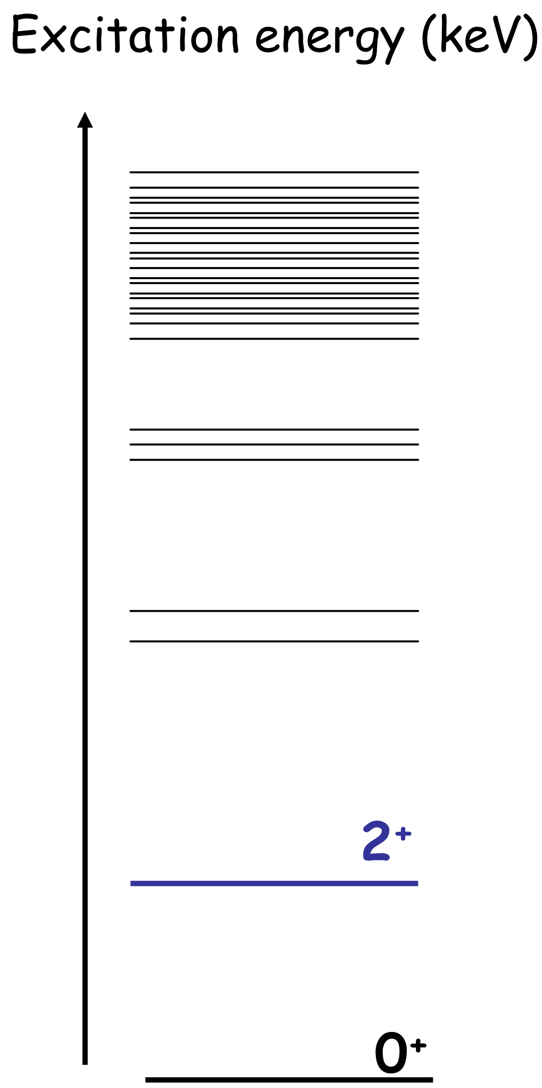
Outline of Lectures

- 1) Overview of nuclear structure 'limits'
  - Some experimental observables, evidence for shell structure
  - Independent particle (shell) model
  - Single particle excitations and 2 particle interactions.

- 2) Low Energy Collective Modes and EM Decays in Nuclei.
  - Low-energy Quadrupole Vibrations in Nuclei
  - Rotations in even-even nuclei
  - Vibrator-rotor transitions, E-GOS curves

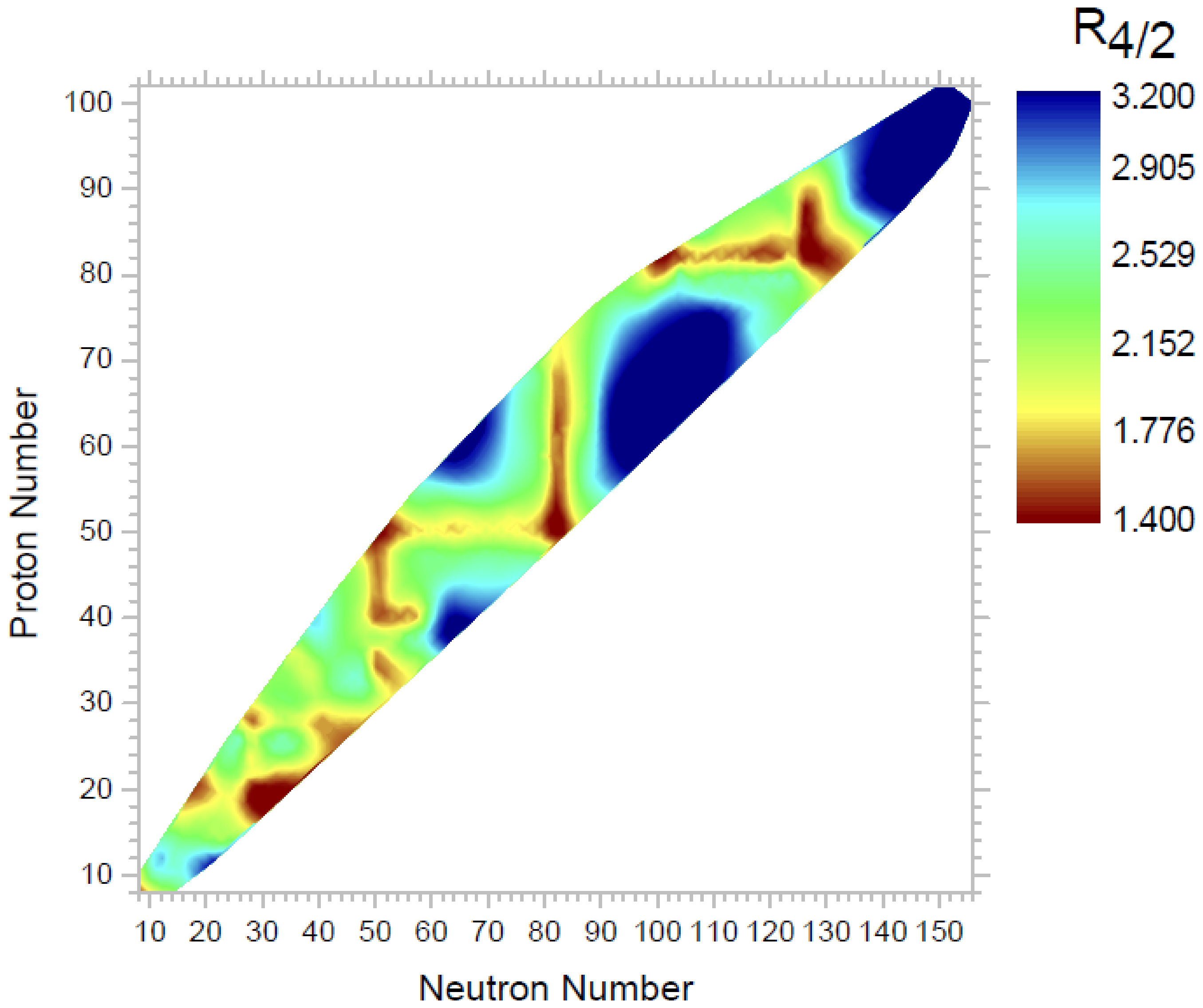
- 3) EM transition rates..what they mean and overview of how you measure them
  - Deformed Shell Model: the Nilsson Model, K-isomers
  - Definitions of B(ML) etc. ; Weisskopf estimates etc.
  - Transition quadrupole moments (Qo)
  - Electronic coincidences; Doppler Shift methods.
  - Yrast trap isomers
  - Magnetic moments and g-factors

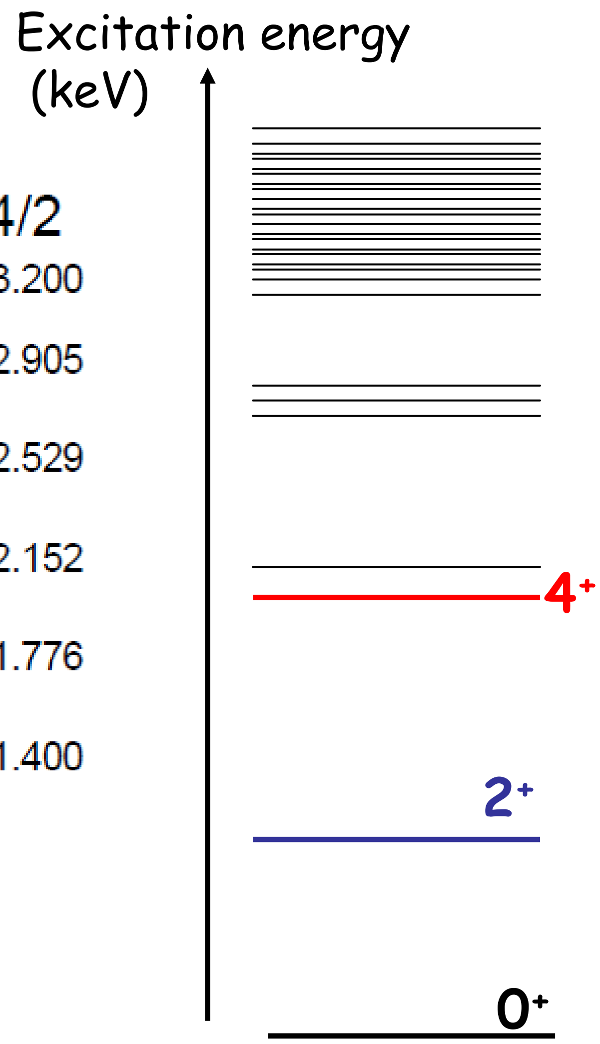




Ground state Configuration.
Spin/parity $I^{\pi}=0^{+}$;
E_x = 0 keV

4+/2+ energy ratio: mirrors 2$^+$ systematics.





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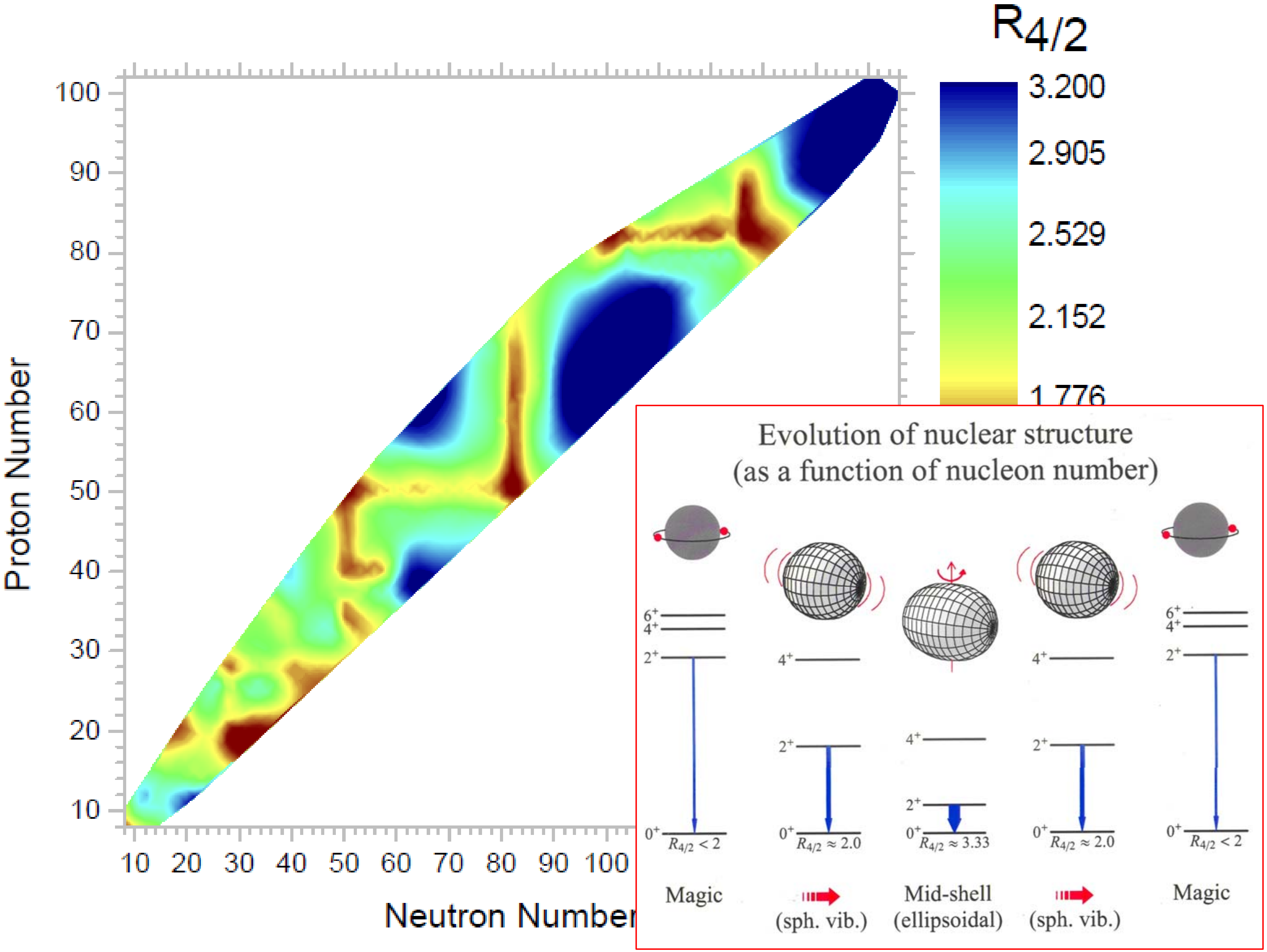
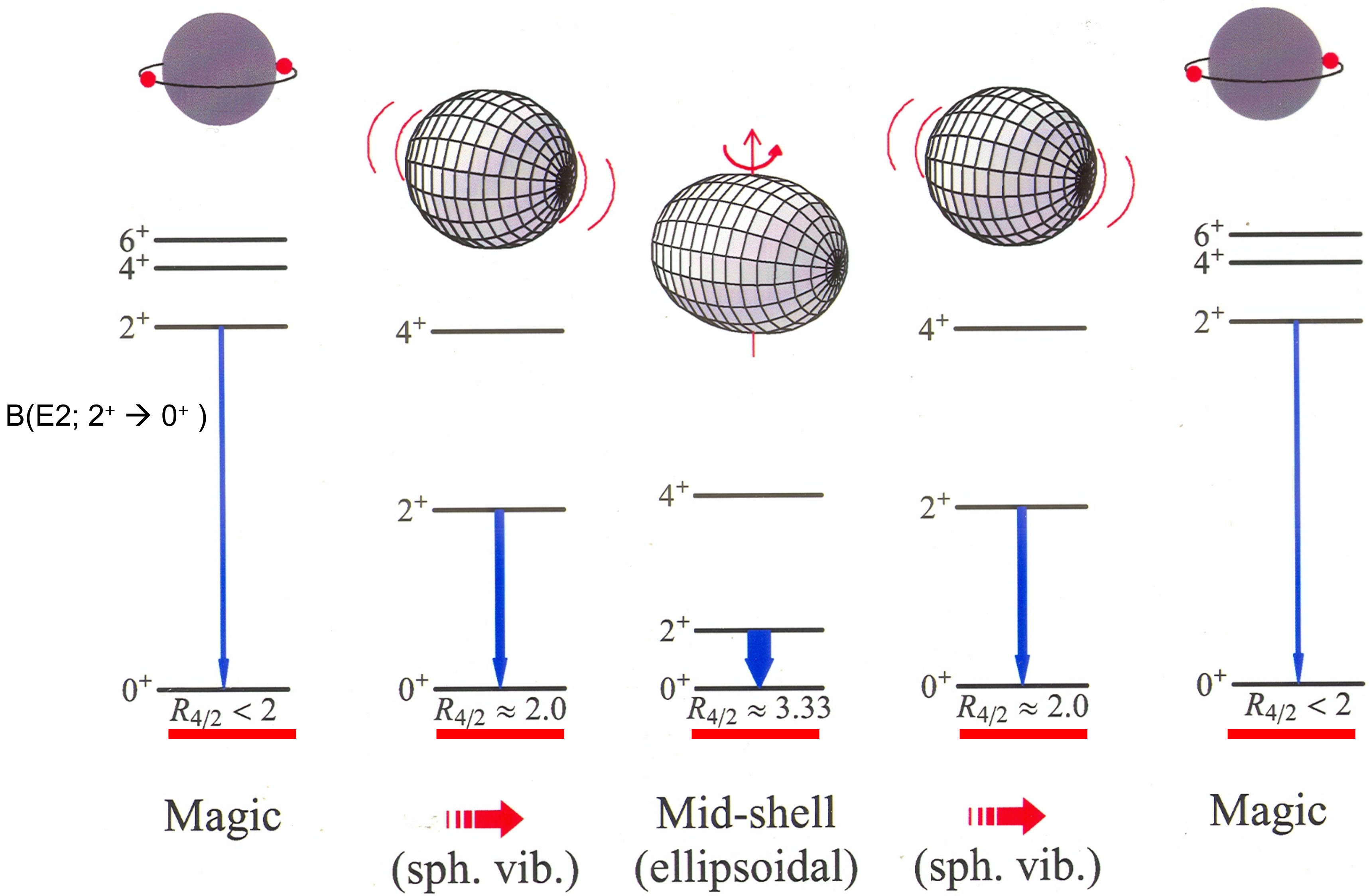


R4/2
3.200
2.905
2.529
2.152
1.776
Proton Number
Neutron Number
100
90
80
70
60
50
40
30
20
10
10 20 30 40 50 60 70 80 90 100
Evolution of nuclear structure
(as a function of nucleon number)
6+
4+
2+
0+
R4/2 < 2
R4/2 ≈ 2.0
R4/2 ≈ 3.33
R4/2 ≈ 2.0
R4/2 < 2
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)
(sph. vib.)
Magic

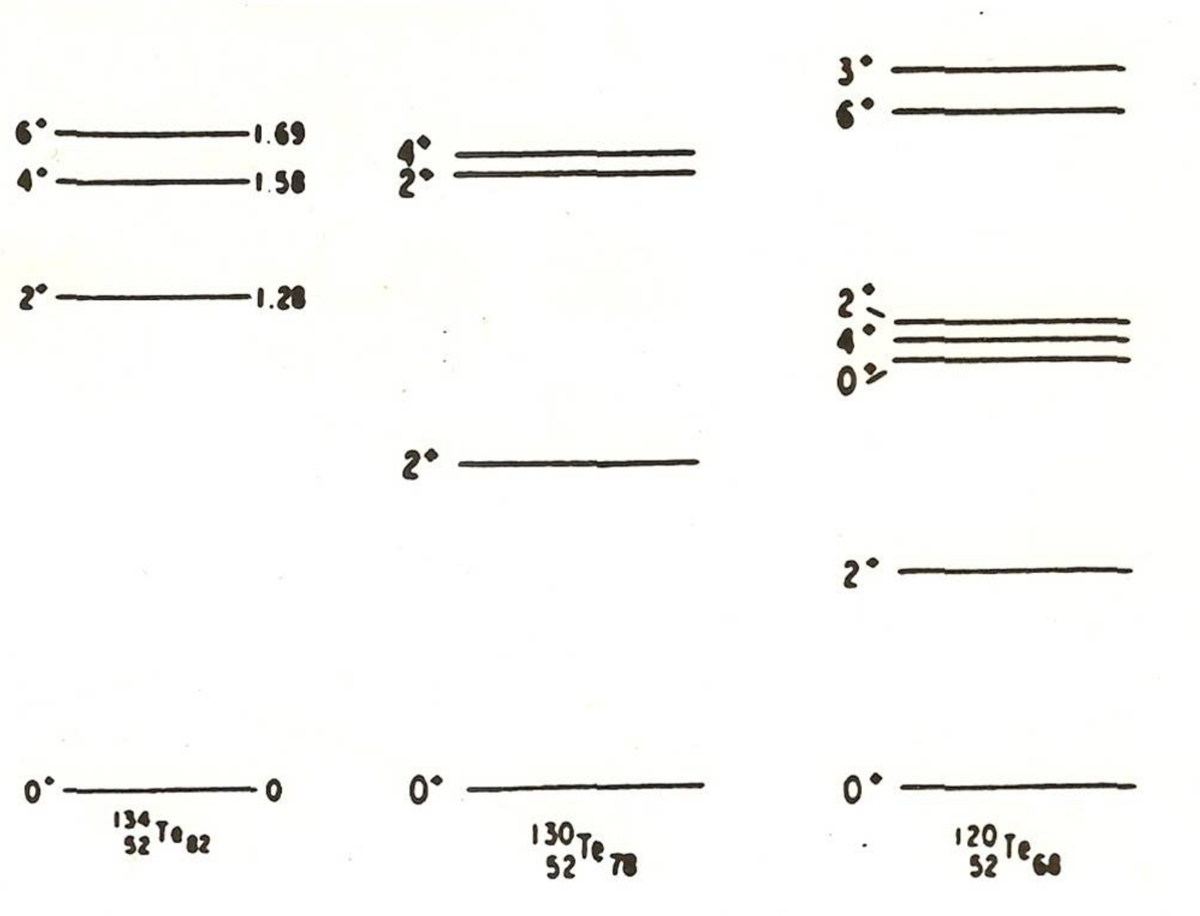
Evolution of nuclear structure

(as a function of nucleon number)



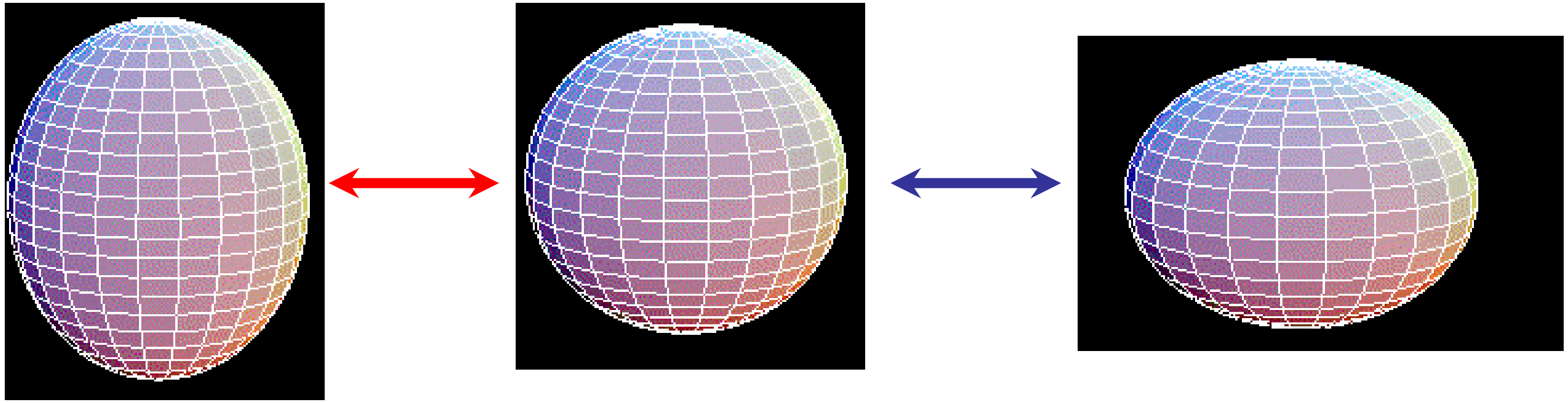
What about <u>both</u> valence neutrons and protons?
In cases of a few valence nucleons there is a lowering of energies, development of multiplets.

