Some Aspects of High(ish) - Spin Nuclear Structure

Paddy Regan
Department of Physics
University of Surrey
Guildford, UK
p.regan@surrey.ac.uk

Outline of Lectures

- 1) Overview of nuclear structure 'limits'
  - Some experimental observables, evidence for shell structure
  - Independent particle (shell) model
  - Single particle excitations and 2 particle interactions.

- 2) Low Energy Collective Modes and EM Decays in Nuclei.
  - Low-energy Quadrupole Vibrations in Nuclei
  - Rotations in even-even nuclei
  - Vibrator-rotor transitions, E-GOS curves

- 3) EM transition rates..what they mean and overview of how you measure them
  - Deformed Shell Model: the Nilsson Model, K-isomers
  - Definitions of B(ML) etc. ; Weisskopf estimates etc.
  - Transition quadrupole moments (Qo)
  - Electronic coincidences; Doppler Shift methods.
  - Yrast trap isomers
  - Magnetic moments and g-factors

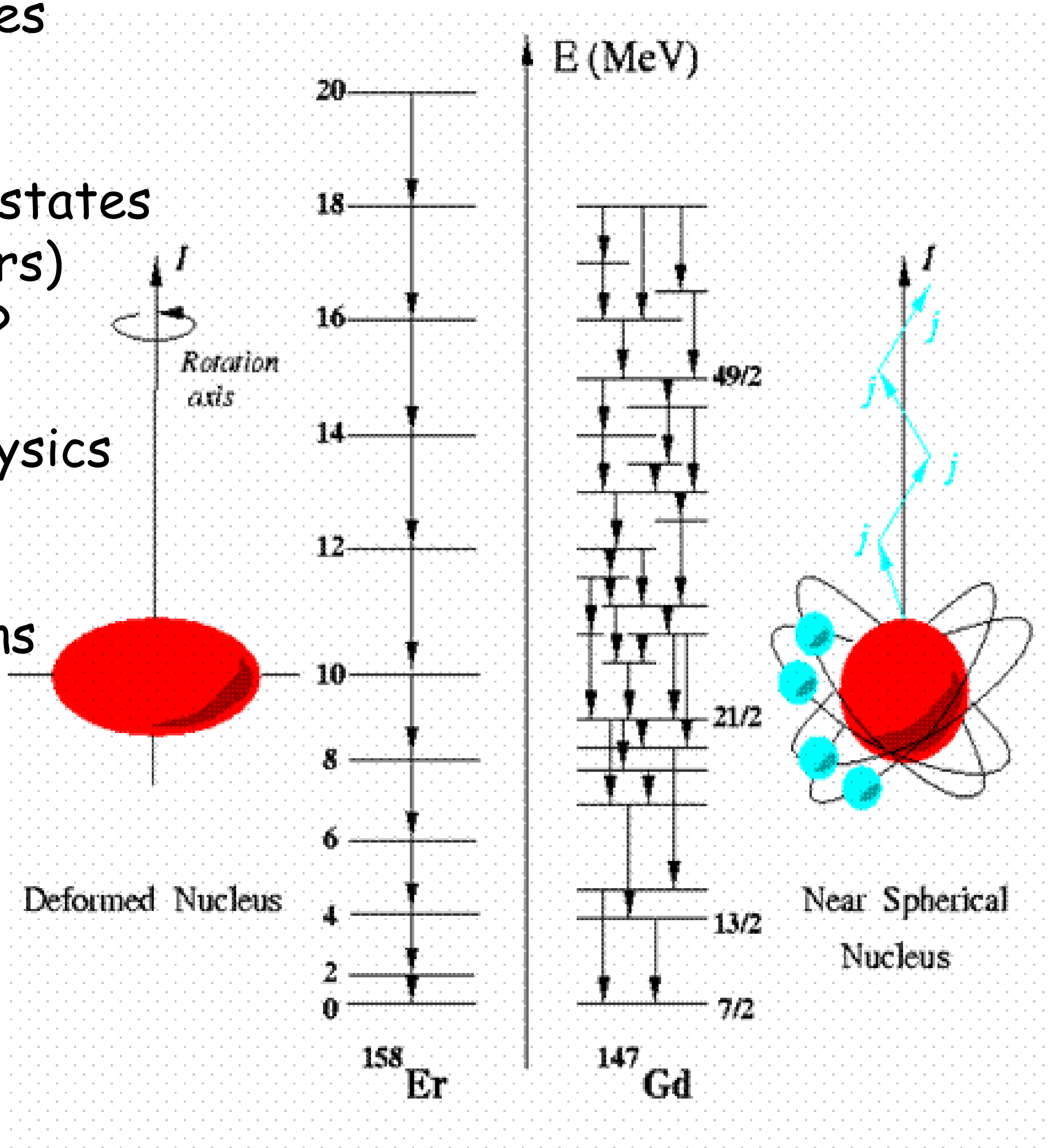
Some nuclear observables?

1) Masses and energy differences
2) Energy levels
3) Level spins and parities
4) EM transition rates between states
5) Magnetic properties (g-factors)
6) Electric quadrupole moments?

Essence of nuclear structure physics

……..

How do these change as functions of N, Z, I, Ex ?



Evidence for Nuclear Shell Structure?

- Increased numbers of stable isotones and isotopes at certain N,Z values.
- Discontinuities in S_n, S_p around certain N,Z values (linked to neutron capture cross-section reduction).
- Excitation energy systematics with N,Z.

More number of stable isotones at N=20, 28, 50 and 82 compared to neighbours…

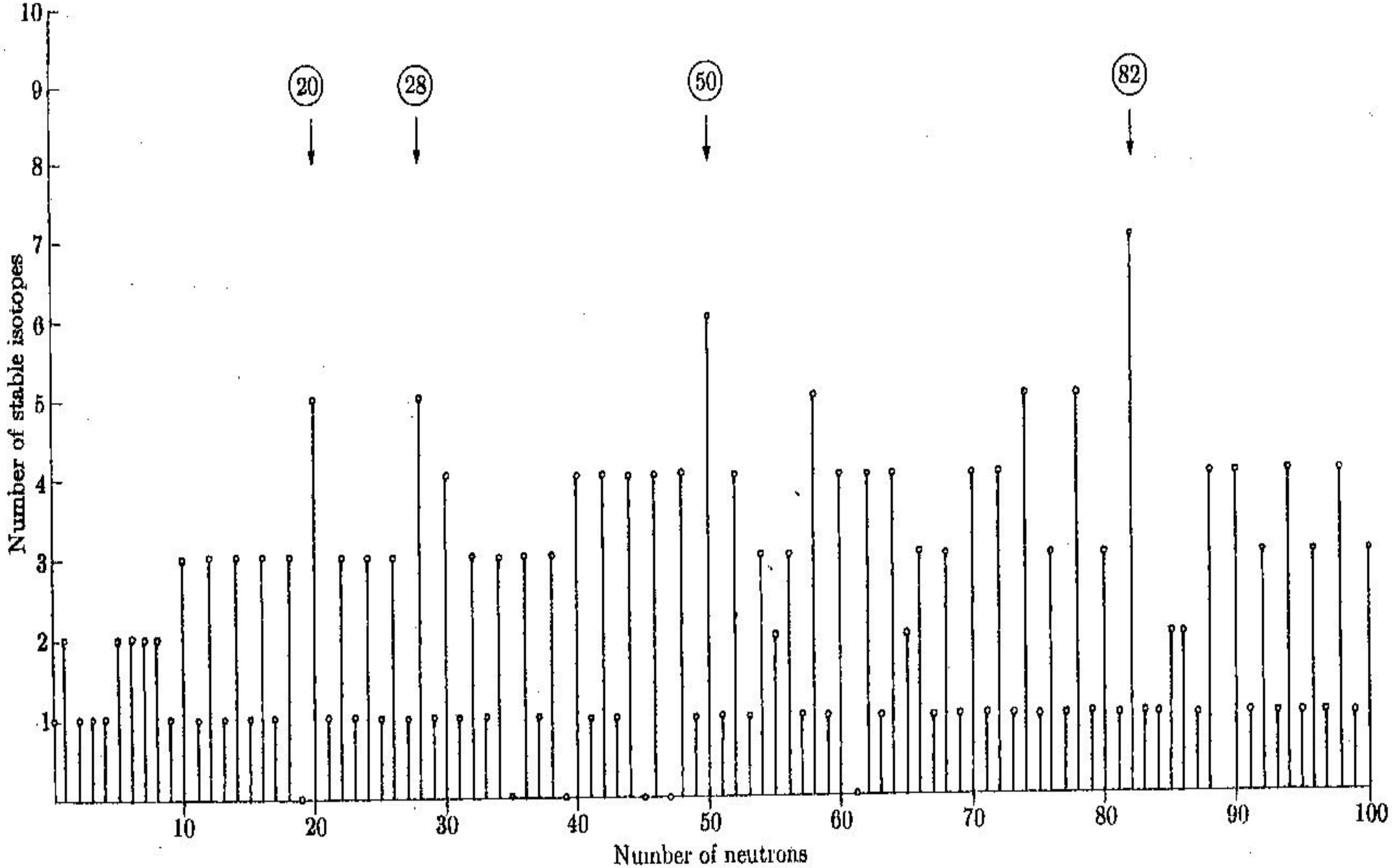


FIG. 17–4. The number of stable nuclides as a function of the neutron number (Flowers[27]).

$S_n = [\ M\ (^{A-1}X_{N-1}) - M(^{A}X_N) + m_n)\ c^2$ = neutron separation energy

$S_p = [\ M\ (^{A-1}X_N) - M(^{A}X_N) + m_p)\ c^2$ = neutron separation energy

NUCLIDE	S_n (MeV)	S_p (MeV)
^{16}O	15.66	12.13
$^{17}O_9$	4.14	13.78
^{17}F	16.81	0.60
^{40}Ca	15.64	8.33
$^{41}Ca_{21}$	8.36	8.89
^{41}Sc	16.19	1.09
^{208}Pb	7.37	8.01
$^{209}Pb_{127}$	3.94	8.15
^{209}Bi	7.46	3.80

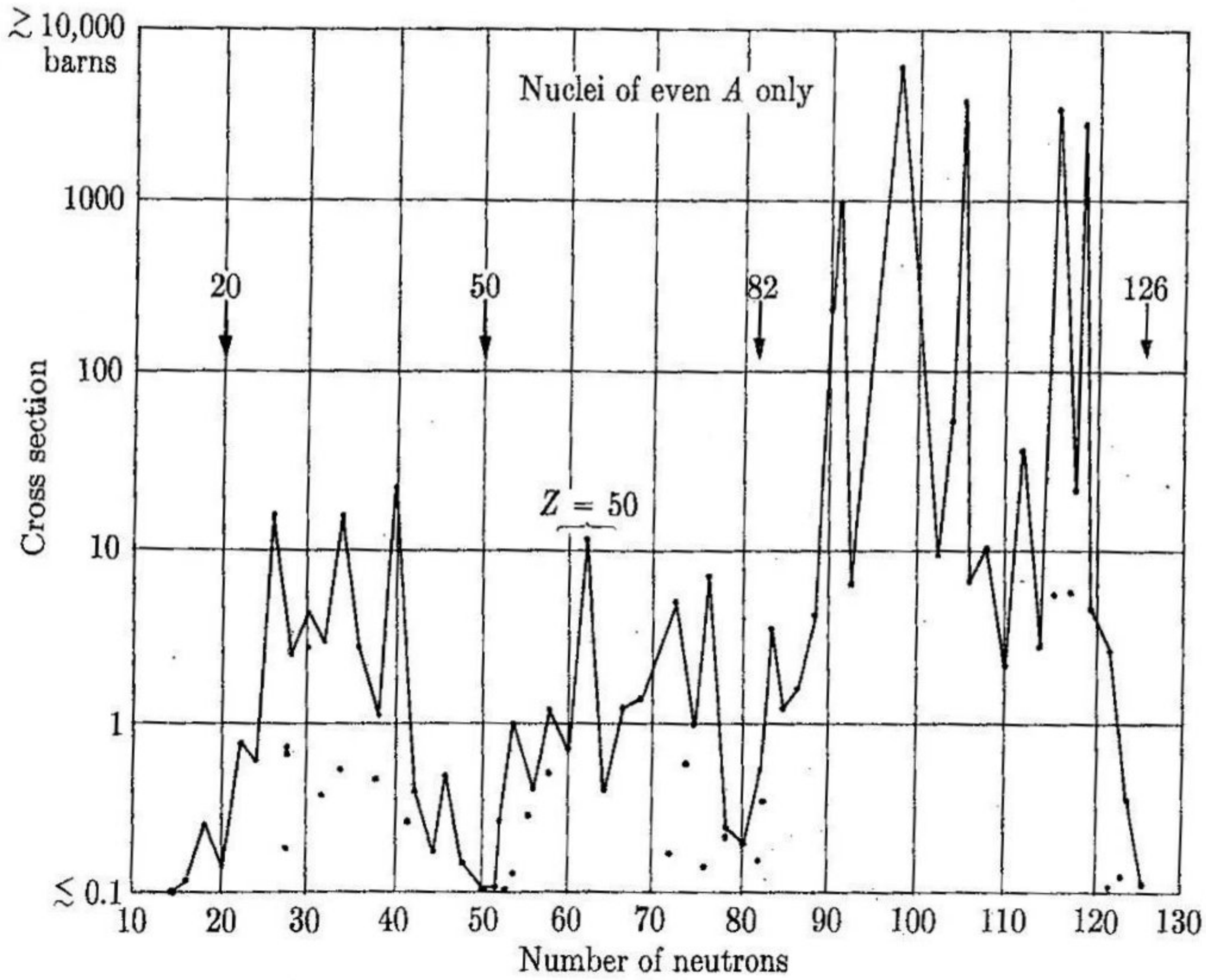
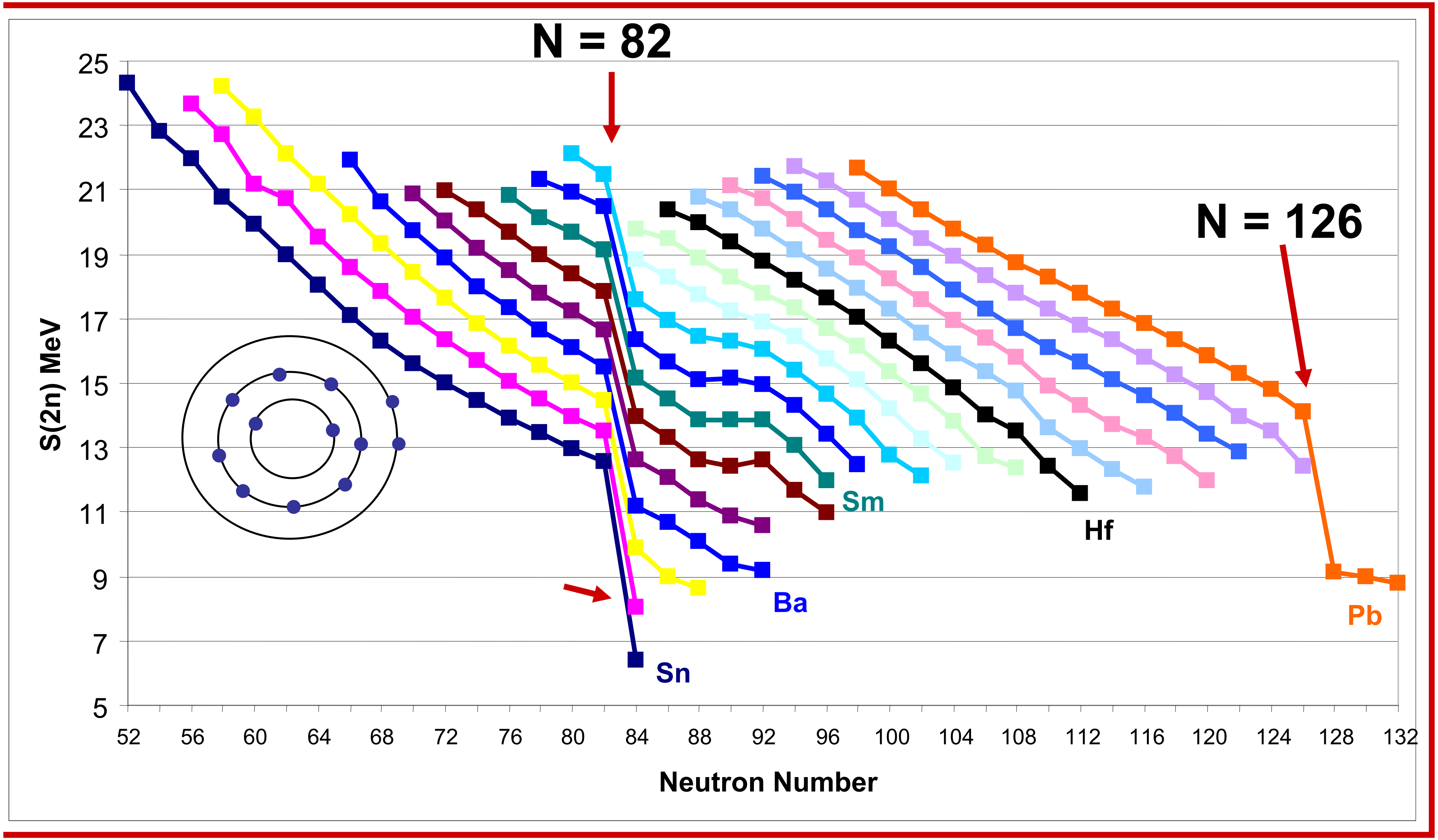


FIG. 17–5. Thermal neutron cross section as a function of the neutron number (Flowers [27]).

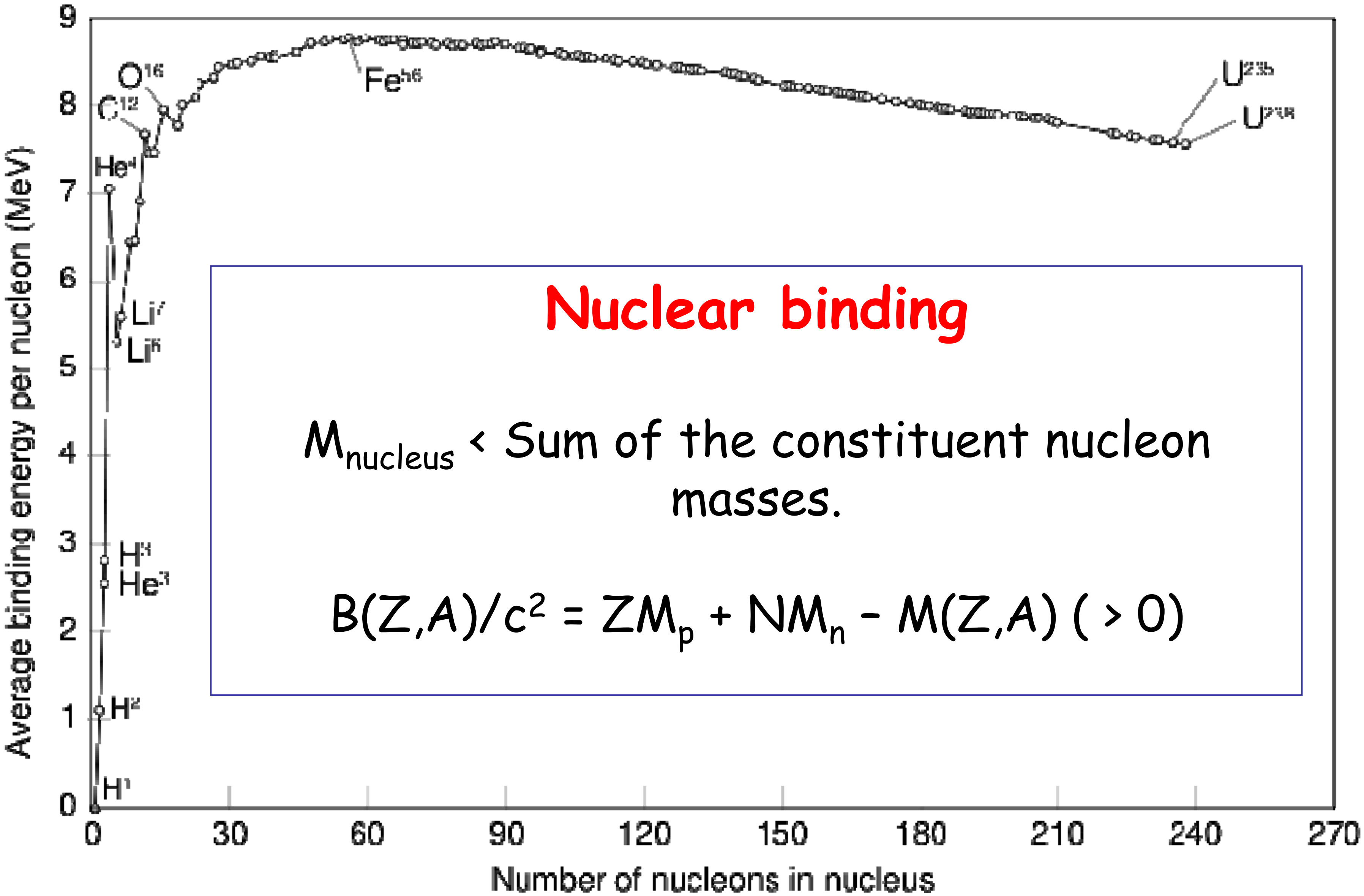
Energy required to remove two neutrons from nuclei

(2-neutron binding energies = 2-neutron "separation" energies)

Discontinuities at N=50, 82 and 126…



A reminder of some undergraduate nuclear physics

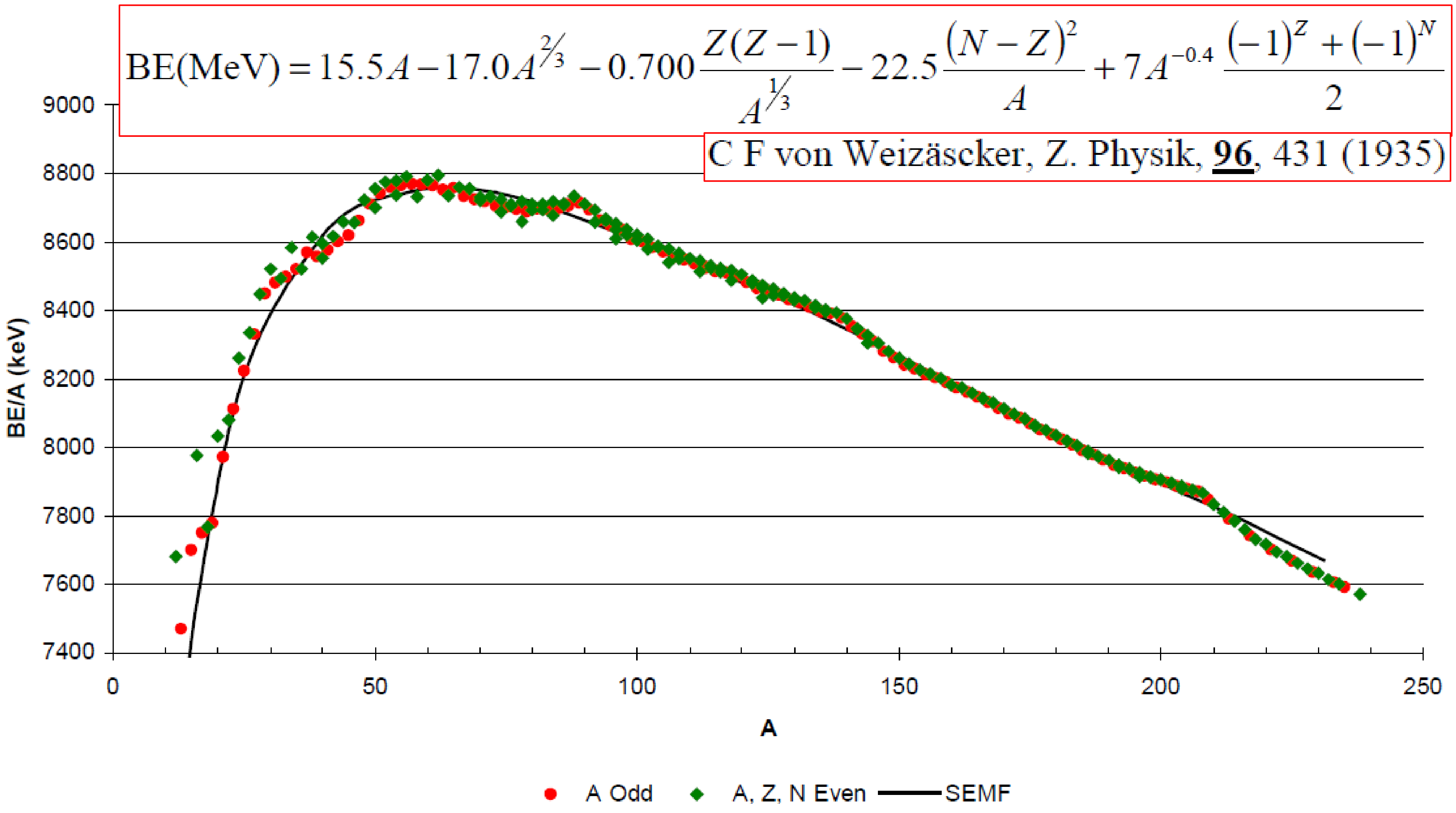


Nuclear binding

$M_{nucleus}$ < Sum of the constituent nucleon masses.

$$B(Z,A)/c^2 = ZM_p + NM_n - M(Z,A) \; (> 0)$$

Semi-empirical mass formula which assumes the nucleus acts like a liquid drop with a well defined surface, reproduces the nuclear binding energy per nucleon curve well, but requires additional corrections around 'magic' N and Z values.



$$BE(\mathrm{MeV}) = 15.5A - 17.0A^{2/3} - 0.700\frac{Z(Z-1)}{A^{1/3}} - 22.5\frac{(N-Z)^2}{A} + 7A^{-0.4}\frac{(-1)^Z + (-1)^N}{2}$$

C F von Weizäscker, Z. Physik, **96**, 431 (1935)

SEMF 'residuals' show effects due to 'magic' proton and neutron numbers.

