Parton showers

* Parton showers evolve the hard process down to the hadronization scale

* → Accellerate particles radiate
  - (colored) quarks radiate gluons
  - (colored) gluons radiate gluons
  - (colored) gluons split into quark pairs

* They generate high-multiplicity final states which can readily converted into hadrons

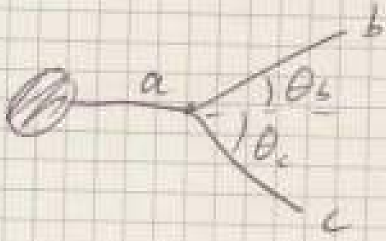
* They are build on the same concept as resummation:

  Logarithmically dominant contributions are <u>universal</u>

Universality of the large logarithms

- Consider the following branching / splitting:

$a \to b\,c$ (can be quarks and/or gluons)



$$z = E_b / E_a \,, \quad t = k_a^2$$

$$\theta = \theta_b + \theta_c = \frac{\theta_b}{1-z} = \frac{\theta_c}{z} = \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}}$$

- It's straight-forward to show that

$$d\sigma_{N+1} = d\sigma_N \frac{dt}{t}\frac{d\varphi}{2\pi}\, dz \frac{\alpha_s}{2\pi} \left| K_{ab}(z,\varphi)\right|^2$$

$$d\bar{\sigma}_{N+1} = d\bar{\sigma}_N \frac{dt}{t}\, dz \frac{\alpha_s}{2\pi} P_{ab}(z)$$

Averaged over φ

Altarelli-Parisi splitting function

- In this relations we have only kept the <u>leading</u> terms in $1/k_T$

relative transverse momentum of partons b, c.

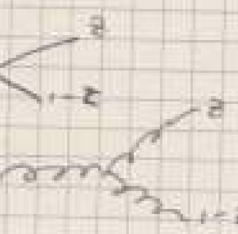
- AP-splitting:

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}$$

$$P_{gq}(z) = C_F \frac{1+(1-z)^2}{z}$$

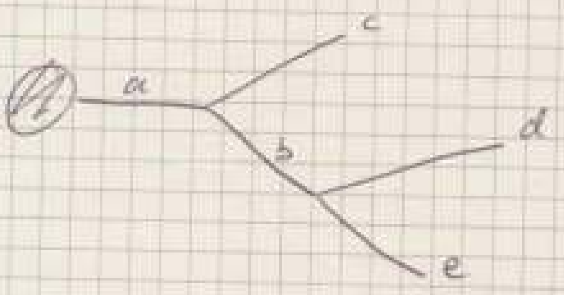
$$P_{qg}(z) = T_R\left(z^2 + (1-z)^2\right)$$

$$P_{gg}(z) = C_A\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$



More than one splitting:

* The process can easily be iterated



$$a(t) \to b(z) + c$$
$$b(t') \to d(z') + e$$

$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \; \frac{dt}{t} \, dz \, \frac{dt'}{t'} \, dz' \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ba}(z) \, P_{db}(z')$$

* It's a <u>Markov</u> process, i.e. a random process in which the probability of the next step depends **only** on the <u>present</u> values of random variables. ("There is no sence of history")

    - In our case the branching depends on the energy fraction z, the virtuality t, and the type of splitting (e.g. $g \to gg$, $q \to qg$, ...)

Following a given line in the branching tree, we have:

$$\underbrace{Q^2}_{\text{scale of the hard process}} \gg t_1 \gg t_2 \gg \dots t_N \gg \underbrace{Q_0^2}_{\text{hadronization scale. pQCD cannot be trusted anymore}}$$

$$\sigma_N \sim \sigma_0\, \alpha_s^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_s^N}{N!} \left(\log \frac{Q^2}{Q_0^2}\right)^N$$

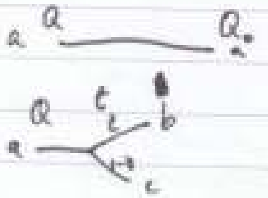
* Each power of α_s comes with a $\log \frac{Q^2}{Q_0^2}$ which can ~~be~~ be a large number. ~~(each~~ for log
* Eventually damped by the $N!$.
  number of resolvable partons

Let $\Phi_a[E, Q^2]$ the ensemble of parton cascades initiated by parton a of ~~the~~ energy E emerging from a hard process with scale Q^2.

Let $\Delta_a(Q_1^2, Q_2^2)$ the probability that a does not branch for virtualities $Q_2^2 < t < Q_1^2$.

We can write a formula that takes into account all the branchings in a given tree:

$$\Phi_a[E, Q^2] = \Delta_a(Q^2, Q_0^2)\, \Phi_a[E, Q_0^2]$$ ← nothing happens: same ensemble but at scale Q_0^2



$$+ \int_{Q_0^2}^{Q^2} \frac{dt}{t} \Delta_a(Q^2, t) \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z)\, \Phi_b[zE, t]\, \Phi_c[(1-z)E, t]$$

branching at scale t, left with two ensembles Φ at scale t, with energies zE and $(1-z)E$.