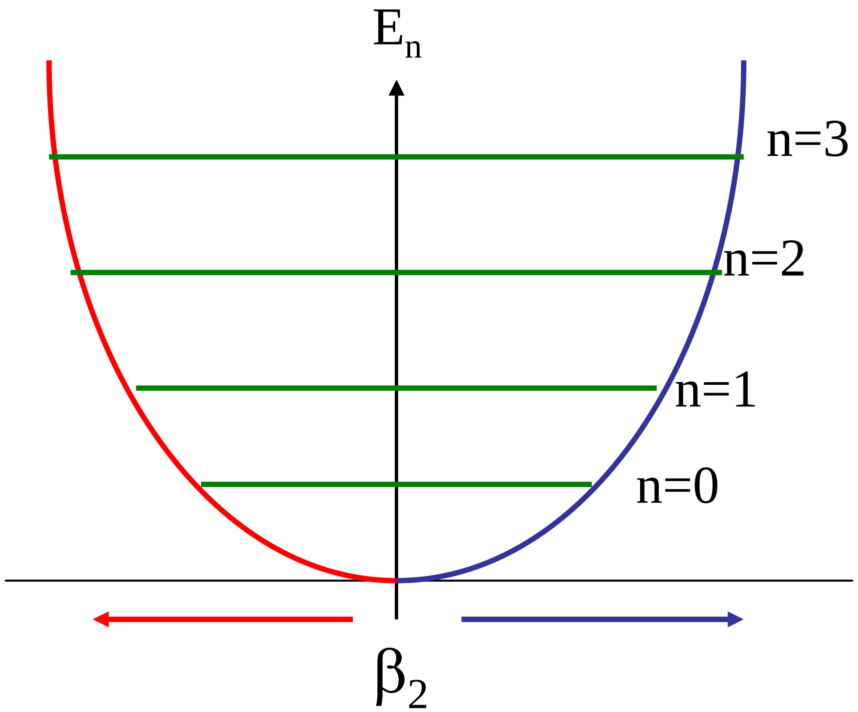
$R_{4/2} \rightarrow$ ~2-2.4



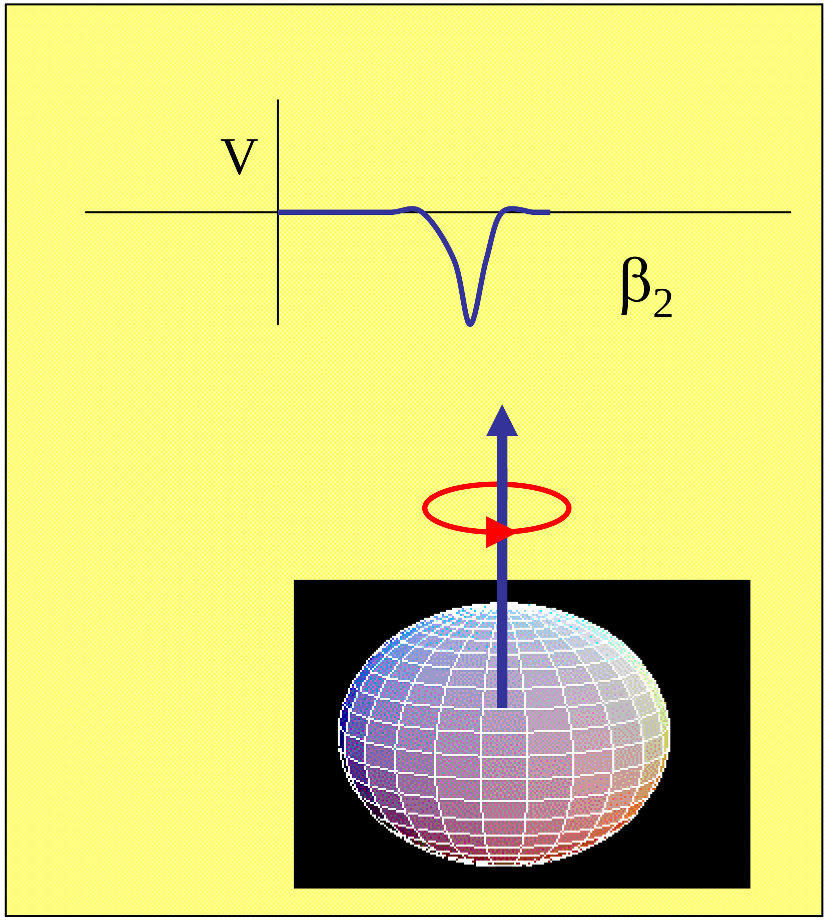
Quadrupole Vibrations in Nuclei ?

- Low-energy quadrupole vibrations in nuclei ?
  - Evidence?
  - Signatures?
  - Coupling schemes ?





http://npl.kyy.nitech.ac.jp/~arita/vib



Low energy vibrations

$\lambda = 1$ is equivalent to motion of center of mass

Lowest physical mode is $\lambda = 2$

Quadrupole

$\Downarrow$

$Y_{2\mu}$

2^+ ________

0^+ ________

We can use the m-scheme to see what states we can make when we couple together 2 quadrupole phonon excitations of order J=2ħ

.

(Note phonons are bosons, so we can couple identical 'particles' together).

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$M = \sum m_i$	J
2	2	4	
2	1	3	
2	0	2	4
2	−1	1	
2	−2	0	
1	1	2	
1	0	1	2
1	−1	0	
0	0	0	0

*Only positive total M values are shown: the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_2 \leq m_1$.

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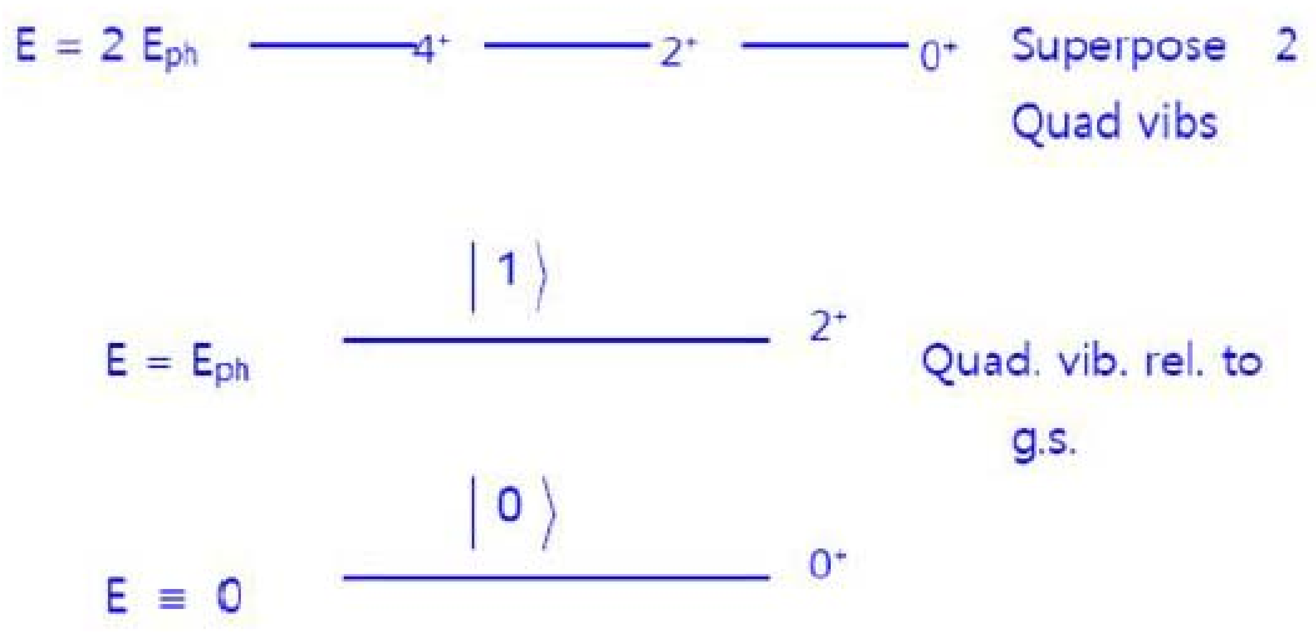
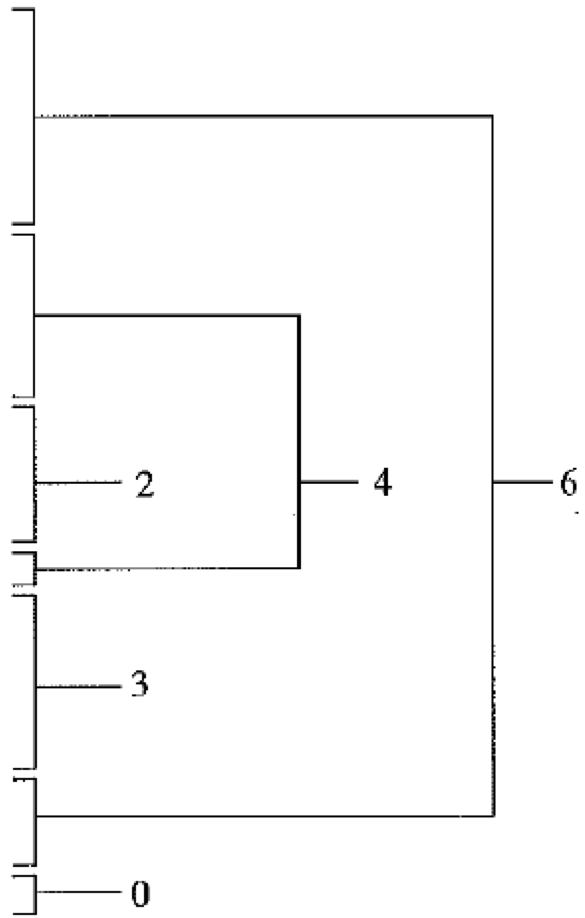


From,
Nuclear Structure From a Simple Perspective, by R.F. Casten, Oxford University Press.

Table 6.2 *m scheme for three-quadrupole phonon states**

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$J_3 = 2$ m_3	M
2	2	2	6
2	2	1	5
2	2	0	4
2	2	−1	3
2	2	−2	2
2	1	1	4
2	1	0	3
2	1	−1	2
2	1	−2	1
2	0	0	2
2	0	−1	1
2	0	−2	0
2	−1	−1	0
1	1	1	3
1	1	0	2
1	1	−1	1
1	1	−2	0
1	0	0	1
1	0	−1	0
0	0	0	0

J



$J = 6, 4, 3, 2, 0$

*Only positive total M values are shown; the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_3 \leq m_2 \leq m_1$.

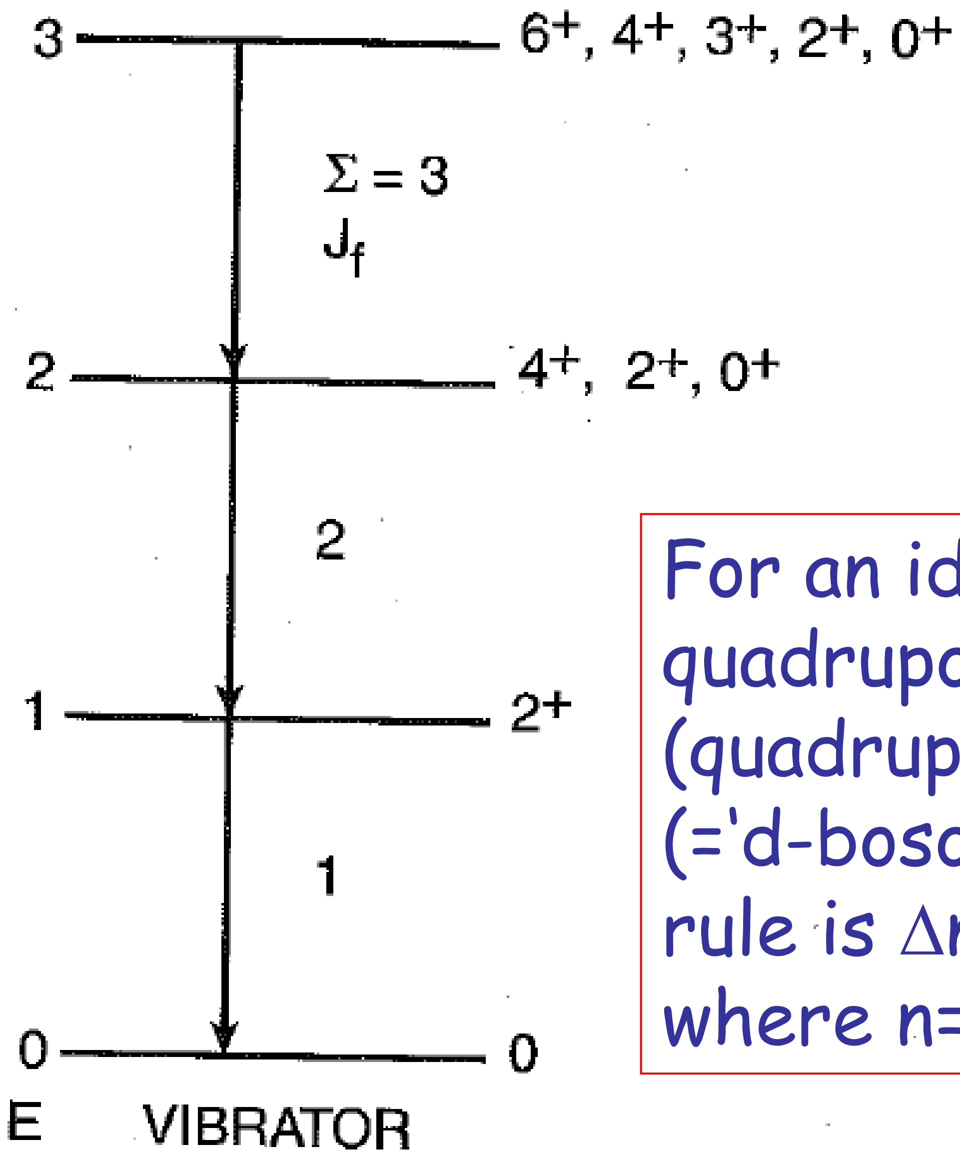


FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

For an idealised quantum quadrupole vibrator, the (quadrupole) phonon (='d-boson') selection rule is $\Delta n=1$,
where n=phonon number.

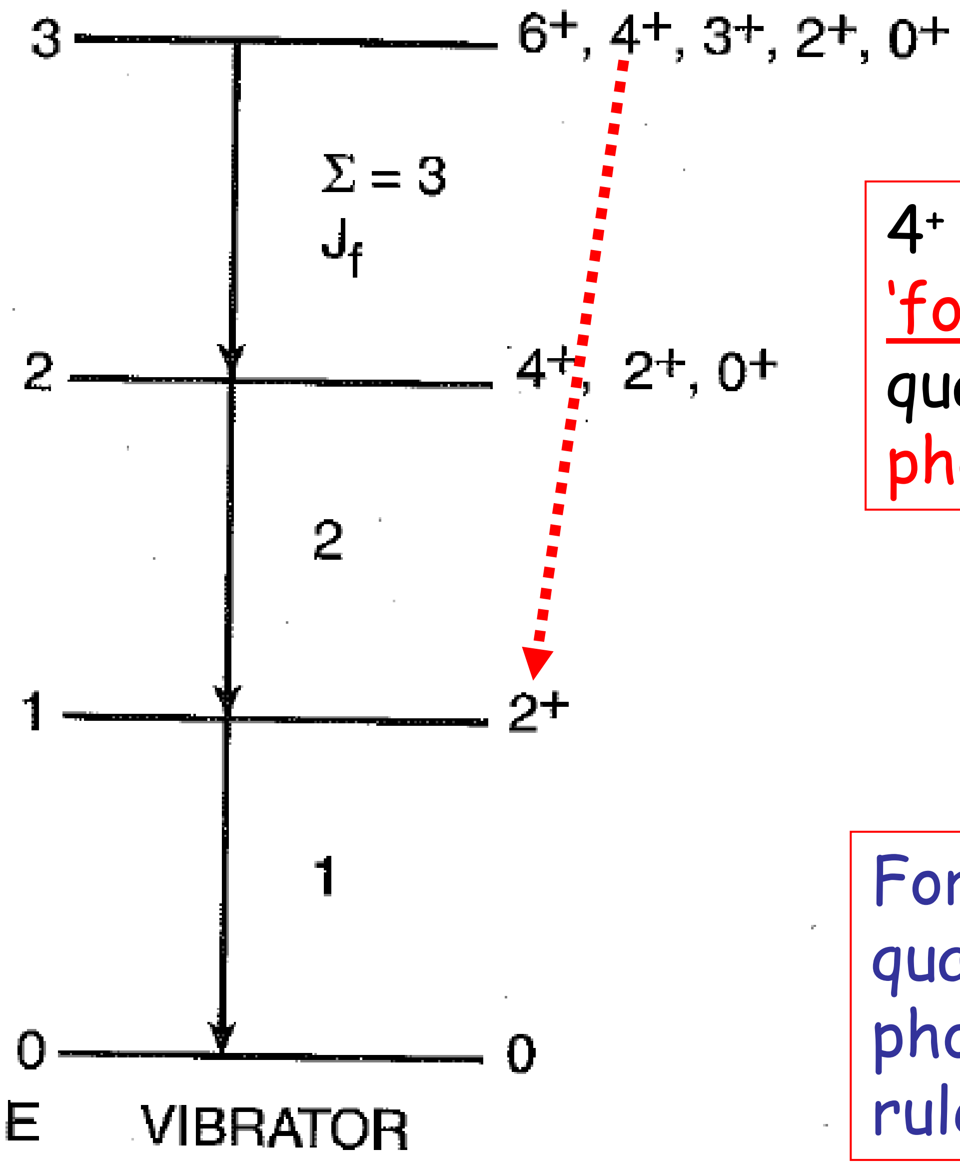


FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

$4^+ \rightarrow 2^+$ E2 from n=3 →n=1 is 'forbidden' in an idealised quadrupole vibrator by phonon selection rule.

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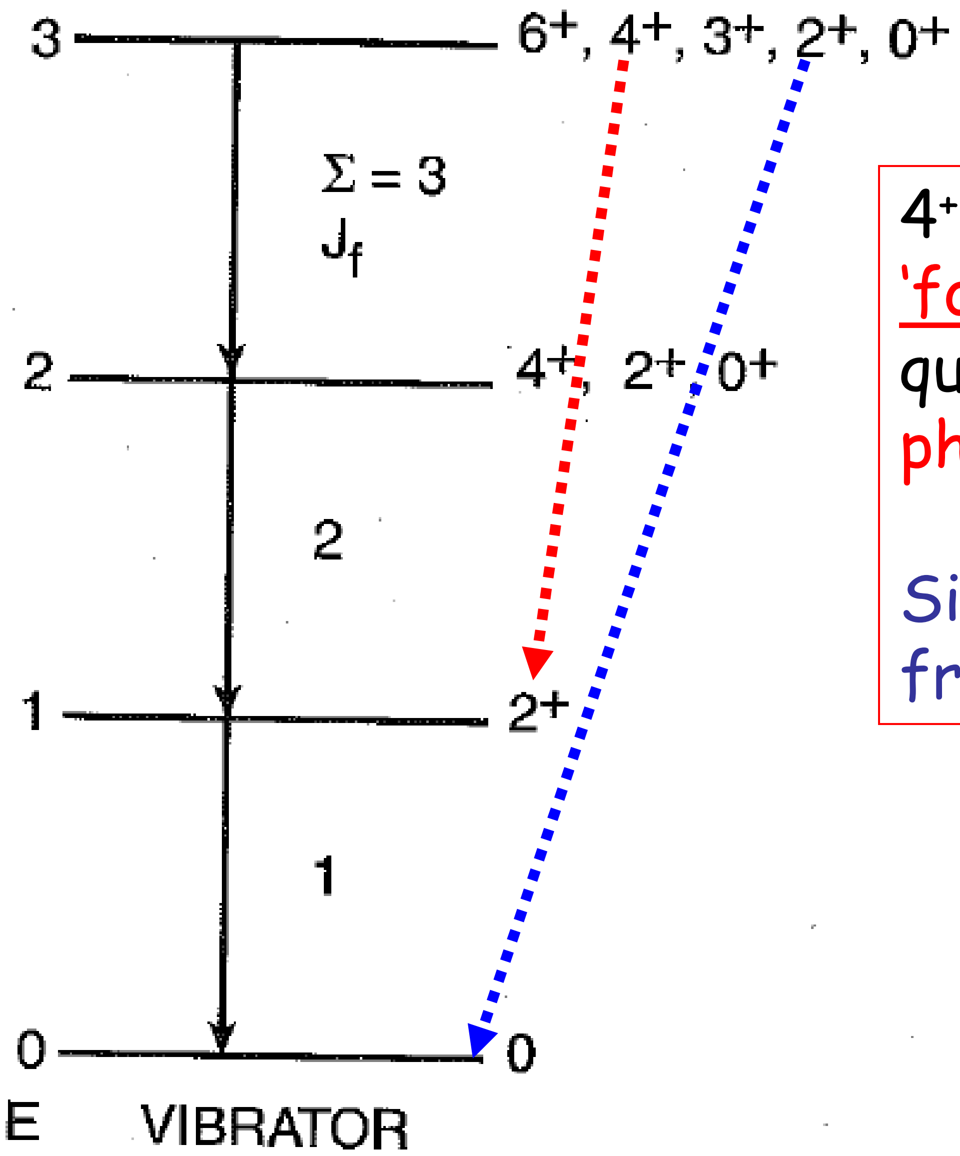

FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

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Similarly, E2 from $2^+ \rightarrow 0^+$ from n=3 →n=0 not allowed.