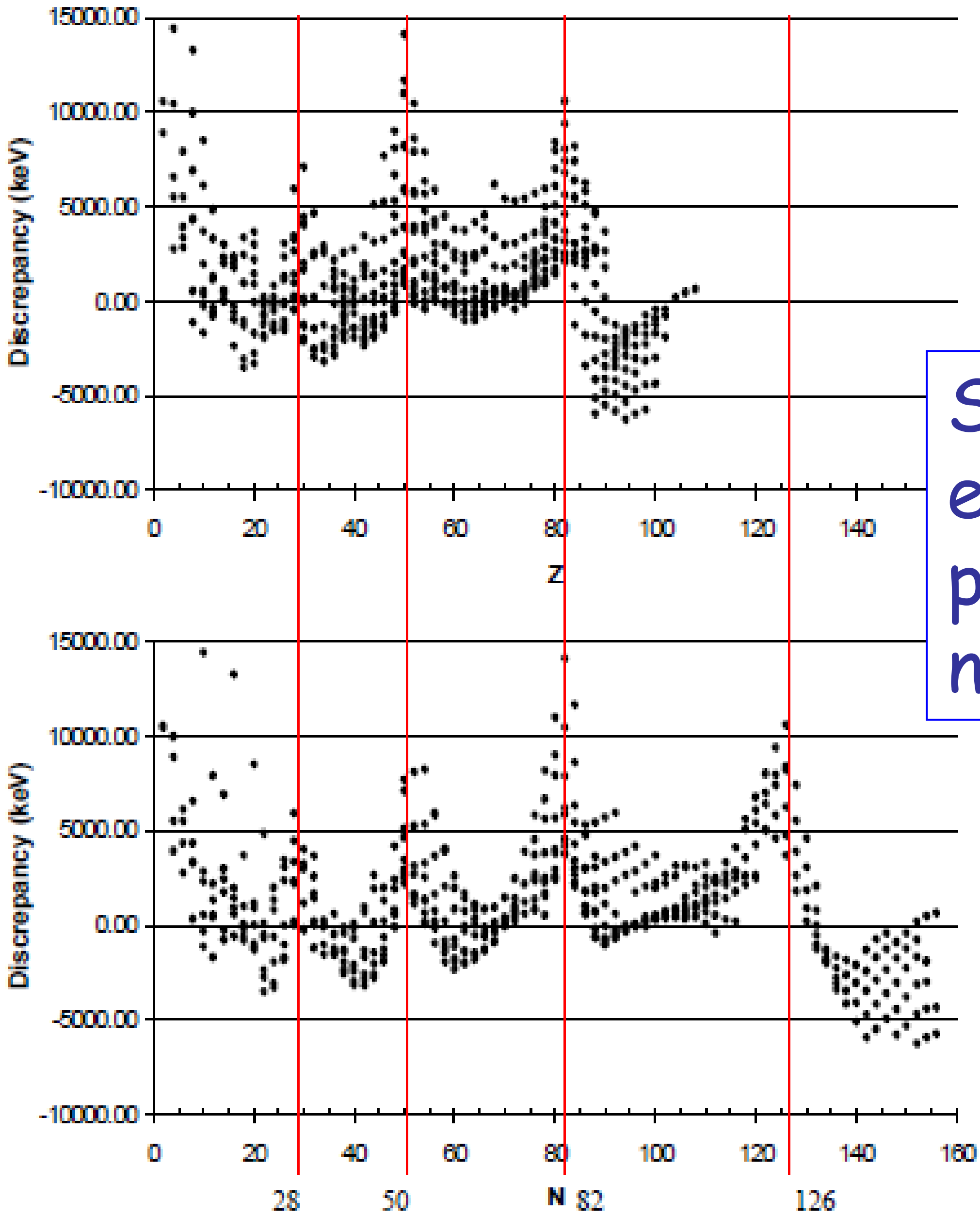
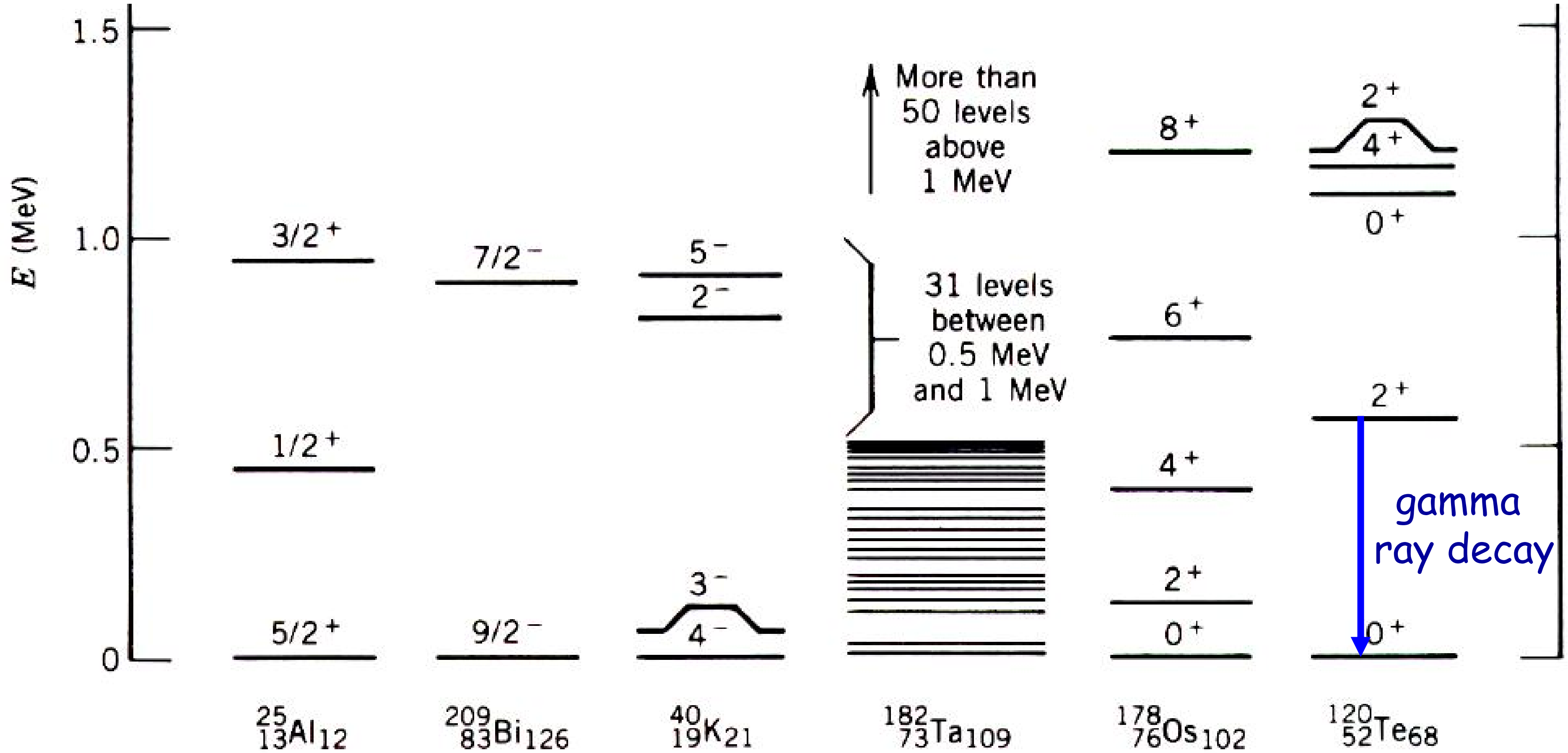


Figure 17 - the residuals for the even-even nuclides, displayed against proton number (top) and neutron number (below).

Nuclear Excited States – Nuclear Spectroscopy .



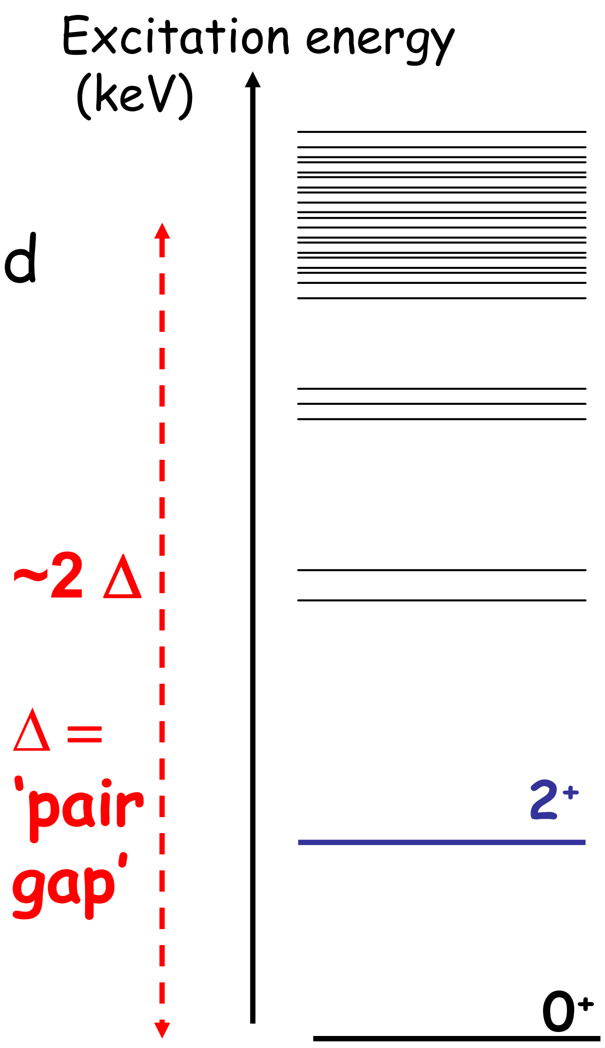
- Nuclear states labelled by spin and parity quantum numbers and energy.
- Excited states (usually) decay by gamma rays (non-visible, high energy light).
- Measuring gamma rays gives the energy differences between quantum states.

Even-Even Nuclei

Excited states spin/parities depend on the nucleon configurations. i.e., which specific orbits the protons and neutrons occupy.

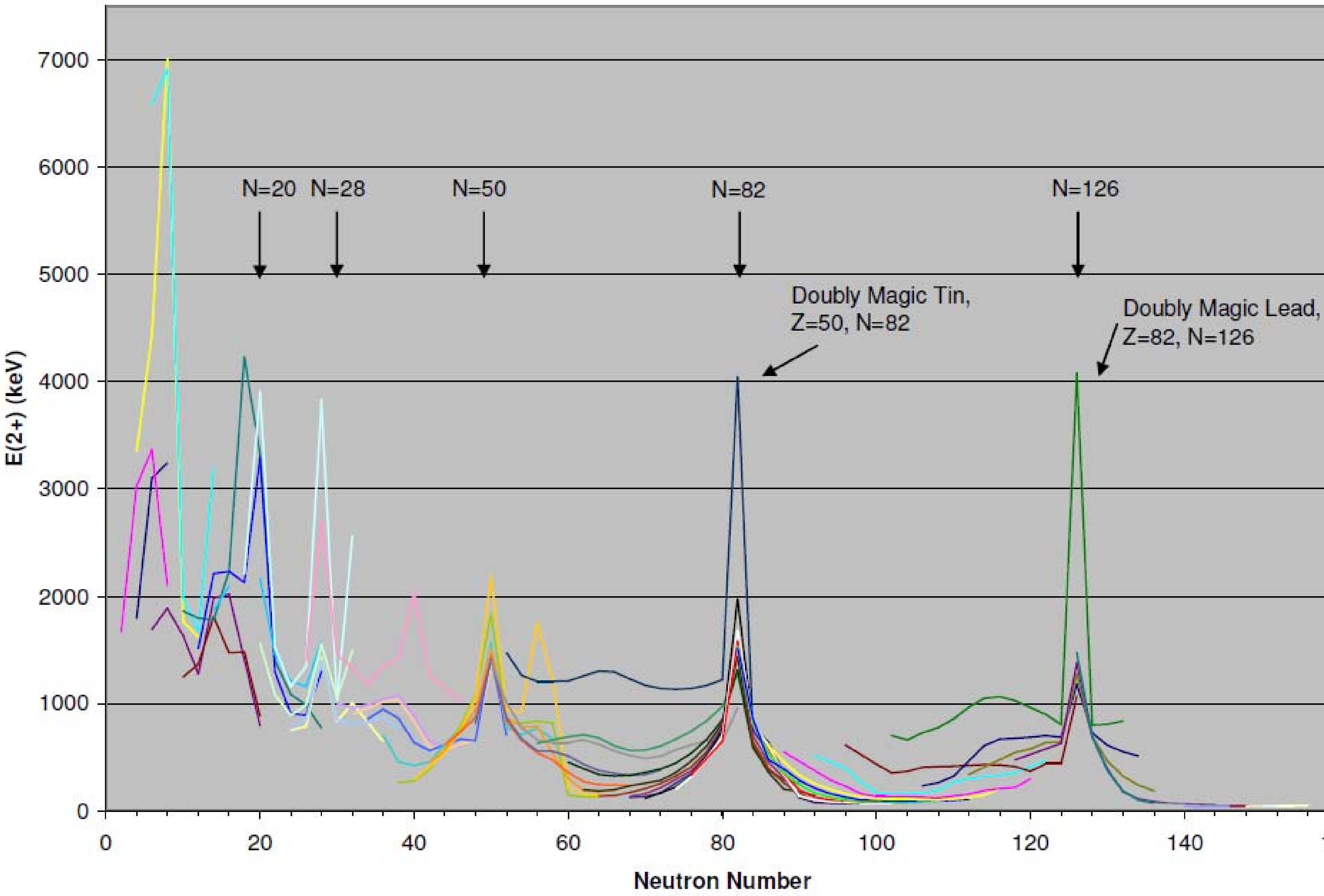
Different orbits costs different amounts of energy → result is a complex energy 'level scheme'.

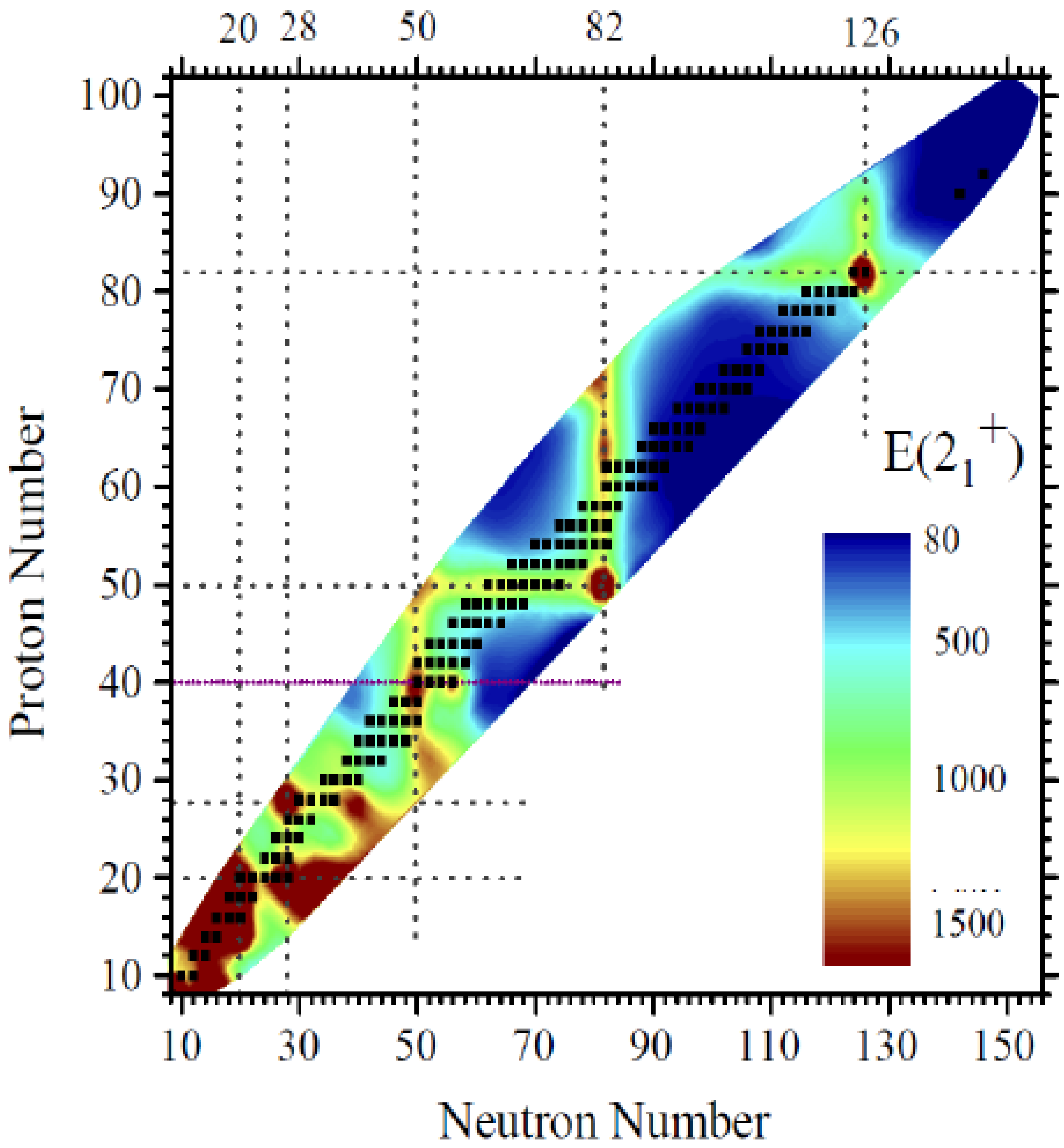
First excited state in (most) even-N **AND** even-Z has $I^{\pi}=2^{+}$

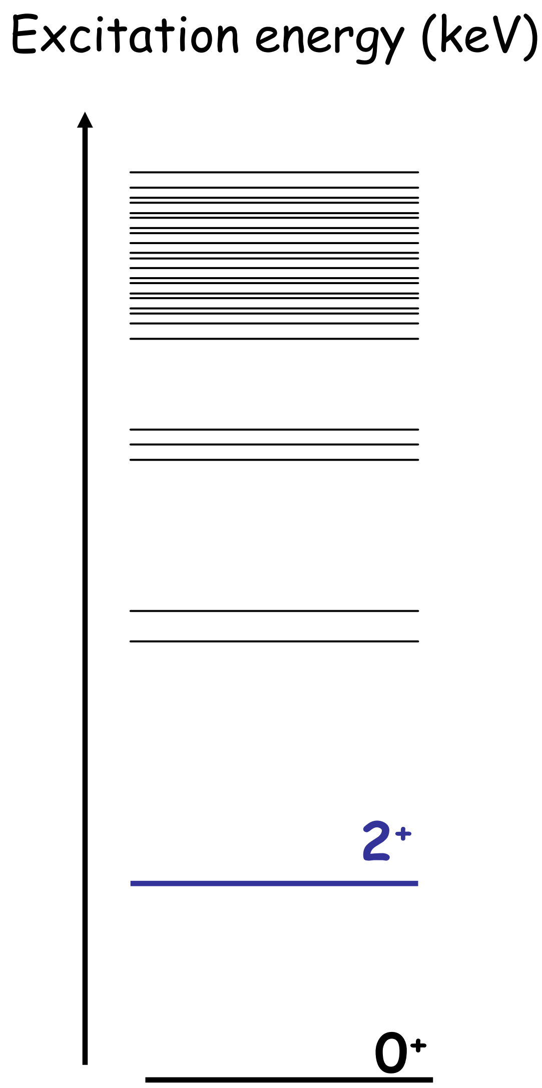


Ground state ($E_x=0$) configuration has $I^{\pi}=0^{+}$

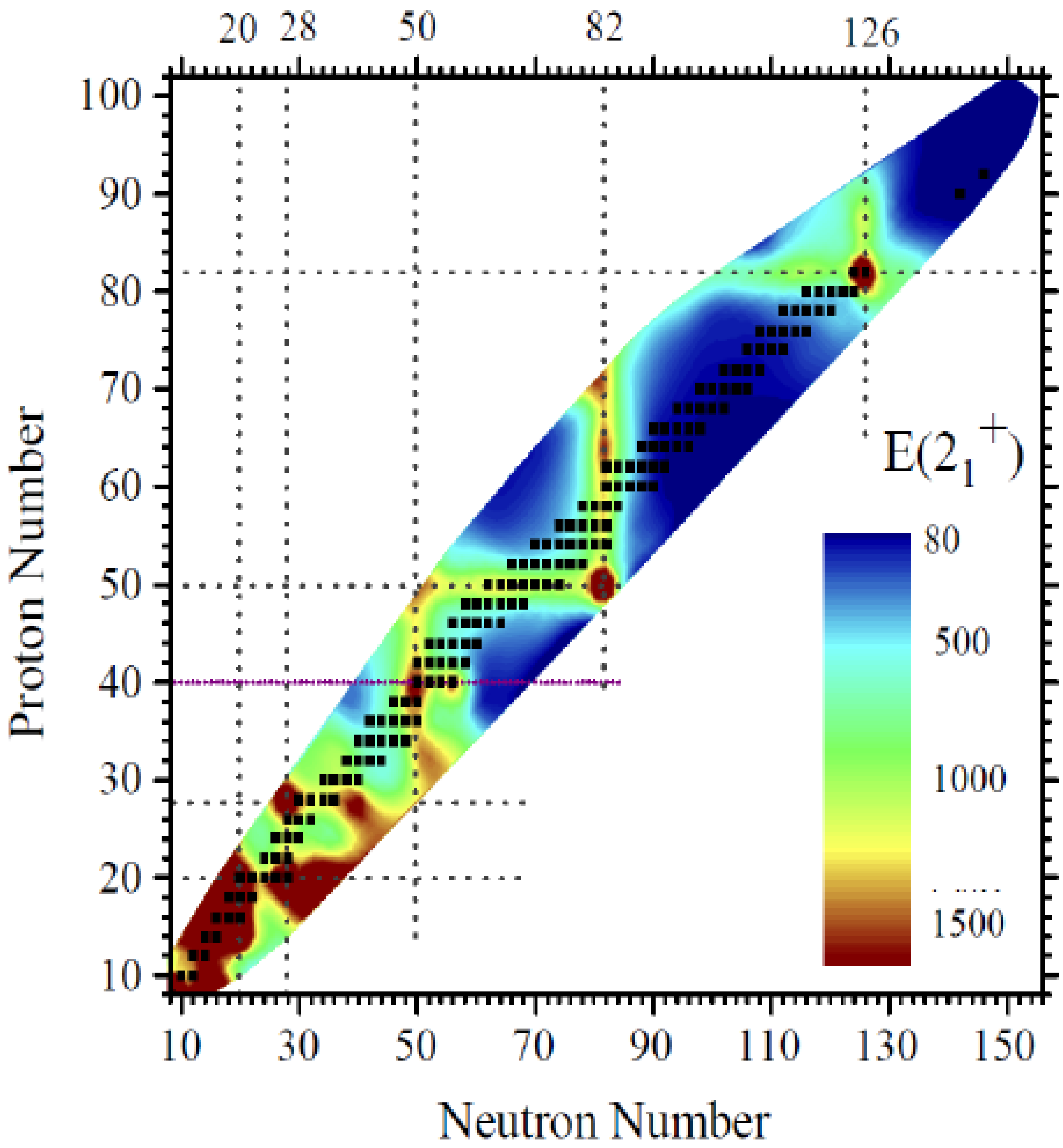
Evidence for nuclear shell structure…..
energy of 1st excited state in even-even nuclei….$E(2^+)$.







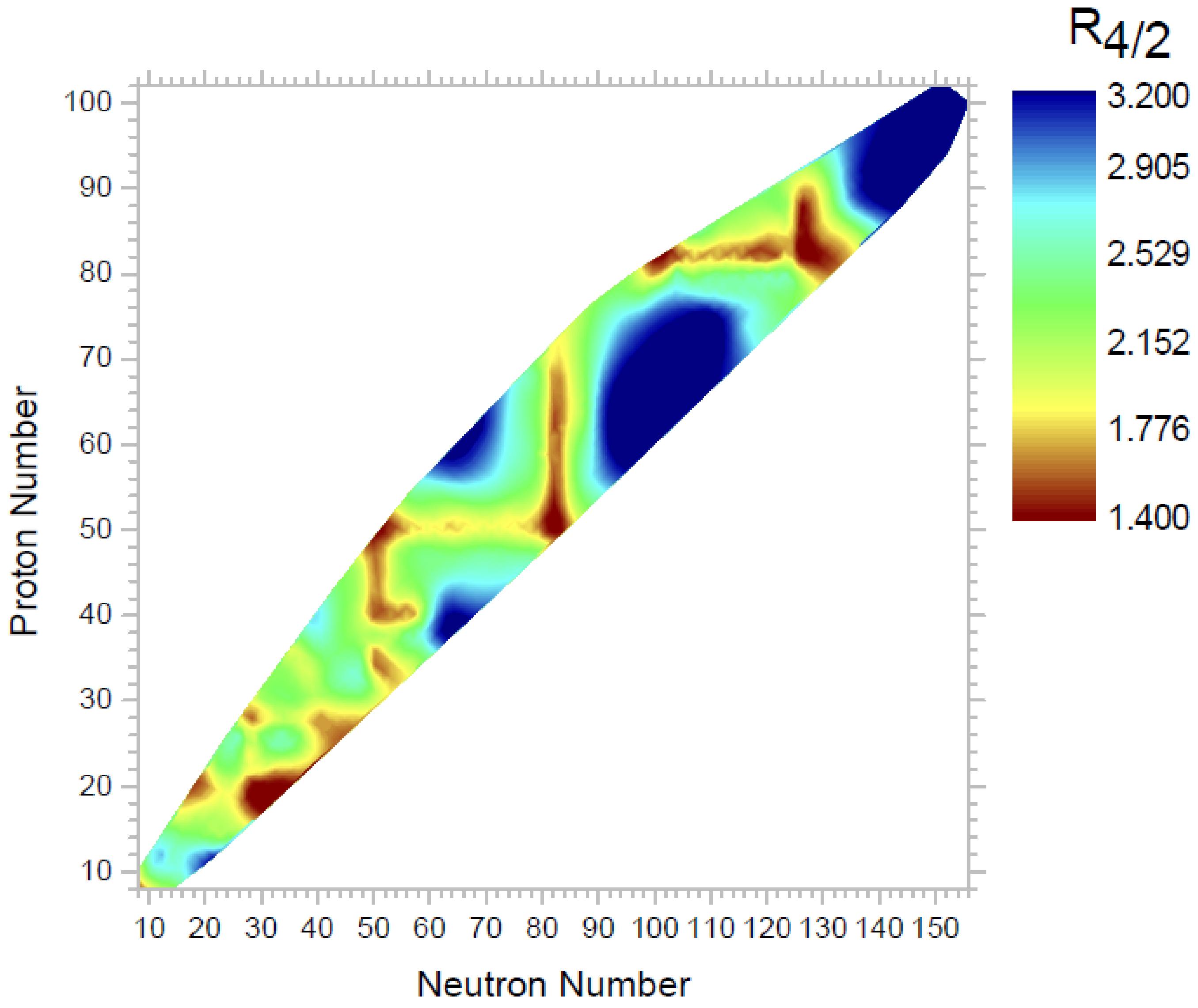
Ground state Configuration.
Spin/parity $I^{\pi}=0^{+}$;
E_x = 0 keV

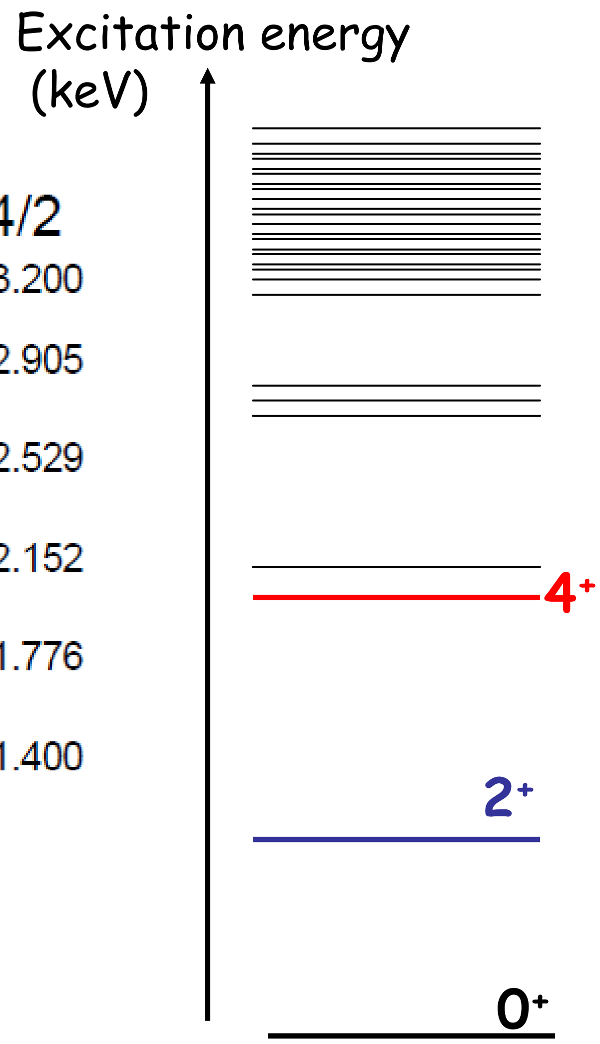


Correlations are observed between the first $I^{\pi}=2^{+}$ energy and (N,Z) values.

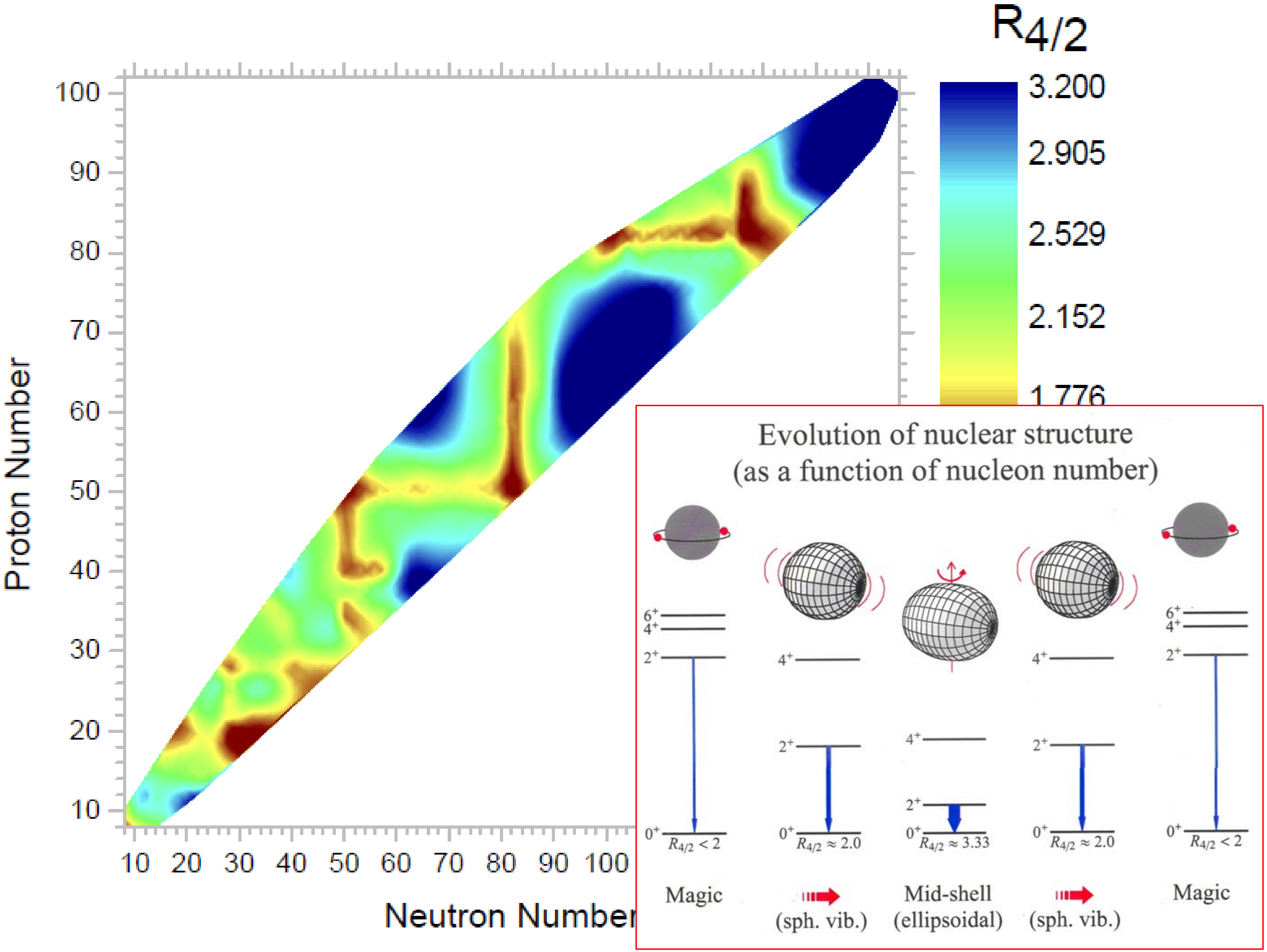
'Magic numbers' which correspond to nuclear shell closures are clear.

