Pictorial view of event at hadron colliders (LHC) ⓪

1. High-Q^2 scattering
   - * where new physics lies
   - * process dependent
   - * first principles description
   - * can be systematically improved (by perturbation theory)
     → higher order corrections

2 Parton Shower
   - * QCD – "known physics"
   - * universal / process independent
   - * first principles description

3 Hadronization
   - * low Q^2 physics
   - * universal / process independent
   - * model dependent → no 1st principle description

4 Underlying event
   - * low Q^2 physics
   - * energy and process dependent
   - * model dependent

M.C. phase space integration

Master formula for hadron collisions:

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1\, dx_2\; f_a(x_1, \mu_F^2)\; f_b(x_2, \mu_F^2) \cdot$$

$$\cdot\; \hat{\sigma}_{ab\to X}\left(x_1, x_2, \alpha_s(\mu_R), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right)$$

two ingredients:

1. P.D.F. (parton distribution function)

   extracted from experiment, using evolution from theory

2. Short distance coëfficients as an expansion in α_s

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \cdot \sigma_2 + \ldots$$

σ_0: L.O.

σ_1: NLO

σ_2: NNLO

etc.

The calculation of cross sections (or decay widths) involve integrations of high-dimensional integrals of very peaked functions:

$$\hat{\sigma} = \frac{1}{2s} \int d\Phi_n \, |M|^2 \quad , \text{ for } n \text{ final state particles}$$

The dimension of $d\Phi_n$ is $\dim[\Phi_n] = 3n-4$

- Due to the high (number of) dimensions, a general and efficient way of determining the integration boundaries and P.S. parametrization are needed.

- Especially when cuts on the final state are applied, analytic methods become too complicated

$\Rightarrow$ Numerical integration is the only way to continue

Setting up the phase-space parametrization

$$d\Phi_n = \left[\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \, \delta^{(4)}\left(p_0 - \sum_{i=1}^{n} p_i\right)$$

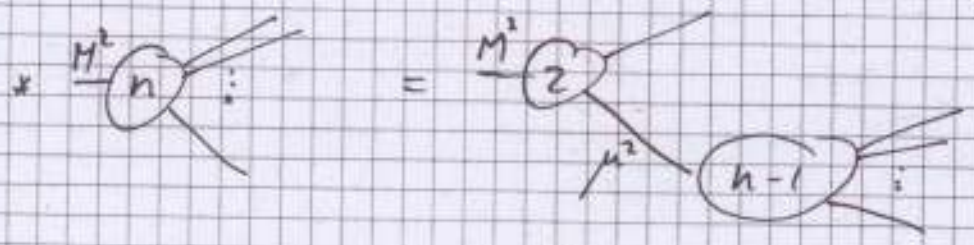
(The overall energy-momentum p_0 are set by the integrals over x_1 and x_2)

→ δ-function : not possible to integrate numerically.

- We need to do the δ-function integral analytically by choosing the set $\{p_i\}$ such that it is already satisfied
- Not so easy if you don't know where to start.

⇒ * Consider 2-body P.S.

$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$, M is the total mass of the system



$$\int_n d\Phi_n = \int_{n-1} \frac{1}{2\pi} \int_0^{M^2} d\mu^2 \int_2 d\Phi_2(M) \, d\Phi_{n-1}(\mu)$$

Integrals via M.C. techniques

$$I = \int_{x_1}^{x_2} dx\, f(x) \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

x_i can be chosen randomly, evenly spaced in $[x_1, x_2]$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2\, dx - I^2$$

V: variance

$$\Rightarrow V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N} [f(x_i)]^2 - I_N$$

then an estimate of the integral I is:

$$I = I_N \pm \sqrt{V_N / N}$$

1. Convergence is slow $\frac{1}{\sqrt{N}}$, but uncertainty can easily be estimated.

2. Errors do <u>not</u> depend on the number of dimensions!

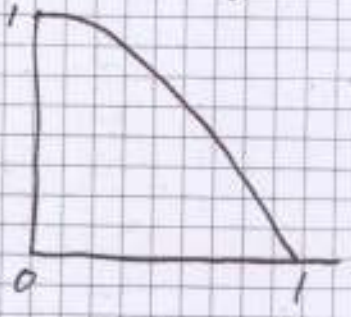
3. Improvement by minimizing V_N
   → smaller number of sampling points N to obtain the same accuracy.

4. Optimal case: $f(x) = \text{constant} \Rightarrow V_N = 0$

Importance sampling

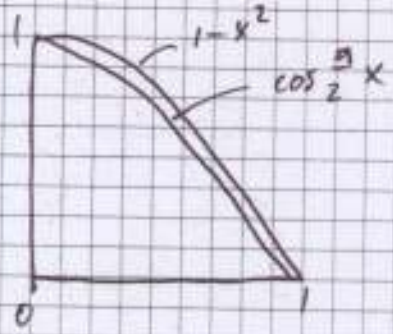
↳ how to minimize V_N ?

e.g. $I = \int_0^1 dx \cos\frac{\pi}{2}x$



$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

The function $(1-x^2)$ is a good approximation of the function $\cos\frac{\pi}{2}x$:



$$I = \int_0^1 dx\ (1-x^2)\ \frac{\cos\frac{\pi}{2}x}{1-x^2} = \int_{\xi_1}^{\xi_2} d\xi \cdot \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2}$$

Almost a constant

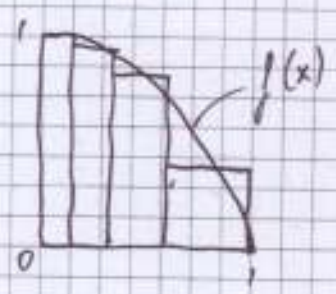
$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

⇒ Gained a factor 100 in sampling points to get a result with the same accuracy

Adaptive importance sampling

Drawback of importance sampling as explained on the previous page is that you need to know a lot about the integrand before starting the integration

idea: learn during the integration about the integrand and use a step-function (numerically) that comes closer and closer to the true integrand.