For an idealised quantum quadrupole vibrator, the phonon (='d-boson') selection rule is $\Delta n_p = 1$

Collective (Quadrupole) Nuclear Rotations and Vibrations

- What are the (idealised) excitation energy signatures for quadrupole collective motion (in even-even nuclei) ?
  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia (I) has rotational energies given by (from $E_{class}(rotor) = \frac{1}{2} L^2/2I$)

$$E_J = \frac{\hbar^2}{2\ell} J(J+1), \qquad \frac{E(4^+)}{E(2^+)} = \frac{4(5) = 20}{2(3) = 6} = 3.33$$

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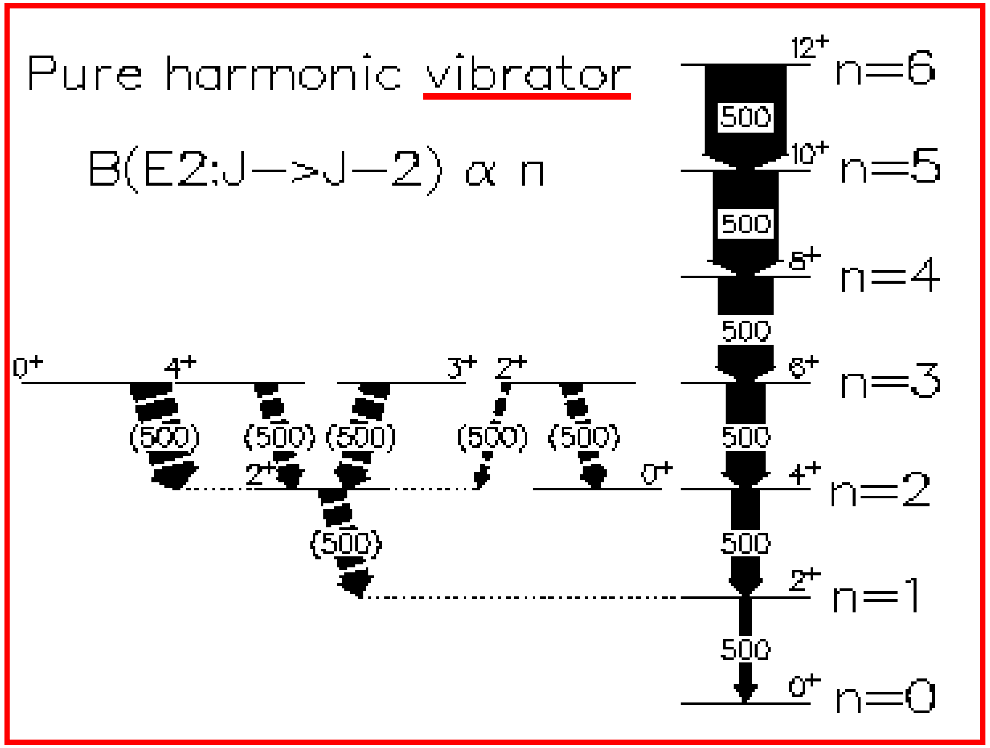
Perfect, quadrupole vibrator has energies given by the solution to the harmonic oscillator potential ($E_{classical} = {}^1/_2 k\Delta x^2 + p^2/2m$).

$$E_N = \hbar\omega N \qquad \frac{E(4^+)}{E(2^+)} = \frac{2}{1} = 2.00$$

Other Signatures of (perfect) vibrators and rotors

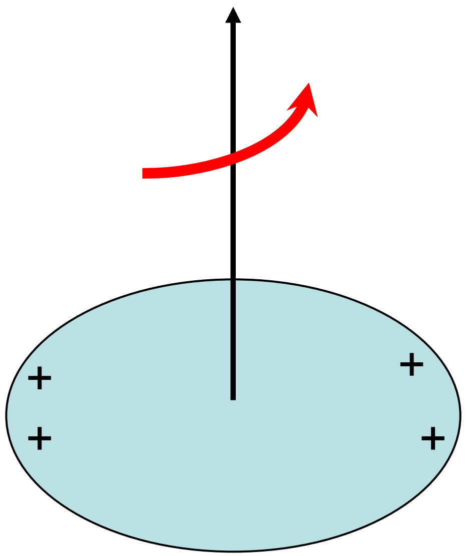
Decay lifetimes give B(E2) values.
Also selection rules important
(eg. $\Delta n=1$).



$E_\gamma=\hbar\omega$; $\Delta E\gamma$ $(J\rightarrow J\text{-}2)=0$

For ('real') examples, see
J. Kern et al., Nucl. Phys. **A593** (1995) 21

Other Signatures of (perfect) vibrators and rotors

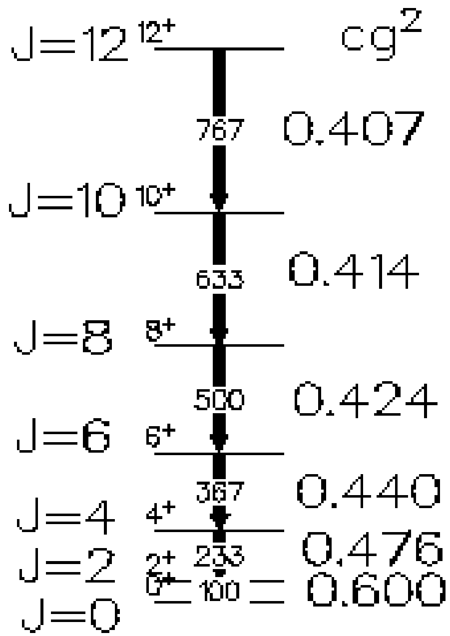


$E_x=(\hbar^2/_{2I})J(J+1)$

Perfect rotor

$B(E2) = kQ^2 \langle J_i K 20 | j_f K \rangle^2$

$$B(E2) \propto \frac{3J(J-1)(J+1)}{(2J-2)(2J-1)(2J+1)} \quad \sim 3/8$$



$E_x=(\hbar^2/_{2I})J(J+1)$, i.e., $E\gamma\ (J\rightarrow J-2)= (\hbar^2/_{2I})[J(J+1) - (J-2)(J-3)] = (\hbar^2/_{2I})(6J-6)$; $\Delta E\gamma=(\hbar^2/_{2I})*12=const.$

Other Signatures of (perfect) vibrators and rotors

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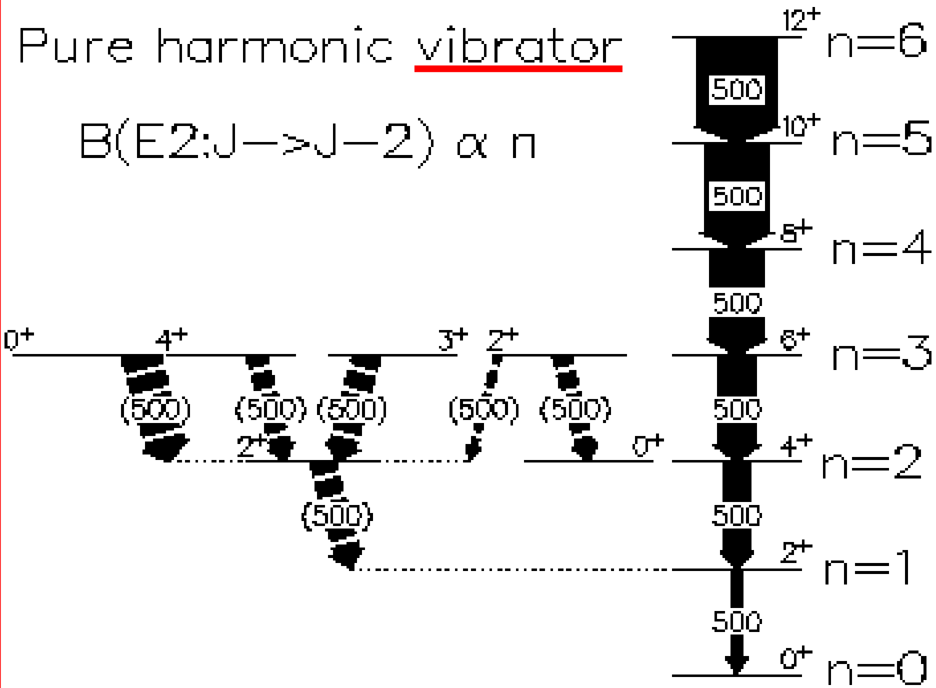
Pure harmonic vibrator

$B(E2: J \to J-2) \propto n$



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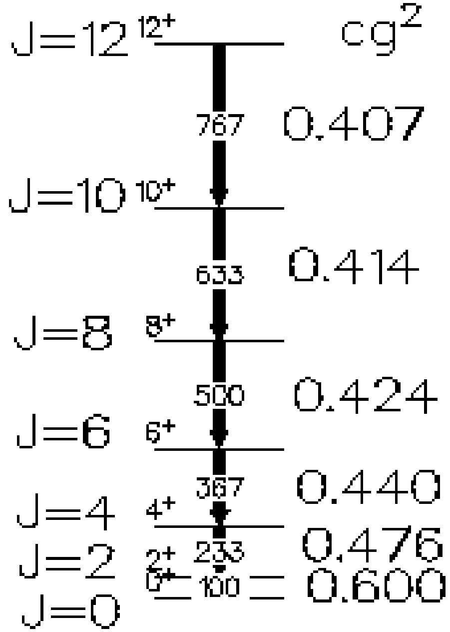

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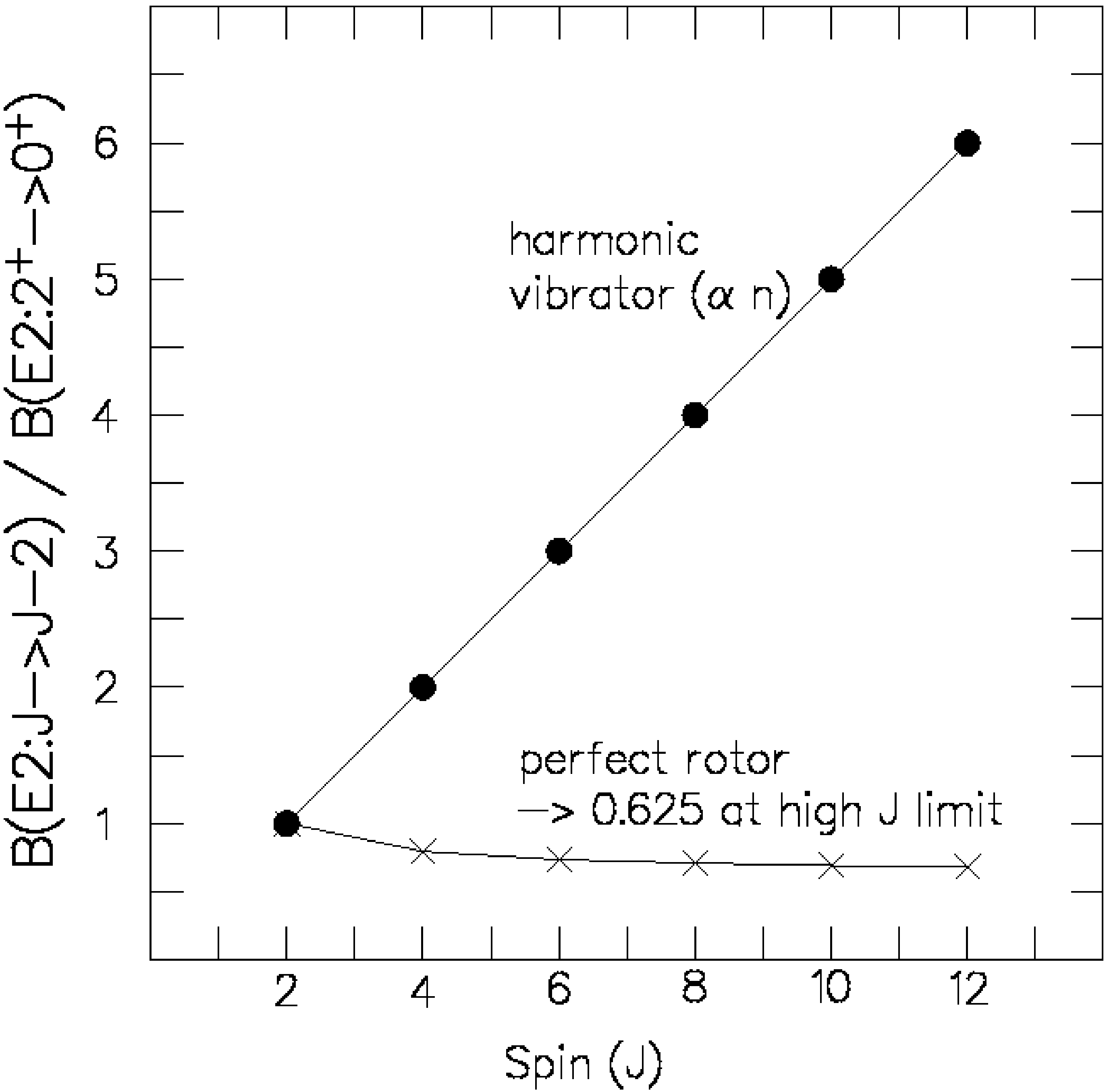
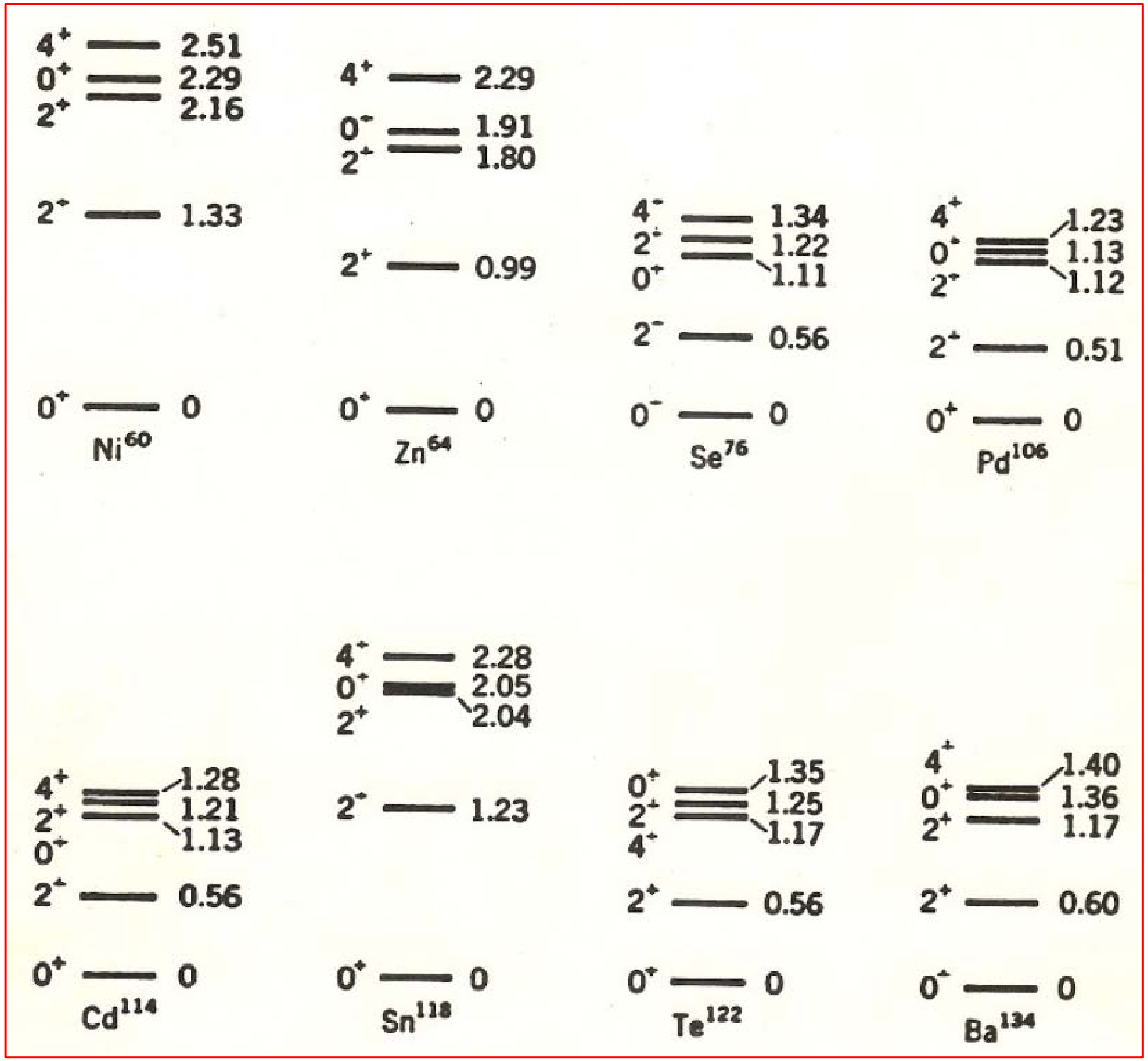


B(E2:J−>J−2) / B(E2:2+−>0+)
6
5
4
3
2
1
2
4
6
8
10
12
Spin (J)
harmonic
vibrator (α n)
perfect rotor
−> 0.625 at high J limit

So, what about 'real' nuclei ?

Many nuclei with R(4/2)~2.0 also show $I^{\pi}=4^+,2^+,0^+$ triplet states at ~2E(2^+).