4+/2+ energy ratio: mirrors 2^+ systematics.





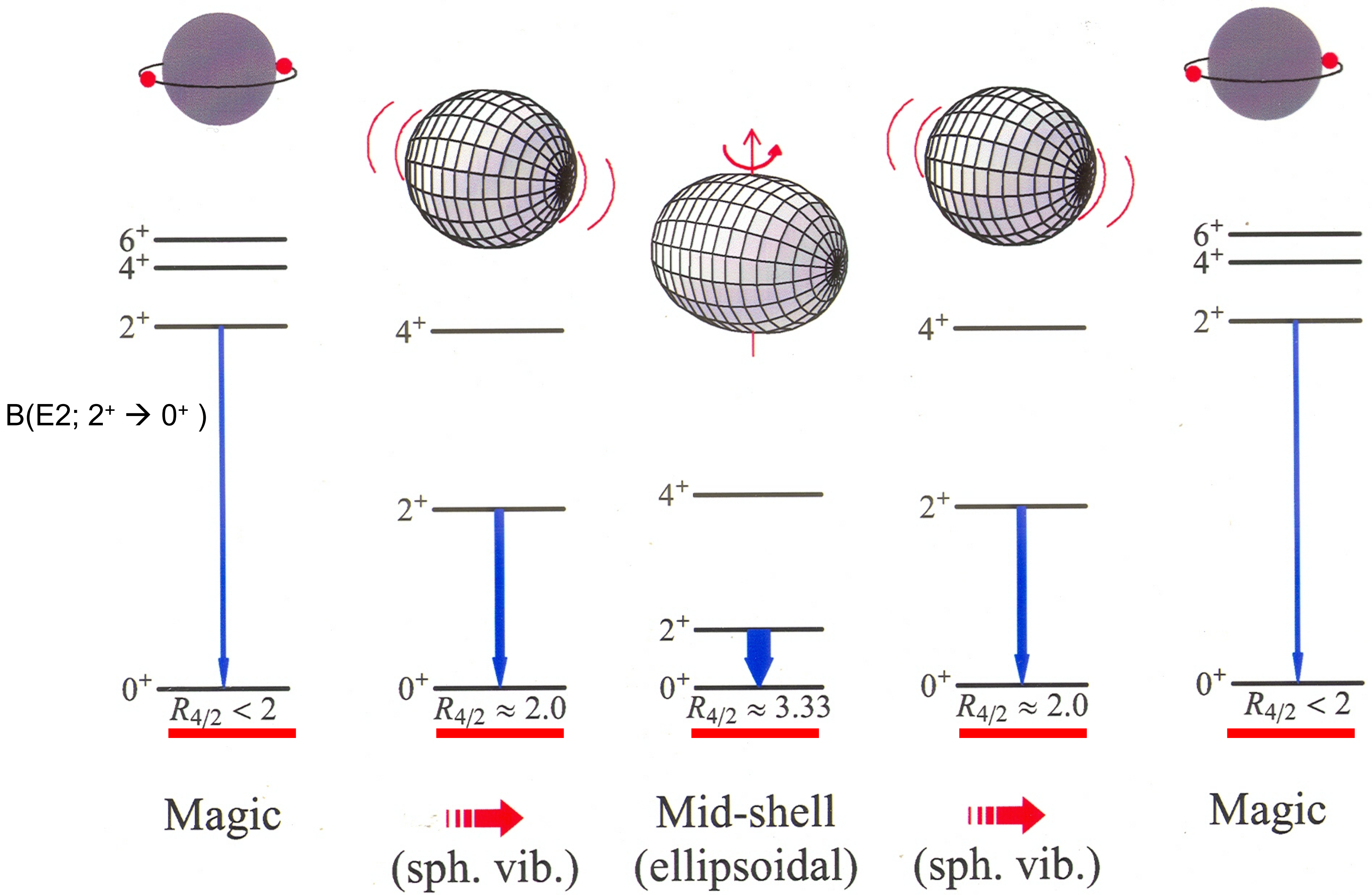
Ground state Configuration.
Spin/parity $I^{\pi}=0^+$;
E_x = 0 keV

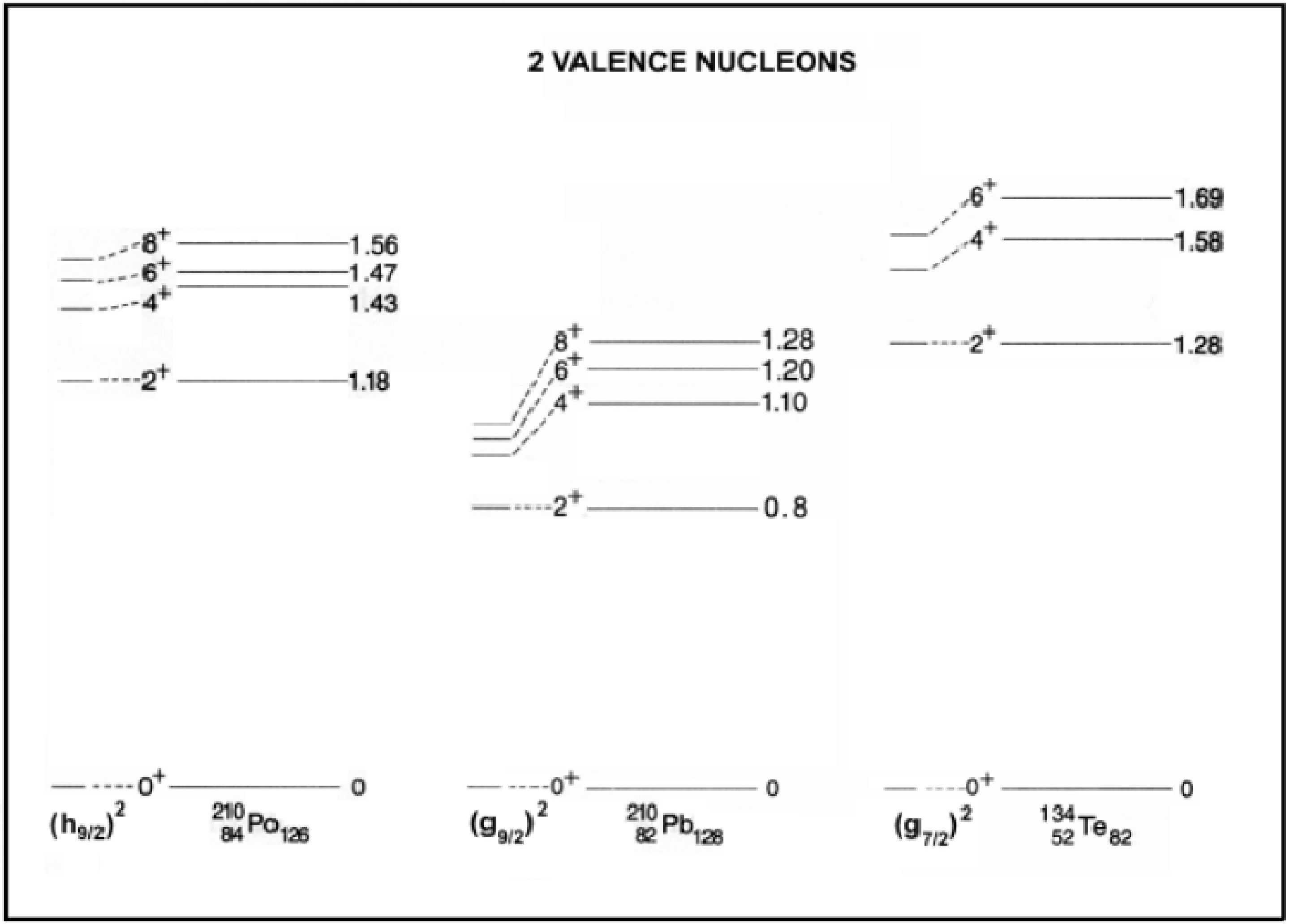


R4/2
3.200
2.905
2.529
2.152
1.776
Proton Number
Neutron Number
100
90
80
70
60
50
40
30
20
10
10 20 30 40 50 60 70 80 90 100
Evolution of nuclear structure
(as a function of nucleon number)
6+
4+
2+
0+
R4/2 < 2
R4/2 ≈ 2.0
R4/2 ≈ 3.33
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)

Evolution of nuclear structure

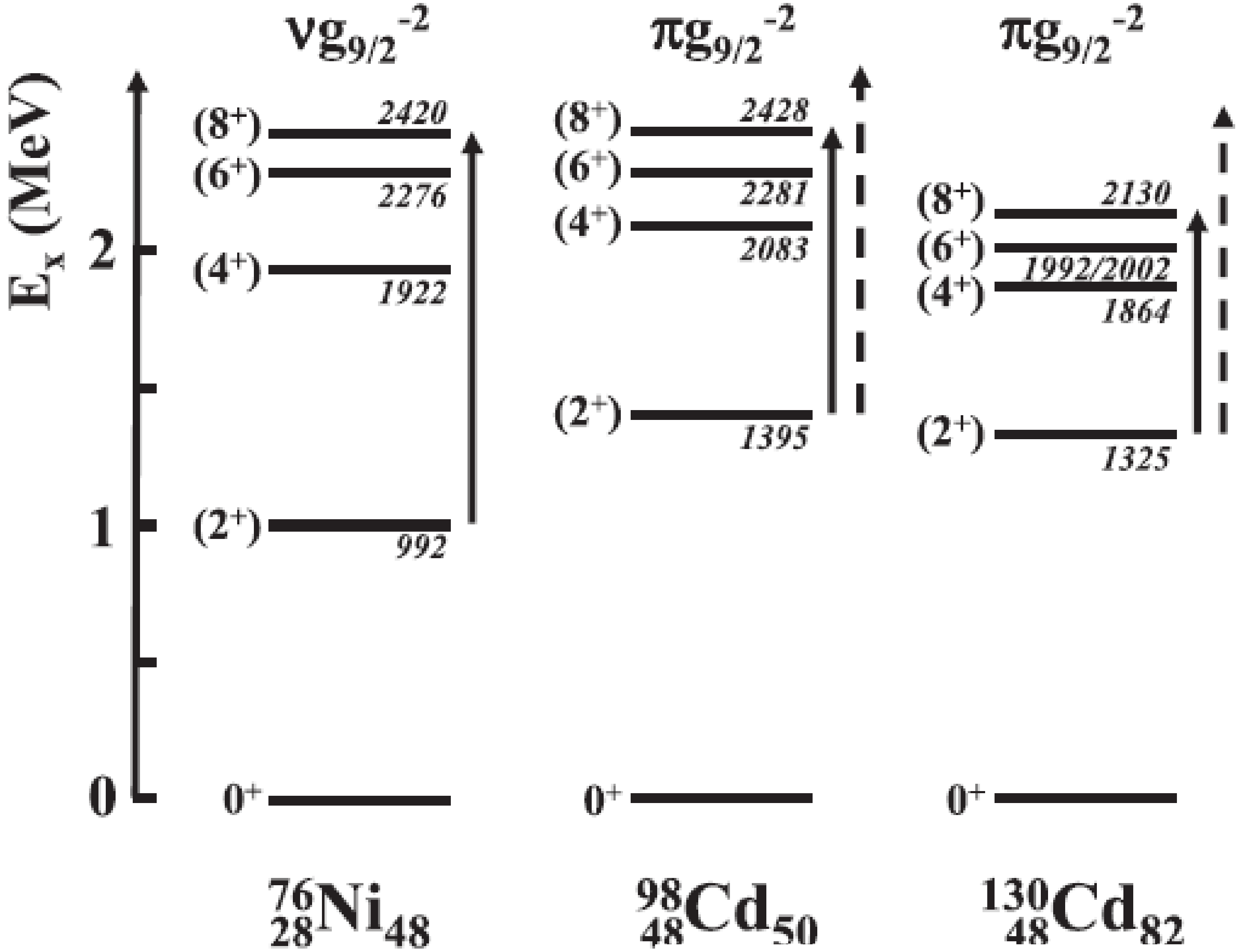
(as a function of nucleon number)





$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)} \sim 1.3 \text{ -ish}$$

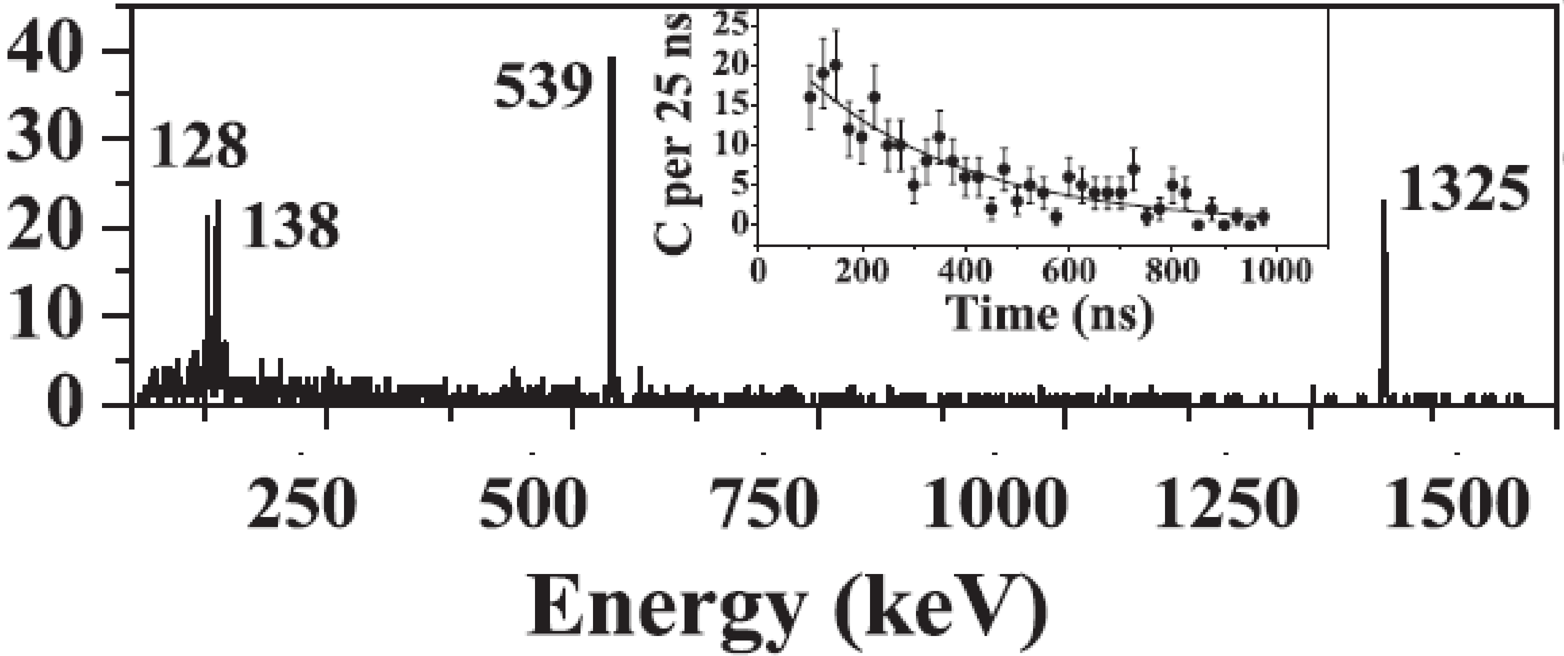
2 neutron or 2 proton **holes** in doubly magic nuclei also show spectra like 2 proton or neutron **particles**.

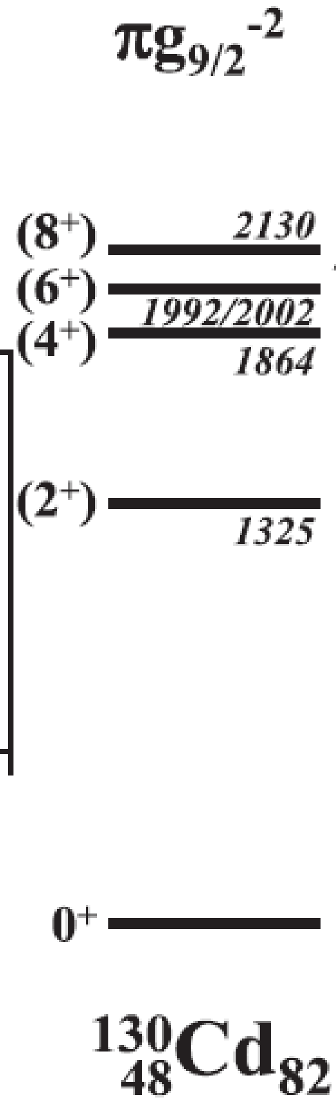


A. Jungclaus et al., PRL **99**, 132501 (2007)

Two particle (or two-hole) nuclei outside magic numbers
Have characteristic decay energy spectra.

These are interpreted as the decays from energy levels.



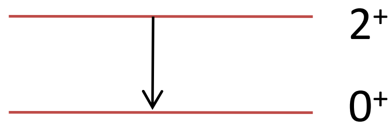


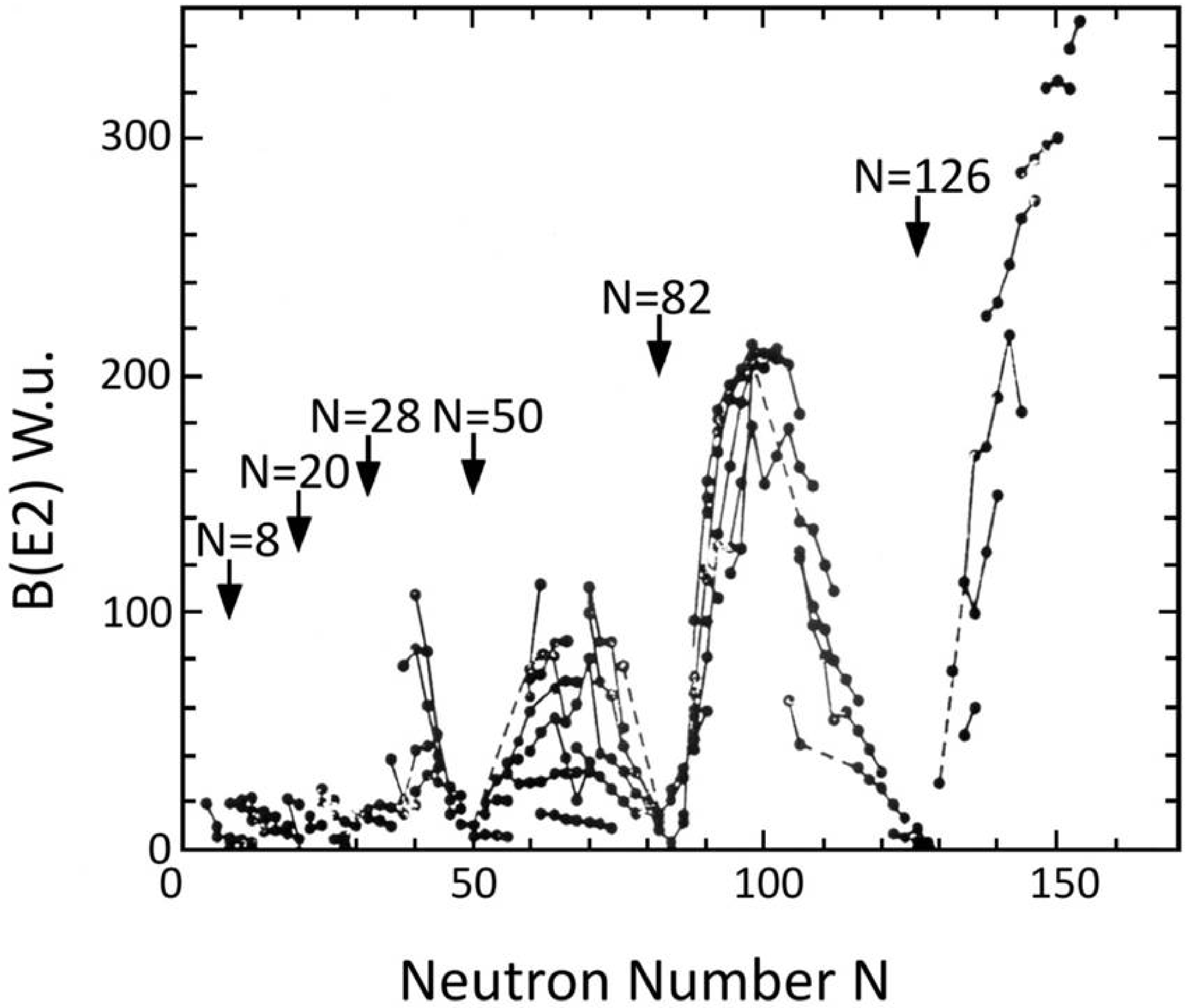
PRL 99, 132501 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Observation of Isomeric Decays in the *r*-Process Waiting-Point Nucleus $^{130}Cd_{82}$

A. Jungclaus,[1] L. Cáceres,[1,2] M. Górska,[2] M. Pfützner,[3] S. Pietri,[4] E. Werner-Malento,[3] H. Grawe,[2] K. Langanke,[2] G. Martínez-Pinedo,[2] F. Nowacki,[5] A. Poves,[1] J. J. Cuenca-García,[2] D. Rudolph,[6] Z. Podolyak,[4] P. H. Regan,[4] P. Detistov,[7] S. Lalkovski,[8,7] V. Modamio,[1] J. Walker,[1] P. Bednarczyk,[2,9] P. Doornenbal,[2] H. Geissel,[2] J. Gerl,[2] J. Grebosz,[2,9] I. Kojouharov,[2] N. Kurz,[2] W. Prokopowicz,[2] H. Schaffner,[2] H. J. Wollersheim,[2] K. Andgren,[10] J. Benlliure,[11] G. Benzoni,[12] A. M. Bruce,[8] E. Casarejos,[11] B. Cederwall,[10] F. C. L. Crespi,[12] B. Hadinia,[10] M. Hellström,[6] R. Hoischen,[6,2] G. Ilie,[13,14] J. Jolie,[13] A. Khaplanov,[10] M. Kmiecik,[9] R. Kumar,[15] A. Maj,[9] S. Mandal,[16] F. Montes,[2] S. Myalski,[9] G. S. Simpson,[17] S. J. Steer,[4] S. Tashenov,[2] and O. Wieland[12]

$$B(E2: 0^+_1 \rightarrow 2^+_1) \propto \langle 2^+_1 || E2 || 0^+_1 \rangle^2$$

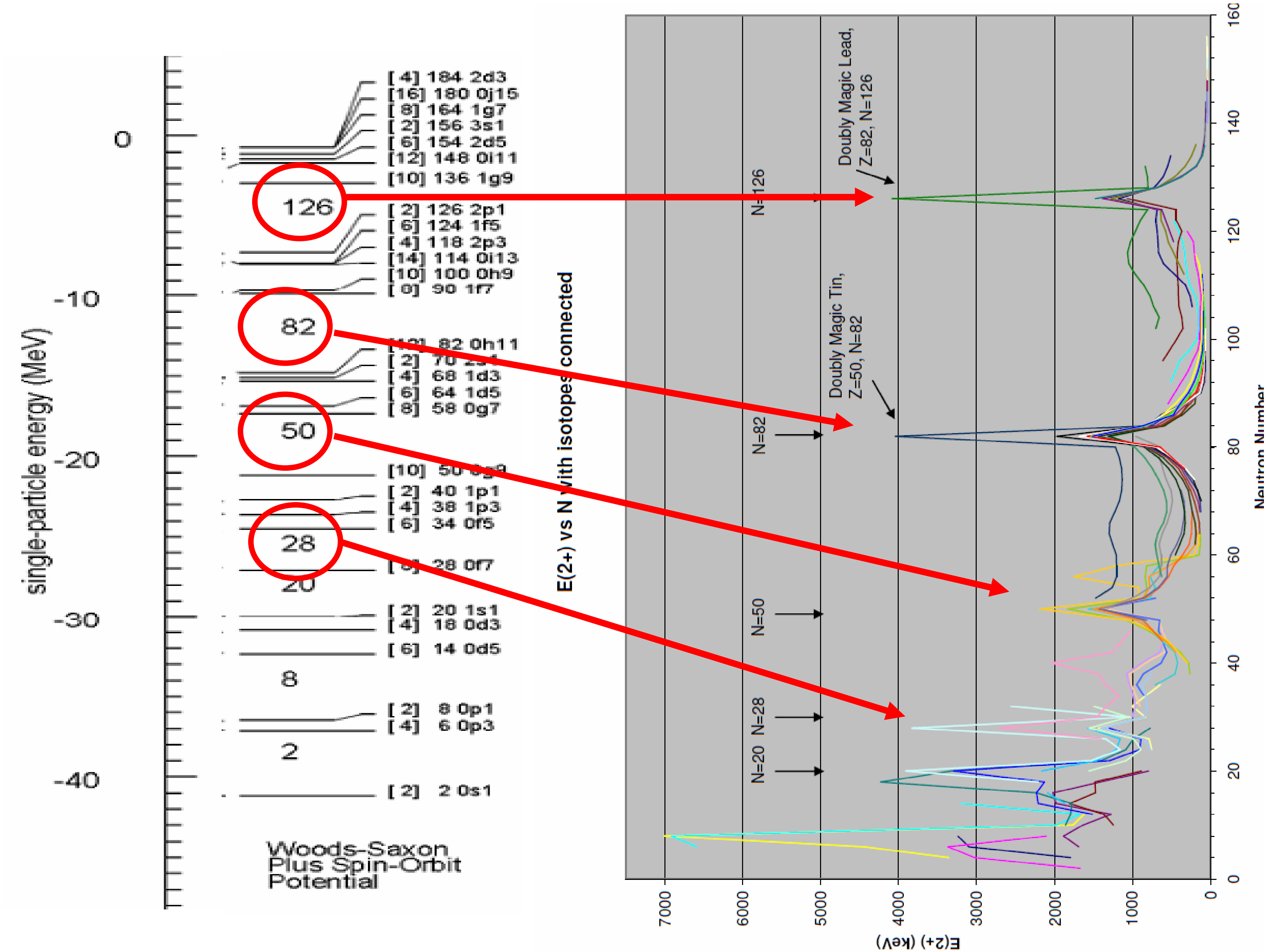




'Magic numbers' observed at

N,Z = 2, 8, 20, 28, (40), 50, (64), 82, 126

Why?



large gaps in single-particle structure of nuclei…MAGIC NUMBERS = ENERGY GAPS

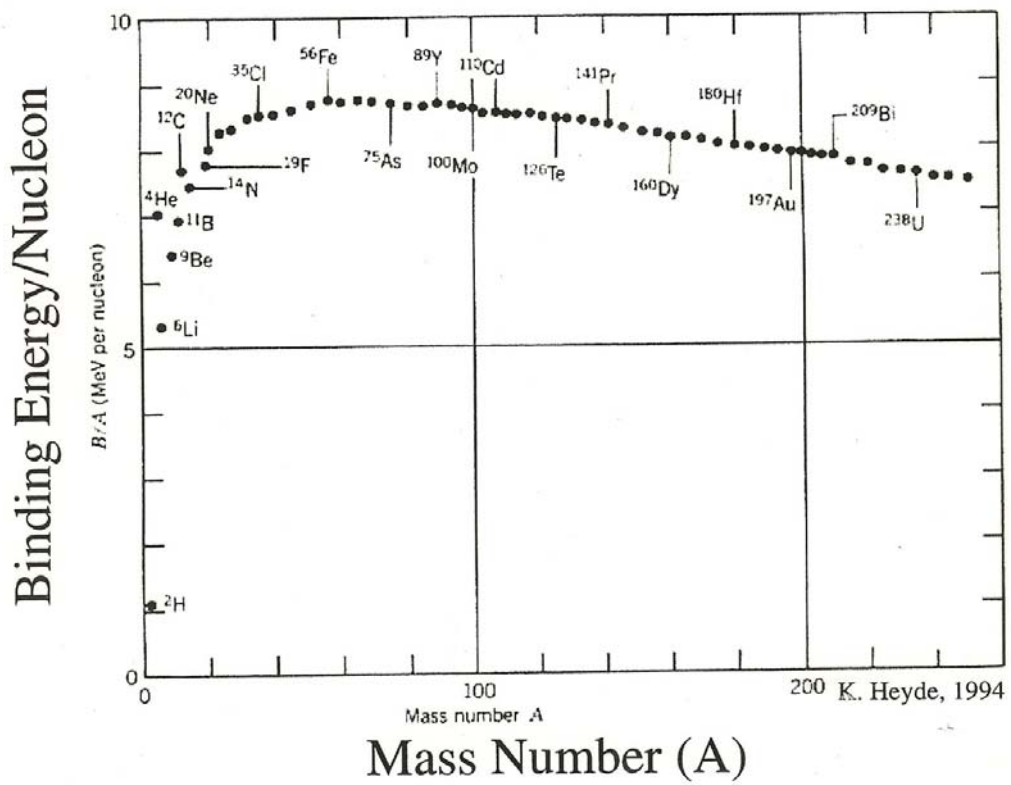
Nuclear Force Saturates

The binding energy **PER nucleon** curve for nuclei saturates at ~8 MeV/u.

Nuclear potential does not interact as A(A-1)…but as A.