⇓

VEGAS

(all bins have equal areas
→ many bins where $f(x)$ is large)

Given the bin-sizes Δx_i:
an approximation for $f(x)$ is

$$p(x) = \frac{1}{N_b \, \Delta x_i} \quad \text{for} \quad \begin{cases} x < x_i \\ x > x_i - \Delta x_i \end{cases}$$

N_b: total number of bins; Δx_i: bin size.

$$I = \int_0^1 dx \, f(x) \quad \Rightarrow \quad I = \int_0^1 dx \, p(x) \, \frac{f(x)}{p(x)}$$

$\frac{f(x)}{p(x)}$ → close to a constant

$p(x)$: absorbed in the "change of variables"
⇒ VEGAS weight (Jacobian)

Adaptive importance sampling in more dimensions

Adaptive importance sampling can easily be generalized to more dimensions:

$$\vec{p}(\vec{x}) = p_x(x) \cdot p_y(y) \cdot p_z(z) \dots$$

* All integration variables are treated independently
  - → peaks need to be aligned with integration axes
  - → correlations between integration variables cannot be properly mapped.

Multi-channel integration

Problems in particle physics are usually so complicated that not all peaks can be aligned with integration variables

→ solution: use different transformations for different peaks

⇒ many "integration channels"

$$p(x) = \sum_{i=1}^{n} \alpha_i \, p_i(x) \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i = 1$$

where each $p_i(x)$ takes care of one peak at a given time

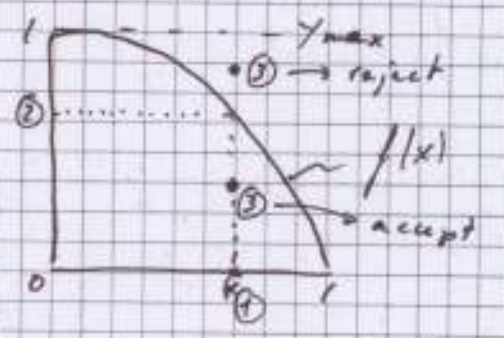
$$I = \int f(\vec{x}) \, d\vec{x} = \sum_{i=1}^{n} \alpha_i \int \left(\frac{f(\vec{x})}{p(\vec{x})} \right) p_i(\vec{x}) \, d\vec{x}$$

→ close to a constant

→ Of course we need to know where the peaks are.
However each of them is treated "independently"
So much simpler as before.

Each sampling point (or Phase Space point) by a MC integration can be seen as an (collision) event in a virtual collider. However the events are not distributed as in nature, but carry a weight.

If we want to simulate a virtual collider, we need to "unweight" them:



1. pick x at random
2. calculate $f(x)$
3. pick $0 < y < y_{max}$ (at random)
4. compare: if $f(x) > y$, accept event and give it weight equal to one

   if $f(x) < y$, throw the event away

* efficiency is $\frac{\text{accepted events}}{\text{total tries}}$

* only works if $f(x)$ is bounded ($\int_0^1 dx \frac{1}{\sqrt{x}}$ does not work)
  → this is always the case at L.O.

* before: same number of events where $f(x)$ is small and $f(x)$ is large
  after: more events where $f(x)$ is large, less where $f(x)$ is small.

* Efficiency can be greatly improved by using a change of variables such that the integrand becomes flat

* if $f(x)$ is constant → 100 % efficiency.

Technical challenges in event generation

e.g. 3-jet production at the LHC:

① Identify all subprocesses (e.g. $q\bar{q} \to ggg$, $gg \to gg$; etc.)

$\Rightarrow$ EASY

② For each of them, find an expression for the matrix elements, e.g. using Feynman diagrams:

$$|M|^2 = \left| \sum_i D_i(\{p\}, \{h\}, \{c\}) \right|^2$$

$\Rightarrow$ STRAIGHT-FORWARD

③ Integrate over the phase-space to obtain total rate and distributions

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_3 \, |M|^2$$

$$\sigma = \sum_{a,b} \int_0^1 f_a(x_1) \, f_b(x_2) \, \hat{\sigma}(x_1, x_2) \, dx_1 \, dx_2$$

$\Rightarrow$ DIFFICULT

④ For simulation of an collider, also need to do "unweighting" of the events

$\Rightarrow$ RELATIVELY EASY, if ③ was done well.

Summary of Event generation at L.O.

1. Monte Carlo matrix element integrators / event generators provide the first estimate of inclusive final states (rates of)

2. Extra radiation is included implicitly:
   - In the initial state in the definition of the P.D.F.
   - In the final state by the definition of assigning a parton to a "jet", and therefore represents all final state evolutions

3. Due to L.O. approximations, the rates (cxn) have a strong dependence on the unphysical renormalization and factorization scales

4. Any tree-level calculation for a final state F can be straightforwardly promoted to the exclusive F+X through a parton shower