First Observation of a Near-Harmonic Vibrational Nucleus

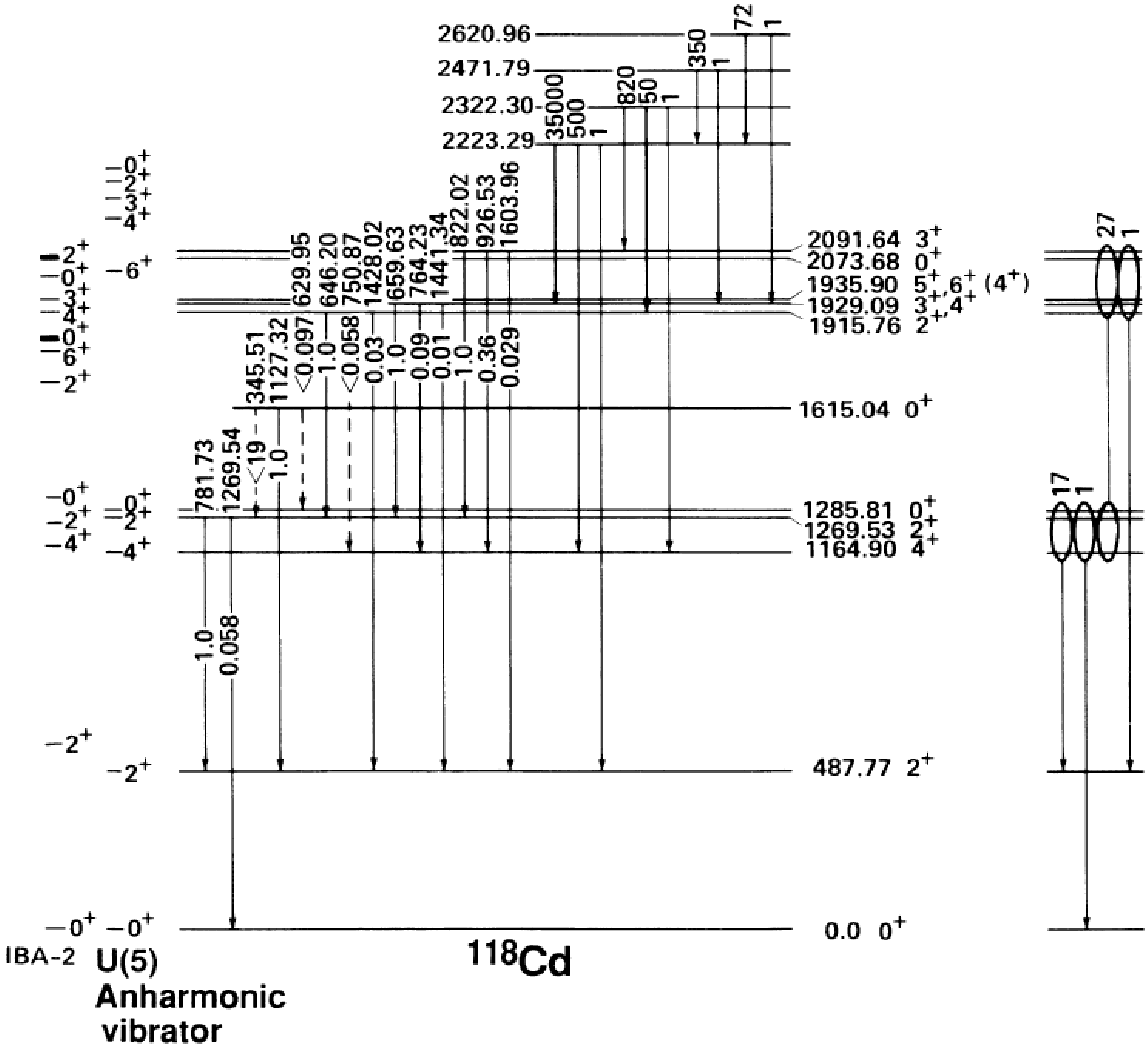
A. Aprahamian
Clark University, Worcester, Massachusetts 01610, and
Lawrence Livermore National Laboratory, Livermore, California 94550

D. S. Brenner
Clark University, Worcester, Massachusetts 01610

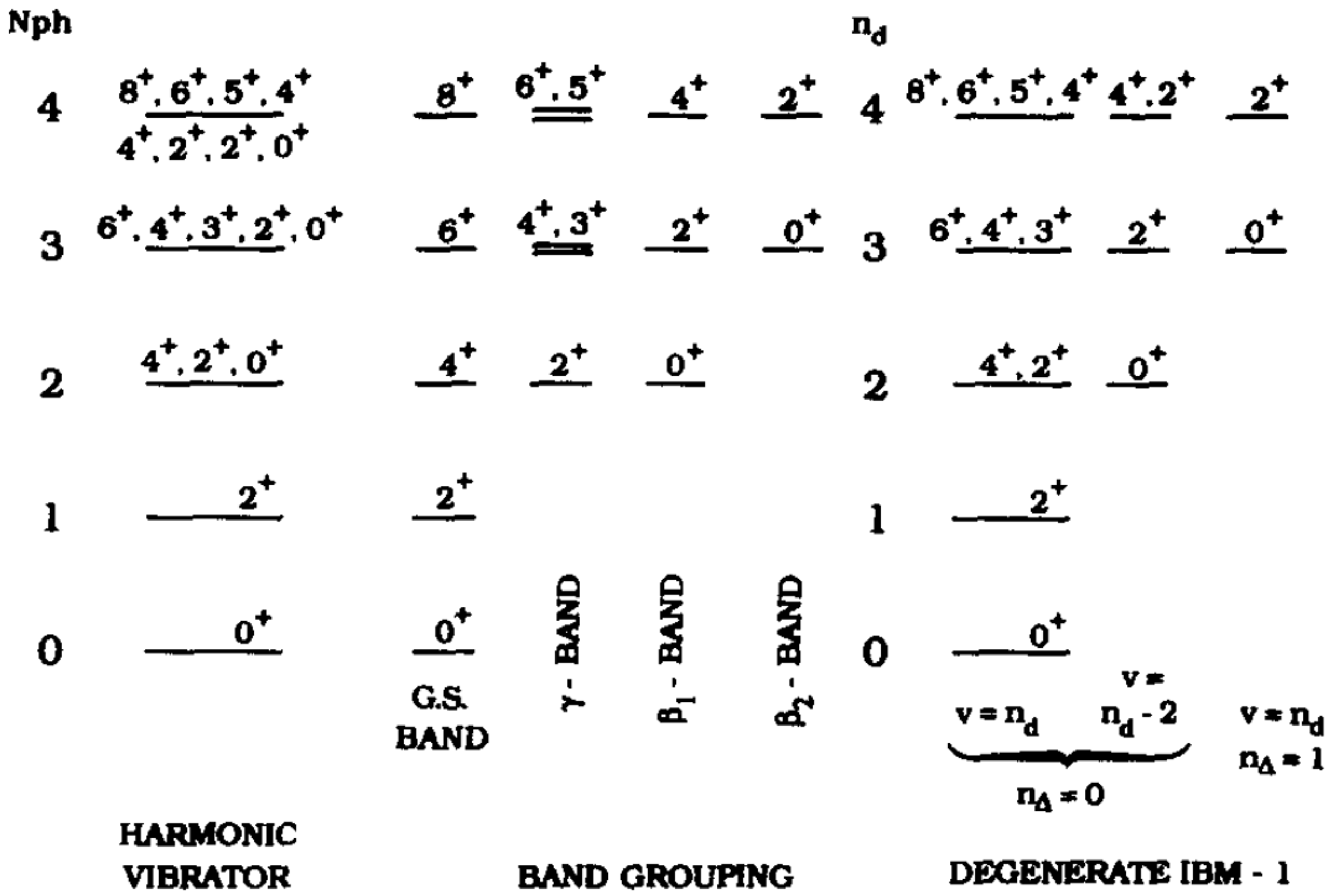
and

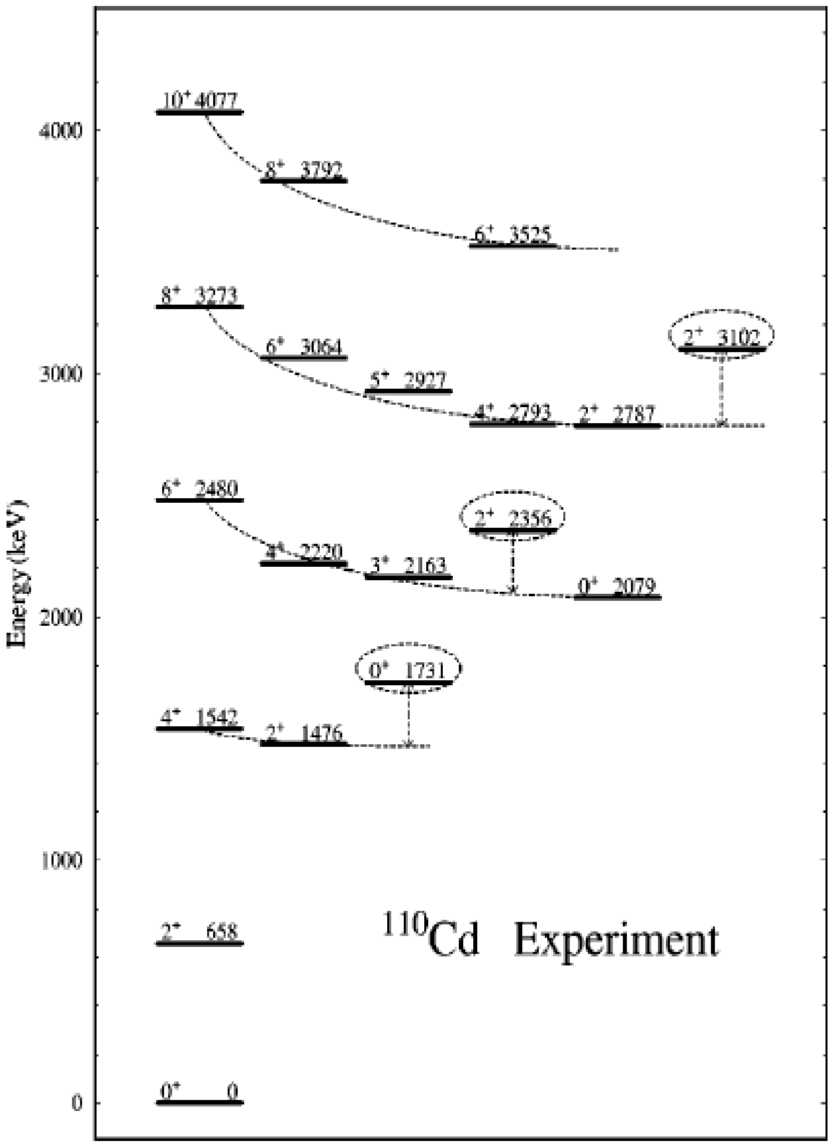
R. F. Casten, R. L. Gill, and A. Piotrowski[(a)]
Brookhaven National Laboratory, Upton, New York 11973
(Received 4 May 1987)

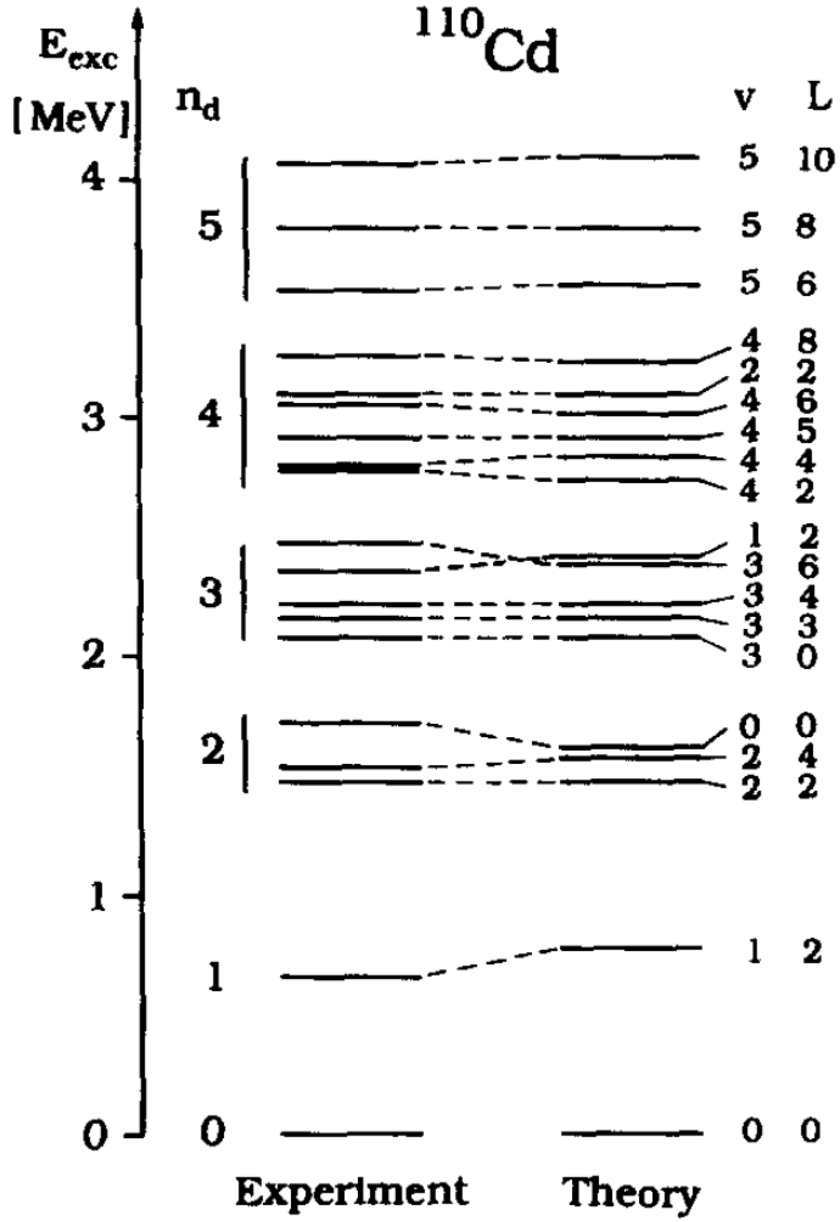
Evidence is presented for five closely spaced states in ^{118}Cd near $E_{ex} \simeq 2$ MeV, which are interpreted as a near-harmonic three-phonon quintuplet. Candidates for even higher-lying multiphonon states are also found. The experimental spectrum is compared with an anharmonic vibrator [or U(5) spectrum] and an interacting-boson-approximation calculation incorporating a two-particle, four-hole intruder configuration.



IBA-2
U(5)
Anharmonic vibrator
118Cd
2620.96
2471.79
2322.30
2223.29
2091.64 3+
2073.68 0+
1935.90 5+,6+ (4+)
1929.09 3+,4+
1915.76 2+
1615.04 0+
1285.81 0+
1269.53 2+
1164.90 4+
487.77 2+
0.0 0+


Nph
4
8+, 6+, 5+, 4+
4+, 2+, 2+, 0+
3
6+, 4+, 3+, 2+, 0+
2
4+, 2+, 0+
1
2+
0
0+
HARMONIC
VIBRATOR
8+
6+, 5+
4+
2+
6+
4+, 3+
2+
0+
4+
2+
0+
2+
0+
G.S.
BAND
γ - BAND
β1 - BAND
β2 - BAND
BAND GROUPING
nd
4
8+, 6+, 5+, 4+
4+, 2+
2+
3
6+, 4+, 3+
2+
0+
2
4+, 2+
0+
1
2+
0
0+
v = nd
v = nd - 2
nΔ = 0
v = nd
nΔ = 1
DEGENERATE IBM - 1


Energy (keV)
4000
3000
2000
1000
0
10+ 4077
8+ 3792
6+ 3525
8+ 3273
6+ 3064
5+ 2927
4+ 2793
2+ 2787
2+ 3102
6+ 2480
4+ 2220
3+ 2163
2+ 2356
0+ 2079
0+ 1731
4+ 1542
2+ 1476
2+ 658
0+ 0
110Cd Experiment

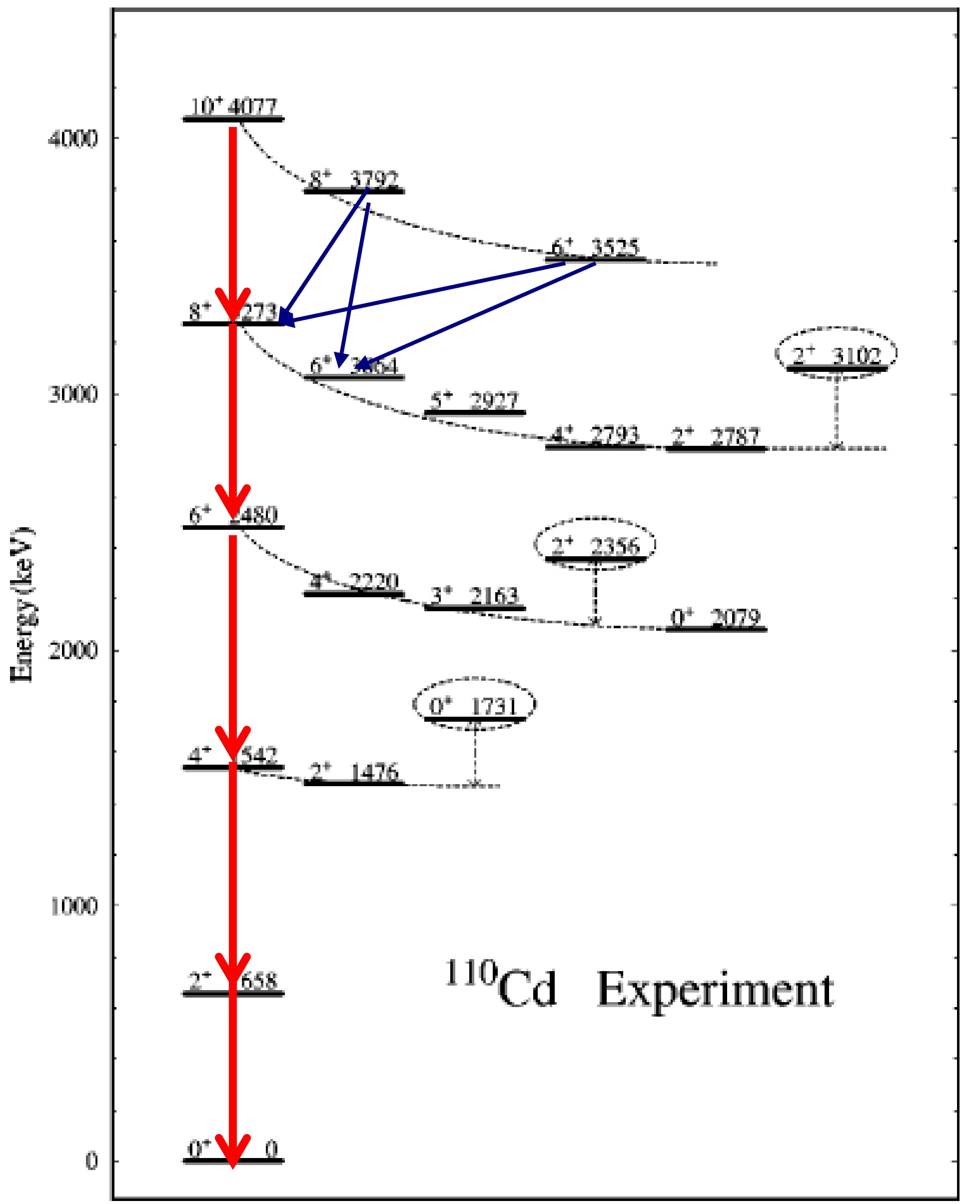

Eexc [MeV]
110Cd
nd
v L
5 10
5 8
5 6
4 8
2 2
4 6
4 5
4 4
4 2
1 2
3 6
3 4
3 3
3 0
0 0
2 4
2 2
1 2
0 0
Experiment
Theory

Note on 'near-yrast feeding' for vibrational states in nuclei.

If 'vibrational' states are populated in very high-spin reactions (such as heavy ion induced fusion evaporation reactions), only the decays between the (near)-YRAST states are likely to be observed.

The effect is to (only?) see the 'stretched' E2 cascade from $J_{max} \rightarrow J_{max}-2$ for each phonon multiplet.

↓ = the 'yrast' stretched E2 cascade.



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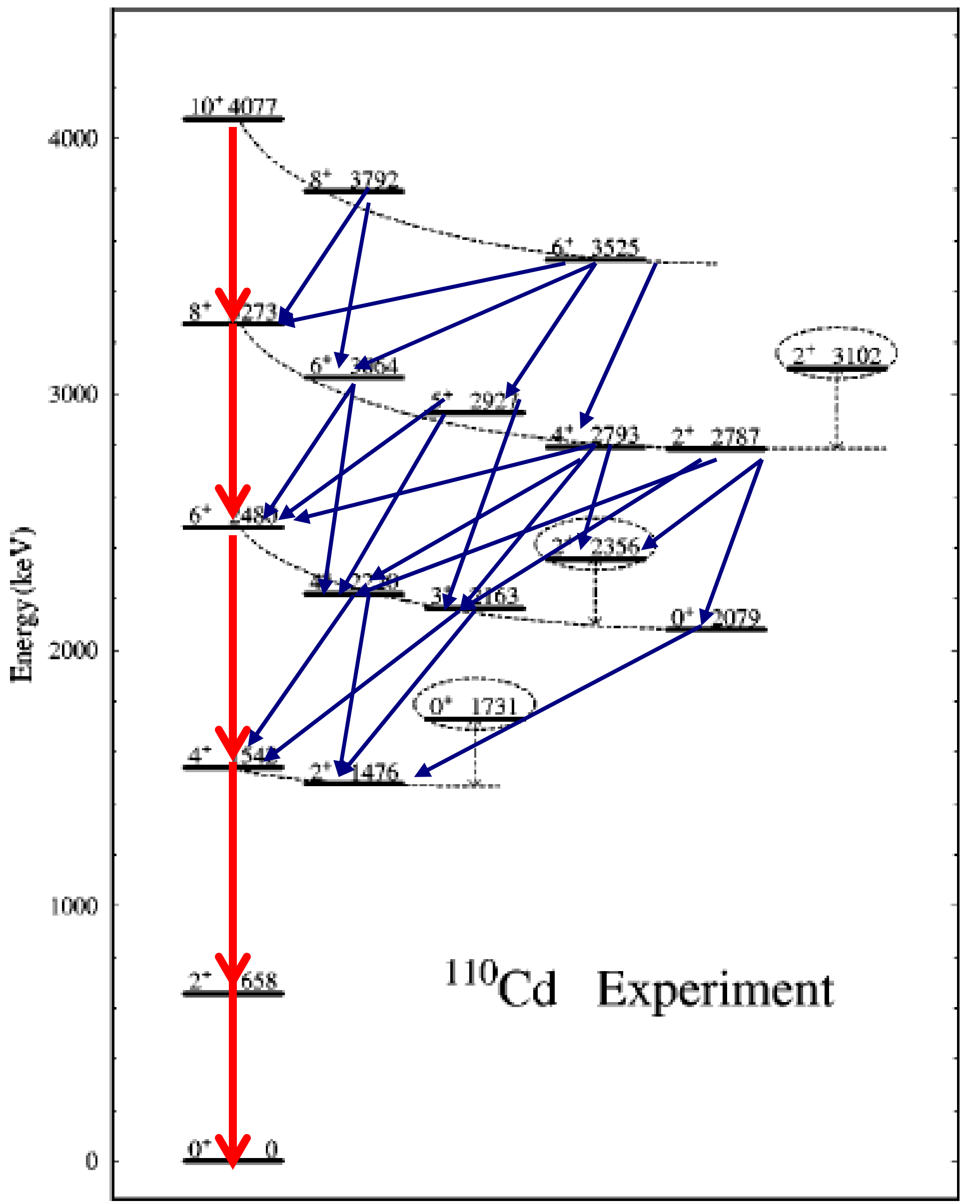
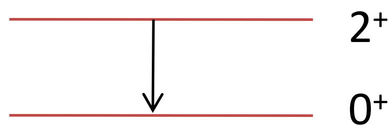