Therefore, nuclear force is short range ~1 nucleon, ~1 fm.

Binding Energies



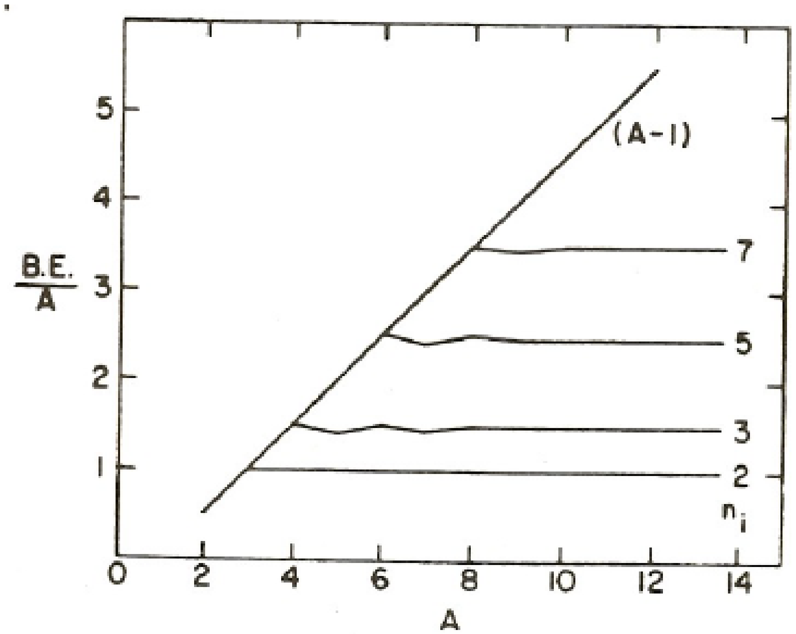
Constant Nuclear Density

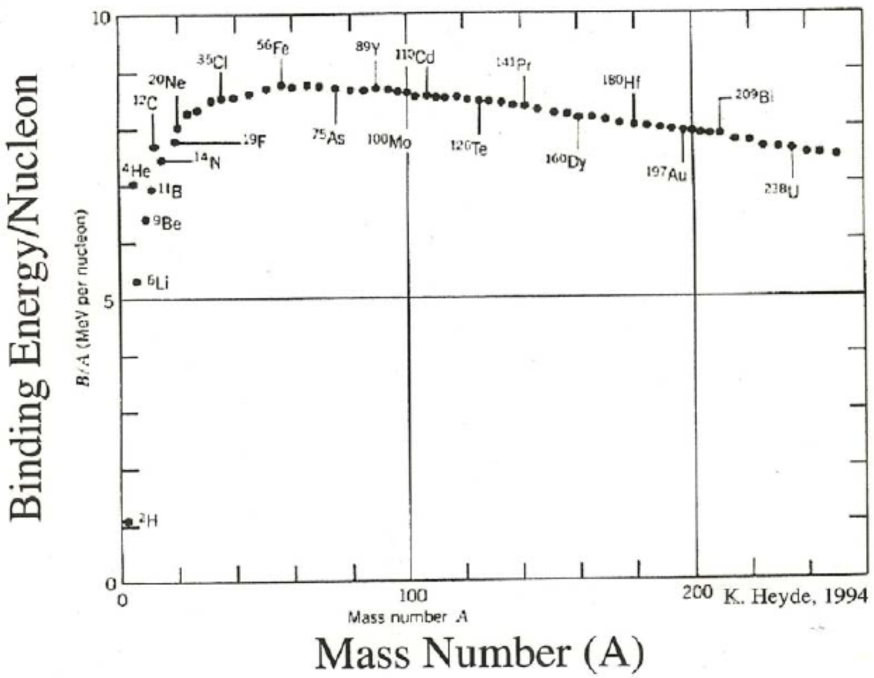
Nuclear Radius: $R = R_o A^{1/3}$ $R_o \sim 1.2$ fm

<u>One possible (wrong!) model of nuclear binding….</u>

Assume that each nucleon interacts with *n* other nucleons and that all such interactions are approximately equal.

The resulting Binding Energy function would then have an A(A-1) energy dependence……
…..but we 'observe' BE ~ A for A>20.





from R.F. Casten book.

Conclusion:

Nuclear force is short range – shorter range than the size of heavy nuclei .

Only really interacts with near neighbours.

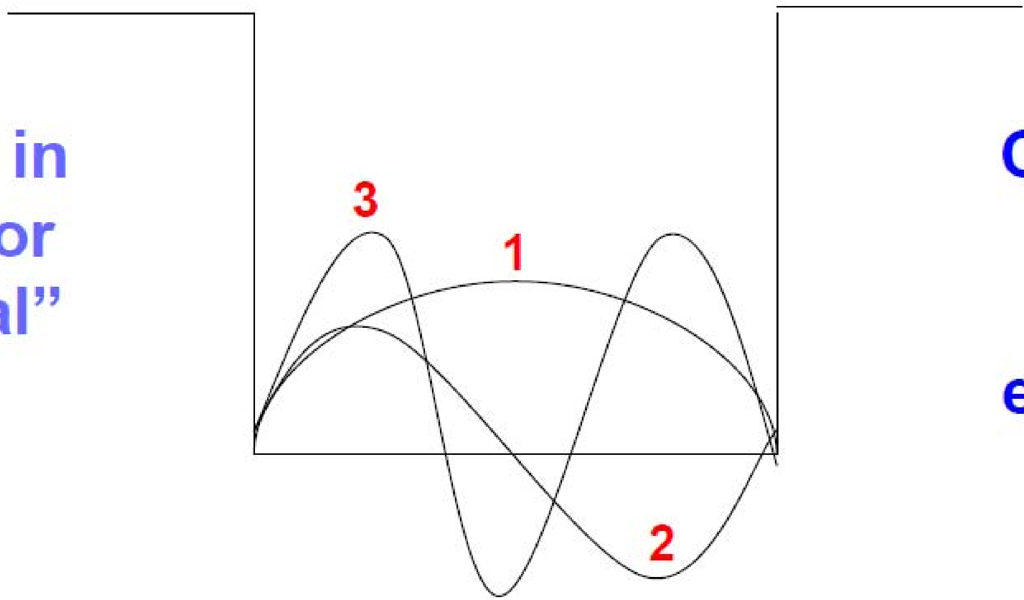
Form of the Nuclear Potential ?

- Assume nuclear 'average' (mean-field) potential can be written to look (a bit like) a square well potential.

- A good first approximation.
  - Constant potential within the nucleus, zero outside the nuclear range.
  - Nearest neighbour approximation valid.

- Add on additional term(s) such as $\underline{l}.\underline{s}$ 'spin-orbit correction' to reproduce the correct answers (i.e. reproduce the magic numbers shell gaps).

Particles in a "box" or "potential" well



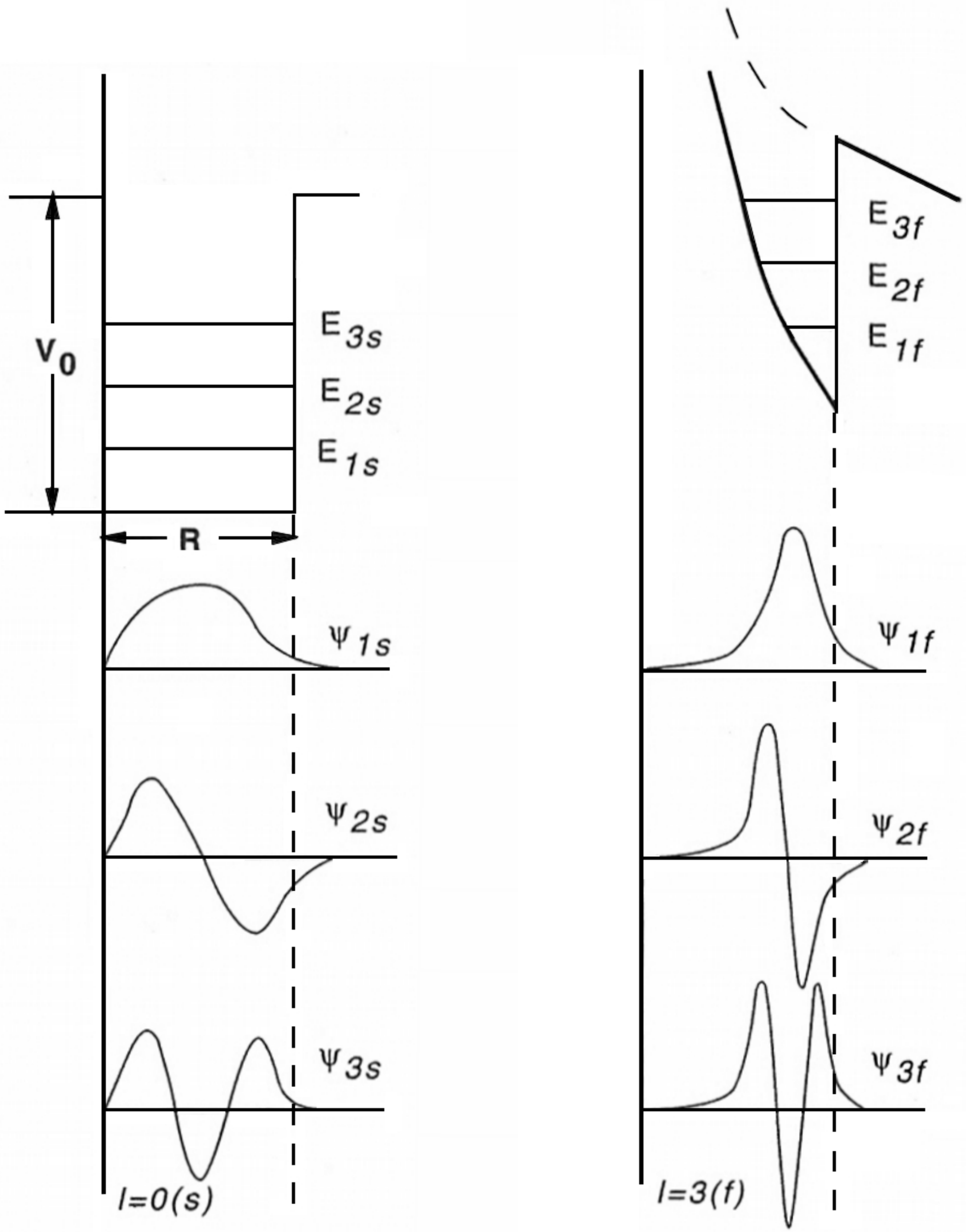
Confinement is origin of quantized energies levels

Energy (E = K+V) depends on wavelength of particle in 3-D box.

Only certain'wavelengths' (i.e. energies) of standing waves are allowed.

These are defined by the principal quantum number, n.

Higher n values correspond to more 'wavelengths' and higher E.



Solving the Schrodinger equation for a Finite Square Well potential gives rise to states of different values of:

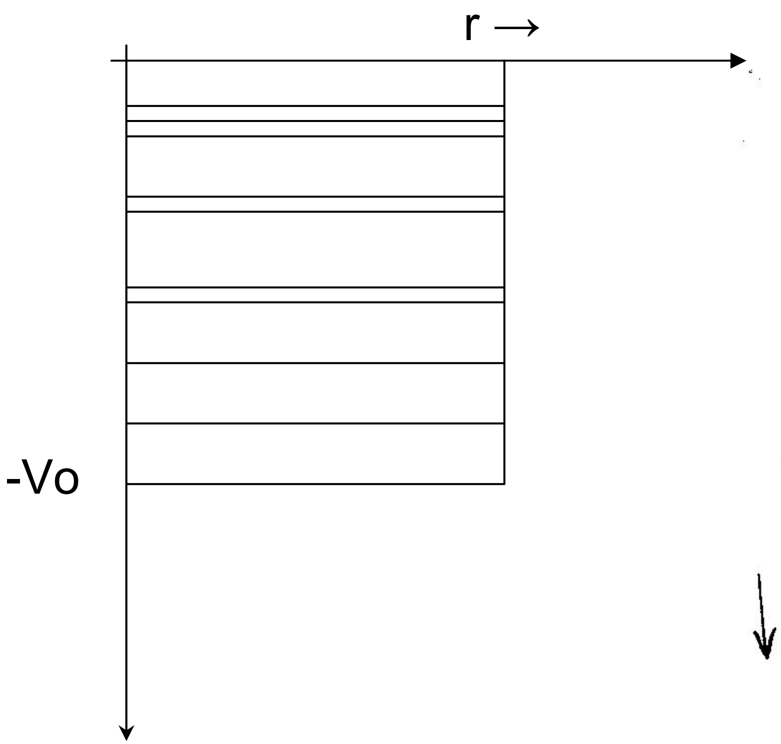
- n (principal quantum number) and
- *l* (orbital angular momentum).

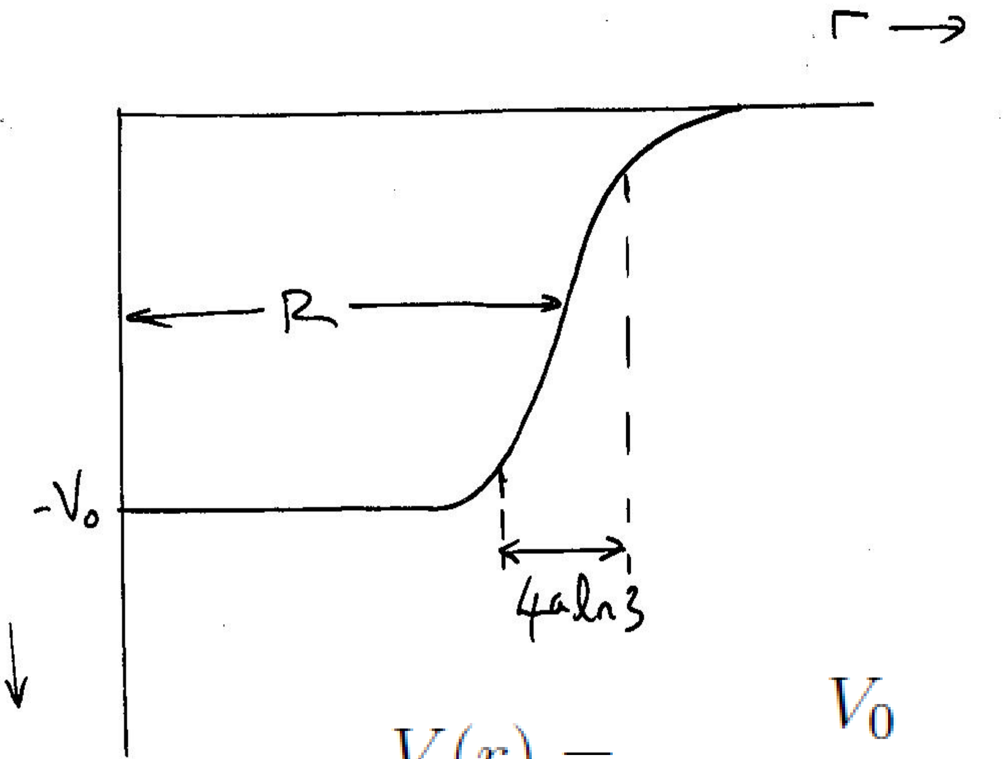
Can tunnel out of the edges,

Also has *l(l+1)* centrifugal term in the potential.

Effect is high – *l* values are on average towards the 'edge' of the nuclear matter.

Assume nuclear 'average' (mean-field) potential can be written to look like a finite square well (defined edge) potential.





$$V(r) = \frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$$

A more 'realistic' potential is something of the Fermi form, Like the Woods-Saxon Potential (looks a bit like an finite square well, but with a more 'diffuse' surface).

Anything else?

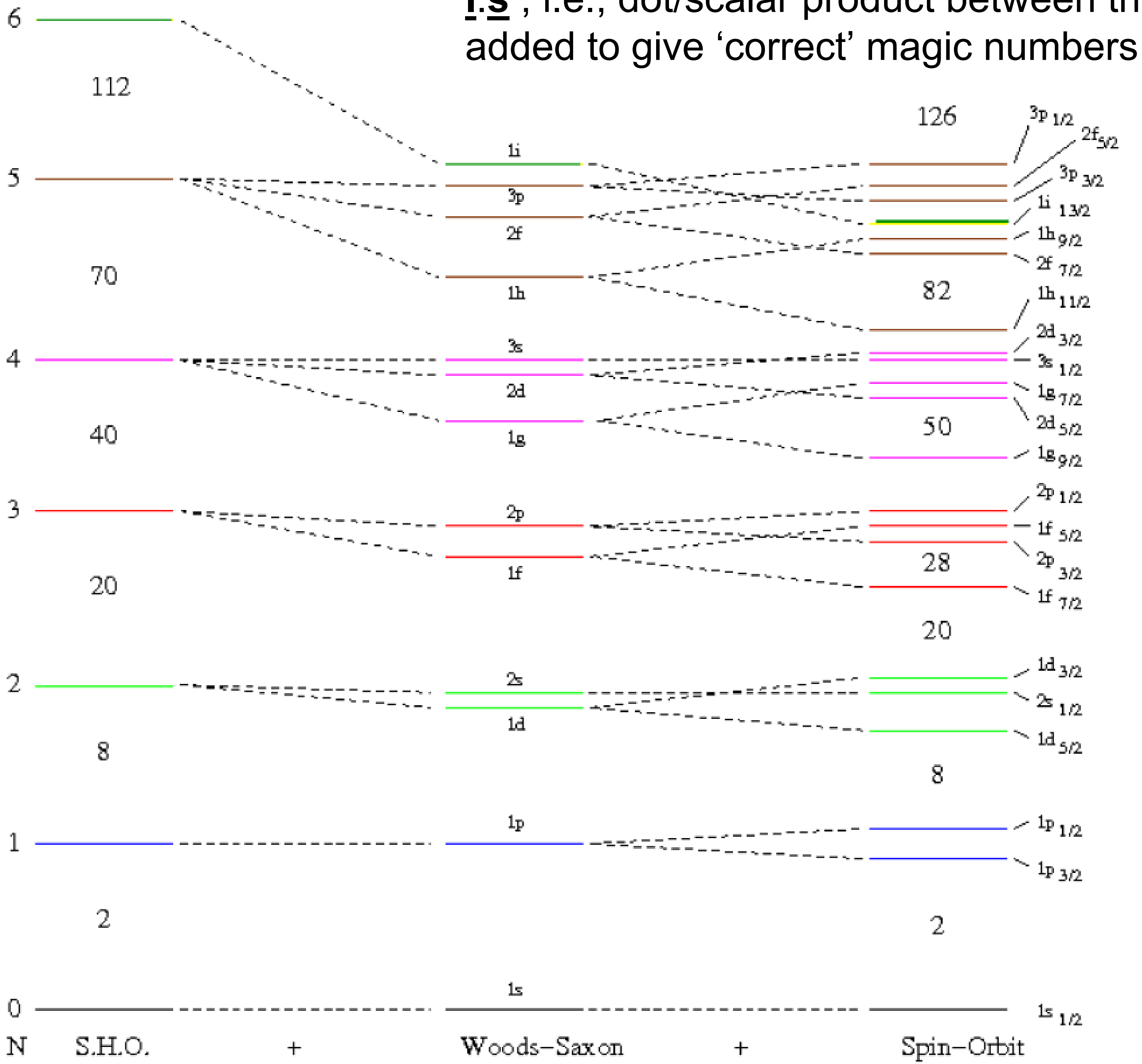
- Pauli exclusion principle, no two nucleons (fermions) can be in the same quantum state.

- Group orbitals in terms of
  - n (principal quantum number)
  - l=orbital angular momentum values allowed for given n. n=0, l=0; n=1, l=1; n=2, l=0,2; n=3, l=1,3;
    n=4, l=0,2,4; n=5, l=1,3,5 etc..

  - Need to account for proton/neutron intrinsic spins
    - j = l + s or l – s (where s = ½ ħ)
    - l values described in spectroscopic notation
      - l = 0 , 1, 2, 3, 4, 5, 6, 7
      - l = s, *p*, *d*, *f*, *g*, *h*, *i*, *k*

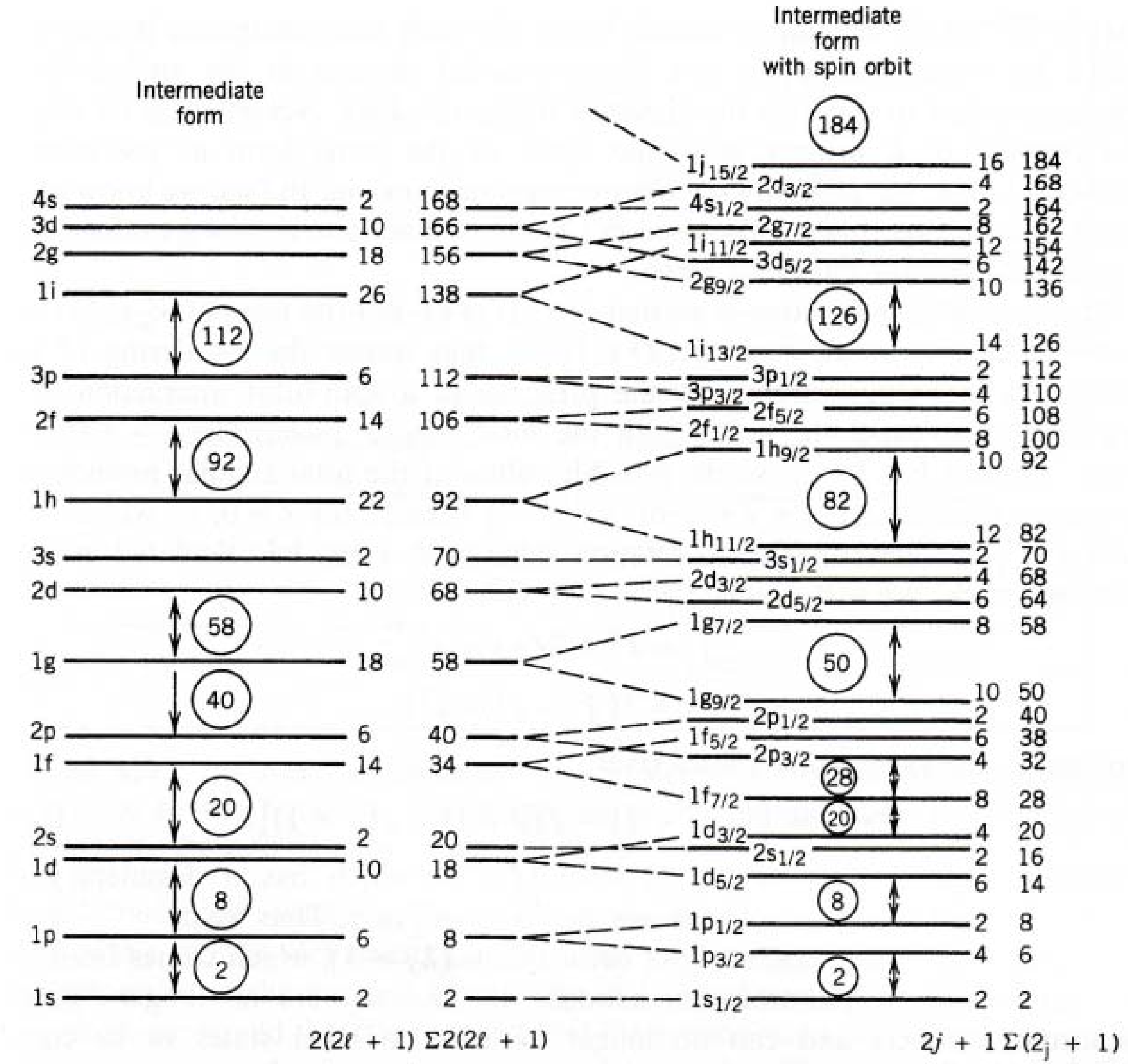
  - Degeneracy of each j level is given by 2j+1, since can have projections of m_j = -j, -j+1, …., j-1, j.

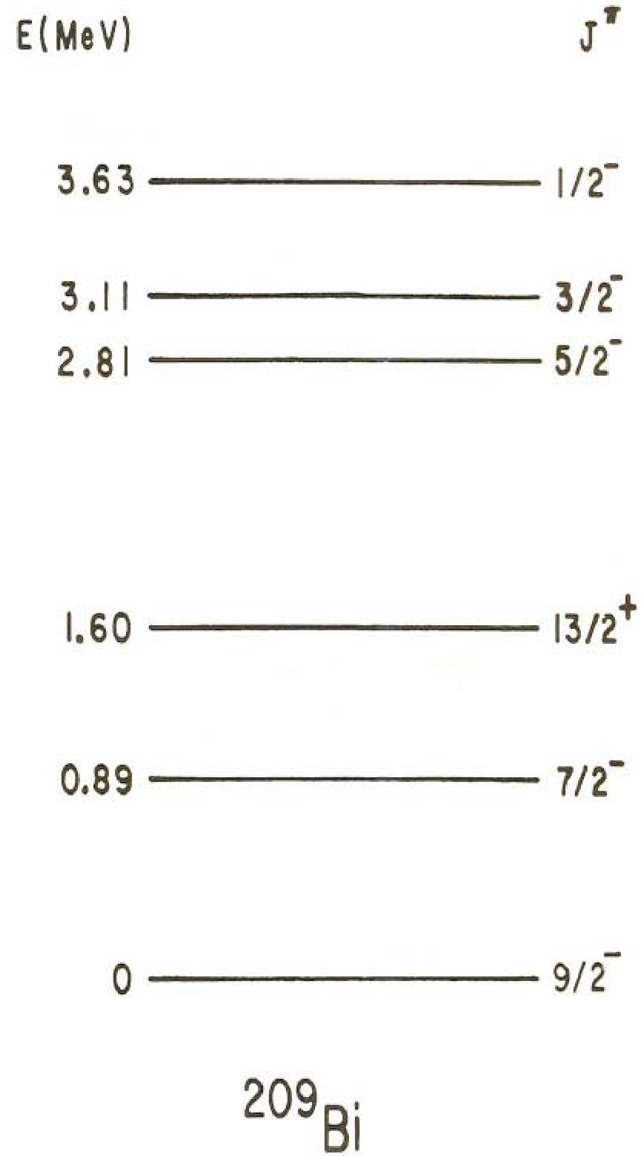
Spin-orbit term, which gives energy correction depending on **l**.**s** , i.e., dot/scalar product between the vectors l and s) is added to give ‘correct’ magic numbers and shell gaps.



