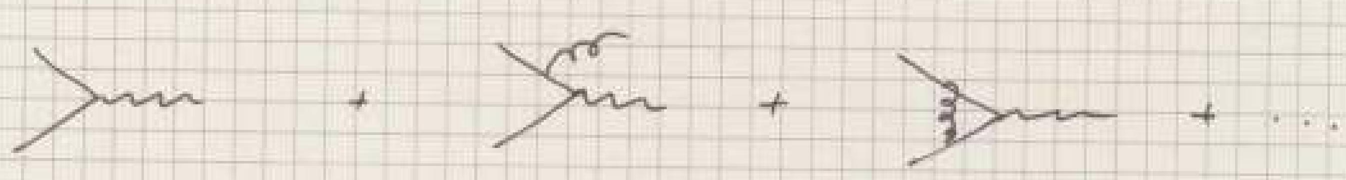
NLO with Parton shower

The other way to improve the parton shower is ~~#~~ to use NLO matrix elements as a starting place for the shower



Problems

* Double counting (The shower can generate some of the same diagram)
* The Diagrams are divergent

⇒ The problems are similar (but not exactly the same) as we have seen in matching higher multiplicity matrix elements

⇒ The solution is completely different

Why do we want NLO corrections with the PS?

Most important are:

- K-factors. The only way to include them consistently and use the information in detector simulations
- Shapes of observables have NLO corrections
  ⇒ impact on acceptance studies in general!
- Theoretical systematics: scale dependence can be computed (and is ~~mean~~ meaningful)
- Predictive Power: the M.C. tool can be used as a tool for "precision" physics

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Why is it so difficult to incorporate NLO matrix elements in the shower?

- KLN cancellation is achieved in partonshowers through unitarity (embedded in the Sudakovs).
  → in Matrix Elements there are IR singularities
- In matching with higher multiplicity matrix elements the IR singularities are cut off. The matching corrects for the bias introduced. ⇒ Unavoidable because only virtual matrix elements ~~cancels~~ can cancel the IR singularities in the real emission.
- In NLOwPS we want to include the virtual matrix elements
  ⇒ This is the difference with the LO matching.

therefore we expect that

1. there is no need to introduce cut-offs
2. An increased complexity in the computations.

Matching NLO with PS : NLOwPS

So, what do we want exactly. Let's define it:

! * Total rates (including observables that are inclusive at the fixed NLO order) accurate at NLO

! * Hard emissions are treated as in NLO computations (i.e. using the matrix elements, not the P.S.)

! * ~~Soft~~ Soft/Collinear emissions are treated as in the P.S.

* NLO results are recovered upon expansion of NLOwPS results in α_s. In other words: No double counting (or under counting)

* The matching between hard and soft/collinear regions is smooth

* Output is a set of events, which are exclusive
  ⇒ the can be passed through a detector simulation.

* that also means that Hadronization models ~~from the~~ ~~Pa~~ from the Parton Shower programs can be used.

Naive proposal for NLO w PS

* A NLO prediction for an observable O is ~~given by~~ by:

$$\left(\frac{d\sigma}{dO}\right)_{NLO} = \sum_{ab} \int dx_1\, dx_2\, d\Phi_{n+1}\, f_a(x_1)\, f_b(x_2) \; \times$$

$$\times \Big[\delta\big(O - O(2 \to n+1)\big)\, M_{ab}^{real} \; +$$

$$\delta\big(O - O(2 \to n)\big) \left(M_{ab}^{\binom{\text{Born + virtual}}{\text{+ integrated counter terms}}} - M_{ab}^{(c.t.)} \right) \Big]$$

counter-terms for real emission.

Note that in the subtraction method the counter terms have <u>Born-like kinematics</u> when plotting these events

* The generation of an set of Events after showering (at L.O.) can be written as follows:

$$F_{MC} = \sum_{ab} \int dx_1\, dx_2\, d\Phi_n\, f_a(x_1)\, f_b(x_2)\, F_{MC}^{(2\to n)}\, M_{ab}^{(Born)}$$

* Matrix elements : ~~tat~~ generate the normalization and hard kinematic configuration

δ-functions, $F_{MC}^{(2\to n)}$ = showers : define the observable final state

Naive proposal for NLO w PS II

→ Naive try: use the NLO kinematic configuration as initial condition for the showers, rather than directly computing the observables.

→ replace:

$$\delta(O - O(2\to n)) \to F_{MC}^{(2\to n)}$$

$$\delta(O - O(2\to n+1)) \to F_{MC}^{(2\to n+1)}$$

$$F_{naive} = \sum_{ab} \int dx_1 dx_2 \, d\Phi_{n+1} \, f_a(x_1) f_b(x_2) \times \left[F_{MC}^{(2\to n+1)} M_{ab}^{(real)} + F_{MC}^{(2\to n)} \left(M_{ab}^{(b+v+I.c.)} - M_{ab}^{(c.t.)} \right) \right]$$

Doesn't work

1. Cancellation between $2\to n+1$ and $2\to n$ configurations occur after the shower ⇒ How can the subtraction method work?

⇒ Unweighting of events impossible.

→ ~~What~~ Hopeless from a practical point of view.