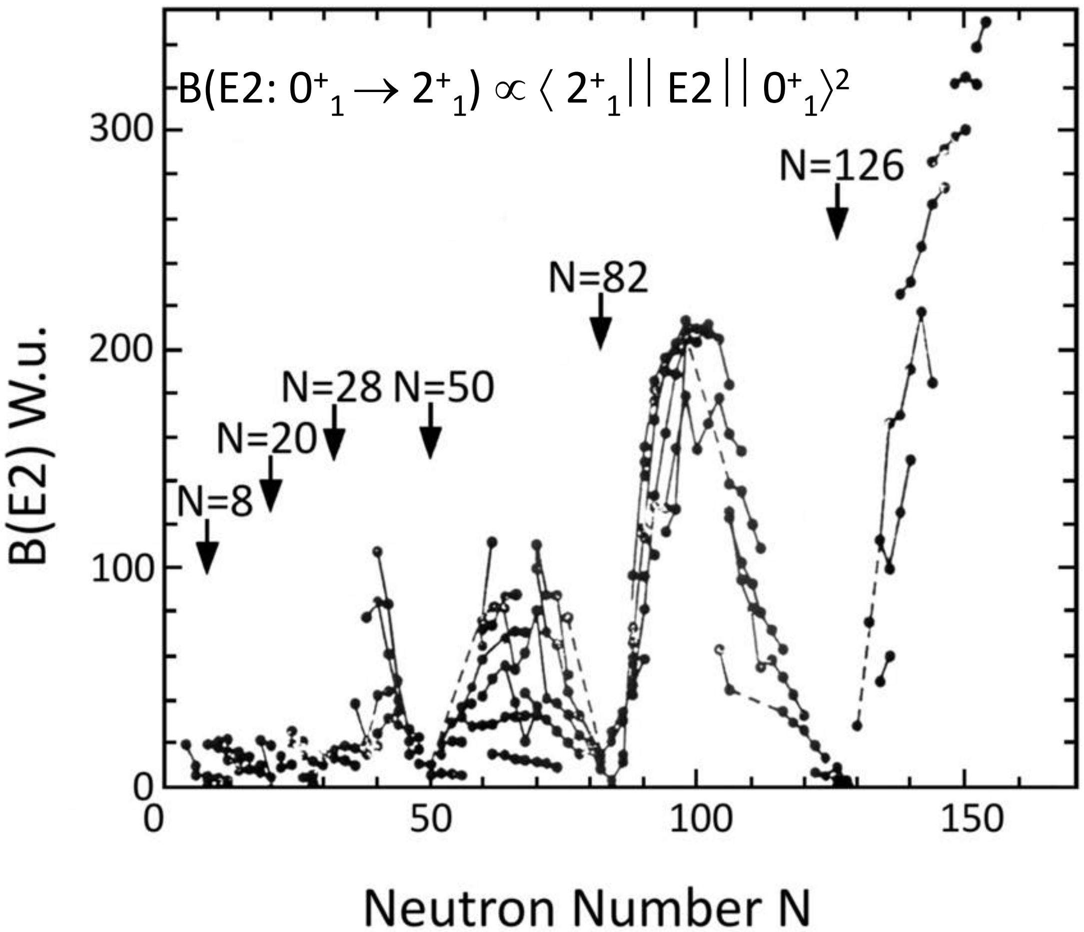
↓ = the 'yrast' stretched E2 cascade.

Nuclear Rotations and Static Quadrupole Deformation

$$T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2)$$

$T(E2)$ = transition probability = $1/\tau$ (secs);
E_γ = transition energy in MeV





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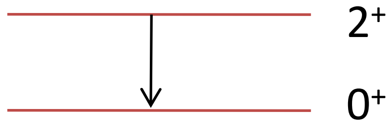
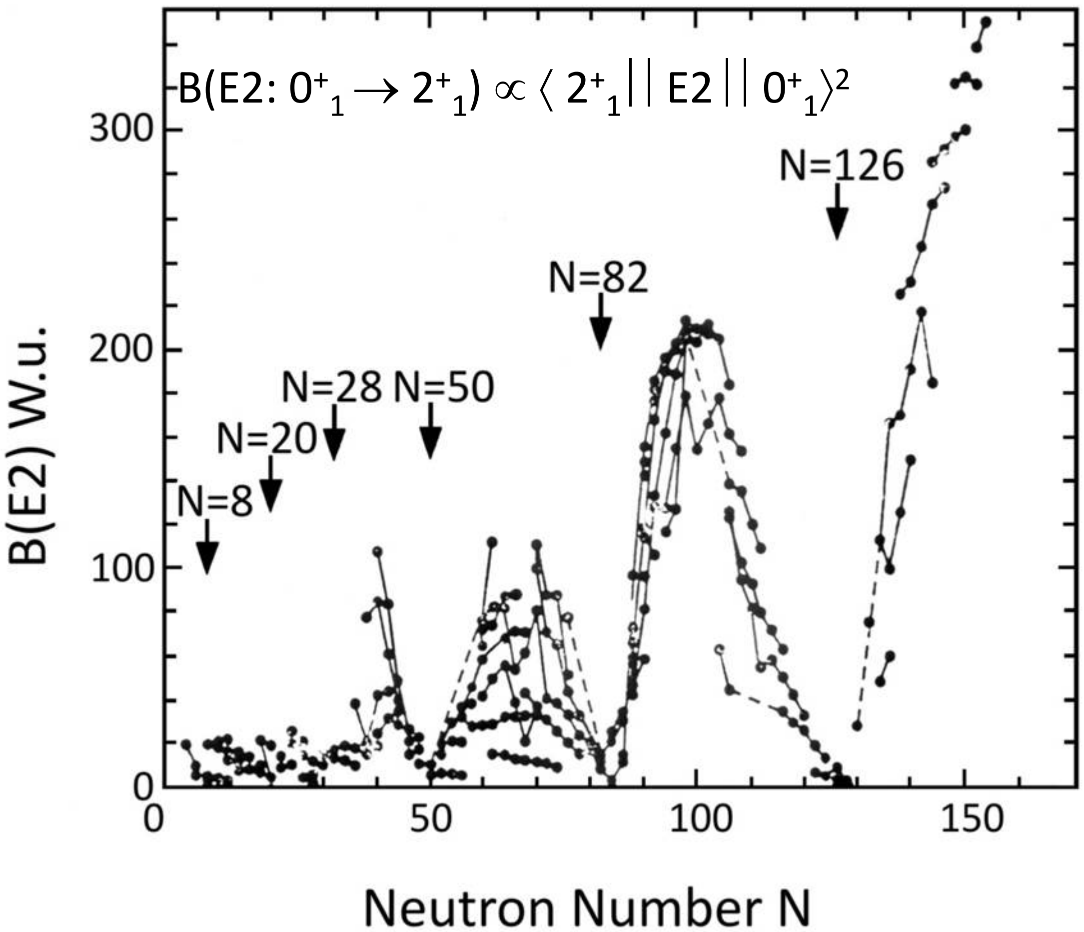




Qo = INTRINSIC (TRANSITION) ELECTRIC QUADRUPOLE MOMENT.

This is intimately linked to the electrical charge (i.e. proton) distribution within the nucleus.

Non-zero Qo means some deviation from spherical symmetry and thus some quadrupole 'deformation'.

Rotational model, B(E2: I→I-2) gives:

$$B(E2) = \frac{5}{16\pi} Q_o^2 \frac{3(I-K)(I-K-1)(I+K)(I+K-1)}{(2J-2)(2J-1)J(2J+1)}$$

Bohr and Mottelson, Phys. Rev. **90**, 717 (1953)

Isomer spin in ^{180}Hf, I^{π}>11 shown later to be $I^{\pi}=K^{\pi}=8^{-}$ by Korner et al. Phys. Rev. Letts. **27**, 1593 (1971)).

K-value very important in understanding isomers.

$$E_x = (\hbar^2/_{2I})*J(J+1)$$

I = moment of inertia.
This depends on nuclear deformation and I~ kMR²
Thus, I ~ kA$^{5/3}$
(since r_{nuc}=1.2A$^{1/3}$fm)

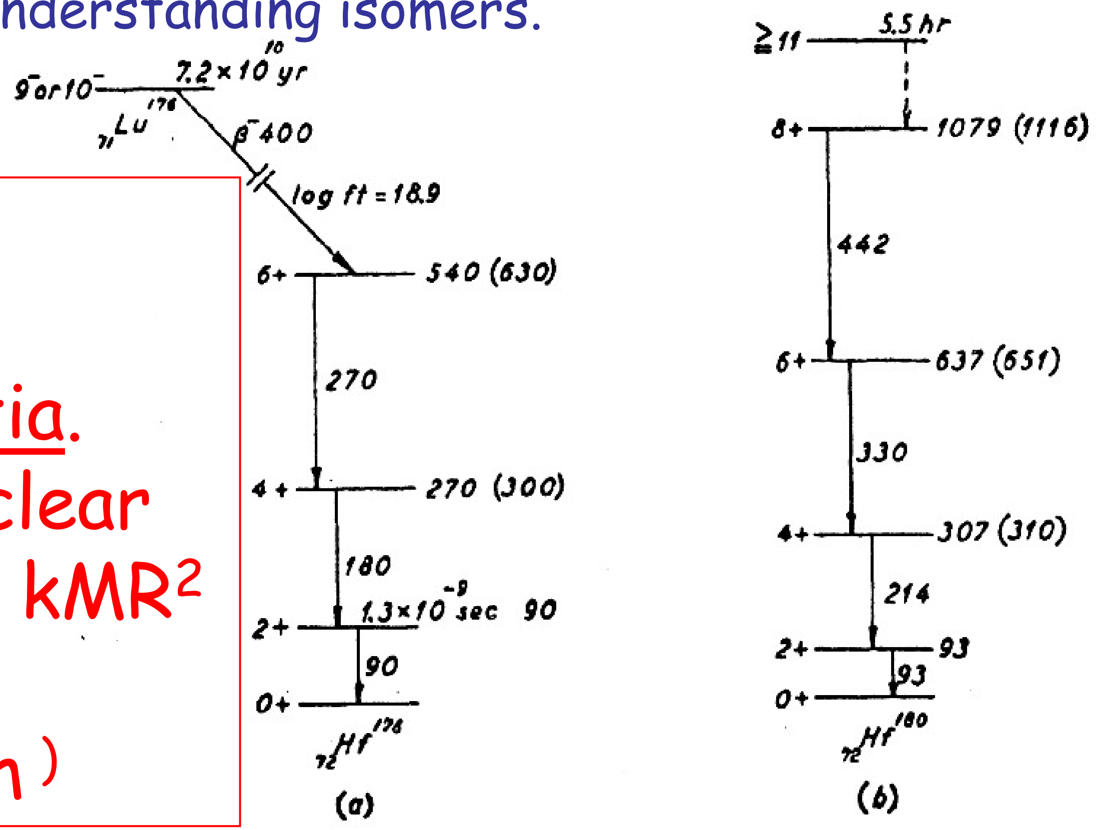
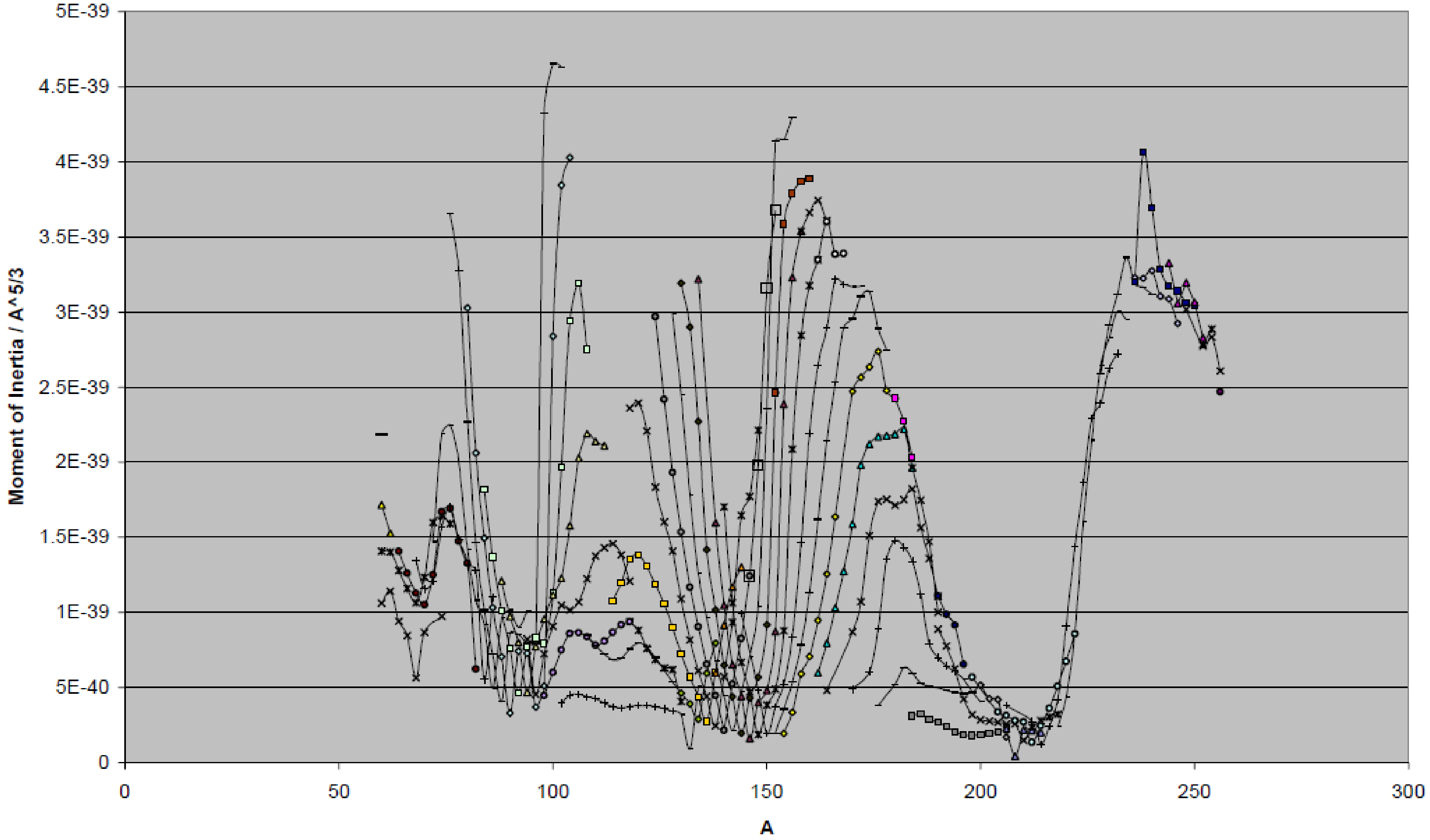
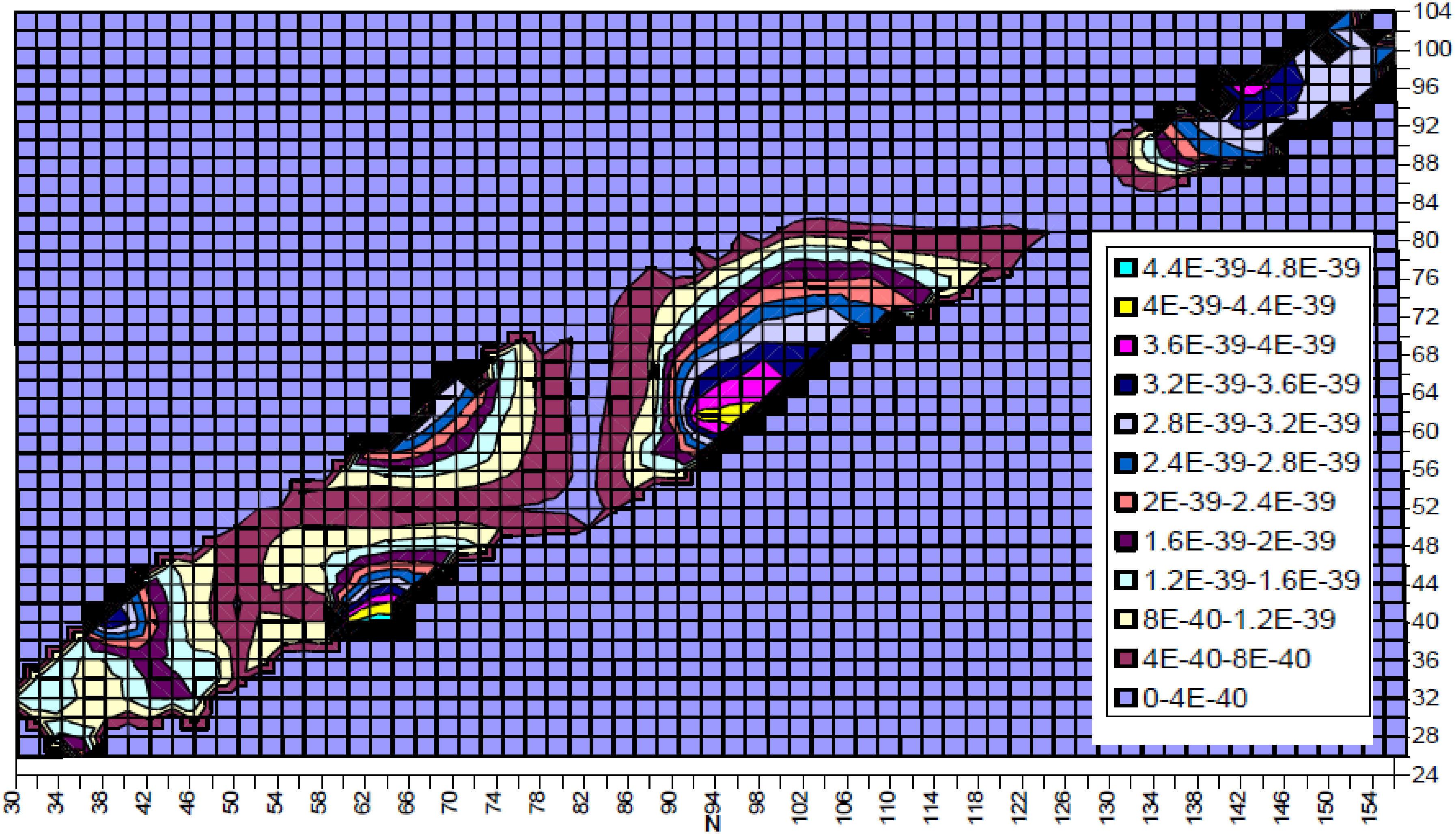


FIG. 2. Suggested decay schemes of $_{71}Lu^{176}$ and $_{72}Hf^{180}$. The gamma-ray energies as well as the β-decay data of Lu^{176} are taken from reference 2. The excitation energies listed in parentheses are obtained from Eq. (1) adjusted to give the energy of the first excited state. All energies are given in kev.

Therefore, plotting the moment of inertia, divided by $A^{5/3}$ should give a comparison of nuclear deformations across chains of nuclei and mass regions….