Description of Doubly-Magic +1 Nuclei

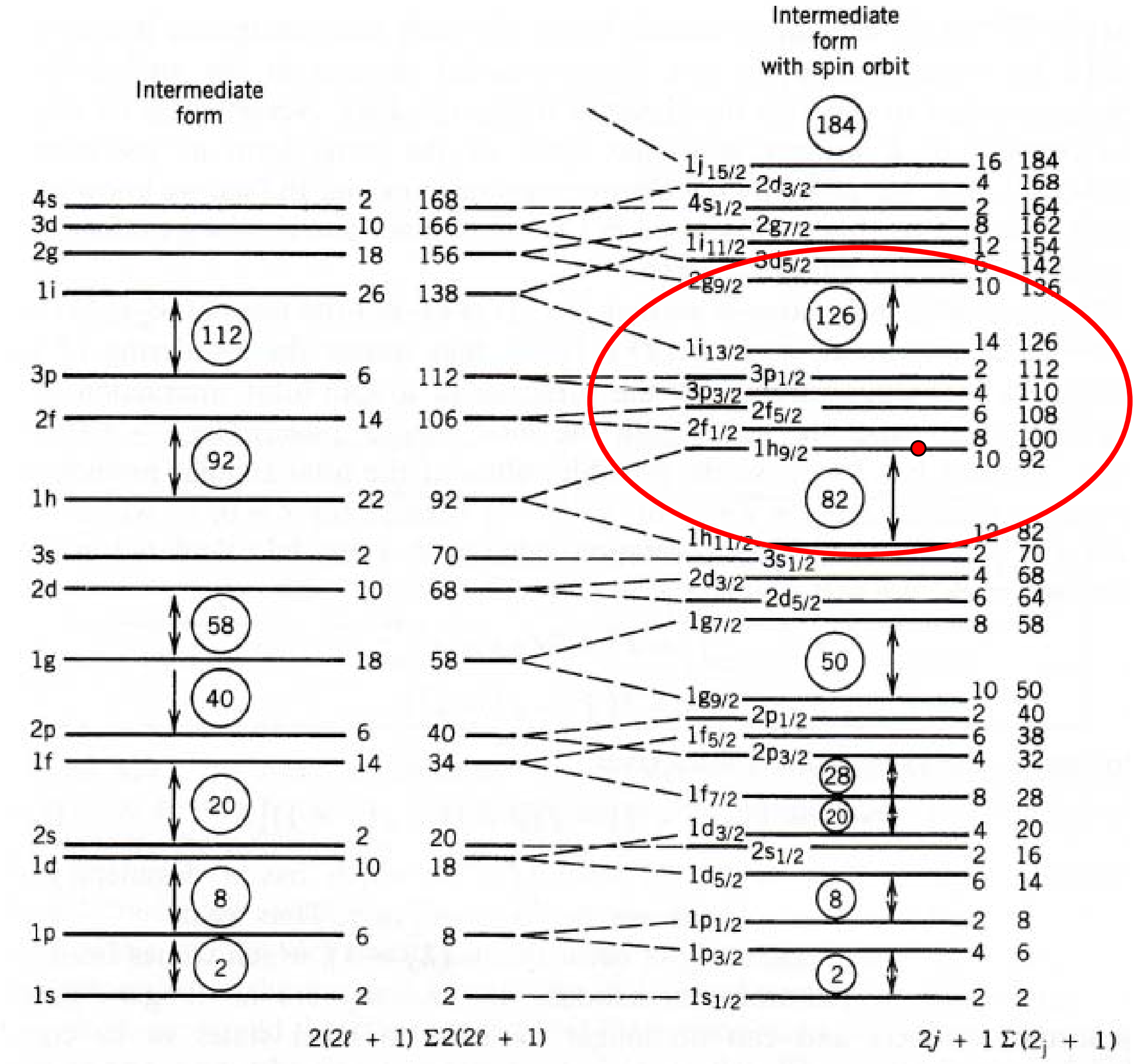
Assume inert core and single, unpaired particle

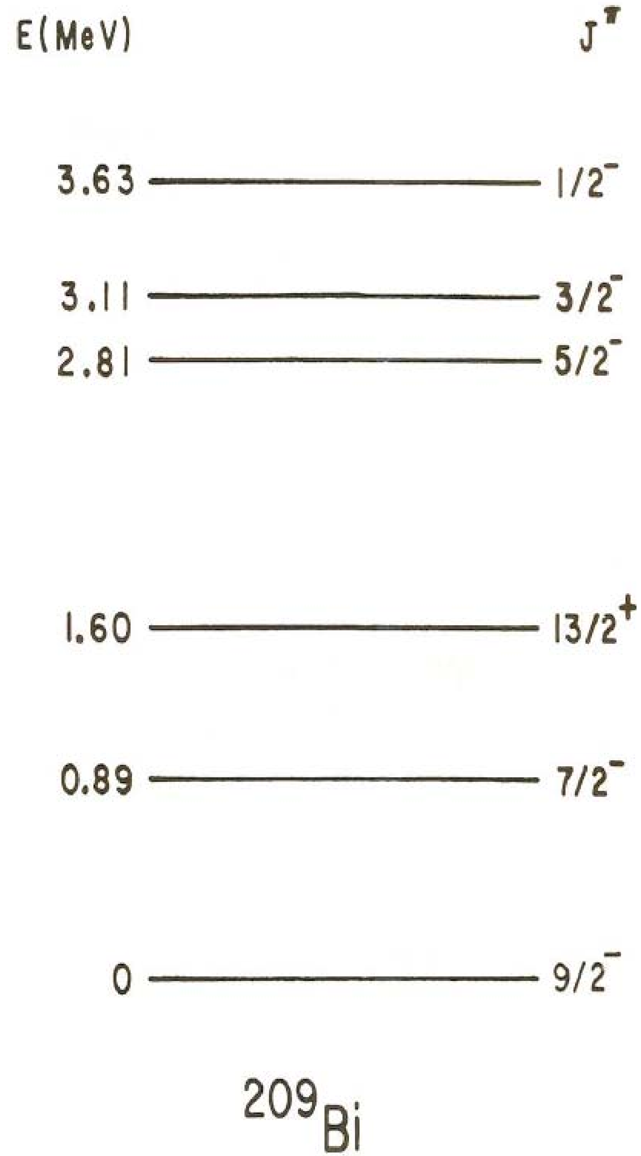




Description of Doubly-Magic +1 Nuclei

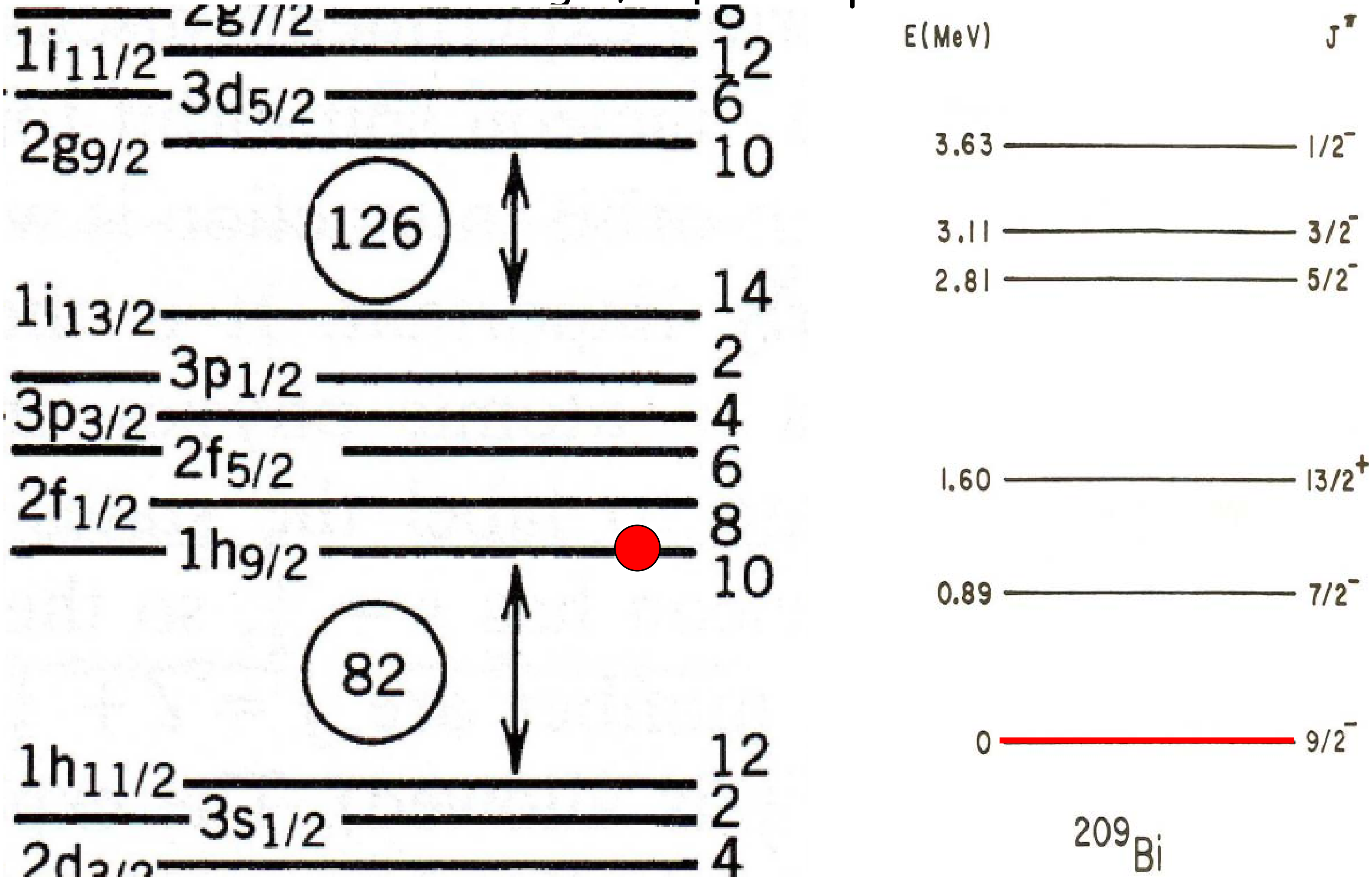
Assume inert core and single, unpaired particle





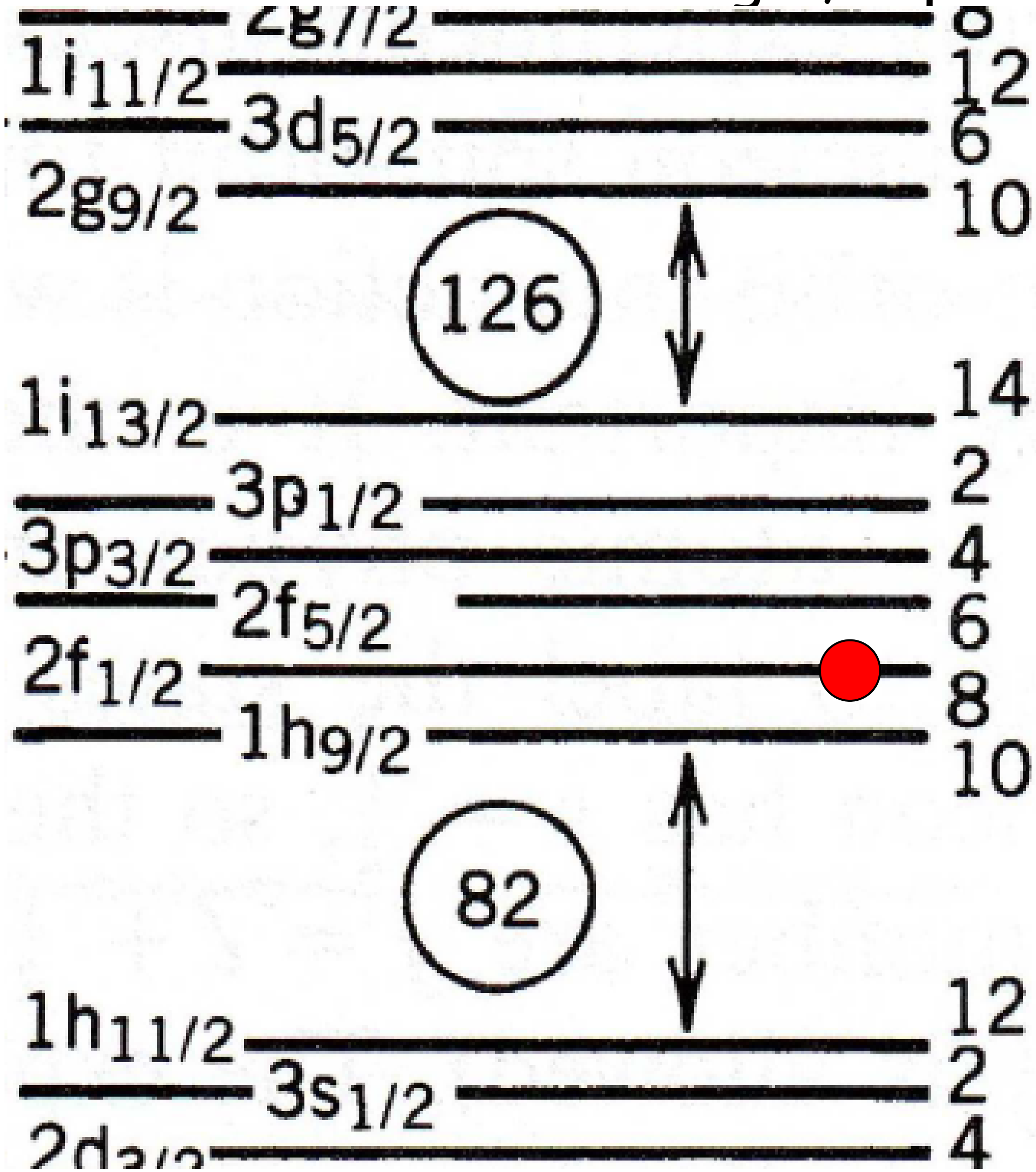
Description of Doubly-Magic +1 Nuclei

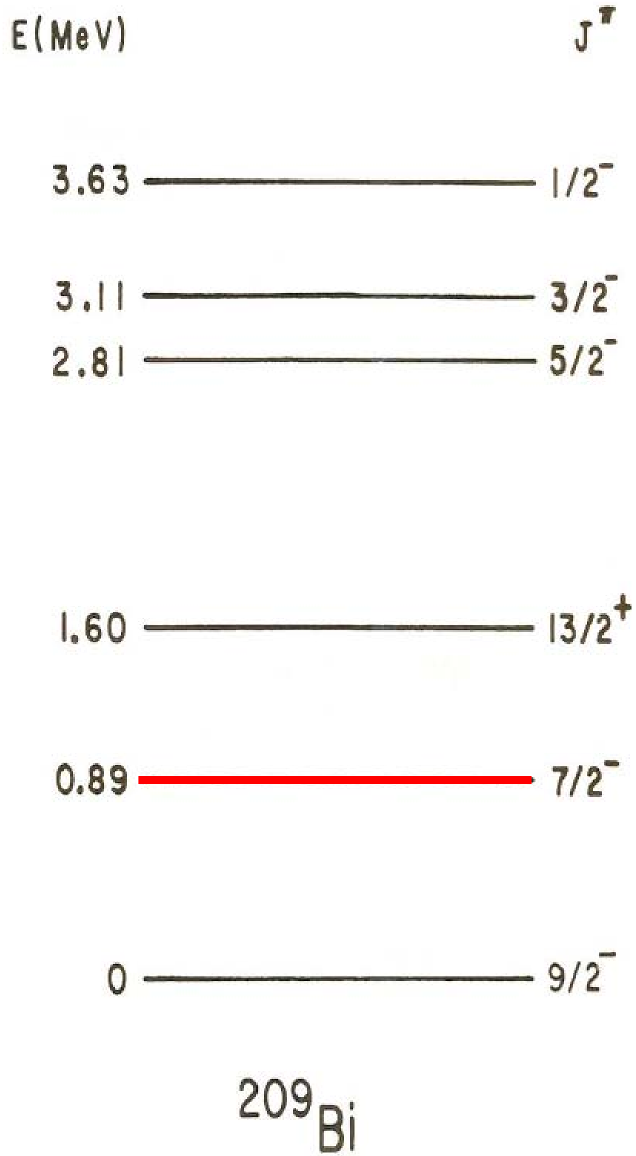
Assume inert core and single, unpaired particle



Description of Doubly-Magic +1 Nuclei

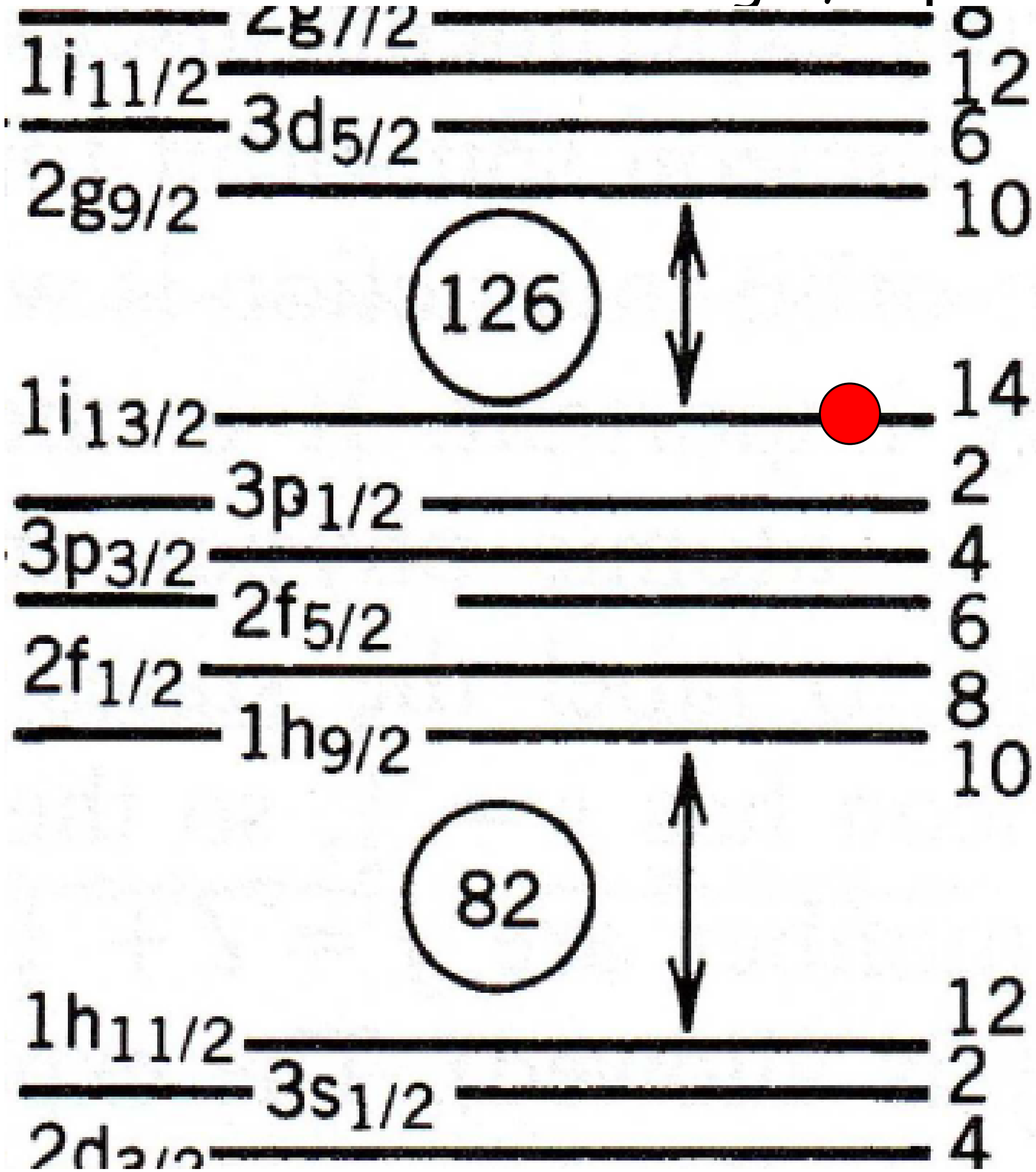
Assume inert core and single, unpaired particle

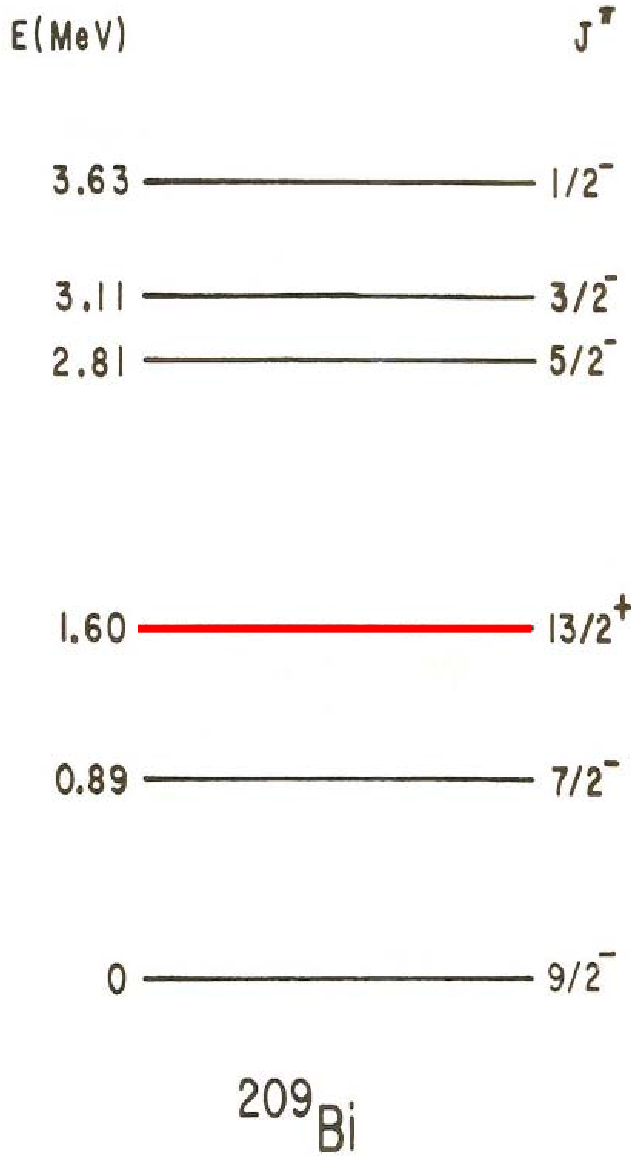




Description of Doubly-Magic +1 Nuclei

Assume inert core and single, unpaired particle





So: We can immediately extend the Shell Model to predict

$J_{g.s.}$ for <u>*any*</u> nucleus which has one nucleon (*p* or *n*) beyond

"doubly magic" core

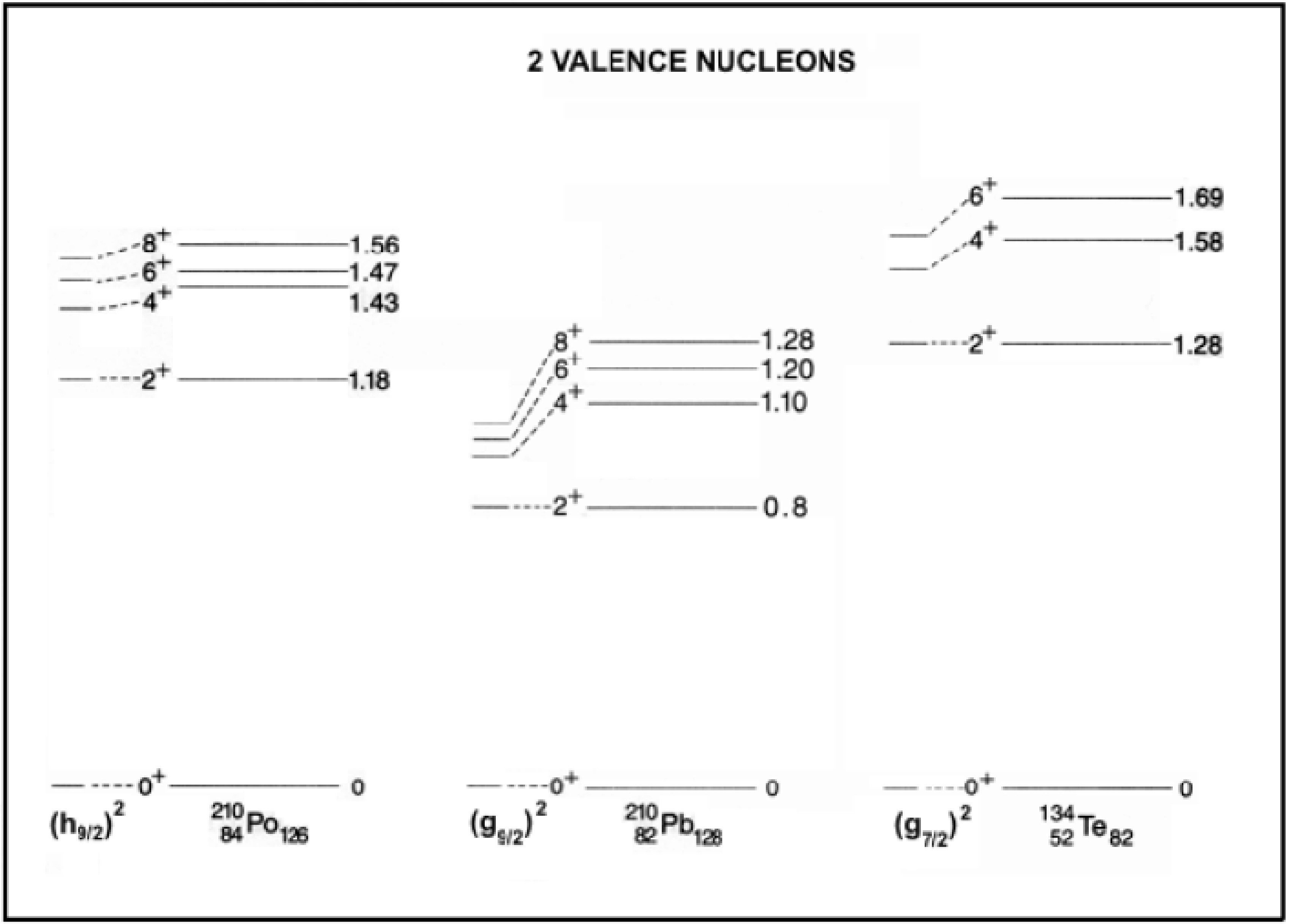
! !

$$J^{\pi}_{g.s.}\ \text{(Doubly Magic + 1)} = j^{\pi}_{\text{last odd nucleon}}$$

Examples: $^{41}_{20}Ca_{21}$ $^{91}_{40}Zr_{51}$ $^{209}_{82}Pb_{127}$ $^{209}_{83}Bi_{126}$

$\frac{7}{2}^{-}$ $\frac{5}{2}^{+}$ $\frac{9}{2}^{+}$ $\frac{9}{2}^{-}$

What about 2 nucleons outside a closed shell ?



Residual Interactions?

- We need to include any addition changes to the energy which arise from the interactions between valence nucleons.

- This is in addition the mean-field (average) potential which the valence proton/neutron feels.

- Hamiltonian now becomes $H = H_0 + H_{residual}$

- 2-nucleon system can be thought of as an inert, doubly magic core plus 2 interacting nucleons.

- Residual interactions between these two 'valence' nucleons will determine the energy sequence of the allowed spins / parities.

What spins can you make?

- If two particles are in identical orbits (j^2), then what spins are allowed?

Two possible cases:

- Same particle, e.g., 2 protons or 2 neutrons = even-even nuclei like ^{42}Ca, 2 neutrons in $f_{7/2}$ = $(\nu f_{7/2})^2$

  We can couple the two neutrons to make states with spin/parity $J^{\pi}=0^{+}$, 2^{+}, 4^{+} and 6^{+}

  These all have T=1 in isospin formalism, intrinsic spins are anti-aligned with respect to each other.

- Proton-neutron configurations (odd-odd)

  e.g., ^{42}Sc, 1 proton and 1 neutron in $f_{7/2}$

  We can couple these two make states with spin / parity 0^{+}, 1^{+}, 2^{+}, 3^{+}, 4^{+}, 5^{+}, 6^{+} and 7^{+}.

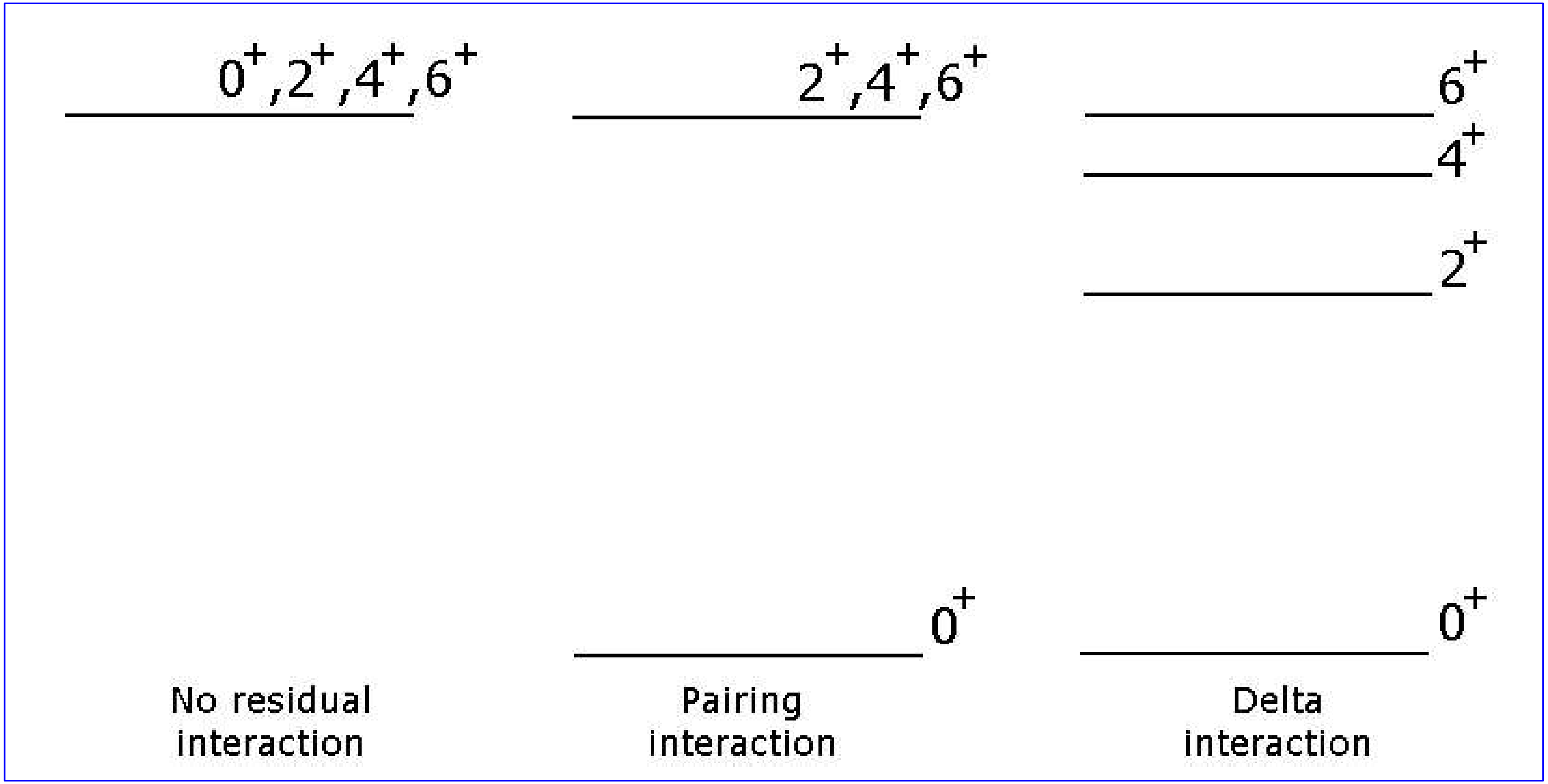
Even spins have T=1 (S=0, intrinsic spins anti-aligned);
Odd spins have T=0 (S=1, intrinsic spins aligned)

m – scheme showing which J_{tot} values are allowed for $(f_{7/2})^2$ coupling of two identical particles (2 protons or 2 neutrons).