2. $$\cancel{M_{ab}^{Born}} F_{MC}^{(2\to n)} M_{ab}^{Born} = \overbrace{(1 + O(\alpha_s)) M_{ab}^{Born}}^{(n+1)\text{-kinematics on } (n\text{-kinematics})}$$

$$\Rightarrow \left(\frac{d\sigma}{dO}\right)_{naive} - \left(\frac{d\sigma}{dO}\right)_{NLO} = O(\alpha_s)$$

We have double counting ⇒ NOT CORRECT.

Note that there is both a branching and a non-branching probability.

Solutions: MC@NLO

There are two general solutions to these problems:

MC@NLO , ~~{~~ Frixione and Webber (2002)

POWHEG , Nason (2004)

Let's start with <u>MC@NLO:</u>

The idea is to remove the spurious $\cancel{\alpha_s}$ $\alpha_s \alpha_s^{\mathrm{Born}}$ terms by hand
$\rightarrow$ MC. counter terms

$$F_{MC@NLO} = \sum_{ab} \int dx_1 dx_2 \, d\Phi_{n+1} f_a(x_1) f_b(x_2) \times$$
$$\Big[F_{MC}^{(2\to n+1)} \left(M_{ab}^{(\mathrm{real})} - M_{ab}^{(MC)} \right) +$$
$$F_{MC}^{(2\to n)} \left(M_{ab}^{(b+v+\mathrm{I\,c.t.})} - M_{ab}^{(c.t.)} + M_{ab}^{(MC)} \right) \Big]$$

where $M_{ab}^{(MC)}$ is the monte-carlo counter term:

$$M_{ab}^{(MC)} = F_{MC}^{(2\to n)} M_{ab}^{\mathrm{BORN}} + \mathcal{O}(\alpha_s^2 \cdot \alpha_s^{\mathrm{Born}})$$

the two MC counterterms remove the spurious NLO terms due to the <u>branching</u> of a final state parton and ~~a~~ the <u>non-branching</u> probability

$\Rightarrow$ By removing these spurious terms, also, the integrands have become <u>finite</u> ~~at the~~ all over phase-space. (i.e. independent of the observable $\Rightarrow$ Fully exclusive).

$\Rightarrow$ Configurations of S- and H- events can be showered independently and added together.

Solutions : POWHEG

Based on the theorem:

> A shower can be defined which has the largest - p_T emission at the first branching, and is equivalent (to Leading Log accuracy) to the angular-ordered shower.

Such a shower goes through the following steps:

1. Do the first branching as usual
   $\rightarrow$ it will define branching variables z and t and a p_T with $t_0 < t < t_{ini}$ and

2. Do a shower from each of the two legs from the first branching with upper scales $z^2 t$ and $(1-z)^2 t$ and veto all emissions with a relative transverse momentum larger than p_T (this is what is called a "vetoed shower").

3. Do a further p_T - vetoed shower with upper scale t_{ini} and lower scale t ("vetoed truncated shower")
   $\rightarrow$ restores coherence

POWHEG

$\Rightarrow$ Exponentiate the full real emission corrections into the sudakov and use that for the first branching. then, proceed as before, with ~~a~~ vetoed and vetoed truncated showers.

- MC must be capable of handling vetoed truncated showers
  $\rightarrow$ may need to use separate package
- Beyond-LL structure changed: Need for retuning?
- $|MC@NLO - POWHEG| = \mathcal{O}(\alpha_s^2)$, so correct.
  But how large are these α_s^2 terms?

"Hardest emission cross section": this needs to be showered by the vetoed/truncated shower

$$d\sigma = \bar{B}(\Phi_B)\, d\Phi_B \left[\Delta_{\hat{R}}(k_{T,min}) + \frac{\hat{R}(\Phi_B, \Phi_R)}{B(\Phi_B)} \Delta_{\hat{R}}(k_T(\Phi_B, \Phi_R))\, d\Phi_R \right]$$

where

$$\bar{B}(\Phi_B) = B(\Phi_B) + V(\Phi_B) + \int d\Phi_R\, R(\Phi_B, \Phi_R) + \underbrace{A(\Phi_{B,X})}_{\text{subtraction terms}}$$

$$\Delta_R(p_T) = \exp\left[-\int d\Phi_R \frac{R(\Phi_B, \Phi_R)}{B(\Phi_B)} \theta(k_T(\Phi_B, \Phi_R) - p_T) \right]$$

$k_T(\Phi_B, \Phi_R)$ tends to the transverse momentum of the emitted parton in the soft and collinear limits.
Emissions with $k_T < k_{T,min}$ are considered unresolvable.

Summary NLO w PS

* Event generators including the typical benefits of NLO computations now exist

* Sensible predictions for total rates and large p_T tails ⇒ can study scale dependence in a realistic environment

* Absence of matching parameters ⇒ increased predictive power with respect to tree-level higher multiplicity matching.

* Multileg NLO results difficult to obtain
  → NLOwPS and tree-level high multiplicity are
  <u>COMPLEMENTARY</u>

* Next step ⇒ CKKW-like procedures at NLO

TWO Methods available MC@NLO and POWHEG.

⇒ Both are being automated

- POWHEG BOX (already available), but needs input from user
- aMC@NLO (not yet available)