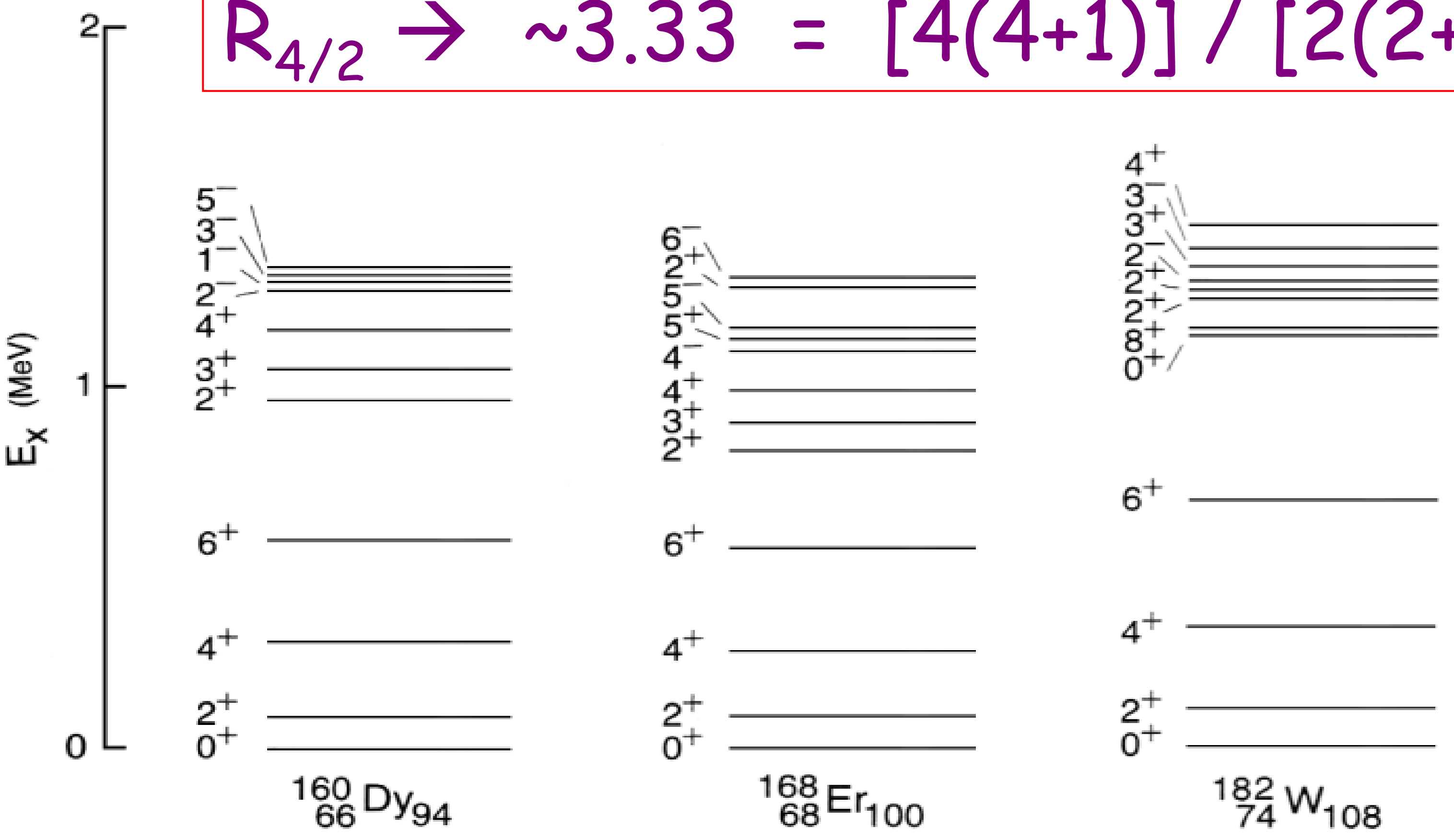


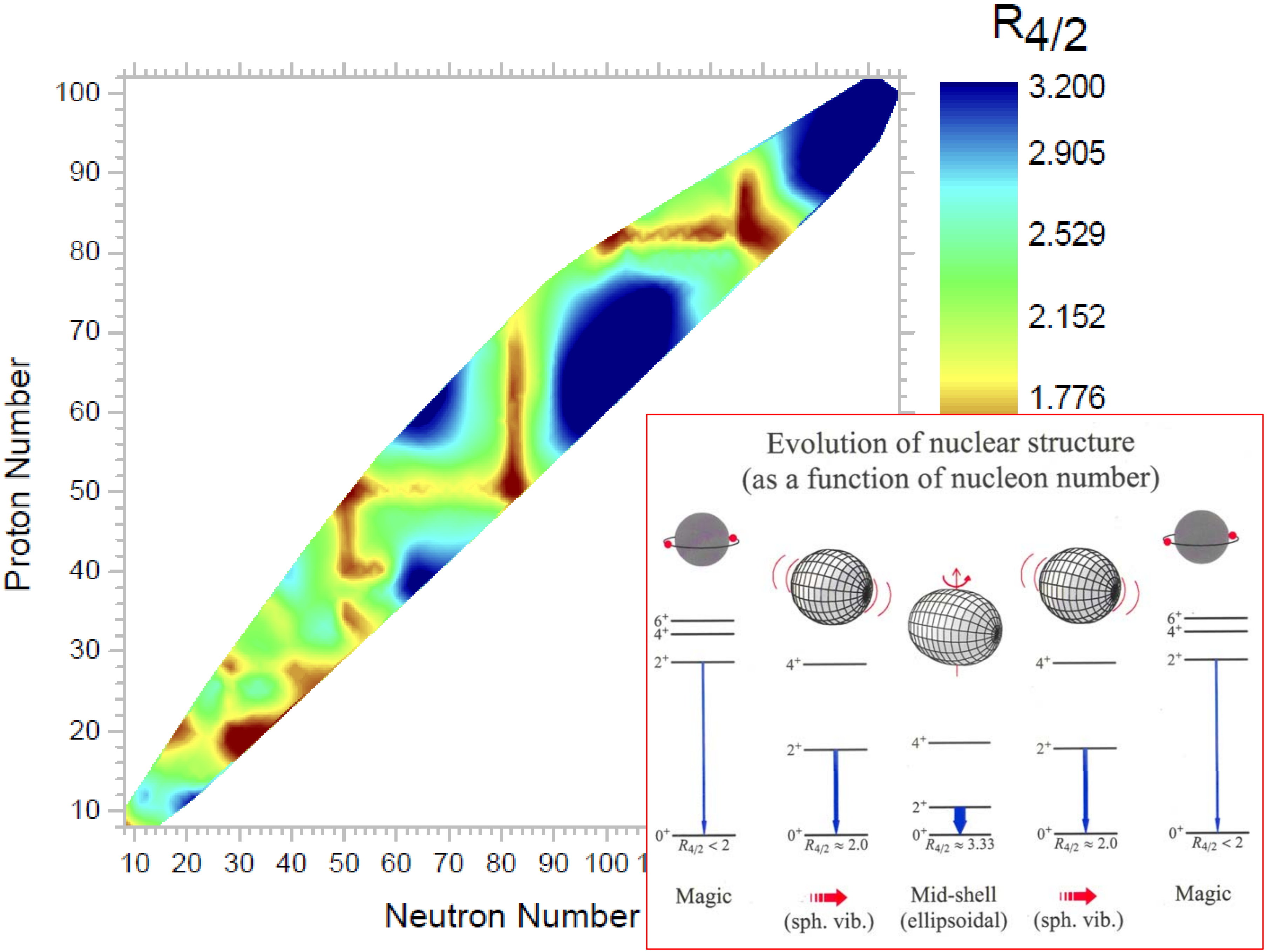


Nuclear static moment of inertia for E(2+) states divided by $A^{5/3}$ trivial mass dependence.
Should show regions of quadrupole deformation.

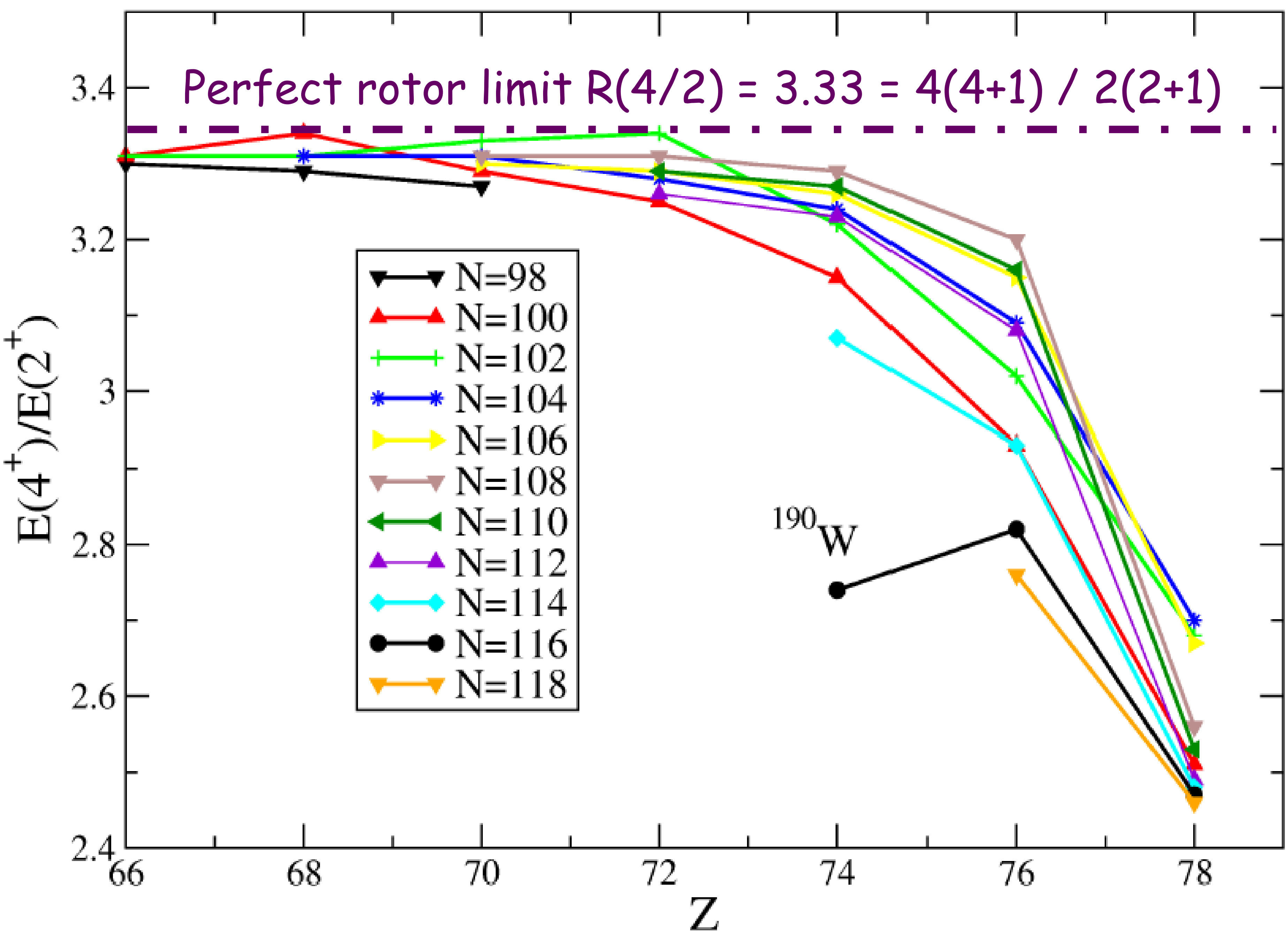
Lots of valence nucleons of both types: emergence of deformation and therefore rotation

$$R_{4/2} \rightarrow \sim 3.33 = [4(4+1)] / [2(2+1)]$$





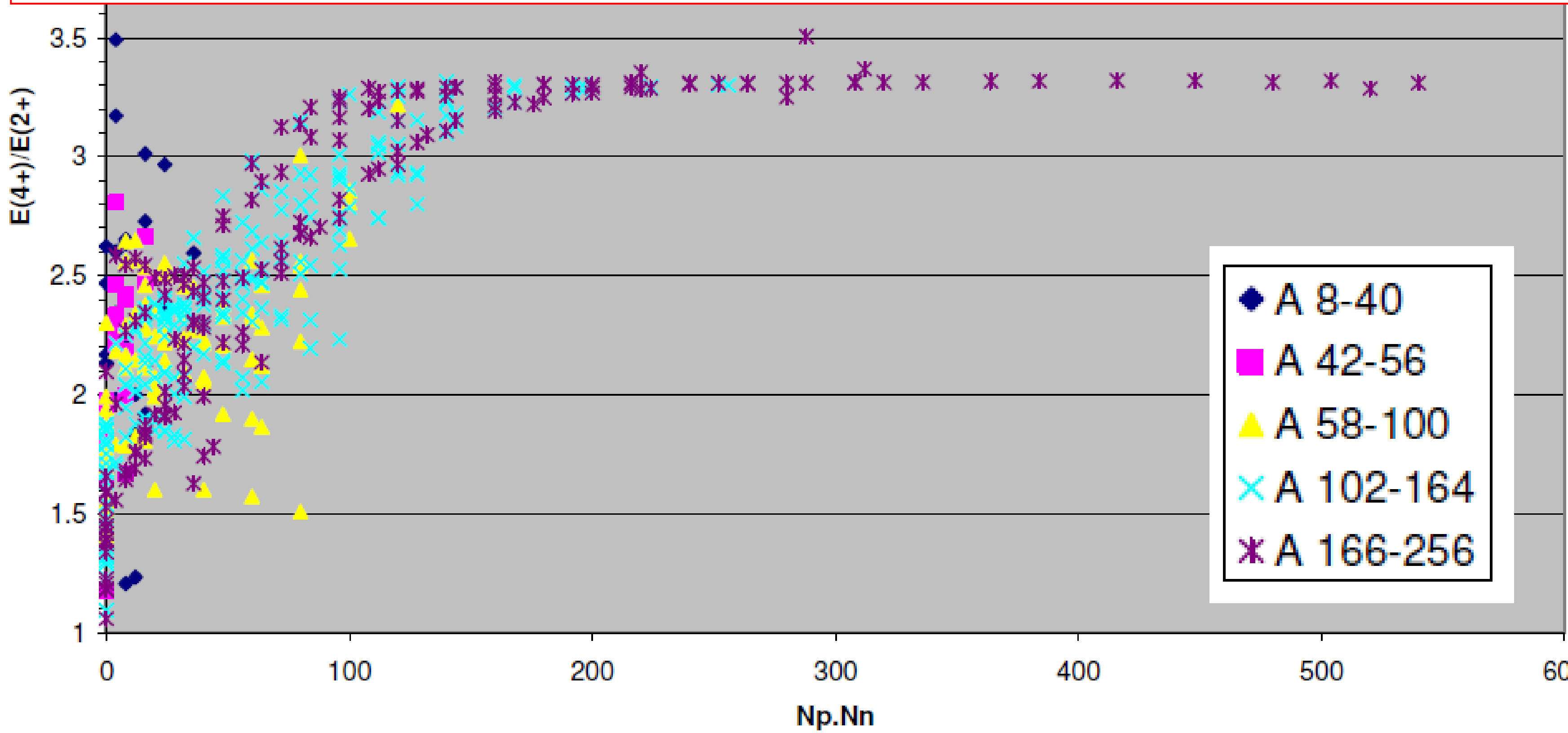
R4/2
3.200
2.905
2.529
2.152
1.776
Proton Number
Neutron Number
Evolution of nuclear structure
(as a function of nucleon number)
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)
(sph. vib.)
Magic



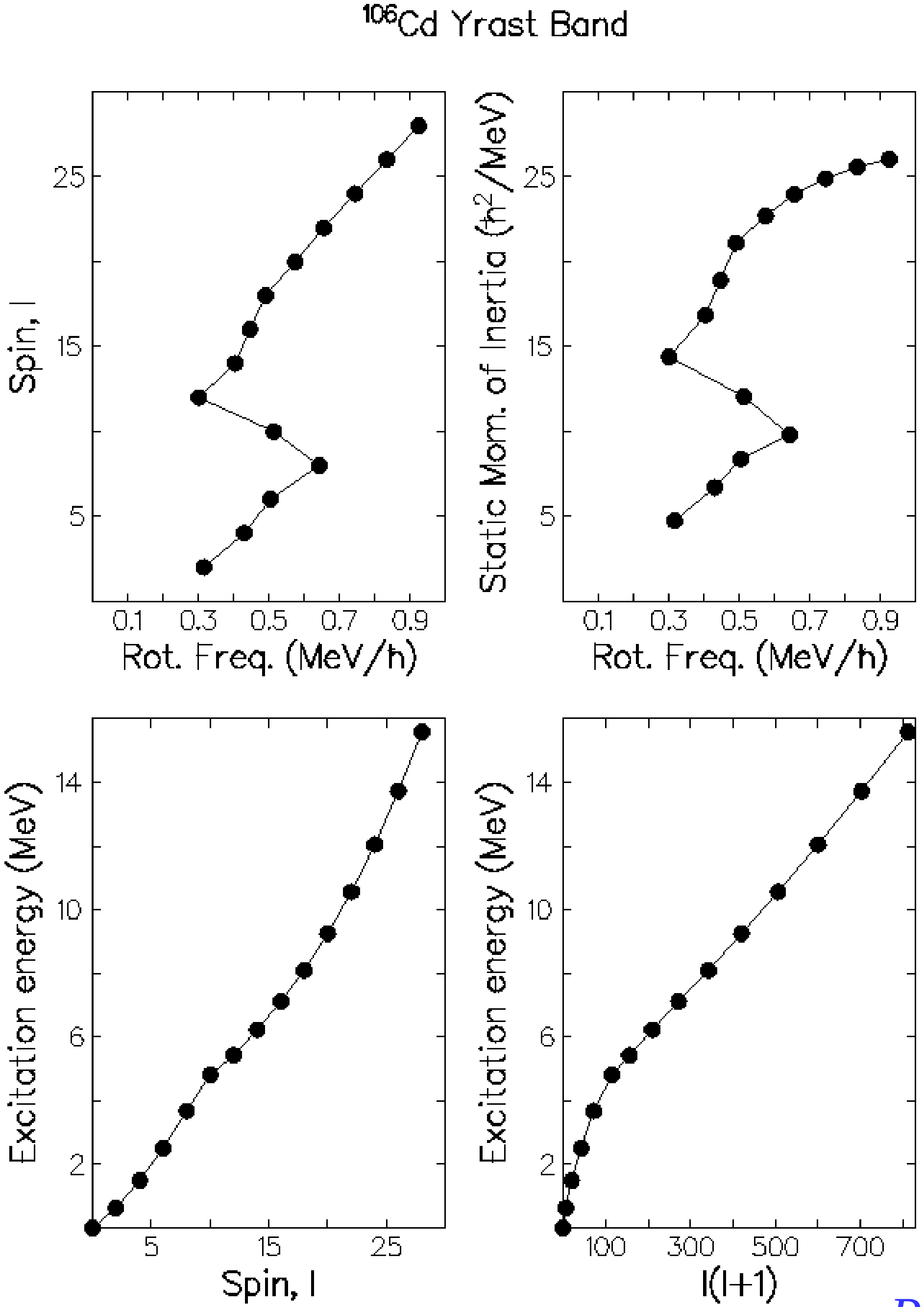
Perfect rotor limit R(4/2) = 3.33 = 4(4+1) / 2(2+1)
N=98
N=100
N=102
N=104
N=106
N=108
N=110
N=112
N=114
N=116
N=118
190W
E(4+)/E(2+)
Z
3.4
3.2
3
2.8
2.6
2.4
66
68
70
72
74
76
78

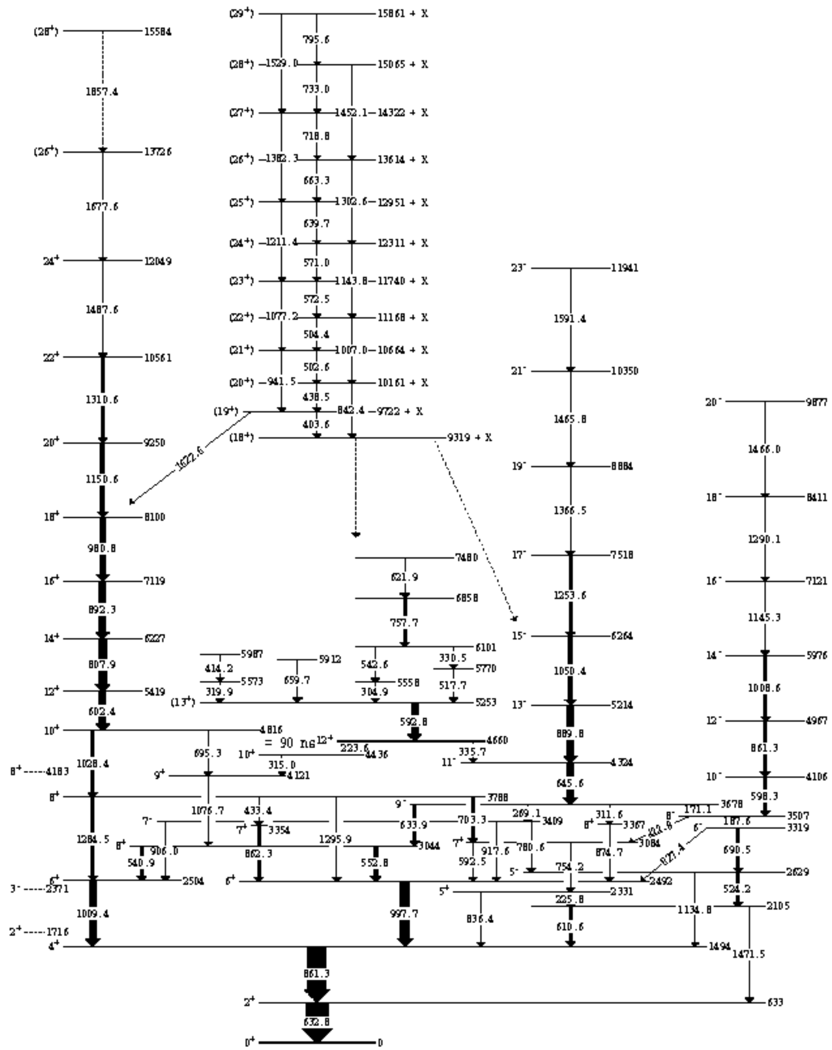
Best nuclear 'rotors' have largest values of $N_\pi.N_\nu$

This is the product of the number of 'valence' protons, N_π X the number of valence neutrons N_ν