$j_1 = 7/2$ m_1	$j_2 = 7/2$ m_2	M	J
7/2	5/2	6	
7/2	3/2	5	
7/2	1/2	4	
7/2	–1/2	3	6
7/2	–3/2	2	
7/2	–5/2	1	
7/2	–7/2	0	
5/2	3/2	4	
5/2	1/2	3	
5/2	–1/2	2	4
5/2	–3/2	1	
5/2	–5/2	0	
3/2	1/2	2	
3/2	–1/2	1	2
3/2	–3/2	0	
1/2	–1/2	0	0

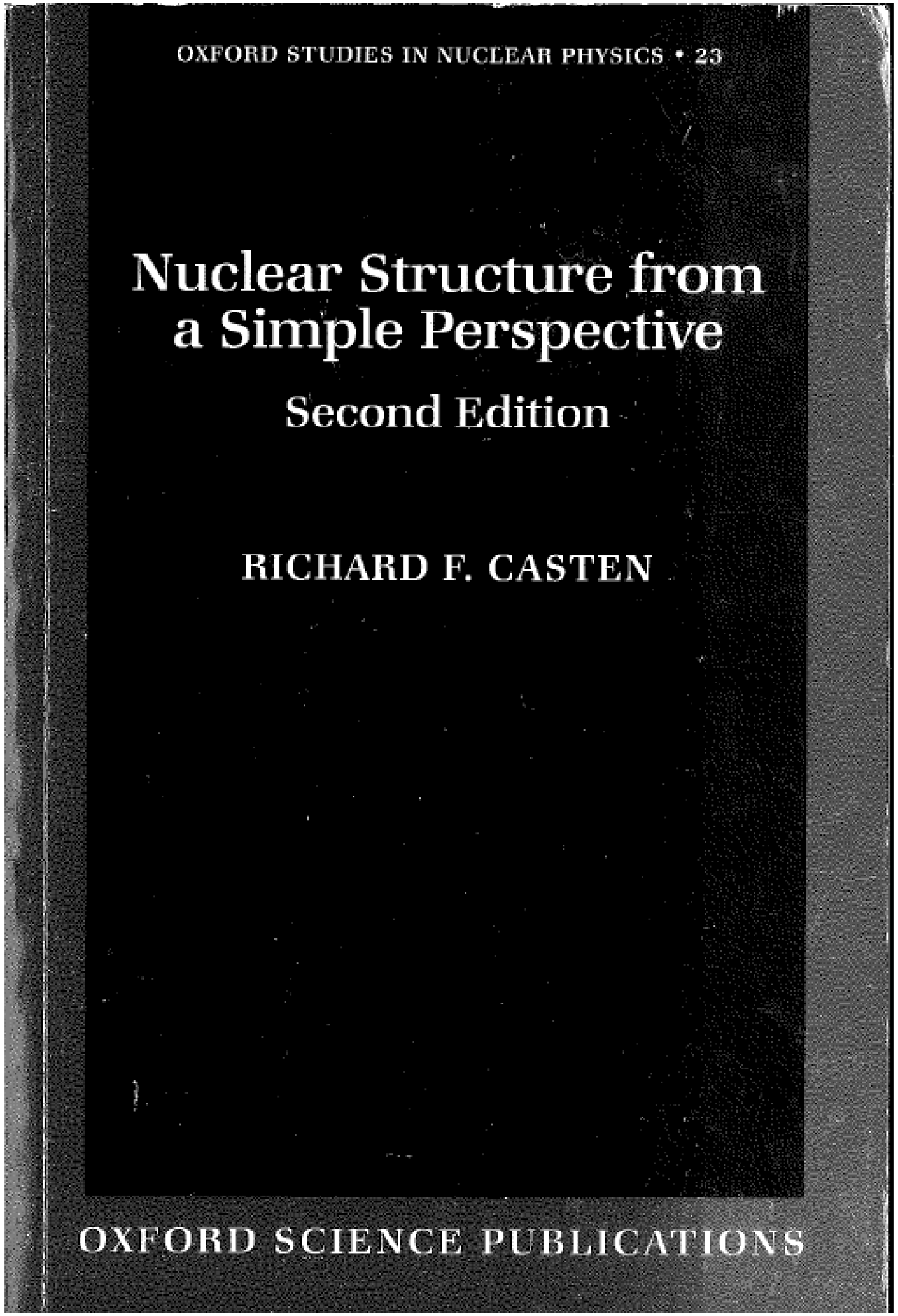
* Only positive total M values are shown. The table is symmetric for $M < 0$.

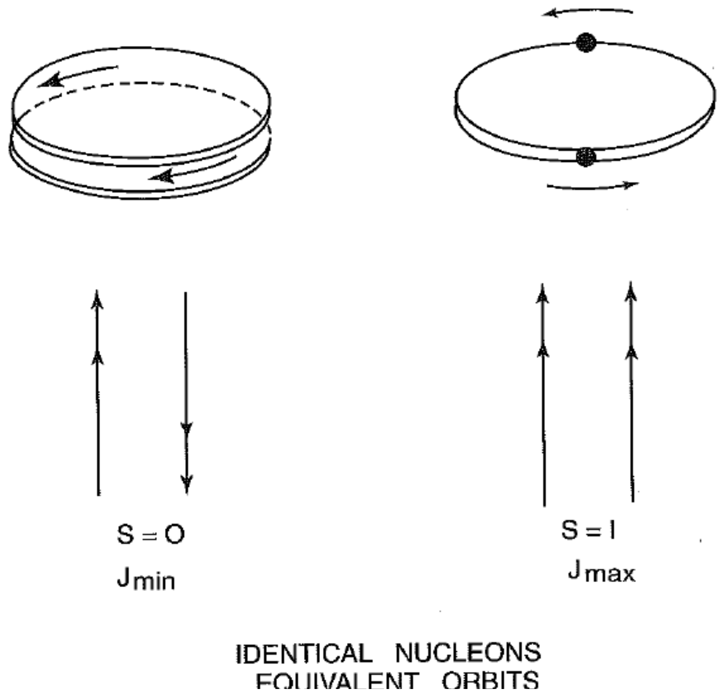
Note, that only even spin states are allowed.



Schematic for $(f_{7/2})^2$ configuration.
4 degenerate states if there are no residual interactions.

Residual interactions between two valence nucleons give additional binding, lowering the (mass) energy of the state.





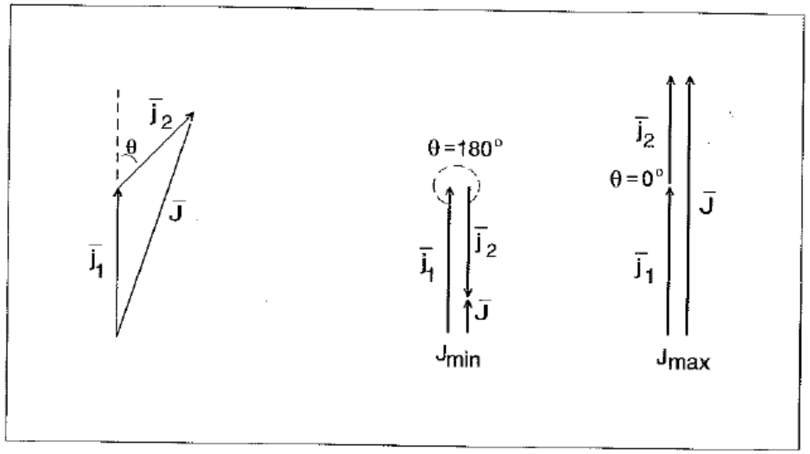
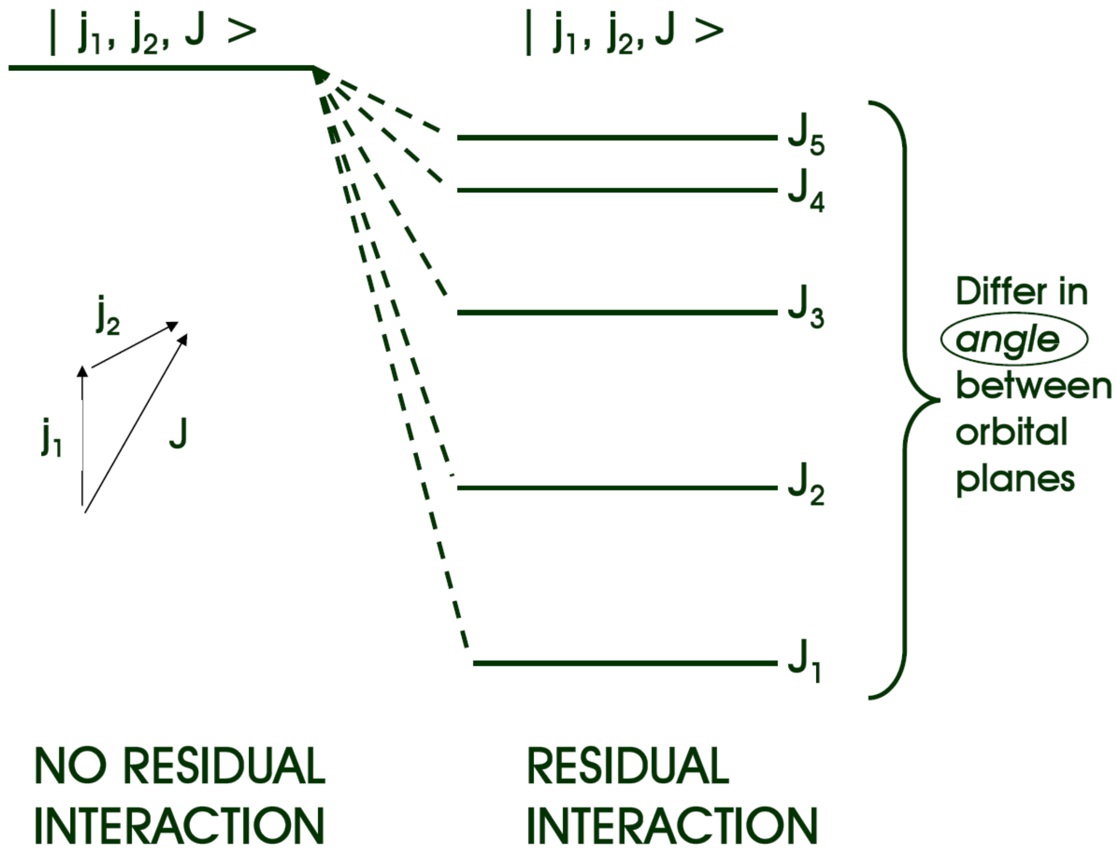


FIG. 4.8. Definition and schematic illustration of some of the ideas used in the geometrical analysis of short-range residual interactions.

Residual Interactions—Diagonal Effects

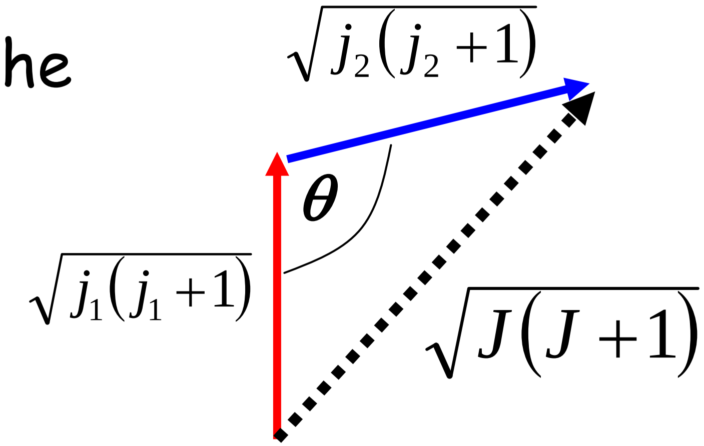
Consider 2 particles, in orbits j_1, j_2 coupled to spin J_i, and interacting with a residual interaction, V_{12}.

2 Identical Nucleons



Geometric Interpretation of the δ Residual Interaction for a j^2 Configuration Coupled to Spin J

Use the cosine rule and recall that the magnitude of the spin vector of spin $j = [\, j\,(j+1)\,]^{-1/2}$



$$J^2 = j_1^2 + j_2^2 - 2 j_1 j_2 \cos(\theta)$$

therefore

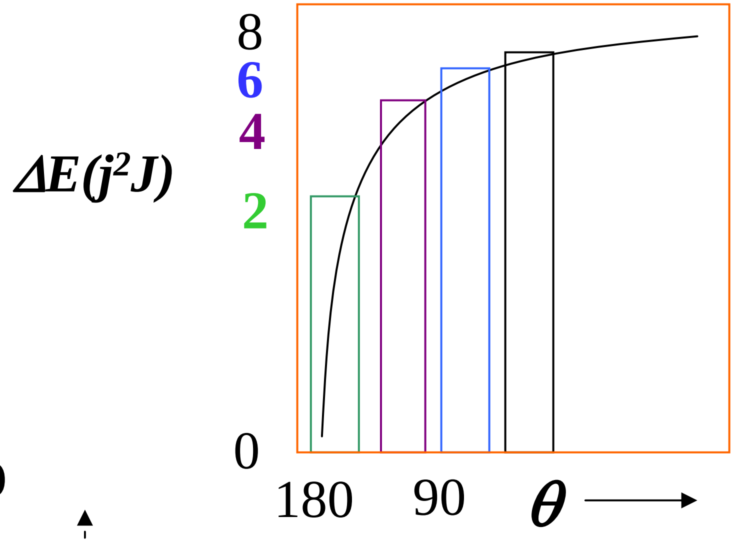
$$J(J+1) = j_1(j_1+1) + j_2(j_2+1) - \sqrt{j_1(j_1+1)}\sqrt{j_2(j_2+1)}\cos(\theta)$$

$$\therefore \text{for } j_1 = j_2 = j \quad \cos^{-1}\left[\frac{J(J+1) - 2j(j+1)}{j(j+1)}\right]$$

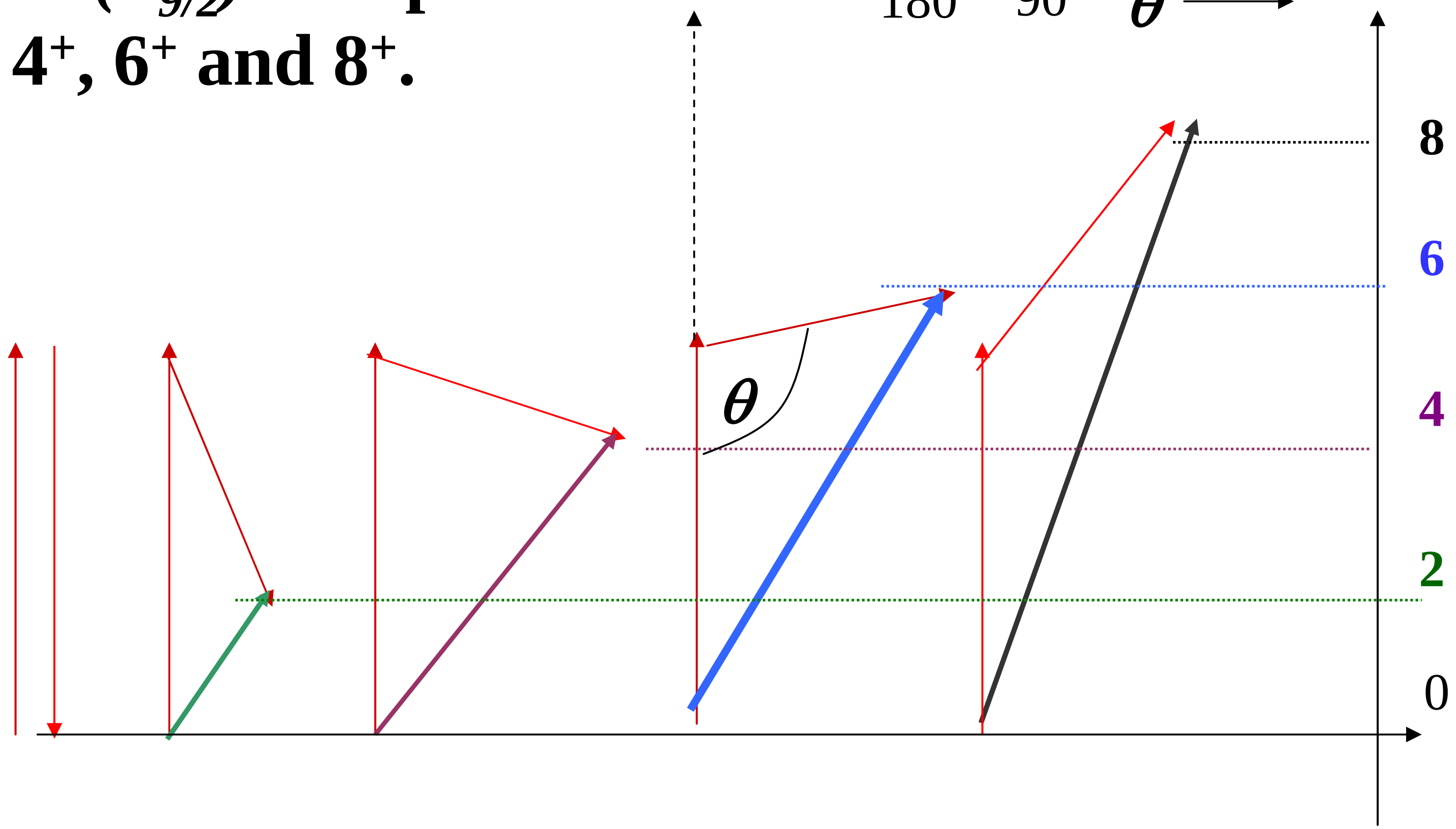
δ–interaction gives nice simple geometric rationale for **Seniority Isomers** from

$\Delta E \sim -V_o F_r \tan(\theta/2)$

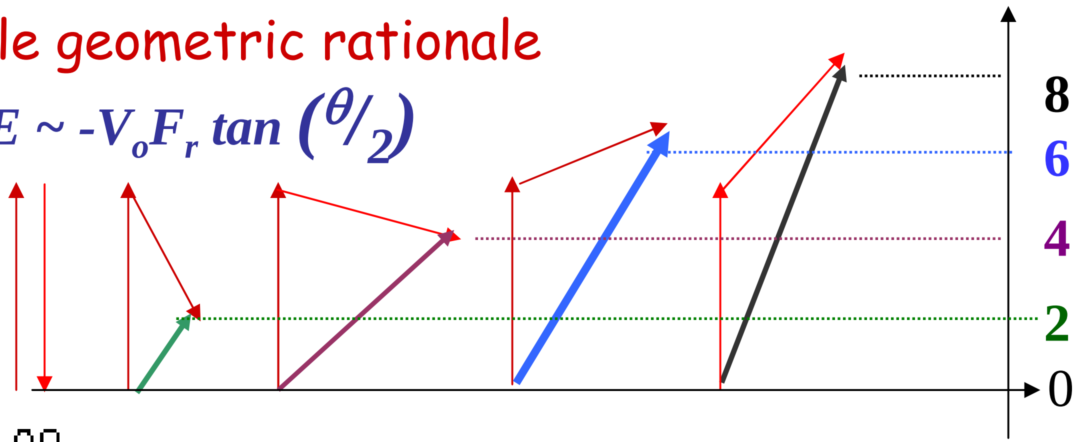
for $T=1$, even J

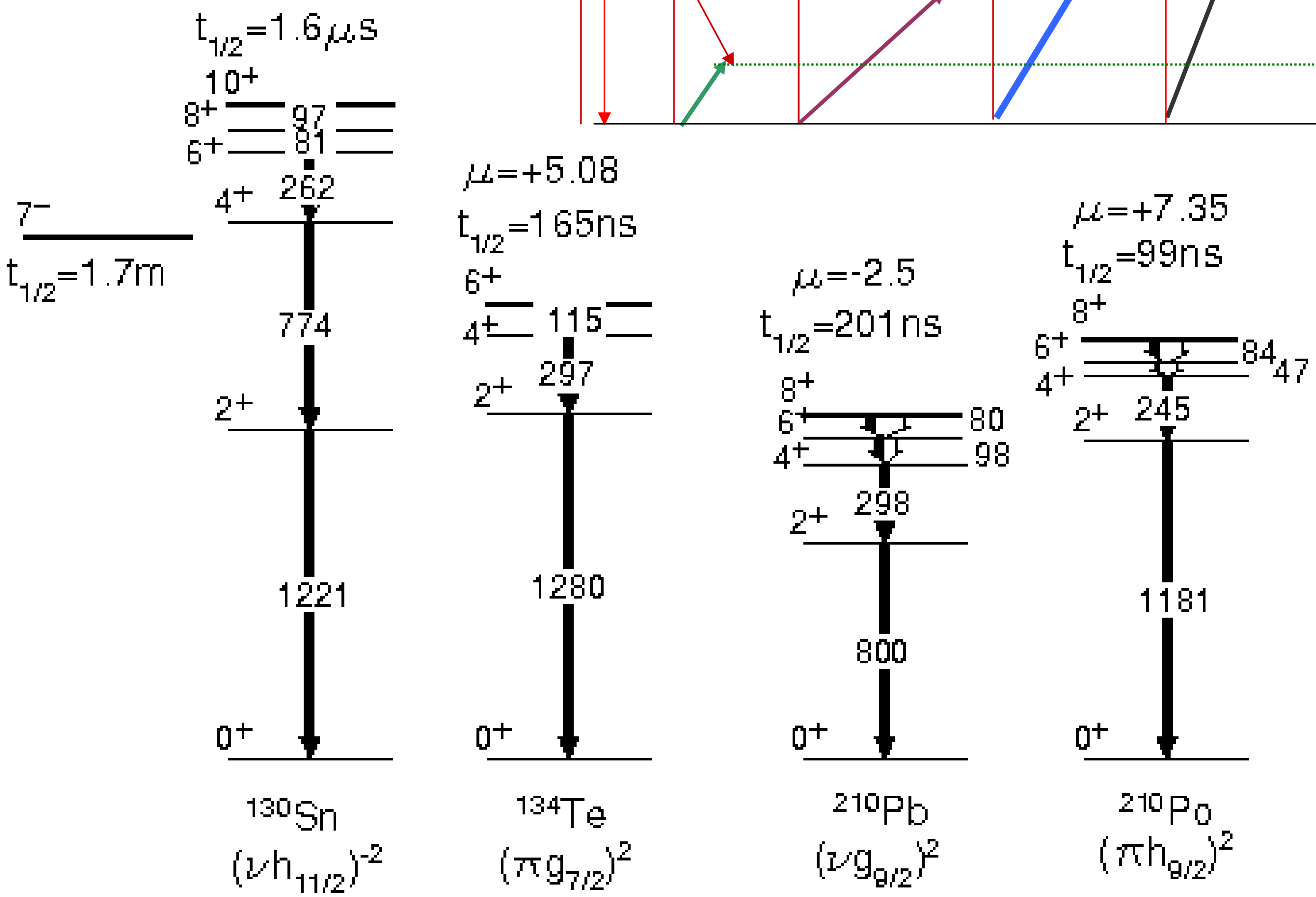


e.g. $J^\pi = (h_{9/2})^2$ coupled to 0^+, 2^+, 4^+, 6^+ and 8^+.



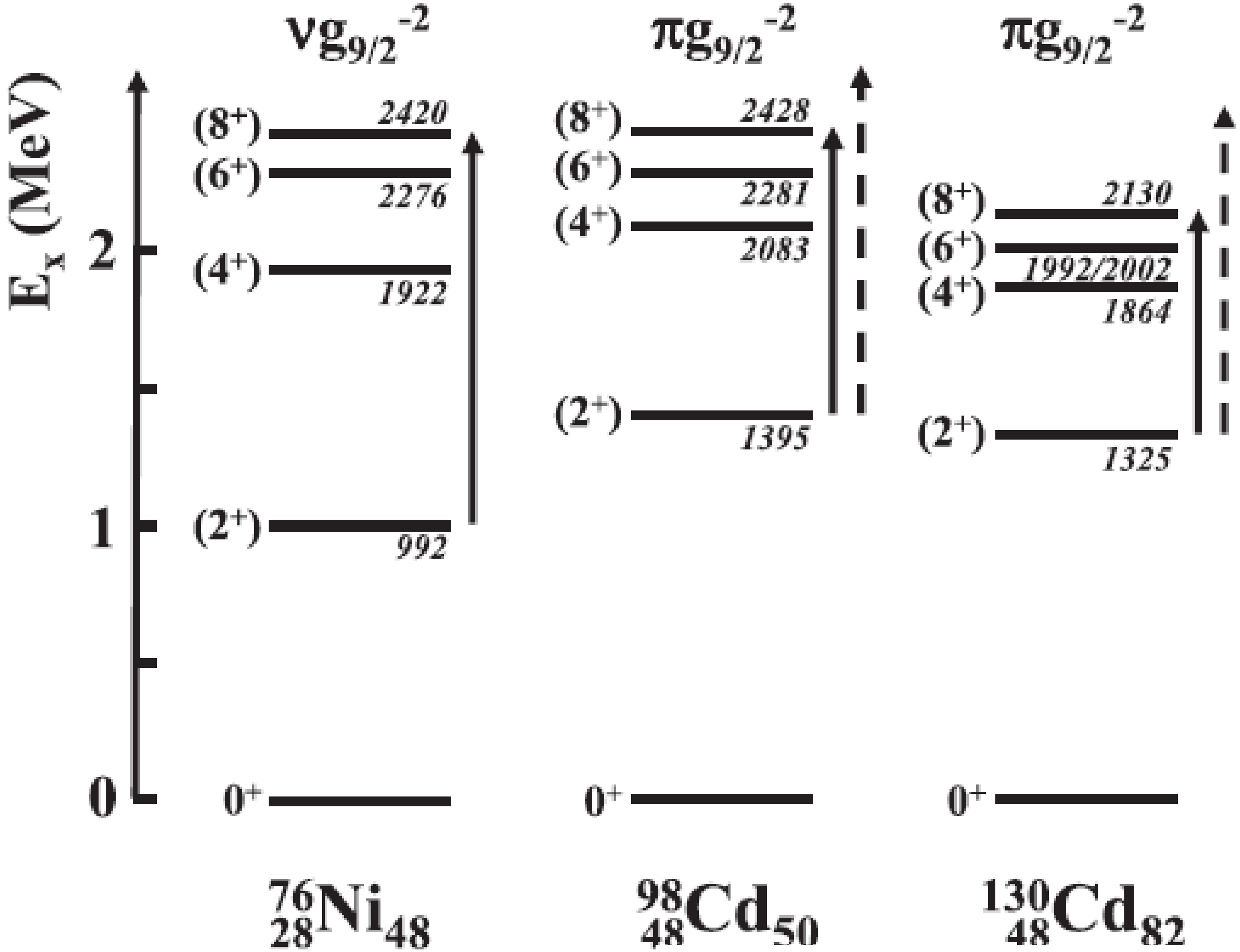
δ – interaction gives nice simple geometric rationale for **Seniority Isomers** from $\Delta E \sim -V_o F_r \tan(\theta/2)$ for *T=1*, even *J*





See e.g., Nuclear structure from a simple perspective, R.F. Casten Chap 4.)

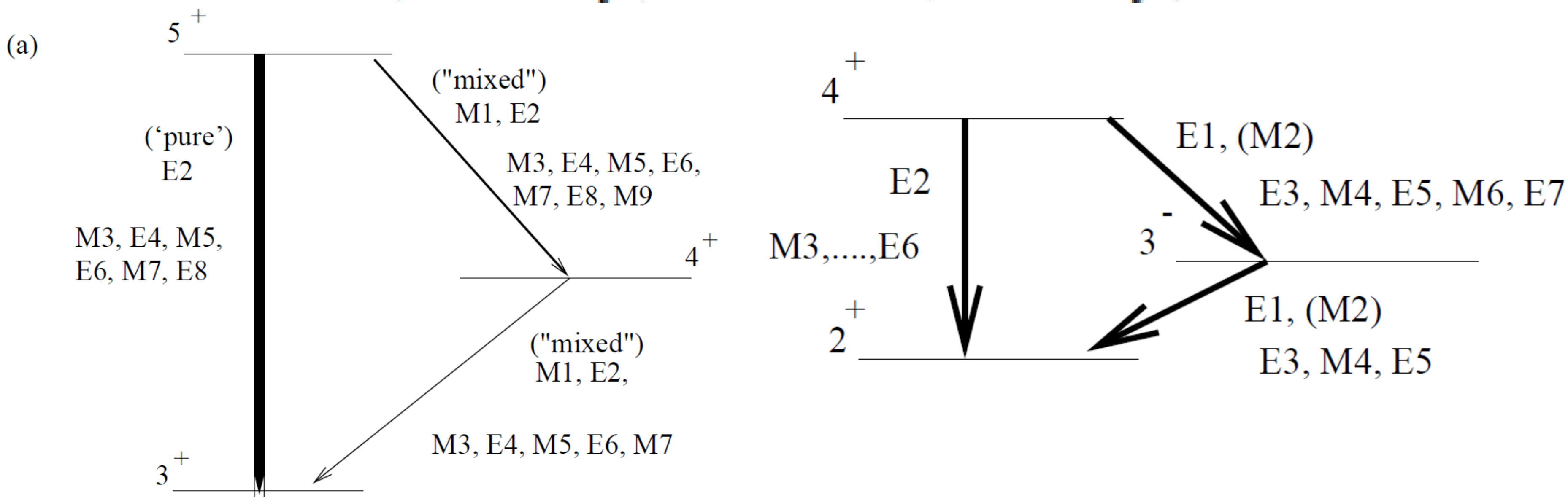
Note, 2 neutron or 2 proton **holes** in doubly magic nuclei show spectra like 2 proton or neutron **particles**.