Sudakov factor

Now, simply imply that no information got lost: the sum ~~of~~ over all ~~possible~~ the probabilities associated with the ~~poss~~ all possible branchings of partons must be one:

$$1 = \Delta_a(Q^2, Q_0^2) + \int_{Q_0}^{Q^2} \frac{dt}{t}\, \Delta_a(Q^2, t) \sum_b \int dz\, \frac{\alpha_s}{2\pi} P_{ba}(z).$$

to find a closed form, derive with respect to $Q_0^2 \Rightarrow$

$$0 = \frac{d\Delta_a(Q^2, Q_0^2)}{dQ_0^2} - \frac{1}{Q_0}\, \Delta_a(Q^2, Q_0^2) \sum_b \int dz\, \frac{\alpha_s}{2\pi} P_{ba}(z)$$

using the boundary condition $\Delta_a(Q^2, Q^2) = 1$, we find immediately:

$$\Delta_a(Q^2, Q_0^2) = \exp\left(-\int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz\, \frac{\alpha_s}{2\pi} P_{ba}(z)\right)$$

This is the Sudakov form factor

Note that: the unitarity condition above implies that <u>some</u> virtual corrections ~~to~~ must be included.

$\Rightarrow$ In a numerical P.S. code we can simply throw a random number R distributed uniformly ~~$~~ in the interval $[0,1]$ and solve for t_{i+1}

$$\Delta(t_i, t_{i+1}) = R$$

to ~~you can~~ determine t_{i+1} ~~for the bran~~ with the correct probability.

Double logs due to the soft ~~behavi~~ limit.

* So far we have only looked at the integral over dt.
* The treatment so far is only valid in the collinear limit. Choices that affect the behavior of the Sudakov outside this limit are equivalent.

For example other choices of the shower evolution parameter t affect the double log structure (from soft divergences)

$$\text{virtuality} \quad t \overset{= k_n^2}{=} z(1-z)\,\theta^2 E^2 \quad \Rightarrow \quad \frac{1}{2}\log^2 t/E^2$$

$$p_T^2 \quad t = z^2(1-z)^2\theta^2 E^2 \quad \Rightarrow \quad \log^2 t/E^2 .$$

$$\text{angle} \quad t = \theta^2 E^2 \quad \Rightarrow \quad \log\frac{t}{\Lambda}\log\frac{E}{\Lambda}$$

$\Rightarrow$ Studying soft emission may give extra information on the proper choice of t.

Angular ordering:

$$d\sigma_g^{soft} = \frac{1}{2s}\left|M_{soft}\right|^2 \frac{d^3k}{(2\pi)^3 2k^0}$$

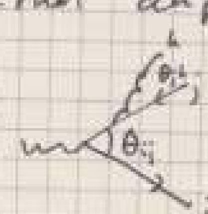
$$= \frac{1}{2s}\left|M_0\right|^2 \frac{-2p_\mu \bar p_\nu}{(pk)(\bar p\cdot k)}\, g^2 C_F \sum \epsilon_\mu \epsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})}\, d\cos\theta$$

asymmth

It's straight-forward to proof that after the integration over the azymuthal angle φ that:

$$\left| \; \right|^2 = \left| \; \right|^2 \Theta(\theta_{ij} - \theta_{jk})$$

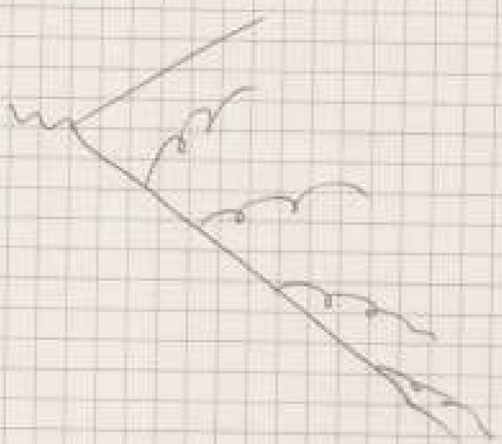


$$+ \left| \; \right|^2 \theta(\theta_{ij} - \theta_{ik})$$

$\Rightarrow$ Radiation happens only for angles smaller than the color connected (antenna) opening

Angular Ordering

* The construction can be iterated : ~~Angl~~ ~~Angels~~ Angles will become smaller and smaller



* The treatment can be generalised to a parton with (color) charge Q_k splitting in i and j : $Q_k = Q_i + Q_j$.

  The results inside the cones $\Rightarrow$ i & j emit as independent charges

  outside $\Rightarrow$ coherent emission from lines $i+j$, it's like emitting from charge Q_k.

* This has an effect on the multiplicity of hadrons in jets (Intrajet radiation), since radiation is more suppressed with respect to the total phase-space available.

* Two ways to implement :

  1. Use angle as evolution value
  2. Use another evolution value, but enforce color ordering

Coherent branching

* the soft stuff we just considered can be combined with the collinear that we did before ⇒
  → All collinear and soft radiation is taken into account at ~~...~~ all orders.

* Most straight-forward by simply replacing the shower variable t with $\zeta = 1 - \cos\theta$.

  → in practice (due to possible massive lines) it's better to use

$$t = E^2 \zeta$$

Three successful implementations:

HERWIG	PYTHIA	SHERPA
$t = E^2 \zeta$	$t = M^2,\ k_T^2$	$t = k_T^2$
hardest not first	hardest first	hardest first
~~...~~ coherent	coherence ~~...~~ forced	coherent
~~...~~ dead zones	no dead zones	no dead zones
all particles recoil	all particles recoil	1 particle recoils.

⇒ Don't be lazy : all have ~~advan~~ pros and cons

USE MORE THAN ONE

Summary Parton showers

* Start from LO process
* let final state partons branch
* branch some more
* until reaching the scale Q_0^2 (hadronization scale)
* 1. use a model to convert partons to hadrons (final state)
  2. further branching to valence quarks and fold with PDF (initial state).
* Add other low p_T stuff (underlying event, ~~multi~~ etc.)

then successful ampl