Alignments and rotational motion in ‘vibrational’ ^{106}Cd (Z=48, N=58),





PHR et al. Nucl. Phys. ***A586*** *(1995) p351*

Some useful nuclear rotational, 'pseudo-observables'…

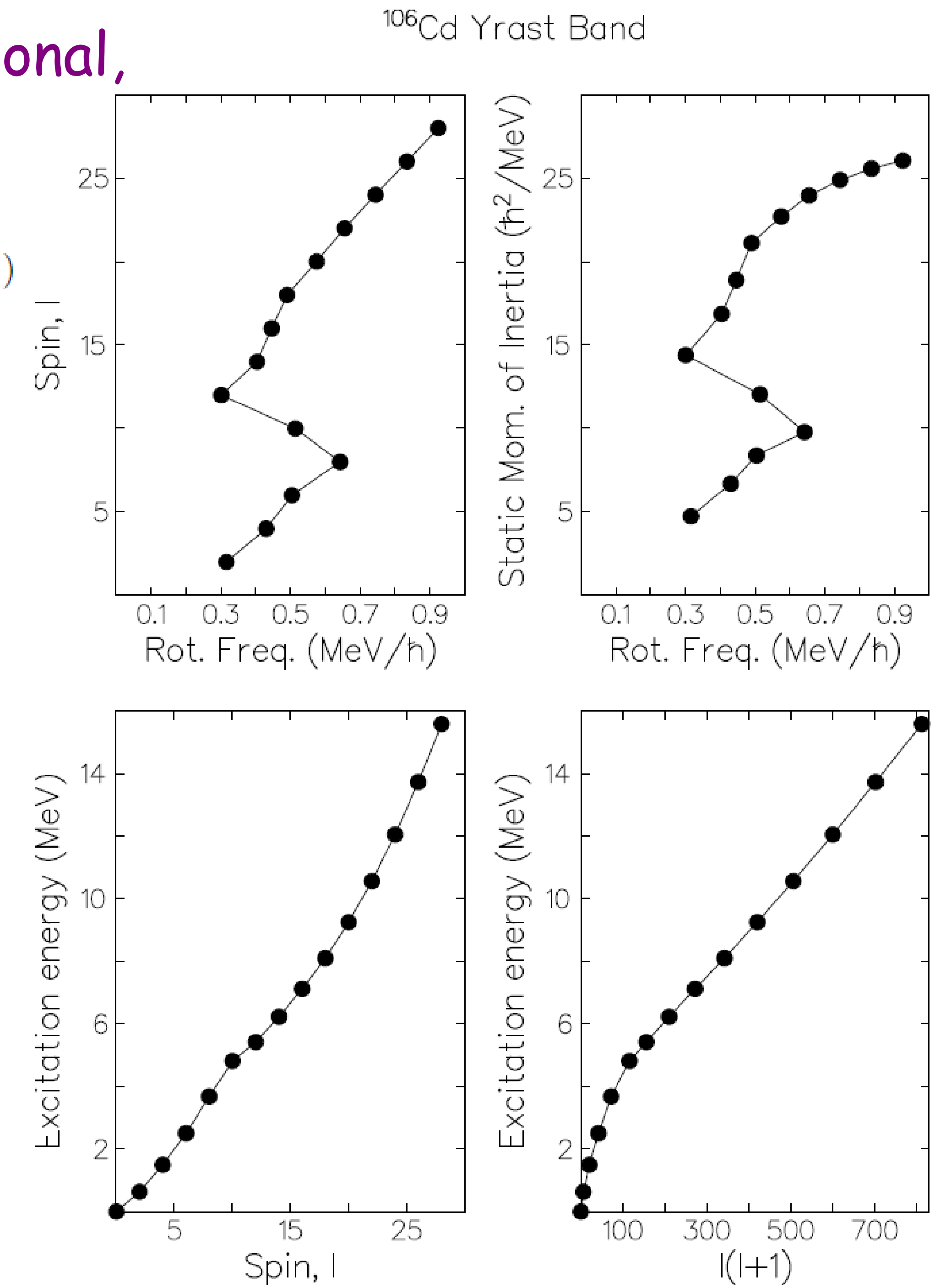
$$E_{rot}(I) = \frac{\hbar^2}{2\mathcal{I}^{(0)}(I)} I(I+1)$$

The kinematic moment of inertia is given by

$$\mathcal{I}^{(1)}(I) = \frac{I}{\omega}$$

while the dynamic moment is given by

$$\mathcal{I}^{(2)} = \frac{dI}{d\omega} \approx \frac{4\hbar}{\Delta E_\gamma}$$



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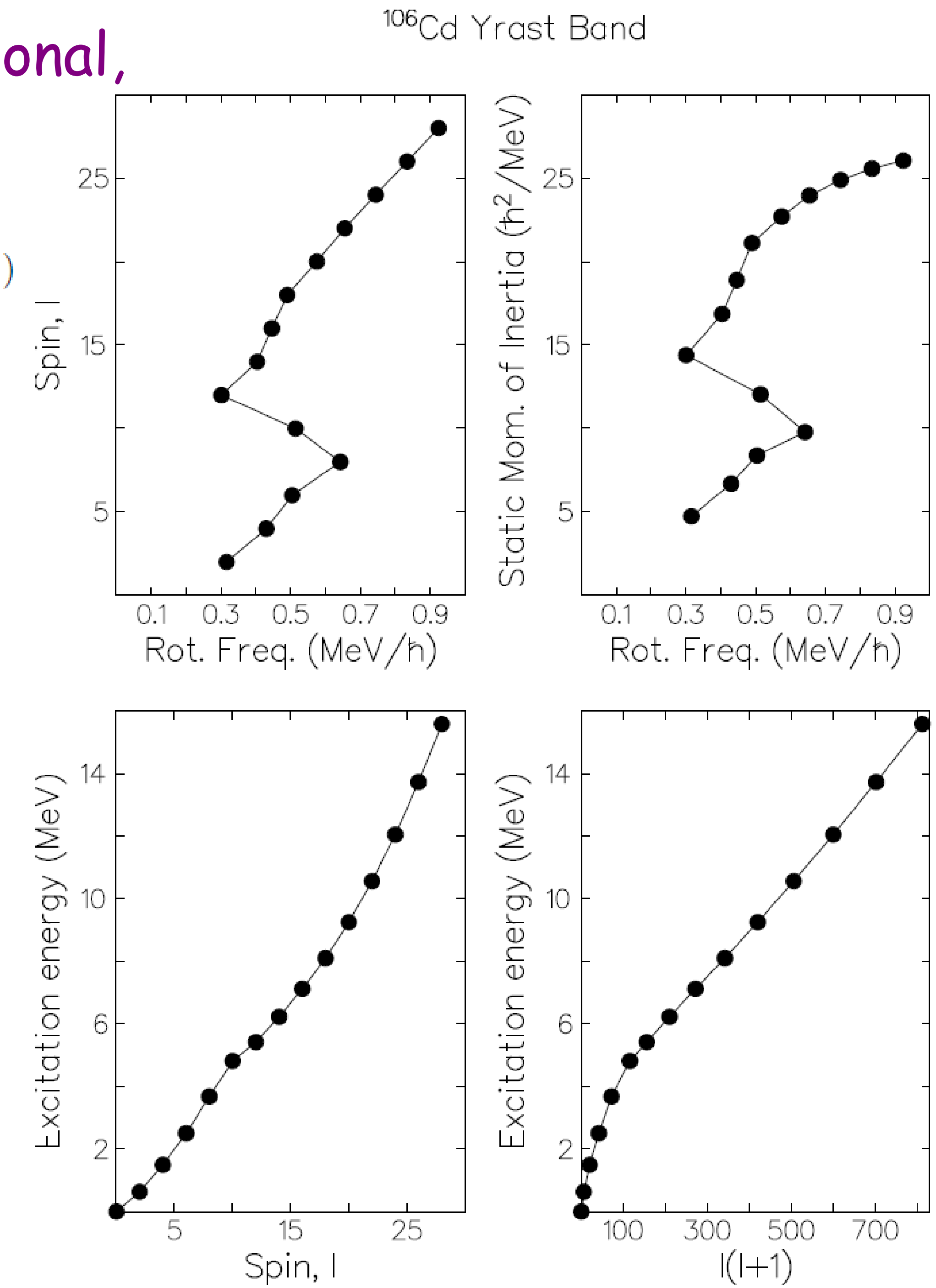
Rotational 'frequency', ω given by,

$$\omega = \frac{dE(I)}{dI_x(I)} \approx \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

$$I_x(I) = \sqrt{I(I+1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2 - K^2}$$

$$\omega \approx \frac{E_\gamma}{\sqrt{(I+\frac{3}{2})^2 - K^2} - \sqrt{(I-\frac{1}{2})^2 - K^2}}$$



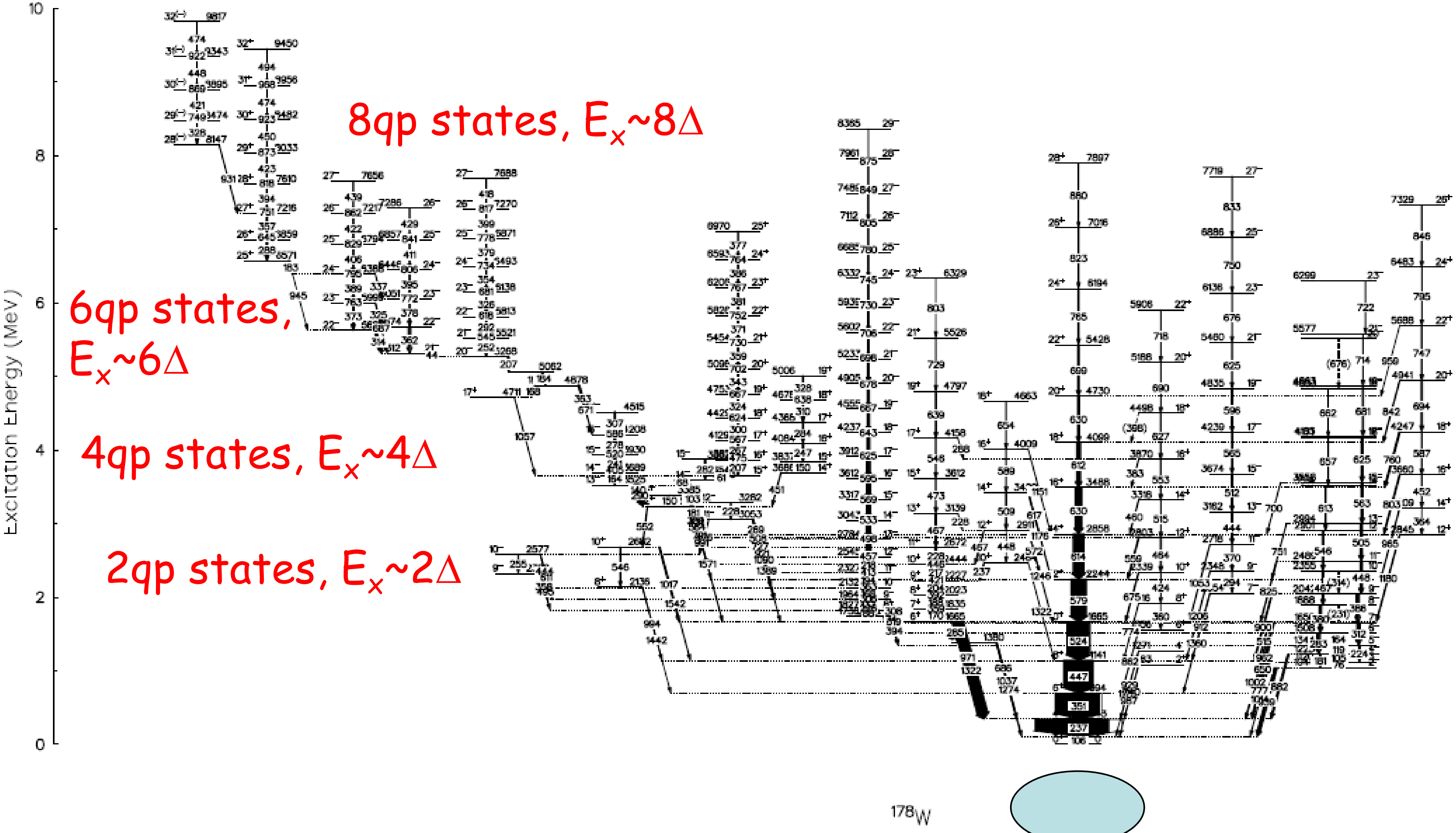
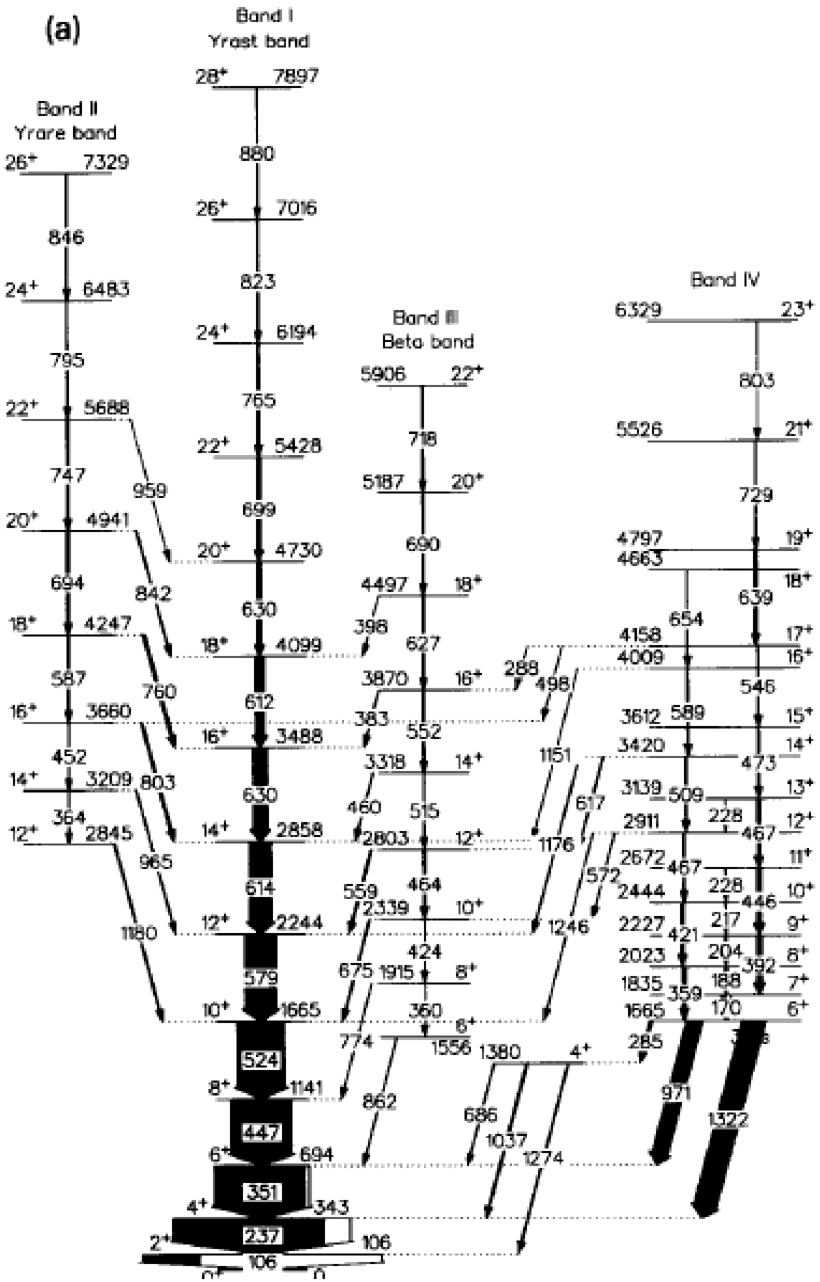


Figure 2.21: Partial decay scheme for ^{178}W showing 0, 2, 4, 6 and 8 quasi-particle structures [9].

C.S.Purry et al., Nucl. Phys. A632 (1998) p229


(a)
Band I
Yrast band
Band II
Yrare band
Band III
Beta band
Band IV

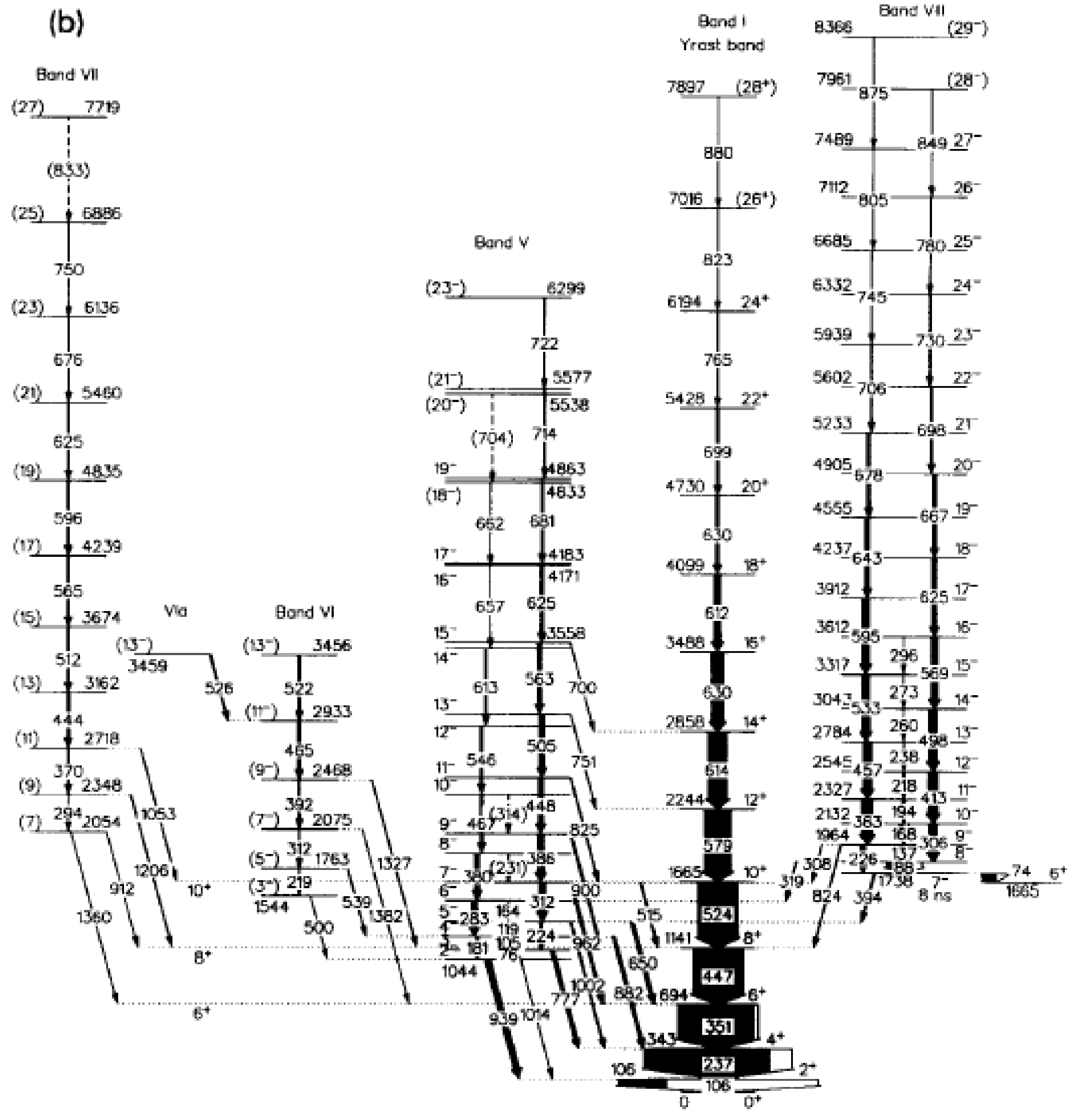


Fig. 1 — continued.

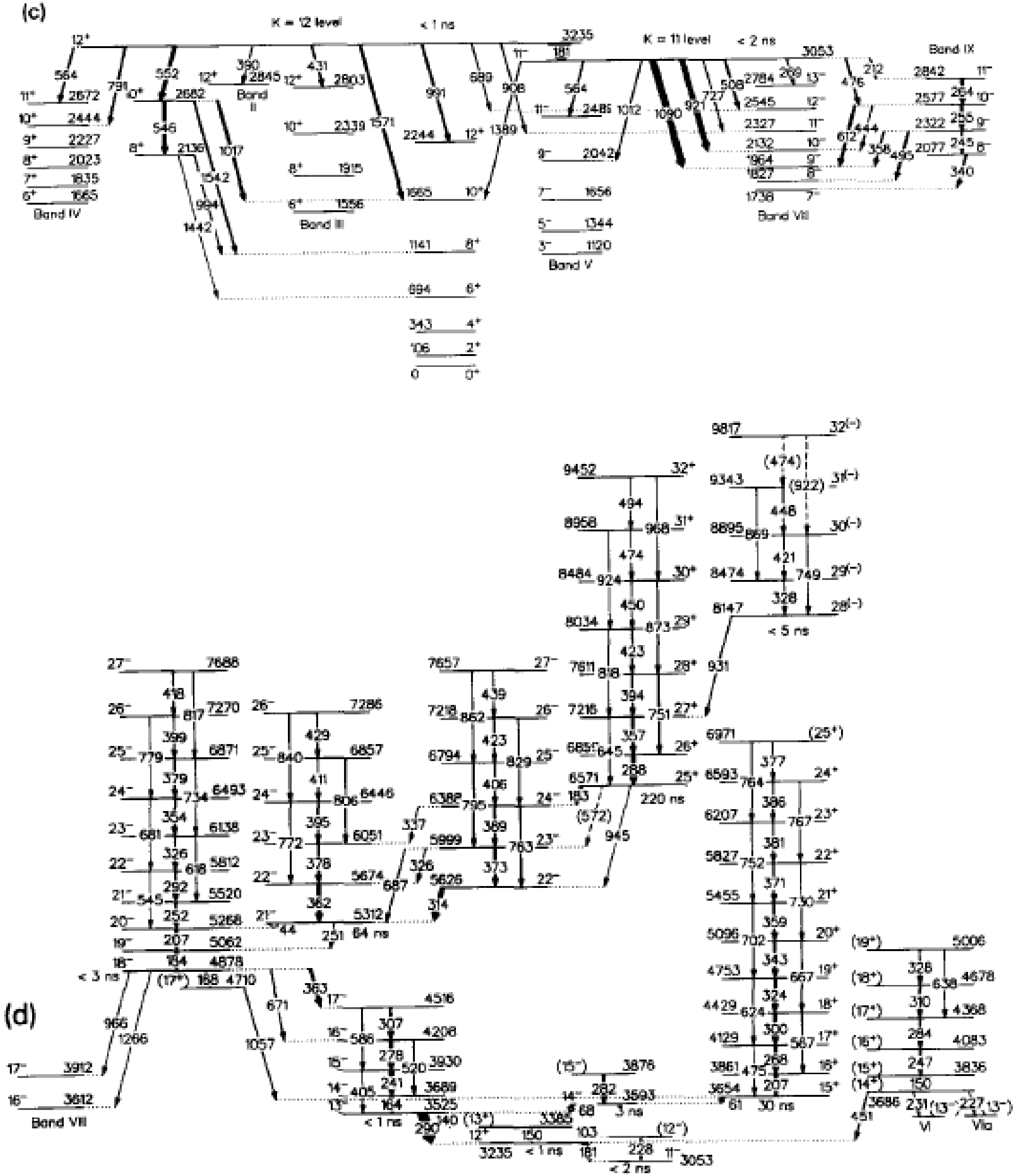


Fig. 1 — continued.