A. Jungclaus et al., PRL **99**, 132501 (2007)

Basic EM Selection Rules?

$$|I_i + I_f| \geq L \geq |I_i - I_f|$$



The transition probability for a state decaying from state J_i to state J_f, separated by energy E_γ, by a transition of multipole order L is given by [1, 7]

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L\left((2L+1)!!\right)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L : J_i \rightarrow J_f) \qquad (1.1.2)$$

where $B(\lambda L : J_i \rightarrow J_f)$ is called the *reduced matrix element.*

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L\left((2L+1)!!\right)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L : J_i \to J_f) \tag{5.0.3}$$

where $B(\lambda L : J_i \to J_f)$ is called the *reduced matrix element.*

Measuring the lifetime (decay probability) of a nuclear state thus gives a value for the $B(\lambda L : J_i \to J_f)$.

For lifetimes, τ in units of seconds where the transition *probability* per unit second, $T = \frac{1}{\tau}$, (E_γ in MeV),

$$T(E1) = 1.587 \times 10^{15} E_\gamma^3 B(E1) \tag{5.0.4}$$

$$T(E2) = 1.223 \times 10^{9} E_\gamma^5 B(E2) \tag{5.0.5}$$

$$T(E3) = 5.698 \times 10^{2} E_\gamma^7 B(E3) \tag{5.0.6}$$

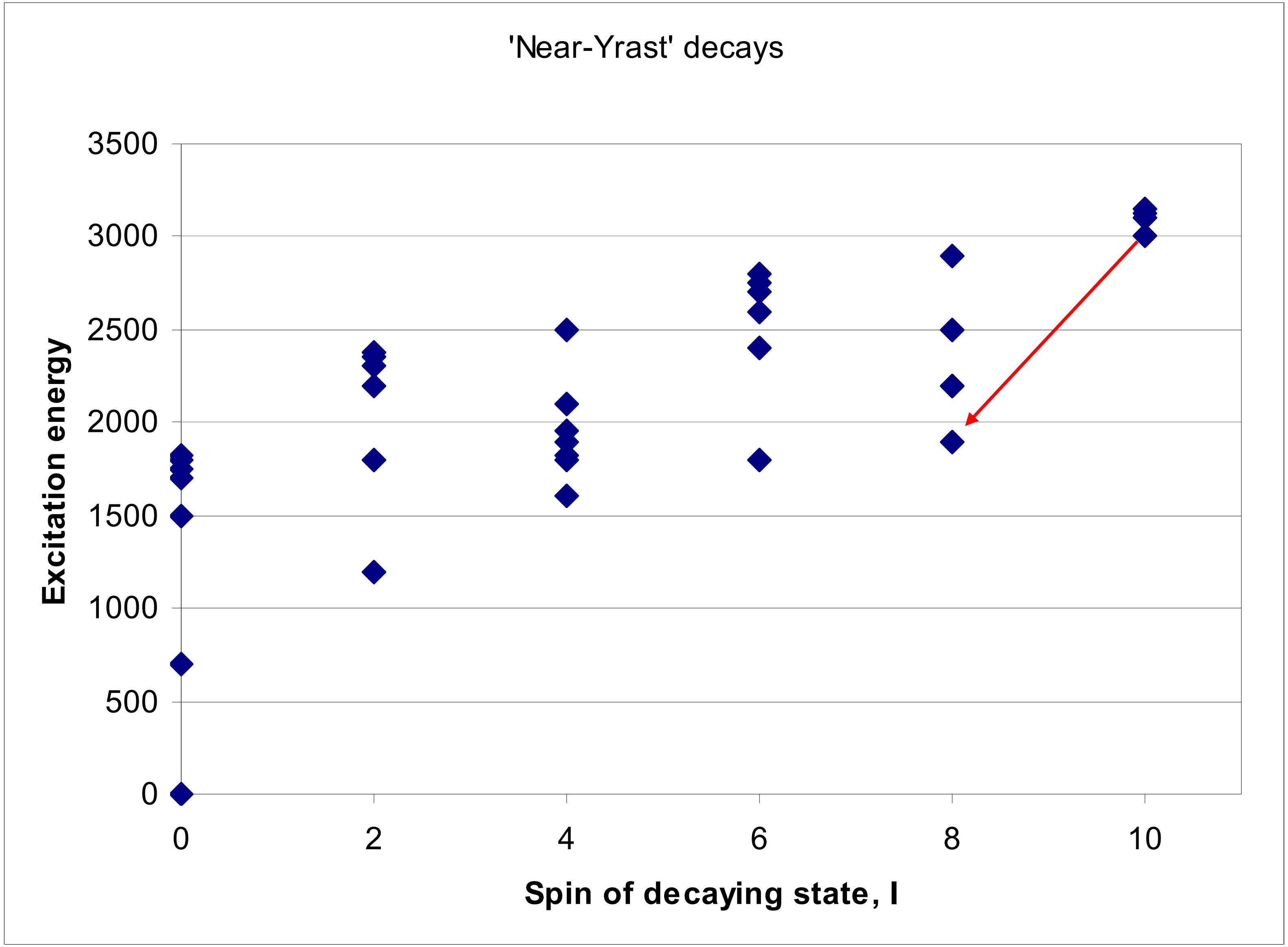
$$T(M1) = 1.779 \times 10^{13} E_\gamma^3 B(M1) \tag{5.0.7}$$

$$T(M2) = 1.371 \times 10^{7} E_\gamma^5 B(M2) \tag{5.0.8}$$

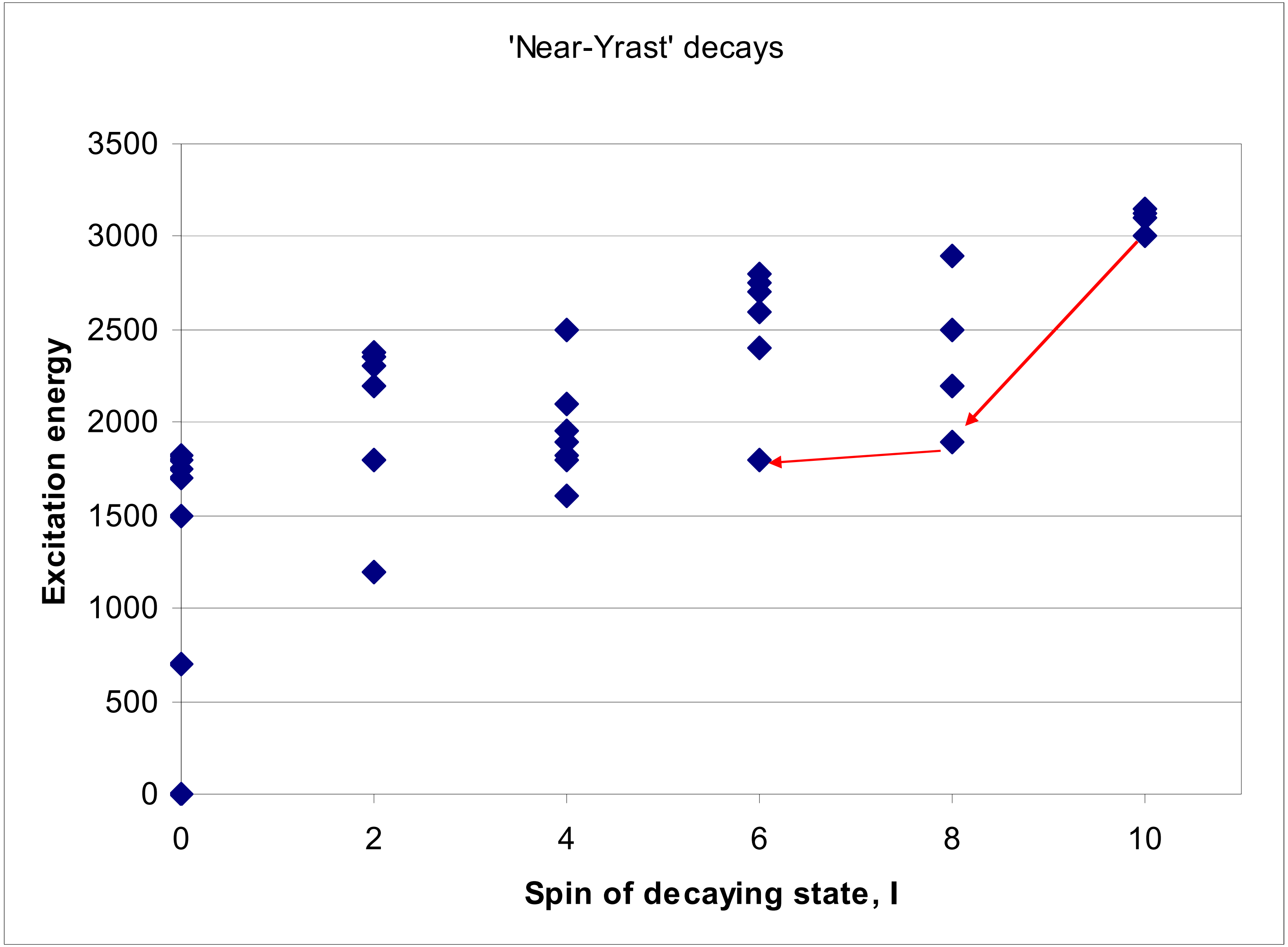
$$T(M3) = 6.387 \times E_\gamma^7 B(M3) \tag{5.0.9}$$

The units of $B(E\lambda)$ are $e^2\mathrm{fm}^{2\lambda}$ and the units of $B(M\lambda)$ are $(e\hbar 2Mc)^2\,(\mathrm{fm})^{2\lambda-2}$.

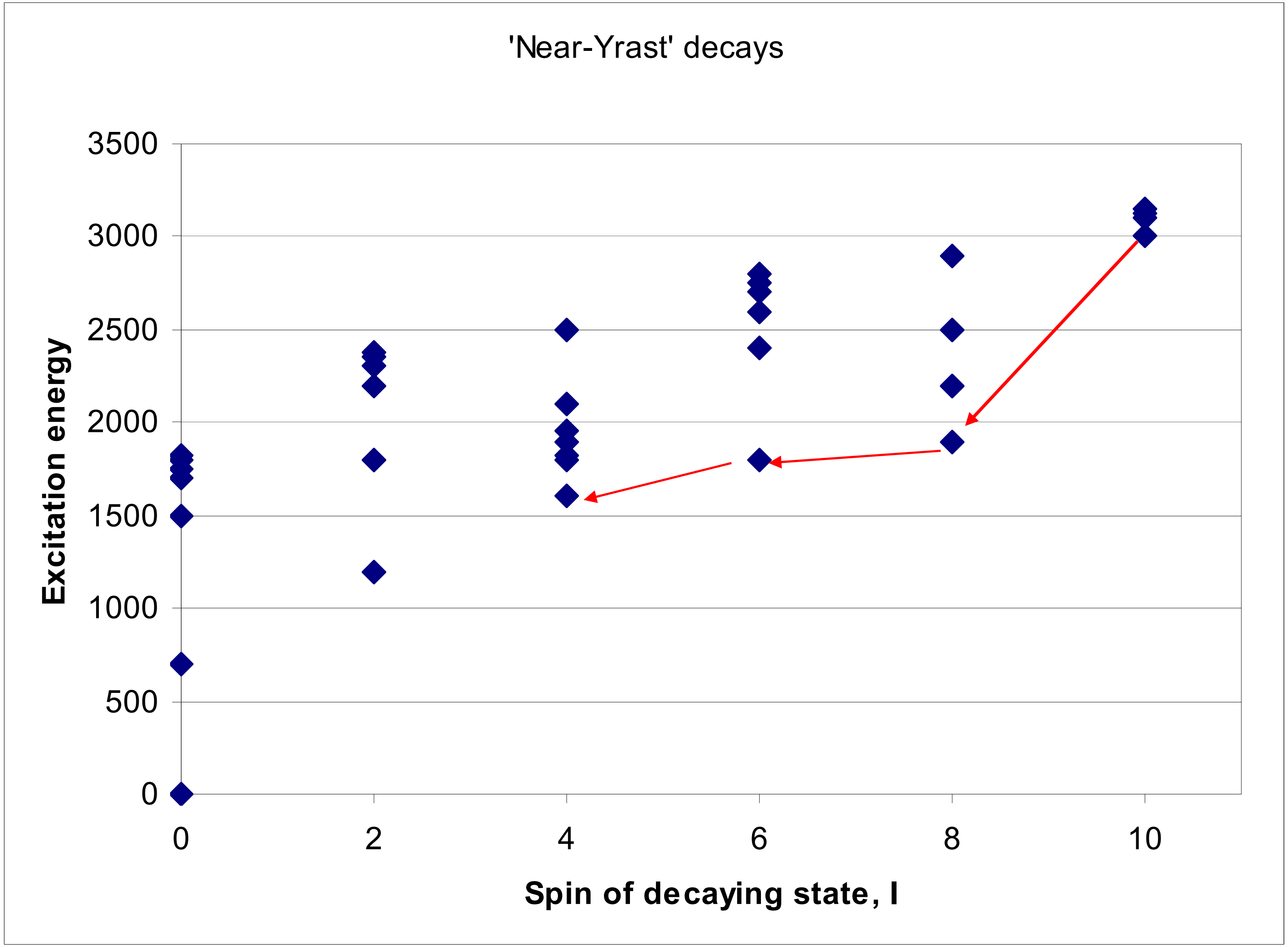
The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



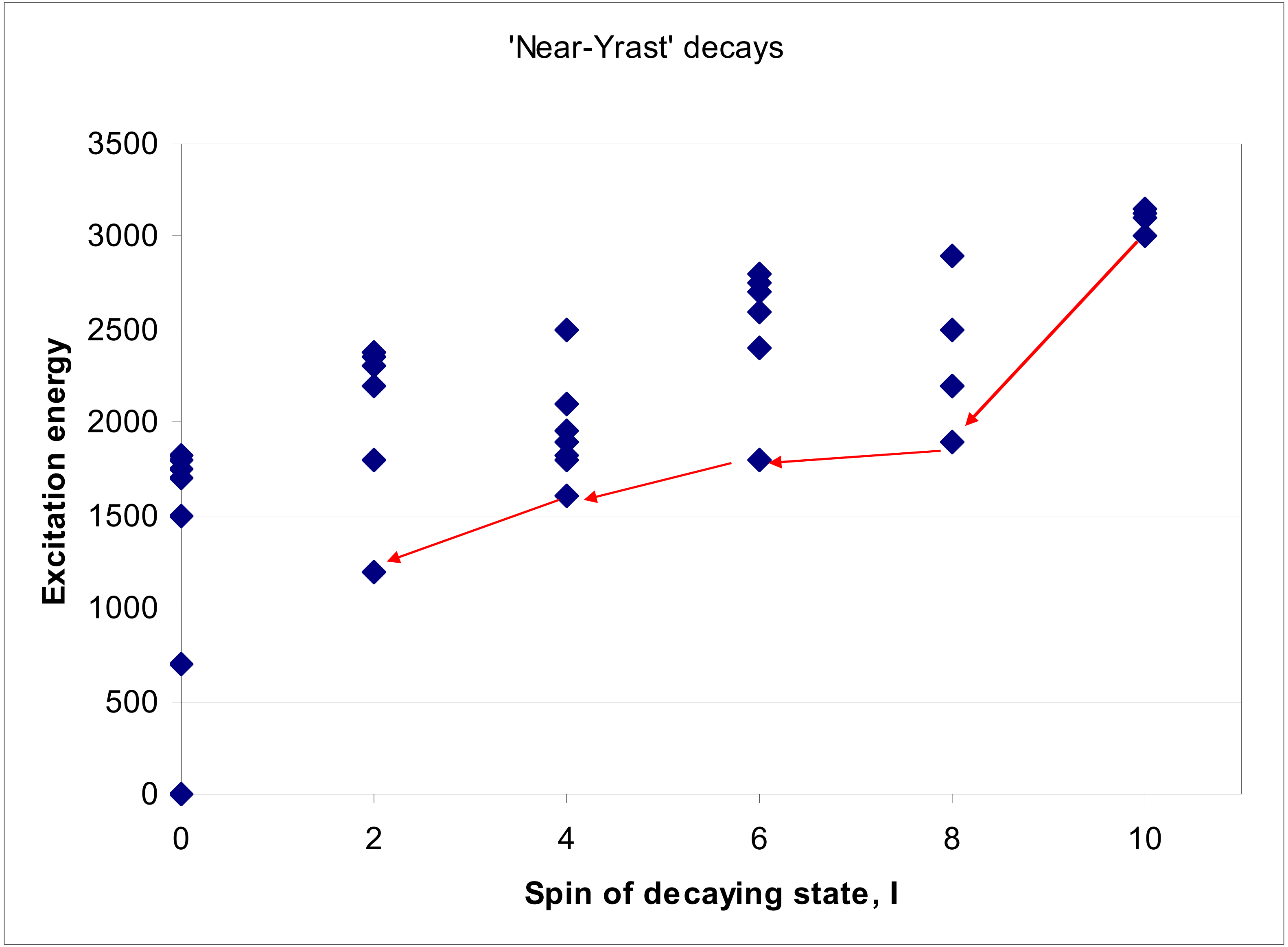
The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



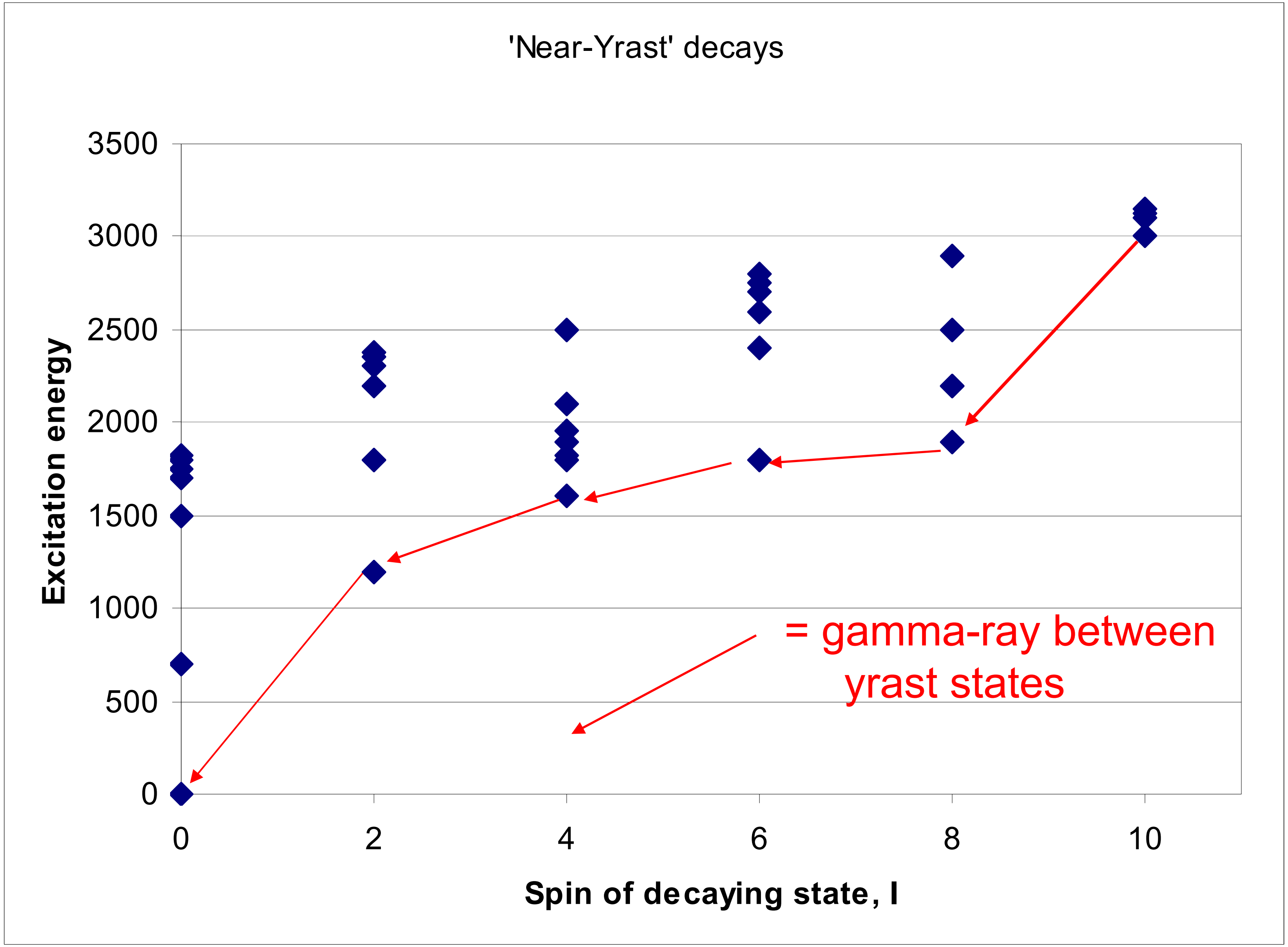
The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



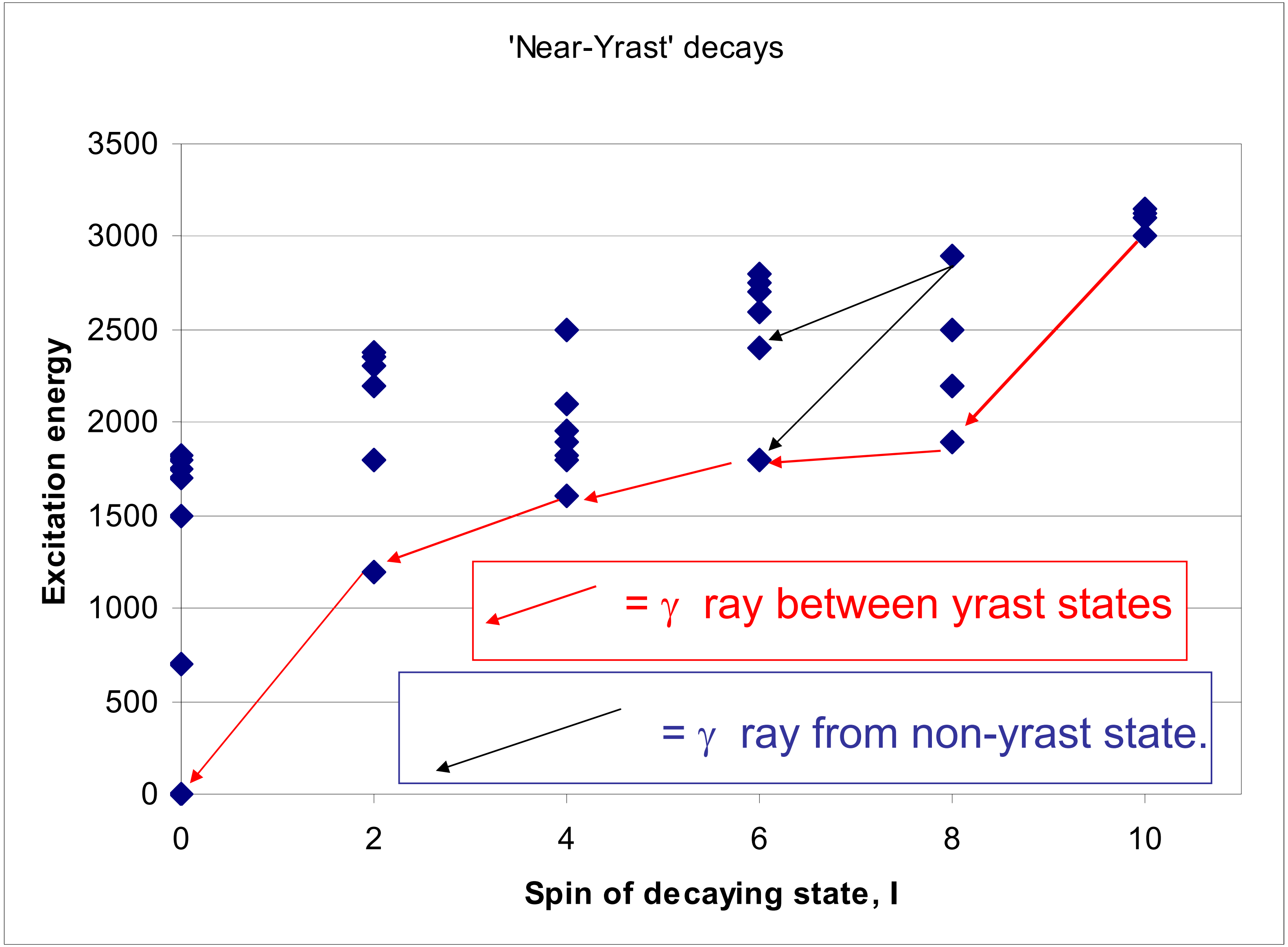
The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



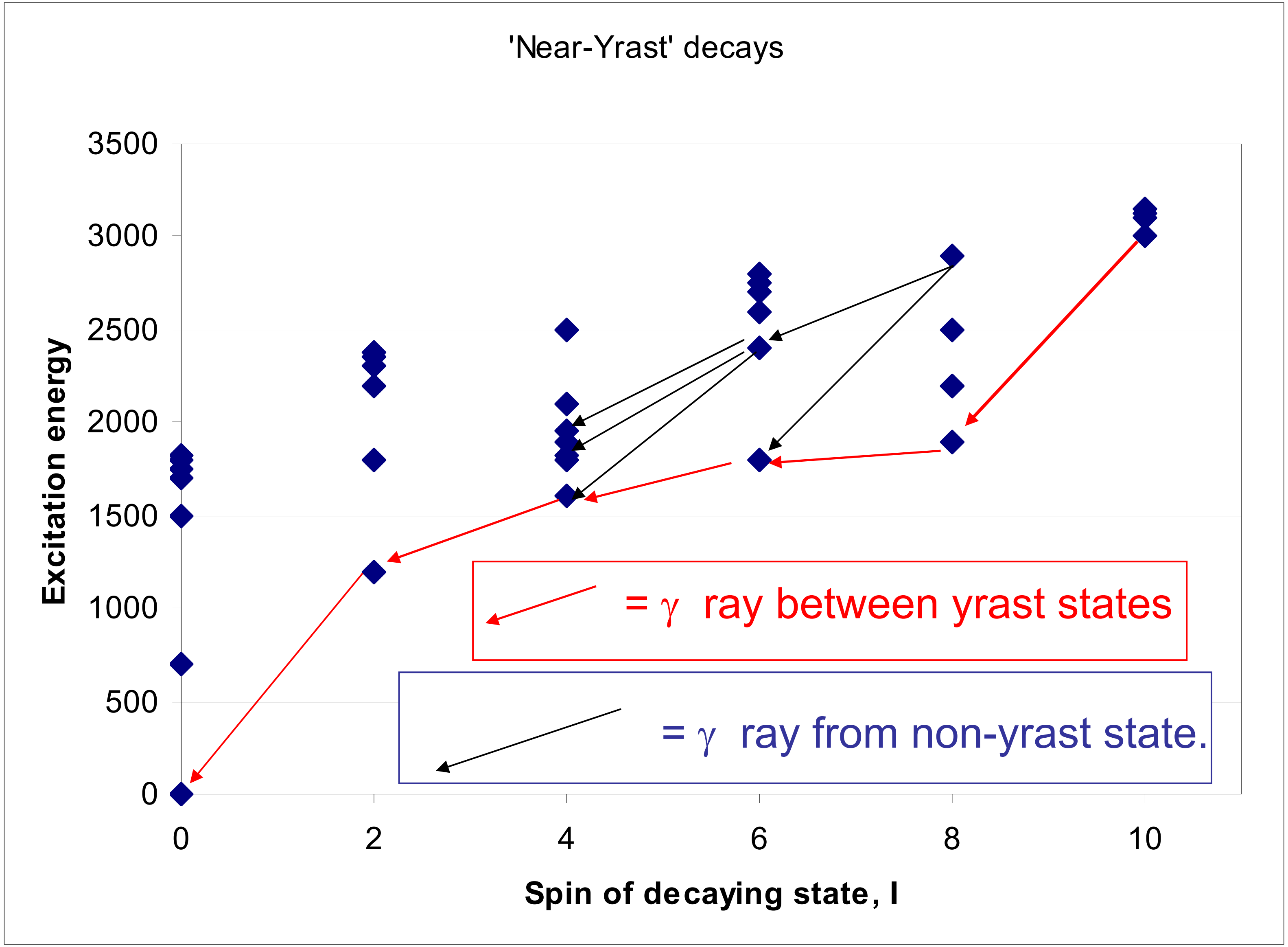
The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



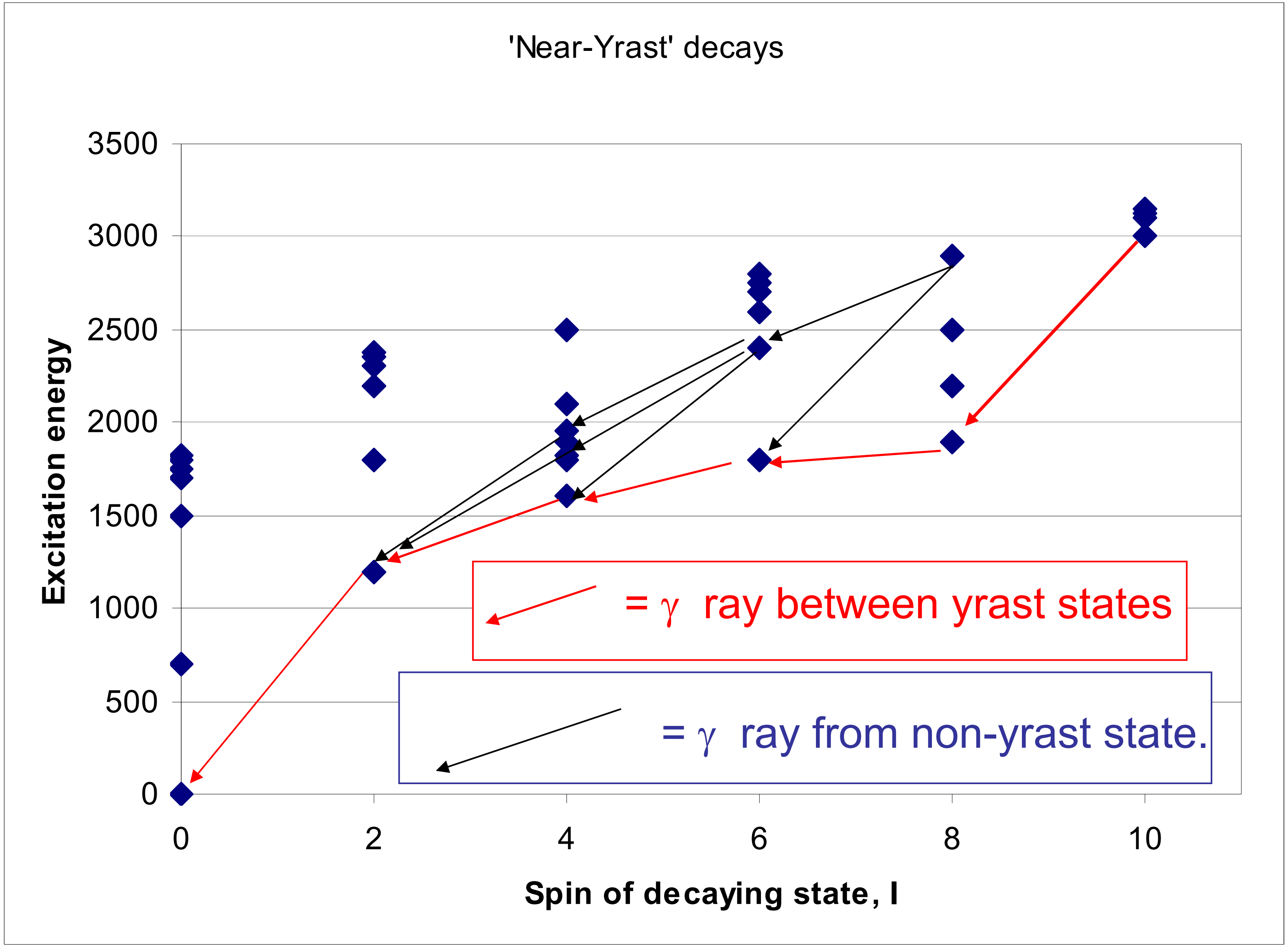
The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays E_γ^5)
Thus, the highest energy transitions for the lowest λ are usually favoured.
Non-yrast states decay to yrast ones (unless very different ϕ , K-isomers)

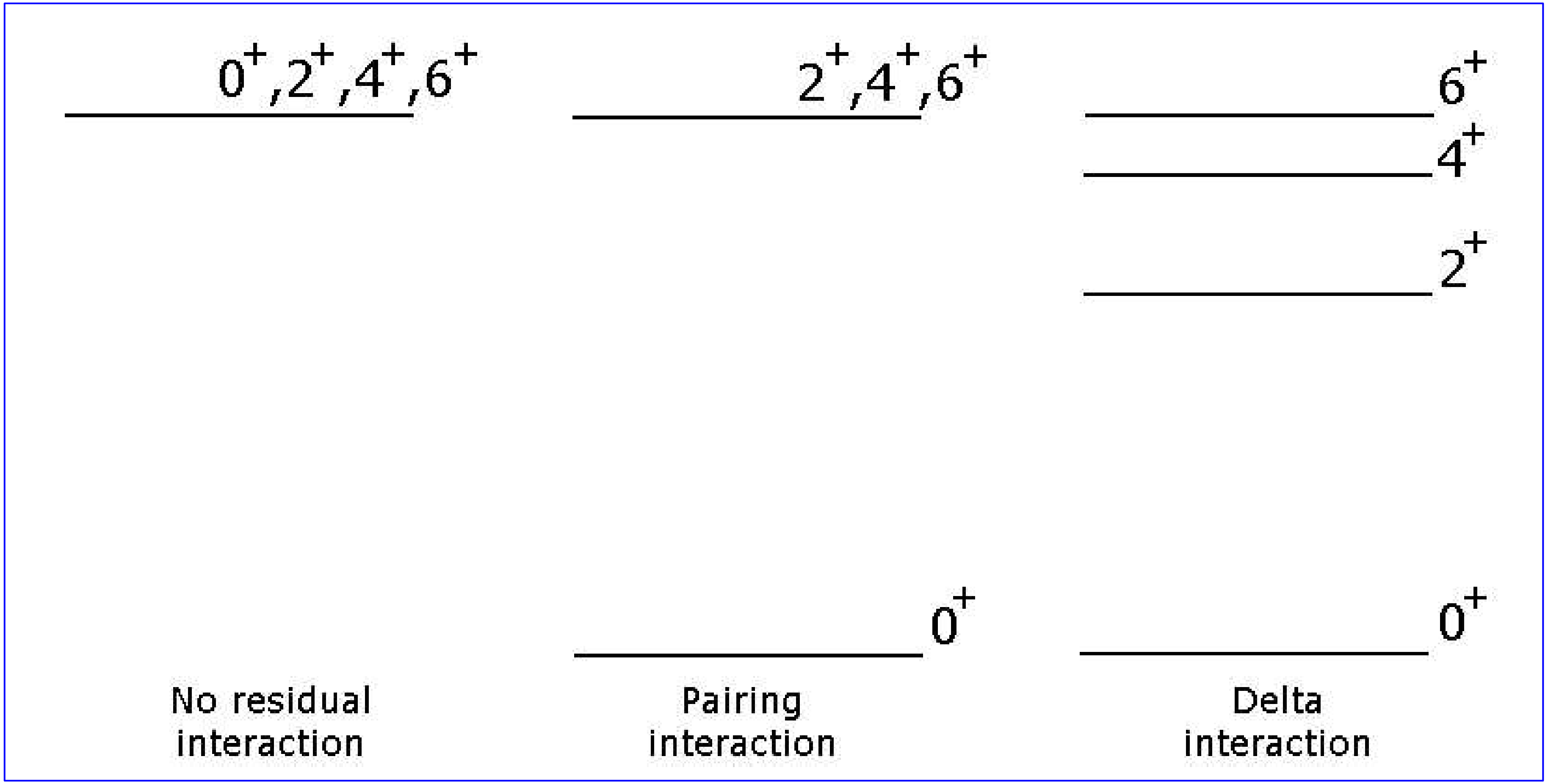


The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays E_γ^5)
Thus, the highest energy transitions for the lowest λ are usually favoured.
Non-yrast states decay to yrast ones (unless very different ϕ , K-isomers)



The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays E_γ^5)
Thus, the highest energy transitions for the lowest λ are usually favoured.
Non-yrast states decay to yrast ones (unless very different ϕ, K-isomers)





Schematic for $(f_{7/2})^2$ configuration.
4 degenerate states if there are no residual interactions.

Residual interactions between two valence nucleons give additional binding, lowering the (mass) energy of the state.

The effective interaction between nucleons deduced from nuclear spectra*

Reviews of Modern Physics, Vol. 48, No. 2, Part I, April 1976
191

John P. Schiffer

Argonne National Laboratory, Argonne, Illinois 60439 and University of Chicago, Chicago, Illinois 60637

William W. True

University of California, Davis, California 95616

B. General features in the experimental matrix elements

Several comparisons of the experimental matrix elements seem worthwhile before embarking on detailed analyses. To gain a qualitative impression of spectra we adopt plots of matrix elements as a function of θ_{12}, the classical angle of orientation between the two angular momentum vectors j_1 and j_2 coupled to J which gives a measure of the overlap of their orbital wavefunctions. θ_{12} is defined by

$$\cos\theta_{12} = \frac{J(J+1) - j_1(j_1+1) - j_2(j_2+1)}{2\sqrt{j_1 j_2 (j_1+1)(j_2+1)}} \quad . \qquad \text{(II.2)}$$

$$\Delta E(j^2 J) \approx \frac{-V_0 F_R}{\pi} \tan\frac{\theta}{2} \quad (T = 1,\ J \text{ even})$$

$$\Delta E\left(j_p j_n J\right)_{j_p = j_n} = \frac{-V_0 F_R}{\pi}\left(\cot\frac{\theta}{2}\right)\left[1 + \frac{1}{\cos^2\left(\frac{\theta}{2}\right)}\right] \quad (T = 0,\ J \text{ odd})$$

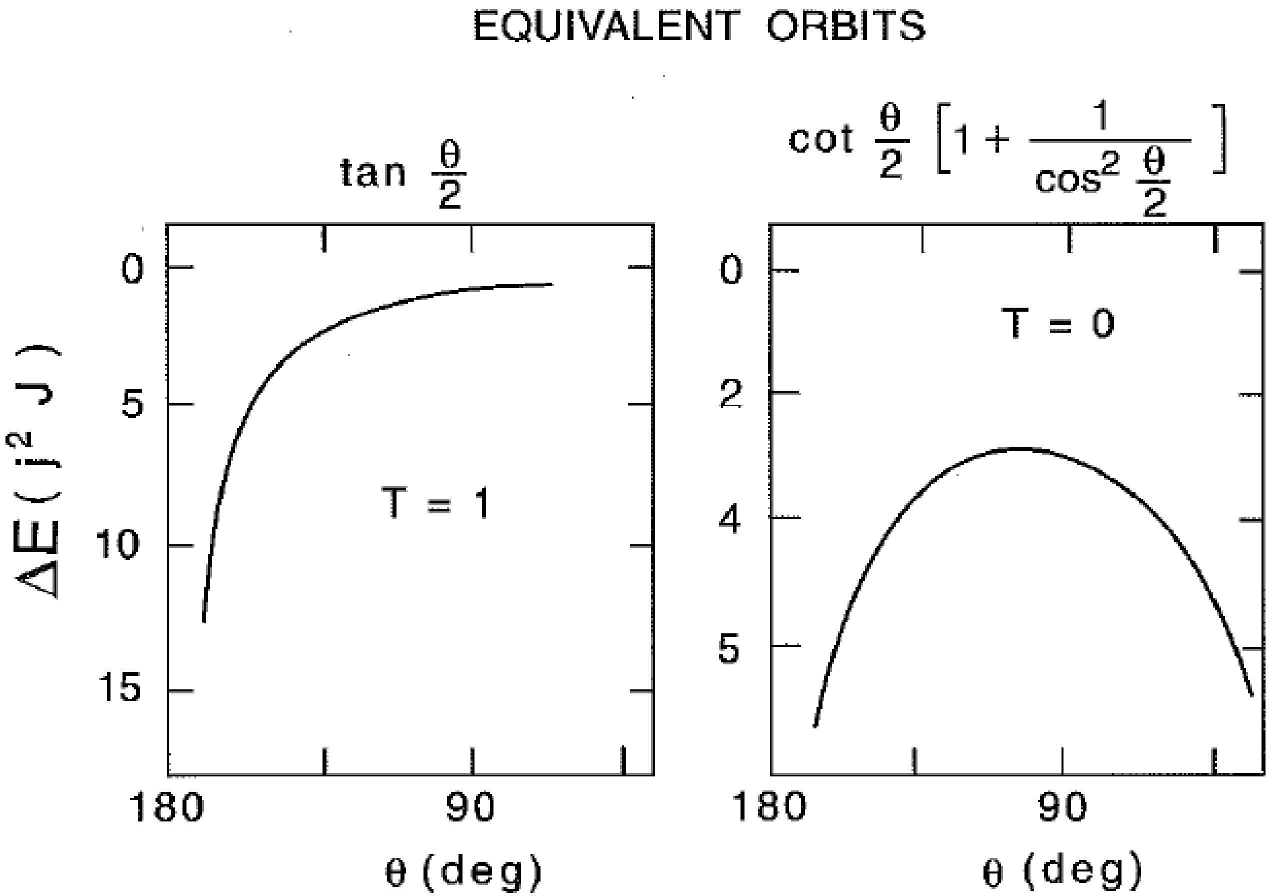


FIG. 4.9. Angular dependence of the δ-function residual interaction strength (lower values correspond to more attractive residual interactions) for two particles in equivalent orbits. (Left) The $T = 1$ (J even) states. (Right) The $T = 0$ (J odd) states. The analytic expressions are indicated above their respective plots.

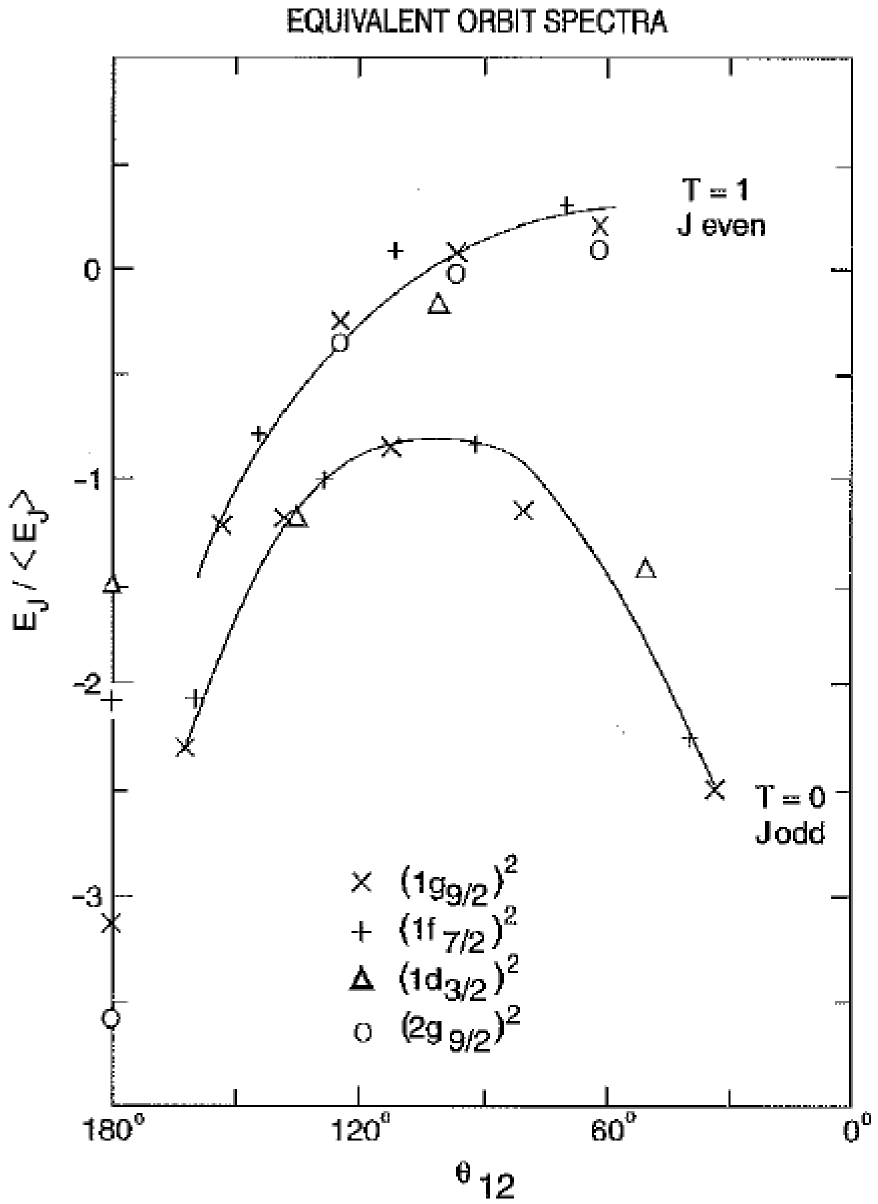


FIG. 4.10. Empirical proton–neutron multiplets for two particle equivalent orbit configurations for comparison with the behavior shown in Fig. 4.9. The curves are drawn through the data (Schiffer, 1971).

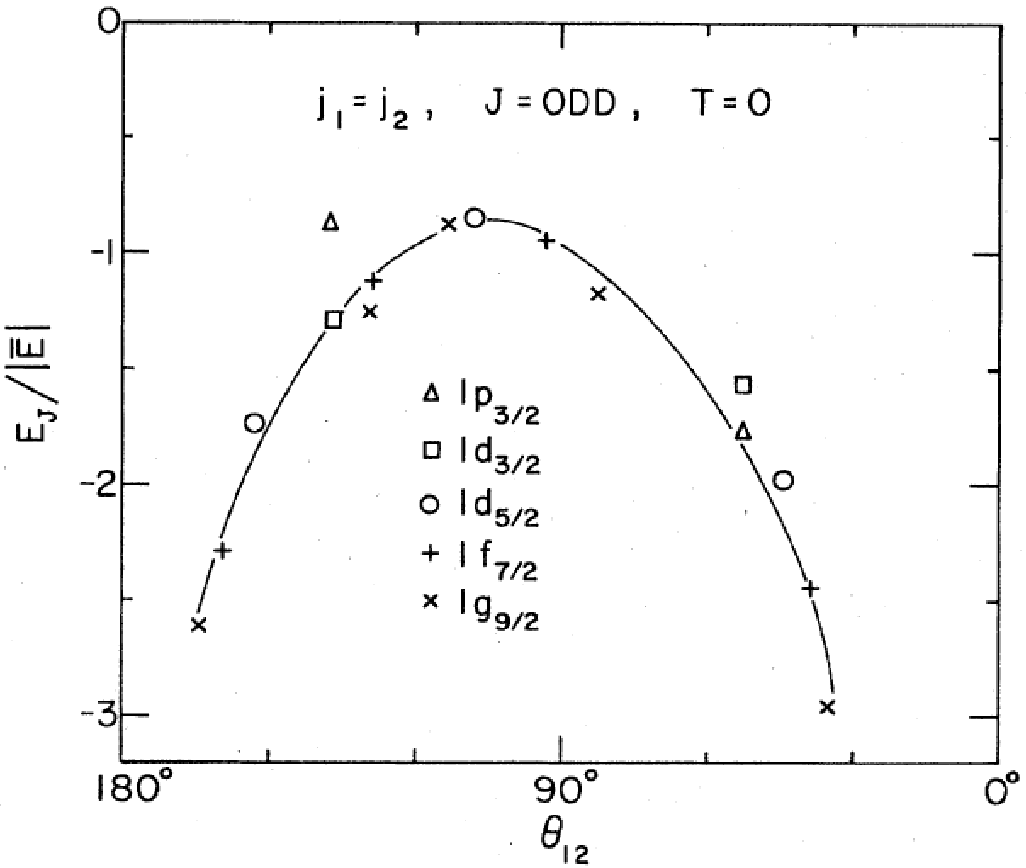


FIG. 2. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 0$. The values of the matrix elements are divided by $\overline{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

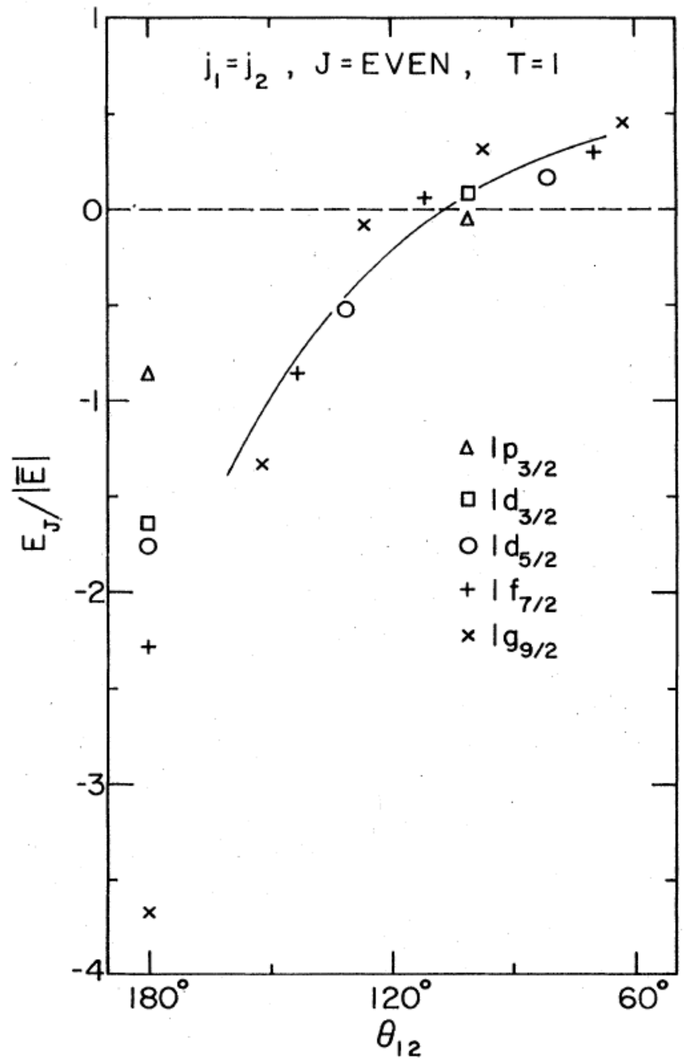


FIG. 3. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 1$. The values of the matrix elements are divided by $\overline{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

J. P. Schiffer and W. W. True: Effective interaction between nucleons
Reviews of Modern Physics, Vol. 48, No. 2, Part I, April 1976

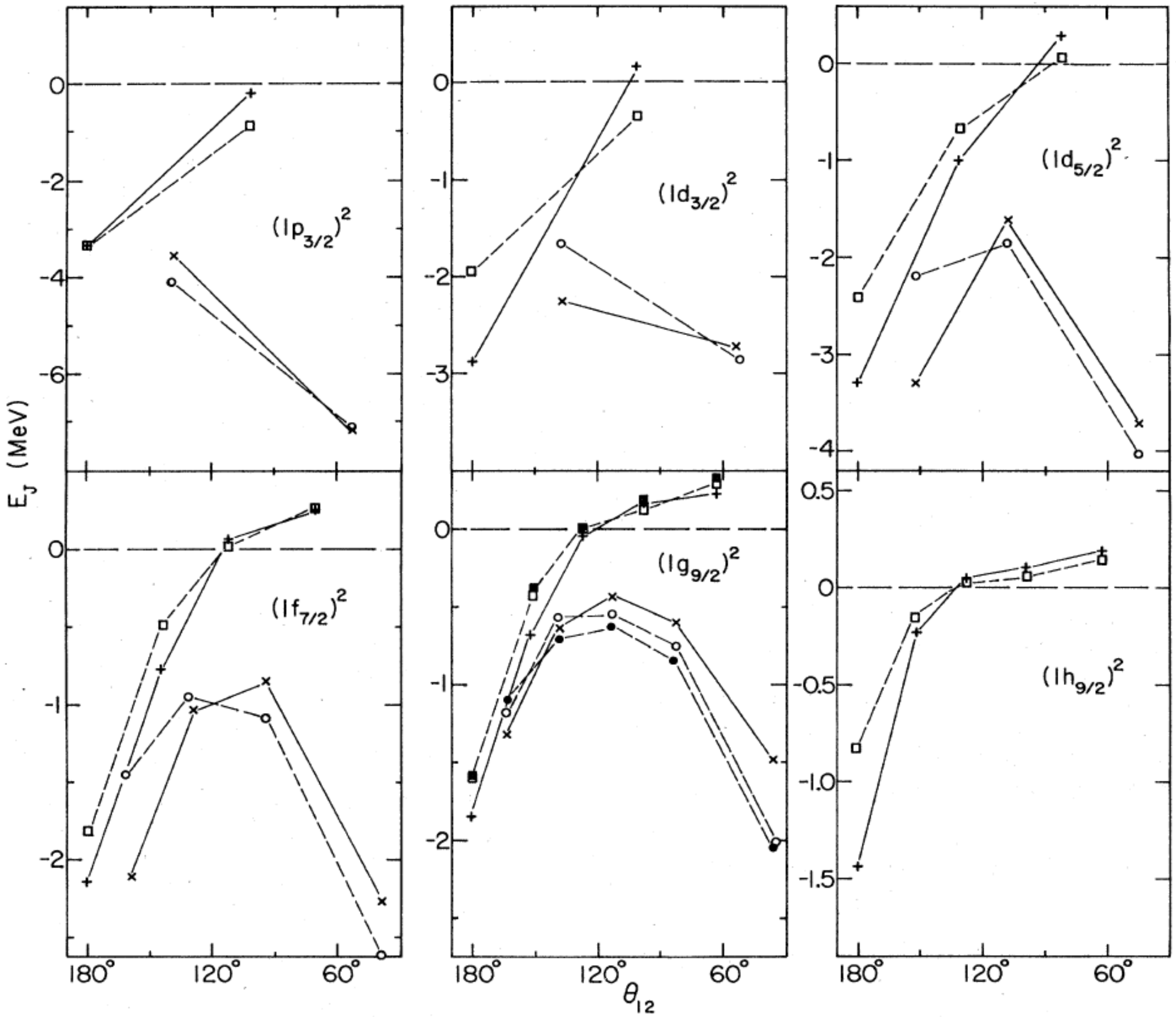


FIG. 4. Plots of the various $j_1=j_2$ multiplets. The data (crosses) are compared to calculations (circles for $T=0$ and squares for $T=1$) with the 12-parameter interaction of Table XVII ($r_2=2.0$ fm). The solid points for the $(1g_{9/2})^2$ multiplet represent calculations with the purely central five-parameter interaction of Table XIII.

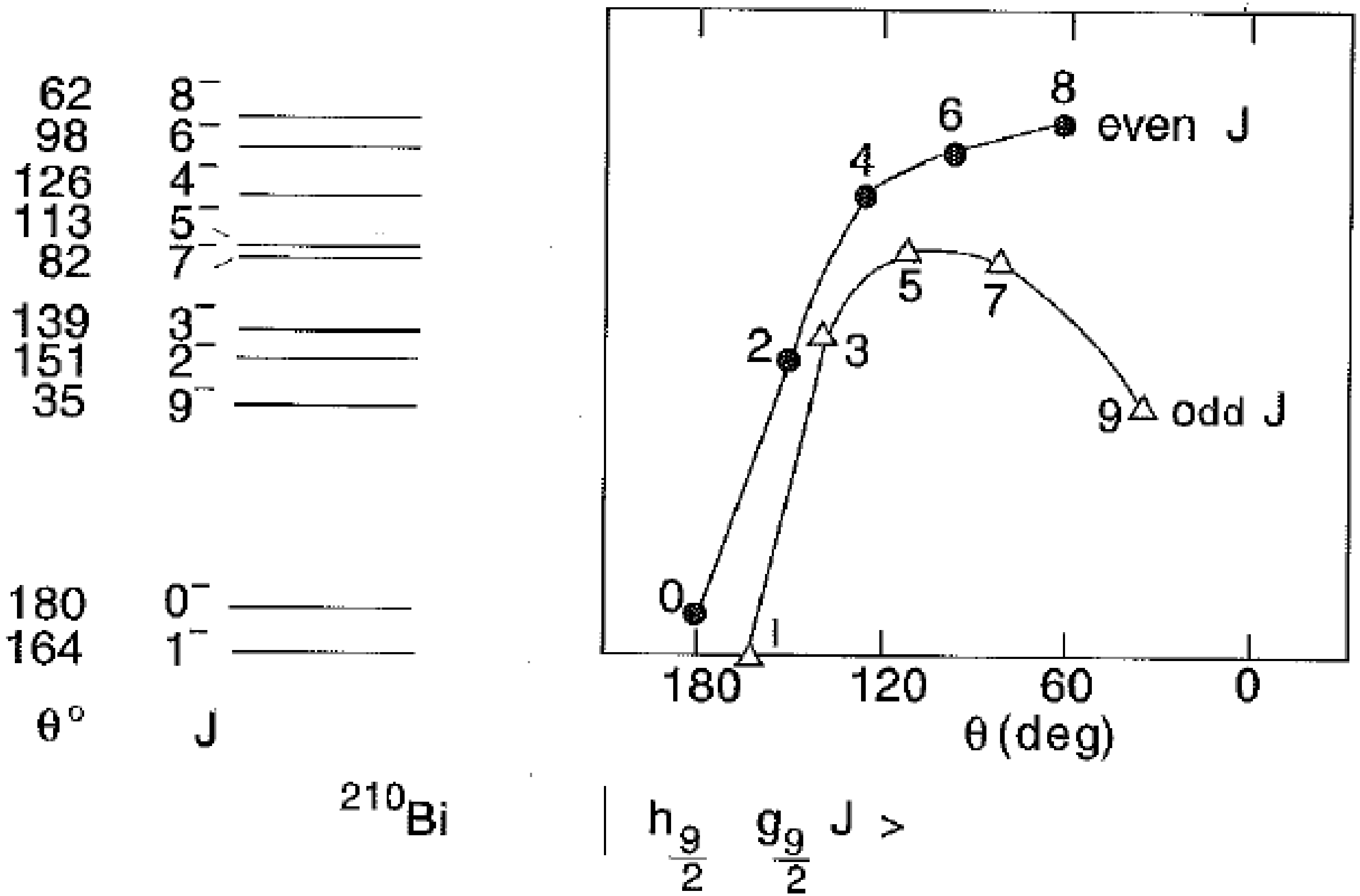


FIG. 4.12. A geometrical analysis of the $|h_{9/2}g_{9/2}J\rangle$ p–n multiplet in ^{210}Bi. The empirical levels are shown on the left along with the semiclassical angle between the orbits of the two nucleons. The right side shows that the levels split into two families, according to J even or J odd. The solid lines are drawn to connect the points.