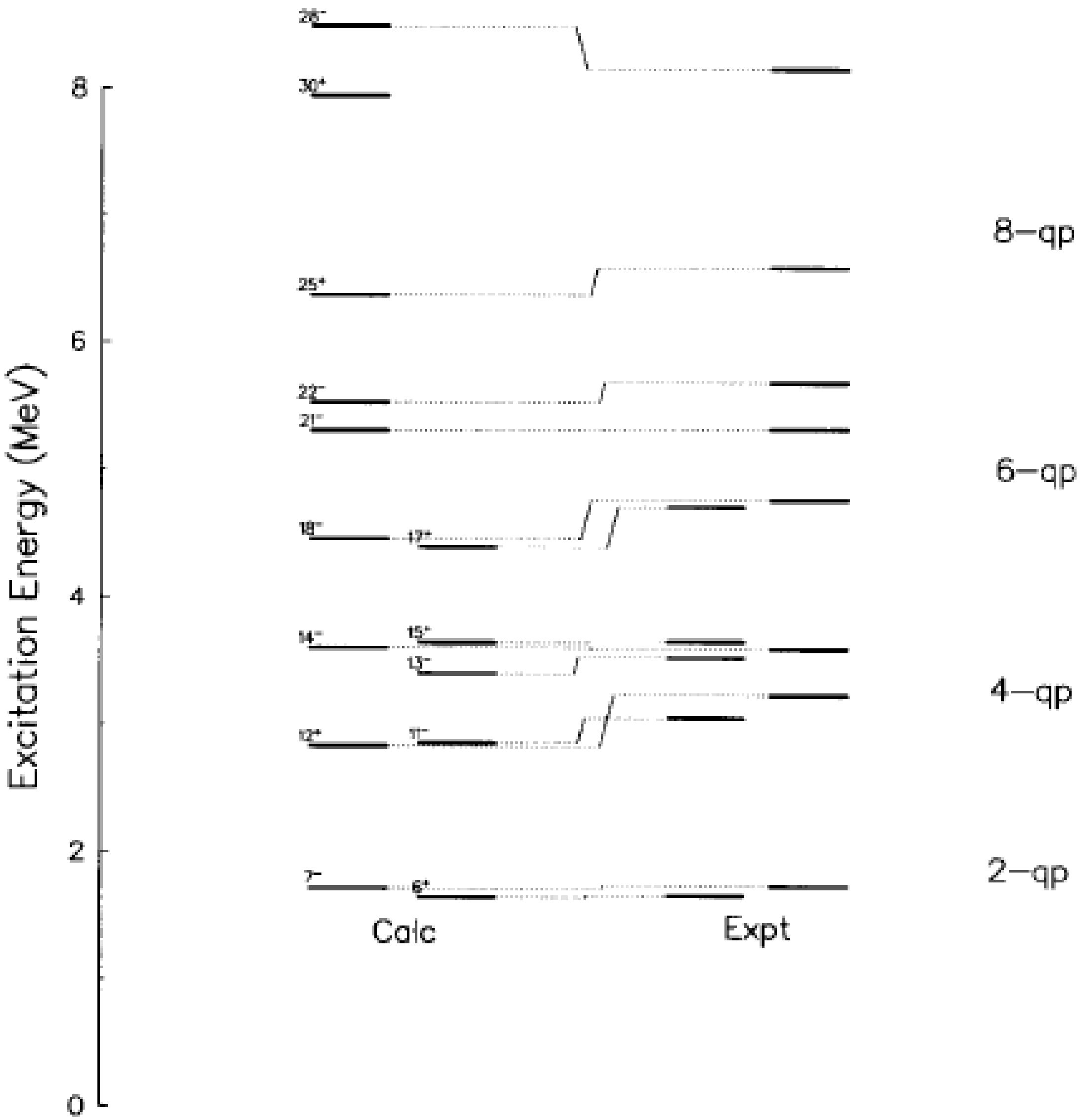


Fig. 14. Experimental and calculated bandhead energies.

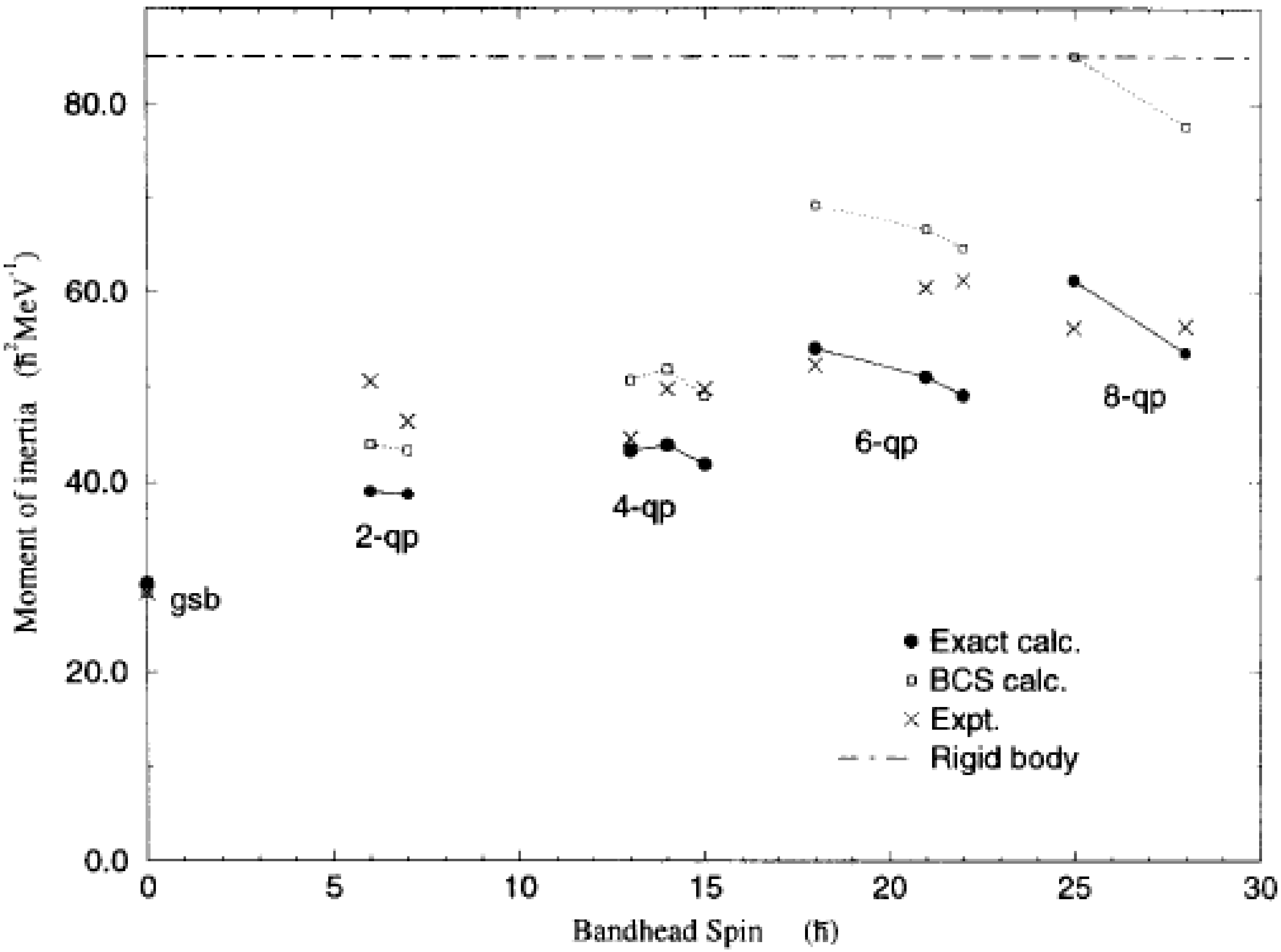


Fig. 17. Comparison of experimental and calculated moments of inertia.

Transitions from Vibrator to Rotor?

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett*. 90** (2003) 152502

$$\text{Vibrator} \quad : \quad E_n = n\hbar\omega = \frac{J}{2}\hbar\omega, \quad E_\gamma = \hbar\omega$$

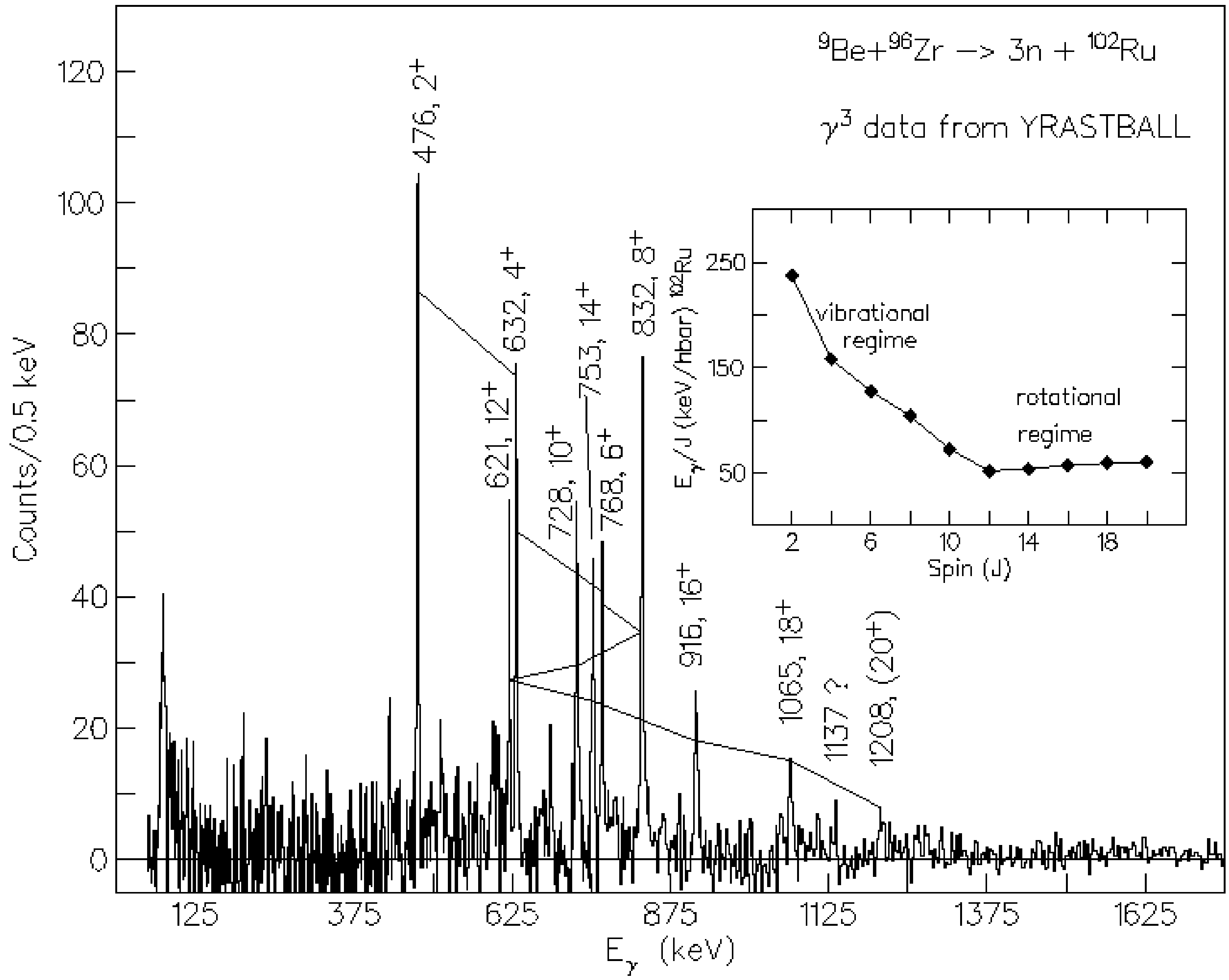
$$\text{Rotor} \quad : \quad E_J = \frac{\hbar^2}{2\ell} J(J+1), \quad E_\gamma = \frac{\hbar^2}{2\ell}(4J-2)$$

$$R = \frac{E_\gamma(J \rightarrow J-2)}{J}$$

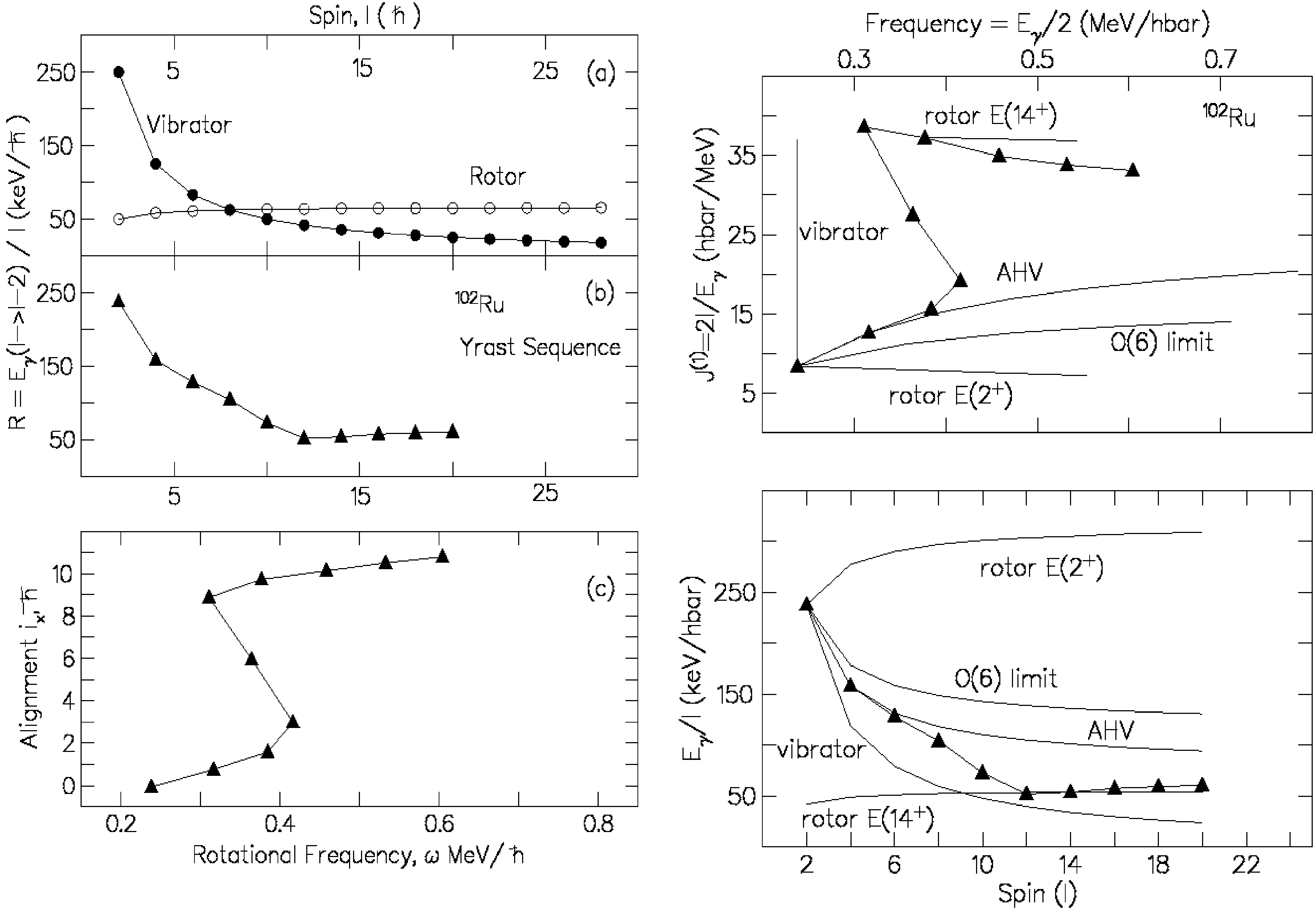
$$\text{Vibrator} \quad : R = \frac{\hbar\omega}{J} \xrightarrow{J\rightarrow\infty} 0$$

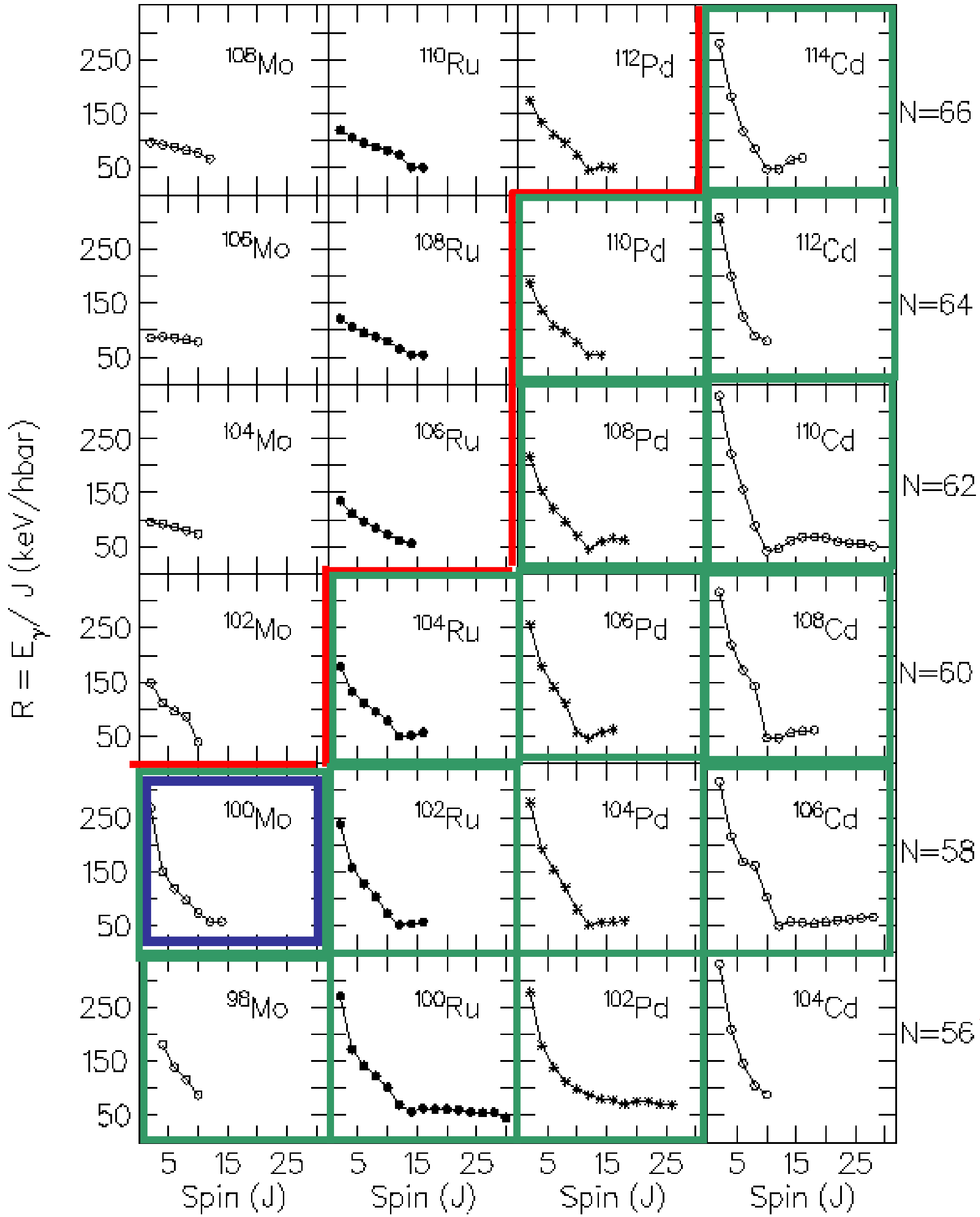
$$\text{Rotor} \quad : R = \frac{\hbar^2}{2\ell}\left(4 - \frac{2}{J}\right) \xrightarrow{J\rightarrow\infty} 4\left(\frac{\hbar^2}{2\ell}\right)$$

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett*. 90** (2003) 152502



PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett*.** **<u>90</u>** (2003) 152502





Vibrator-Rotator phase change is a feature of near stable (green) A~100.

'Rotational alignment' can be a crossing between quasi-vibrational GSB & ***deformed rotational*** sequence.
(stiffening of potential by population of high-j, equatorial ($h_{11/2}$) orbitals).

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett.*** **90** (2003) 152502